

# Gauge/Gravity Duality, Black Holes and Strong Coupling Quantum Field Theory

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Main message of this talk is that:

- Strongly coupled QFT is dual to a Einstein type theory of gravity
- Under certain circumstances under which Newton's coupling  $G_N^{-1} \sim$  (effective degrees of freedom) is small the strong coupling quantum field theory can be solved by semiclassical gravity

Space Time-1 (Galileo, Newton)



*t* is 'universal time' the same for all observers traveling at a constant velocity relative to each other.

Simultaneity of events is independent of the 'state of motion'.



$$\vec{v}' = \vec{u} + \vec{v}$$

Transformation properties of space and time are reflected in the covariance of the equations of motion.

e.g.

Newton's laws:  $m\ddot{\vec{x}}_{\alpha} = \vec{F}_{\alpha}(\vec{x}_1, \cdots, \vec{x}_n)$ ,

 $\alpha = 1, \cdots, n$ 

Navier-Stokes eqns.:

$$\begin{pmatrix} \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \end{pmatrix} \vec{v} = \nu \nabla^2 \vec{v} - \vec{\nabla} P$$

$$\nabla \cdot \vec{v} = 0, \quad \vec{v}(\vec{x}, t) = \text{ fluid velocity}$$

$$P(\vec{x}, t) \text{ is the pressure}$$

Space Time-2 (Einstein Special Relativity)

Speed of light is finite!

Simultancity of events is no more independent of the state of relative motion.

Space and time become space-time as they transform into each other.



Once again transformation properties of space time are reflected in the covariance of the equations of electro-magnetism

 $\frac{\partial}{\partial x^{\mu}}F^{\mu\nu}(x) = J^{\nu}(x) \text{ (Maxwell's equations)}$ 

(Historically Maxwell's equations came before special relativity.)

Space Time-3 (Einstein's General Relativity)

Newton's law of gravitation

$$\vec{F}_{12} = -G_N \frac{m_1 m_2}{r^2} \hat{r}$$



Gravitational force is instantaneous

Inconsistent with special relativity

Einstein (1907): Principle of equivalence

"The effect of a constant gravitational field is equivalent to a uniformly accellerated frame." One important consequence of this is that a clock slows down as the strength of the gravitational field increases or equivalently a photon looses energy as it 'climbs' in a gravitational field ( $\nu$  decreases,  $\lambda$  increases)



$$\Delta \tau = \sqrt{1 - \frac{2MG_N}{c^2 r}} \Delta t$$

## General Relativity (1915):

Explains 'gravity' as a local warping of spacetime.

The metric  $G_{\mu\nu}(x,t)$  of space-time responds to matter according to the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R - \wedge G_{\mu\nu} = -8\pi G_N T_{\mu\nu}$$



## Black Holes:

Einstein's equations predict black holes.

(Schwarzschild, Oppenheimer and Snyder, Chandrasekhar)

 $M > 3M_{\odot}$  gravitational collapse to form a black hole

$$ds^{2} = G_{\mu\nu}dx^{\mu}dx^{\nu}$$
  
=  $-\left(1 - \frac{2GM}{c^{2}r}\right)dt^{2} + \frac{1}{\left(1 - \frac{2GM}{c^{2}r}\right)}dr^{2} + r^{2}d\Omega^{2}$ 

The most important feature of a black hole space-time is the fact that it has a 'horizon'.

$$\left(1 - \frac{2GM}{c^2r}\right) = 0$$

 $r_H = \frac{2GM}{c^2}$ 

Black Hole Horizon



$$\Delta \tau = \sqrt{1 - \frac{2GM}{c^2 r}} \Delta t$$

classical general relativity  $\Rightarrow$  black hole is black. No light can escape from  $r < r_h$ . Matter falling into a black hole cannot come out. As you approach  $r \rightarrow r_h +$ , clocks become infinitly slow:  $\Delta \tau \rightarrow 0$ . Black hole horizon, Quantum Mechanics and Thermodynamics

In quantum mechanics if a black hole absorbs it has to emit

 $\langle i|H_{\rm int}|f\rangle = \langle f|H_{\rm int}|i\rangle^{\star}$ 

Black holes are characterized by a temperature

$$T = \frac{\hbar c}{8\pi GM} = \frac{\hbar}{4\pi cr_h}$$

(Schwarzschild black hole)

and an entropy consitent with the first law of thermodynamics

$$S = \frac{A_h c^3}{4\hbar G_N} = \frac{A_h}{4\ell_p^2}, \qquad \qquad \ell_p^2 = \frac{\hbar G_N}{c^3}$$

(Bekenstein-Hawking)

The notion of entropy makes sense only in a finite quantum theory of gravity.

The black hole behaves like a thermodynamic object and satisfies the laws of thermodynamics.

1st Law :  $dM = TdS + \Omega dJ + \phi dQ$ 

2nd Law :  $(S_{bh} + S_{rad.}) \ge 0$ 

*D*-branes: The Building Blocks of String Theory

*D*-branes are constituent objects of string theory.

(Polchinski)

'Analogous' to quarks of QCD.

Dp brane is a p-dim. object in 10 dim. string theory

 $p = 0, 1, 2, \cdots, 9$ 

e.g. D0 brane is a point like object

D1 brane is a string like object

D2 brane is a 2-dim. membrane

Properties of *D*-branes:

(i) Dp branes carry charge and they couple to gauge fields

$$e_p \int C_{p+1}, \quad F_{p+2} = dC_{p+1}$$

(ii) The brane tension (mass/vol.)

$$au_p = e_p \sim rac{1}{g_s} \gg 1, \ g_s 
ightarrow 0$$

 $g_s$  is the string coupling.

Gravitational potential  $\sim \tau_p G_N \sim g_s \rightarrow 0$ .

(iii) **D**-branes emit and absorb open strings.



Massless modes of the open string are gauge fields in 10-dim.

Interacting brane systems are described, in the infra-red, by non-abelian gauge theories in their world volume.

$$g_{YM}^2 = g_s$$

(iv) Gravitational potential of 'N' D-branes

$$N\tau_p G_N \sim N \frac{1}{g_s} g_s^2 \sim N g_s$$

finite for  $g_s \ll 1$  provided  $N \gg 1$ 

 $g_s N = \lambda$  fixed ('t Hooft coupling)

 $\lambda \gg 1$  leads to a sizable gravitational field.

It turns out that for large N and large  $\lambda$ , Dbranes source supergravity fields  $\leq$  spin 2.

 $G_N \sim g_s^2 \sim \frac{\lambda^2}{N^2} \ll 1$ 

#### Black Hole Thermodynamics

Statistical mechanics of D-branes  $\Rightarrow$  Black hole thermodynamics

$$S_{BH} = \frac{A_h}{4G_N} = k_B \ell n \Omega = S_{\text{Boltzmann}}$$

(Strominger-Vafa)

Hawking radiation could be calculated using *D*-branes

(Dhar, Mandal, SRW, Das, Mathur)

Maldacena: D3-branes and the AdS/CFT correspondence

Under special circumstances the dynamics of a large number of D-branes is either given by a non-abelian gauge theory or by supergravity.

#### N coincident D3 branes:

In the long wavelength limit ( $\gg \ell_s$ ), massless modes of the open strings are gauge fields



- (i) in the 3+1 dim. space-time occupied by the D3 branes there is a 3+1 dim. SU(N) gauge field  $A^{ab}_{\mu}$
- (ii) in the dim. transverve to the brane the gauge fields behave like scalars  $\phi^I_{ab}$  in 3+1 dim.

The interactions of these low lying modes are governed by a SU(N) non-abelian gauge theory containing the fields:  $A_{\mu}, \Phi^{I}, \psi^{I}$  (required by supersymmetry).

$$\mathcal{L} = -\frac{1}{4g_{YM}^2} \operatorname{tr} \left( F_{\mu\nu} F^{\mu\nu} \right) - \frac{1}{2} \operatorname{tr} \left( \nabla_\mu \Phi_I \nabla^\mu \Phi^I \right)$$
$$- \sum_{I < J} \operatorname{tr} \left( [\Phi_I, \Phi_J]^2 \right) + O.\mathsf{T}.$$

The trace is over SU(N) indices

It turns out that this theory is super symmetric and conformally invariant in 3+1 dim. for all values of the coupling constant  $g_{YM}$ . A critical line in the space of 'open string couplings'.

The space-time symmetry group is the conformal group in 3+1 dim. SO(2,4). The internal symmetry is SO(6), because we have 6-scalars.

Precise statement of AdS/CFT correspondence

 $\mathcal{N} = 4$  supersymmetric Yang-Mills theory in 3+1 dim. is dual to type IIB string theory in a space-time with  $AdS_5 \times S^5$  boundary conditions.

Correspondence of gauge theory and string theory parameters:

(YM coupling)  $g_{YM}^2 = g_s$  (string coupling)

(t'Hooft coupling)  $\lambda = g_s N = \left(\frac{R}{\ell_s}\right)^4$ 

R is the AdS radius and  $\ell_s$  is the string length

(Newton's coupling)  $G_{10} = R^5 G_5 = g_s^2 \ell_s^8$ 

$$G_{10} = \frac{1}{N^2} \left(\frac{R}{\ell_s}\right)^8 \ell_s^8$$

String theory  $\rightarrow$  Supergravity

Strong Coupling implies small AdS curvature  $\lambda = \left(\frac{R}{\ell_s}\right)^4 \gg 1$ 

Weak Coupling Gravity :  $G_{10} \sim N^{-2} \ll 1$ 

AdS<sub>5</sub>:  $ds^{2} = R^{2} \left[ e^{2y} \left( -dx_{0}^{2} + dx_{1}^{2} + dx_{2}^{2} + dx_{3}^{2} \right) + dy^{2} \right]$   $y \to \infty \text{ is the 4-dim. boundary of the space time}$ 

 $ds^{2} = R^{2}e^{2y}\left(-dx_{0}^{2} + dx_{1}^{2} + dx_{2}^{2} + dx_{3}^{2}\right)$ 

Gravitational potential energy of a mass m



 $y \rightarrow \infty$  is the boundary of Minkowski spacetime in 3+1 dim.

$$dS_{\text{boundary}}^2 \approx R^2 e^{2y} \left( -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \right)$$

Scale invariance of the metric:

$$x_{\mu} \rightarrow \alpha x_{\mu}, \ y \rightarrow y - \ell n \ \alpha$$

The extra dim. can be interpreted as the scale of the QFT. Scaling  $x_{\mu}$  to larger values corresponds to going deeper into the interior of AdS<sub>5</sub> space.

 $E_{CFT} = R^2 e^{2y} E_{proper}$  $y \to +\infty \quad E_{CFT} \to \infty$  $y \to -\infty \quad E_{CFT} \to 0$ 

Short distance physics is coded near the boundary.

Long distance physics is coded deep in the interior of  $AdS_5$ .

#### Applications of the AdS/CFT correspondence

Near extremal D3 branes in  $AdS_5$  at  $y = y_0$ 



$$ds^{2} = R^{2} \left[ e^{2y} \left( -hdx_{0}^{2} + d\vec{x}^{2} \right) + \frac{dy^{2}}{h} \right] + R^{2} d\Omega_{5}^{2}$$
$$h = 1 - e^{-4(y-y_{0})}$$

 $y = y_0$  is the horizon where h = 0. The D3 brane near the boundary  $y \to \infty$  is AdS<sub>5</sub>. It is a solution of the 5-dim. Einstein eqns. with a -ve cosmological constant

$$R_{\mu\nu} = -\frac{4}{R^2} G_{\mu\nu}$$

#### Thermodynamics of the D3 black brane

The holographic dual of a translationally invariant black brane in  $AdS_5$  with a horizon at  $y = y_0$  is the  $\mathcal{N} = 4$  gauge theory in 3+1 dim. at finite temperature T (Witten).

Horizon implies a temperature  $T = \frac{e^{y_0}}{\pi R}$  and entropy

$$s = \frac{S}{V_3} = \frac{\pi^6 R^8 T^3}{4G_{10}}$$

(Berkenstein-Hawking)

$$AdS/CFT \Rightarrow R^{4} = \frac{\sqrt{8\pi G_{10}}}{2\pi^{5/2}}N$$
$$\Rightarrow s = \frac{S}{V_{3}} = \frac{\pi^{2}}{2}N^{2}T^{3}$$

 $\epsilon = 3P$  (conformal invariance)

 $\epsilon + P = Ts$  $P = \frac{1}{4}Ts = \frac{\pi^2}{8}N^2T^4$ 

We have calculated the equation of state of a strongly coupled gauge theory!

## Fluctuating horizons and Fluid dynamics

Generalize black brane (hole) thermodynamics to fluid dynamics

Local thermal equilibrium

Parameters of the metric, e.g. the horizon become slowly varying functions of the boundary coordinates  $(\vec{x}, t)$ .  $r_h = r_h(\vec{x}, t)$ 



A ripple on the horizon of a black hole is absorbed in a characteristic time by the black brane.

$$\begin{aligned} r_h &\Rightarrow \delta r_h(\vec{x}, t) + r_h \\ T &\Rightarrow T + \delta T(\vec{x}, t) \\ \frac{\partial}{\partial x^{\mu}} \frac{\delta T}{T} \sim \frac{1}{LT} \ll 1 \end{aligned}$$

Hydrodynamic description of the gauge theory is valid for time and length scales  $L \gg \frac{1}{T}$ 

Ripples can be analyzed in terms of quazi normal modes which have a complex frequency:  $w = w_R + iw_I$ ,  $w_I \propto T$ . Wave falling into the horizon gives rise to time asymmetry and energy loss.

In the gauge theory this corresponds to dissipation of an initial disturbance due to viscous effects, even though the underlying microscopic theory is unitary and non-dissipative.

Horizons imply viscosity. A very physical resolution of the information paradox. AdS/CFT and non-linear Fluid Dynamics at Strong ('t Hooft) Coupling

 $u^{\mu}(\vec{x},t), \ u^{\mu}u_{\mu} = -1, \ T(\vec{x},t)$ 

4 independent variables, the 4-velocity  $u^{\mu}(\vec{x},t)$ and the local temperature  $T(\vec{x},t)$ .

AdS/CFT gives a precise meaning to these variables at strong coupling as collective coordinates associated with boosts and scale transformations. D3 brane in terms of "infalling" Edington-Finklestein coordinates

 $ds^2 = 2dtdv - r^2 f(r)dt^2 + r^2 d\vec{x}^2$ 

↓ boosted metric
well behaved solution

 $ds^2 = -2u_{\mu}dx^{\mu}dv - r^2f(br)u_{\mu}u_{\nu}dx^{\mu}dx^{\nu}$ 

 $+r^2 P_{\mu\nu} dx^{\mu} dx^{\nu}$ 

 $P_{\mu\nu} = (\eta_{\mu\nu} + u_{\mu}u_{\nu}).$ 

 $(u_{\mu}, b)$  4 parameters of  $\frac{SO(3,1)}{SO(3)} \times D$ .

slowly varying  $u^{\mu}(\bar{x},t)$ ,  $b(\bar{x},t)$  (Nambu-Goldstone modes)

Solve Einstein eqn. in AdS<sub>5</sub> iteratively by correcting the metric. This happens provided the equations of relativistic fluid-dynamics are satisfied. (S. Bhattacharya, V. Hubeny, S. Minwalla, M. Rangamani)

$$\partial_{\mu}T^{\mu\nu} = 0$$
  
$$T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} + P\eta^{\mu\nu}$$
  
$$-2\eta \left(P^{\mu\alpha}P^{\mu\beta} \left[\partial_{\alpha}u_{\beta} + \partial_{\beta}u_{\alpha}\right] \frac{1}{2} - \frac{1}{3}P^{\mu\nu}\partial_{\alpha}u^{\alpha}\right)$$

$$\epsilon = 3P, \ P = \frac{\pi^2}{8}N^2T^4, \ \eta = \frac{\pi}{8}N^2T^3$$

Hence solutions of relativistic fluid dynamics give rise to solutions of Einstein equations that describe ripples on the horizon of the D3-brane.

# Viscosity-Entropy ratio

Since 
$$s = \frac{\pi^2}{2}N^2T^3$$

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \quad (\text{Policastro-Son-Starinets})$$

Strongly coupled fluid behaves more like a liquid than a gas.

Experimentally for QGP  $\frac{\eta}{s} \gtrsim 1$ .

Systematic higher order corrections can be calculated in the derivative expansion. There are more terms than proposed by Israel and Stewart.

(see also done by R. Baier, P. Romatschke, D.T. Son, A. Starinets and M.A. Stephanov)

Non-Relativistic Navier-Stokes Equation from Gravity (Bhattacharya, Minwalla, SRW)

Speed of sound in fluid:  $v_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{c^2}{3}$  $u^0 = \frac{1}{\sqrt{1 - \vec{V}^2/c^2}}, \ u^i = \frac{V^i}{\sqrt{1 - \vec{V}^2/c^2}}, \ \frac{|\vec{V}|}{v_s} \ll 1$ 

 $\epsilon \rightarrow 0$  scaling limit to project out sound mode:

$$\vec{V}(\vec{x},t) = \epsilon \vec{v}(\epsilon \vec{x}, \epsilon^2 t)$$
$$P(\vec{x},t) = P_0 + \epsilon^2 p(\epsilon \vec{x}, \epsilon^2 t) \rho_e$$

implies

$$\partial_i v^i = 0$$
 and  $\partial_t v^i + v^j \partial_j v^i = \nu \partial^2 v^i - \partial^i p$   
 $\nu = \frac{\eta}{4P_0}$ 

Navier-Stokes eqns. for incompressible fluid!

Inherits a reduced 14 parameter symmetry group from the relativistic theory.

Non-conformal fluid dynamics in 1+1 dim. from Gravity

(Justin David, M. Mahato and SRW)

N D1 branes  $\Rightarrow (T \neq 0)$ .

SU(N) gauge theory in 1+1 dim. with 16 supersymmetries, at finite temp. T.

Fluid dynamics:

 $T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} + P\eta^{\mu\nu} - \xi P^{\mu\nu}\partial_{\lambda}u^{\lambda}$ (no shear term in 1+1 dim.)

 $\xi > 0 \quad \eta_{\mu
u}T^{\mu
u} 
eq 0$ 

(If  $\eta_{\mu\nu}T^{\mu\nu} = 0$ , no dissipative dynamics).

Dual gravity description in 2 regimes:

 $\sqrt{\lambda}N^{-2/3} \ll T \ll \sqrt{\lambda}$  non-extremal D1  $\sqrt{\lambda}N^{-1} \ll T \ll \sqrt{\lambda}N^{-2/3}$  fundamental string  $\lambda = g^2_{\rm MM}N$  Dispersion relation for gauge invariant quasinormal mode  $\Rightarrow$ 

$$\omega = \frac{q}{\sqrt{2}} - \frac{i}{8\pi T} q^2$$
$$\omega = v_s q - i \frac{\xi}{2(\epsilon + P)} q^2$$
$$\Rightarrow v_s = \frac{1}{\sqrt{2}} \text{ and } \epsilon = 2P$$
$$\frac{S}{L} \equiv s = N^2 T^2 \sqrt{\frac{2\pi \ell_s^2}{g_s}}$$
$$\epsilon + P = Ts = N^2 T^3 \sqrt{\frac{2\pi \ell_s^2}{g_s}}$$

$$\xi = \frac{N^2 T^2}{4\pi} \sqrt{\frac{2\pi \ell_s^2}{g_s}}$$

$$\Rightarrow \qquad \frac{\xi}{s} = \frac{1}{4\pi}$$

for a large class of bulk geometries

# Charged Black Brane and a New Term in Fluid Dynamics

(N. Banerjee, J. Bhattacharya, S. Bhattacharya, S. Dutta, R. Loganayagam, P. Sarowka; Erdmenger, Haack, Kaminski, Yarom)

Einstein-Maxwell-Chern-Simons system

$$S = \frac{1}{16\pi G_5} \int d^5 x \sqrt{-g_s} (R + 12 - F_{AB} F^{AB} + \frac{4\kappa}{3} \epsilon^{LABCD} A_L F_{AB} F_{CD})$$
(for  $\mathcal{N} = 4$  YM,  $\kappa = -\frac{1}{2\sqrt{3}}$ )

Black-brane solution with mass and charge.

 $ds^{2} = -2u_{\mu}dx^{\mu}dr - r^{2}V(r,m,a)u_{\mu}u_{\nu}dx^{\mu}dx^{\nu}$  $+ r^{2}P_{\mu\nu}dx^{\mu}dx^{\nu}$ 

 $A = \frac{\sqrt{3}q}{2r^2} u_{\mu} dx^{\mu}$ 

$$V(r, m, q) = 1 - \frac{m}{r^4} + \frac{q^2}{r^6}$$

Charged black brane is dual to a fluid at finite temp. T and chemical potential  $\mu$ 

$$T_{\mu\nu} = P(\eta_{\mu\nu} + 4u_{\mu}u_{\nu}) - 2\eta\sigma_{\mu\nu} + \cdots$$
$$J_{\mu} = nu_{\mu} - DP_{\mu}^{\nu}D_{\nu}n + \xi\ell_{\mu}$$
$$P = \frac{R^{4}}{16\pi G_{5}}; \quad \eta = \frac{R^{3}}{16\pi G_{5}} = \frac{s}{4\pi}, \quad n = \frac{\sqrt{3}q}{16\pi G_{5}},$$
$$D = \frac{1+m/R^{4}}{4m/R^{3}}, \quad \xi = \frac{3\kappa q^{2}}{16\pi G_{5}m}$$

 $\ell_{\mu} = \epsilon_{\mu\nu\rho\sigma} u_{\nu} \omega_{\rho\sigma}, \ \omega_{\rho\sigma} = \partial_{\rho} u_{\sigma} - \partial_{\sigma} u_{\rho}$  (vorticity)

Chern-Simons term  $\Rightarrow$  Anomaly in field theory

Reflected in the presence of  $\ell_{\mu}$  term in  $J_{\mu}$ 

(Son, Surowka)

May be relevant to explain bubbles of strong parity violation observed at RHIC.

# A List of possible Applications:

# Quantum Gravity

- The AdS/CFT correspondence provides us with a well defined quantum theory of gravity in terms of the dual gauge theory.
- An immediate consequence is the resolution of the black hole information paradox.
- Gauge theory provides a framework to resolve the singularities of black holes. (Alvarez-Gaume, Gomez, Basu, Liu, Marino, SRW)
- Cosmological singularity.
- Cosmological models. (Kachru, Kallosh, Linde, Trivedi, McAlistar, Maldacena ....)

• Black holes in the sky: GRS 1915 + 105

Extreme Kerr  $\frac{J}{GM} > .98$  (McClintock et. al.)

AdS/CFT methods to compute properties of this black hole using the near horizon geometry NHEK (Strominger et. al.)

# Quantum Chromodynamics (QCD)

Why are 'quarks confined'?



- QCD string connecting quarks in the boundary conformal field theory is a hologram of a fundamental string in a warped geometry in the extra dimension
- Physics of strongly coupled plasmas in heavy ion collisions
- Gluon scattering amplitudes

Strong coupling calculations in condensed matter systems

Boundary condensed matter system (QFT)

Fluid dynamics limit

Conservation laws

 $\partial_{\mu}T^{\mu\nu} = 0, \ \partial_{\mu}J^{\mu} = 0 \text{ ect.}$ 

 $S = S_{QFT} + \int dx g_{\mu\nu} T^{\mu\nu} + \int A_{\mu} J^{\mu} + \cdots$ 

Conserved currents source the 'gauge' fields  $g_{\mu\nu}, A_{\mu}, \cdots$  in the gravity theory in 1 higher dimension. Tranport coefficients like viscosity, charge and heat conductivity can be calculated using AdS/CFT by solving linear equations in an appropriate black hole/brane background

Quantum Critical phenomena and black holes

- Anomalous Nernst effect
- AC conductivity calculations
- de Haas-van Alphen oscillations
- Non-Fermi Liquids
- Cold Atoms (conformal fluid)

A partial list of reviews:

1. Large N field theories, string theory and gravity

Ofer Aharony, Steven S. Gubser, Juan Martin Maldacena, Hirosi Ooguri, Yaron Oz Phys.Rept.323:183-386,2000. e-Print: hep-th/9905111

2. Microscopic formulation of black holes in string theory

Justin R. David, Gautam Mandal, Spenta R. Wadia Phys.Rept.369:549-686,2002. e-Print: hep-th/0203048

 Lectures on holographic methods for condensed matter physics
 Sean A. Hartnoll, Mar 2009. 85pp. Class.Quant.Grav.26:224002,2009. e-Print: arXiv:0903.3246 [hep-th]

- 4. Holographic duality with a view toward many-body physics
   John McGreevy, Sep 2009. 64pp.
   e-Print: arXiv:0909.0518 [hep-th]
- Lectures on Holographic Superfluidity and Superconductivity
   Christopher P. Herzog
   J. Phys. A42:343001,2009.
   e-Print: arXiv:0904.1975 [hep-th]
- Condensed matter and AdS/CFT Subir Sachdev, Feb 2010. 39pp.
   e-Print: arXiv:1002.2947 [hep-th]