Quarks & Leptons: Universality versus Complementarity

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Two fundamental issues

Leptons vs Quarks

Understanding Fermion Masses

Unification of particles and forces GUT's, Strings...
1. Confronting parameters of leptons and quarks
Correspondence:

$u_r, u_b, u_j \leftrightarrow \nu$

$\color{red}{d_r, d_b, d_j \leftrightarrow e}$

Symmetry:

Leptons as 4th color

Unification:

form multiplet of the extended gauge group, in particular, 16-plet of SO(10)

Accidental?

On the other hand …
Mixing

<table>
<thead>
<tr>
<th>Mixing</th>
<th>Quarks</th>
<th>Leptons</th>
<th>Complementarity</th>
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</thead>
<tbody>
<tr>
<td>1-2, $\theta_{12}$</td>
<td>13°</td>
<td>34°</td>
<td>$\theta_{12} + \theta_{C} = 46.7° \pm 2.4°$</td>
</tr>
<tr>
<td>2-3, $\theta_{23}$</td>
<td>2.3°</td>
<td>42°</td>
<td>$\theta_{23} + V_{cb} = 43.9° \pm 5.1/-3.6°$</td>
</tr>
<tr>
<td>1-3, $\theta_{13}$</td>
<td>$\sim 0.5°$</td>
<td>&lt; 8°</td>
<td></td>
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</table>

$\nu_\mu - \nu_\tau$ permutation symmetry?
2-3 symmetry: remark

\( \nu_\mu - \nu_\tau \) permutation symmetry

Matrix for the best fit values of parameters (in meV)

\[
\begin{pmatrix}
A & B & B \\
B & C & D \\
B & D & C
\end{pmatrix}
\]

\[
\begin{pmatrix}
3.2 & 6.0 & 0.6 \\
24.8 & 21.4 & 30.7
\end{pmatrix}
\]

\[\sin^2 \theta_{13} = 0.01 \quad \sin^2 \theta_{23} = 0.43\] (Bari group)

Substantial deviation from symmetric structure
Even stronger deviations are possible within the error bars

Structure of mass matrix is sensitive to small deviations of 1-3 mixing from zero and 2-3 mixing from maximal
**Tri/bimaximal mixing**

- maximal 2-3 mixing
- zero 1-3 mixing
- no CP-violation

\[
U_{tbm} = U_{23}(\frac{\pi}{4}) U_{12}
\]

\[
U_{tbm} = \begin{pmatrix}
\sqrt{2/3} & \sqrt{1/3} & 0 \\
-\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\
\sqrt{1/6} & -\sqrt{1/3} & \sqrt{1/2}
\end{pmatrix}
\]

\[v_3 \text{ is bi-maximally mixed}\]
\[v_2 \text{ is tri-maximally mixed}\]

\[
\sin^2 \theta_{12} = 1/3 \text{ in agreement with 0.315}
\]

Mixing parameters - some simple numbers 0, 1/3, 1/2

Clebsch Gordan coefficients

Relation to group matrices?

S₃ group matrix

In flavor basis...
Relation to masses?
No analogy in the Quark sector?
Implies non-abelian symmetry
Hierarchy of masses:

- **Quarks**
  
  \[ |m_2 / m_3| > 0.18 \]
  \[ |m_\mu / m_\tau| = 0.06 \]

- **Leptons**
  
  \[ |m_c / m_t| \sim 0.005 \]
  \[ |m_s / m_b| \sim 0.02 - 0.03 \]
  \[ |m_2 / m_3| > 0.18 \]
  \[ |m_\mu / m_\tau| = 0.06 \]

Neutrino mass hierarchy is the weakest one.

- **Upper and down fermions have different mass hierarchies**
  
  \[ m_u : m_c : m_t = \lambda^4 : \lambda^2 : 1 \]
  \[ m_d : m_s : m_b \sim \lambda^2 : \lambda : 1 \]
Mass Ratios

An interplay of regularities and randomness?

Regularities?

\[ m_u m_t = m_c^2 \]

Gatto-Sartori-Tonin relation

\[ \sin \theta_c \sim \sqrt{m_d/m_s} \]

Koide relation
Additional structure?

Similar gauge structure, correspondence

Very different mass and mixing patterns

Partial symmetries in leptonic (neutrino) sector?

Q-L complementarity?

Lepton sector

Quark sector

Additional structure exists which produces the difference.

Symmetry correspondence

Is this seesaw?
Something beyond seesaw?
2. Quark-Lepton universality

Can approximate quark lepton universality be realized in spite of strong difference of mass and mixing patterns?
The idea is

\[ M_f = M_0 + \delta M_f \]

universal mass matrix

corrections depend on flavor

\[ f = u, d, l, \nu \]

Weak universality:

two universal mass matrices for up and down fermions:

\[ M_0^{(up)} \]
\[ M_0^{(down)} \]

allows to explain different mass hierarchies

The Majorana mass matrix the RH neutrino can have different structure
is realized in terms of the mass matrices (matrices of the Yukawa couplings) and not in terms of observables – mass ratios and mixing angles.

Universal structure for mass matrices of all quarks and leptons in the lowest approximation:

\[
\begin{align*}
Y_U &= Y_D = Y_{\nu D} = Y_L = Y_0 \\
\end{align*}
\]

Small perturbations:

\[
\begin{align*}
Y_f &= Y_0 + \Delta Y_f \\
(Y_0)_{ij} &\gg (\Delta Y_f)_{ij}
\end{align*}
\]

\(f = u, d, L, D, M\)
Small perturbations allow to explain large difference in mass hierarchies and mixings of quarks and leptons. Important example:

$$Y_0 = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix}$$

Unstable with respect to small perturbations

$$Y_{f_{ij}} = Y_0^{i_j} \left(1 + \varepsilon_{f_{ij}}^i\right)$$

Form of perturbations is crucial

$$\lambda \sim 0.2 - 0.3$$

Perturbations

$$\varepsilon \sim 0.2 - 0.3$$
Nearly singular matrix of RH neutrinos leads to
- enhancement of lepton mixing
- flip of the sign of mixing angle,
  so that the angles from the charged leptons
  and neutrinos sum up
Froggatt- Nielsen mechanism, U(1) family symmetry

\[ a_{ij} f_i f_j^c H \left( \frac{\sigma}{M_F} \right)^{q(i)+q(j)} \]

q(i) is the U(1) charge of i-family, \( a_{ij} = O(1) \),
\( \sigma \) is the F-N scalar whose VEV violates U(1)
\( M_F \) is the scale at which F-N operators are formed

\[ (Y^f)_{ij} = a_{ij} \lambda^{q(i)+q(j)} \]
\[ \lambda = <\sigma>/M_F \]

\[ a_{ij} = a_0 (1 + \varepsilon_{ij}) \]

nearly universal

For the simplest prescription \( q(1) = 2, q(2) = 1, q(3) = 0, q(\sigma) = -1 \)
the required structure is reproduced
In some (universality) basis in the first approximation all the mass matrices but $M_l$ (for the charged leptons) are diagonalized by the same matrix $V$:

$$V^+ M_f V = D_f$$

For the charged leptons, the mass is diagonalized by $V^*$

$$V^T M_l V^* = D_l$$

Diagonalization:

- $V$ for $u, d, \nu$
- $V^*$ for $l$

In the first approximation

Quark mixing: $V_{CKM} = V^+ V = I$

Lepton mixing: $V_{PMNS} = V^T V$

SU(5) type relation

$M_l = M_d^T$

Another version is when neutrinos have distinguished rotation:

- $V$ for $u, d, l$
- $V^*$ for $\nu$
In general, up and down fermions can be diagonalized by different matrices $V'$ and $V$ respectively:

$$V_{CKM} = V'^+ V$$

$$V_{PMNS} = V^T V'$$

$$V_{PMNS} = V^T V_{CKM} + V_{PMNS} V_{CKM}^+ = V_{PMNS}^0 V_{CKM}^+$$

Quark and lepton rotations are complementary to $V^T V$:

- symmetric, characterized by 2 angles;
- close to the observed mixing for $\theta/2 \sim \phi \sim 20 - 25^\circ$;
- 1-3 mixing near the upper bound;
- gives very good description of data;
- predicts $\sin \theta_{13} > 0.08$.

$V_{PMNS}^0 = V^T V$

$V_{PMNS}$ (with CKM corr.)
Features of zero order mixing

\[ V_{PMNS}^0 = V^T V = P U P \]

\[ P = \text{diag. matrix of phases} \]

\[ U = \begin{pmatrix}
\cos\theta & -\cos\phi \sin\theta & -\sin\phi \sin\theta \\
\vdots & \sin^2\phi - \cos^2\phi \cos\theta & -\sin\phi \cos\phi (1 + \cos\theta) \\
\vdots & \vdots & \cos^2\phi - \sin^2\phi \cos\theta
\end{pmatrix} \]

symmetric matrix \quad phase \quad \alpha = 0
Universal mixing and universal matrices

\[ M_{u,v} \sim m \ D^* A D^* \]
\[ M_d \sim m \ D^* A D \]
\[ M_l \sim m \ D A D^* \]

\[ D = \text{diag}(1, i, 1) \]
\[ A \text{ is the universal matrix: } \]
\[ A \sim \begin{pmatrix}
\varepsilon_1^2 & \varepsilon_2^2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1^2 & \varepsilon_2 & \varepsilon_1 \\
\varepsilon_1 & 1
\end{pmatrix} \]
\[ \varepsilon_i \sim 0.2 - 0.3 \]

Can be embedded in to SU(5) and SO(10) with additional assumptions
3. Quark-Lepton complementarity
Difficult to expects exact equalities but qualitatively

- 2-3 leptonic mixing is close to maximal because 2-3 quark mixing is small
- 1-2 leptonic mixing deviates from maximal substantially because 1-2 quark mixing is relatively large
``Lepton mixing = bi-maximal mixing – quark mixing''

- Quark-lepton symmetry
- Existence of structure which produces bi-maximal mixing
- Mixing matrix weakly depends on mass eigenvalues

In the lowest approximation:

\[ V_{\text{quarks}} = I, \quad V_{\text{leptons}} = V_{\text{bm}} \]
\[ m_1 = m_2 = 0 \]

Appears in different places of theory
\[ U_{\text{bm}} = U_{23}^m U_{12}^m \]

Two maximal rotations

\[ U_{\text{bm}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \]

\[ U_{\text{PMNS}} = U_{\text{bm}} \]

- maximal 2-3 mixing
- zero 1-3 mixing
- maximal 1-2 mixing
- no CP-violation

Contradicts data at $(5-6) \sigma$ level

As dominant structure? Zero order?
QLC1: $V_{bm}$ from neutrinos

**Leptons**

- $V_\nu = V_{bm}$
- $V_l = V_{CKM}$

\[ V_{PMNS} = V_l^+ V_\nu = V_{CKM}^+ V_{bm} \]

**Quarks**

- $V_u = I$
- $V_d = V_{CKM}$

\[ V_{quarks} = V_u^+ V_d = V_{CKM} \]

(or both rotations from neutrinos)

**Predictions:**

\[ \sin \theta_{12} = \sin(\pi/4 - \theta_C) + 0.5 \sin \theta_C (\sqrt{2} - 1 - V_{cb}) \]
\[ \sin \theta_{13} = \sin \theta_{23} \sin \theta_C = 0.16 \]
\[ D_{23} = 0.5 \sin^2 \theta_C + \cos^2 \theta_C V_{cb} \cos \alpha = 0.02 +/- 0.04 \]

\[ \sin^2 \theta_{12} = 0.330 \]

- large!

H. Minakata, A.S.
QLC1: \[ \sin \theta_{12} = \sin(\pi/4 - \theta_C) + 0.5\sin \theta_C \left( \sqrt{2} - 1 - V_{cb} \cos \alpha \right) \]

Tri-bimaximal: \[ \sin \theta_{12} = 1/\sqrt{3} \]

\[
\begin{align*}
\text{QLC1} & \quad V_{CKM}^{+} V_{bm} \\
& \sim V(\theta_C)^{+} V_{23}(45^0) V_{12}(45^0)
\end{align*}
\]

\[ \sin^2 \theta_{12} = 0.330 \]

\[
\begin{align*}
\text{Tri-bimaximal} & \quad V_{23}(45^0) V_{12}(\arcsin 1/\sqrt{3}) \\
& \quad \text{or formula for the Cabibbo angle:} \\
& \quad \sin \theta_C = \sqrt{2} \cos \theta_C - 2\sqrt{3}
\end{align*}
\]

Two independent combinations of matrices give the same value of 1-2 mixing

At least one of the approaches is wrong

M. Schmidt, A.S.
QLC2: \( V_{bm} \) from charged leptons

**Leptons**

- \( V_\nu = V_{CKM}^+ \)
- \( V_l = V_{bm}^+ \)
- \( V_{leptons} = V_{bm} V_{CKM}^+ \)

**Quarks**

- \( V_u = V_{CKM}^+ \)
- \( V_d = I \)
- \( V_{quarks} = V_u^+ V_d = V_{CKM} \)

**Predictions:**

- \( \sin \theta_{sun} \sim \sin (\pi/4 - \theta_C) \)
- \( \sin \theta_{13} = - \sin \theta_{sun} V_{cb} = 0.03 \)
- \( D_{23} = \cos \theta_{sun} V_{cb} \cos \delta < 0.03 \)

H. Minakata, A.S.
1-2 mixing

\[ U_{tbm} = U_{tm} U_{m13} \]
\[ U_{QLC1} = U_C U_{bm} \]

3ν analysis does change bft but error bars become smaller

\[ \theta_{12} + \theta_C \sim \pi/4 \]
Non-zero central value (Fogli, et al): Atmospheric neutrinos, SK spectrum of multi-GeV e-like events
2-3 mixing

SK (3ν) - no shift from maximal mixing

\[ \sin^2 2\theta_{23} > 0.93, \ 90\% \ C.L. \]

1). in agreement with maximal
2). shift of the bfp from maximal is small
3). still large deviation is allowed:

\[ \frac{0.5 - \sin^2 \theta_{23}}{\sin \theta_{23}} \sim 40\% \]
Q & L: strong difference of mass and mixing pattern;
- ti-bimaximal mixing;
- possible presence of the special leptonic (neutrino) symmetries

Still approximate quarks-leptons universality can be realized. No additional structures apart from seesaw is required.

Data may indicate existence of the Q-L complementarity which implies generation of bi-maximal mixing and Q-L symmetry

Coincidences: real or accidental?
Koide relation

\[
\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}
\]

Koide relation

\[
\tan \theta_C = \sqrt{3} \frac{\sqrt{m_\mu} - \sqrt{m_e}}{2\sqrt{m_\tau} - \sqrt{m_\mu} - \sqrt{m_e}}
\]

Both relations can be reproduced if

\[
m_i = m_0 (z_i + z_0)^2
\]

\[
\Sigma_i z_i = 0
\]

\[
z_0 = \frac{\sqrt{\Sigma_i z_i^2}}{3}
\]


with accuracy 10^{-5}

Another representation which is closely connected to circulant symmetry

C A Brannen

Neutrinos
General picture

Standard Model

Planck scale physics
Screening of Dirac structure

Double (cascade) seesaw

\[ m = \begin{pmatrix} 0 & m_\nu & 0 \\ m_\nu^T & 0 & M_\nu^T \\ 0 & M_\nu & M_S \end{pmatrix} \]

Additional fermions

\[ m_\nu = m_D M_D^{-1} M_S M_D^{-1} m_\nu \]

If \( M_D = A^{-1} m_D \)

\[ m_\nu = A^2 M_S \]

\[ A \sim v_{EW}/M_{GU} \]

\( m_D \) similar (equal) to quark mass matrix - cancels

Structure of the neutrino mass matrix is determined by \( M_S \)

\( \rightarrow \) physics at highest (Planck?) scale immediately

M. Lindner
M. Schmidt
A.S.
JHEP0507, 048 (2005)

R. Mohapatra
PRL 56, 561, (1986)

R. Mohapatra.
J. Valle

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R. Mohapatra
PRL 56, 561, (1986)

R. Mohapatra.
J. Valle

A.S.
PRD 48, 3264 (1993)
Seesaw provides scale and not the flavor structure of neutrino mass matrix.

Structure of the neutrino mass matrix is determined by:

- Origin of "neutrino" symmetry
- Origin of maximal (or bi-maximal) mixing
- \( M_S \sim M_{Pl} \) leads to quasi-degenerate spectrum if e.g. \( M_S \sim I \),
- Q-L complementarity

Reconciling Q-L symmetry and different mixings of quarks and leptons.
Smallness of neutrino masses

For the third generation:

\[
\frac{m_3}{m_\tau} \sim (0.3 - 1) \times 10^{-10}
\]

\[
\frac{m_\tau}{m_+} \sim 10^{-2}
\]

\[
m_3 \sim m_+ \frac{m_\tau}{M}
\]

Scale of new physics

\[
m_\nu = - m_D^T M_R^{-1} m_D
\]

\[
m_D = Y <H>
\]

\[
\begin{bmatrix}
0 & m_D \\
M_D^T & M_R
\end{bmatrix}
\]

If \( M_R \gg m_D \)

\[
\nu \quad \text{N}
\]

P. Minkowski
T. Yanagida
M. Gell-Mann, P. Ramond, R. Slansky
S. L. Glashow
R.N. Mohapatra, G. Senjanovic
Neutrino symmetry?

Maximal 2-3 mixing

Neutrino mass matrix in the flavor basis:
For charged leptons: \( D = 0 \)

\[
\begin{pmatrix}
A & B & B \\
B & C & D \\
B & D & C
\end{pmatrix}
\]

Zero 1-3 mixing

\( \nu_\mu - \nu_\tau \)

permutation symmetry

Can both features be accidental?

Often related to equality of neutrino masses

Discrete symmetries \( S_3, D_4 \)

Can this symmetry be extended to quark sector?

Are quarks and leptons fundamentally different?
Universal 2-3 symmetry?

2-3 symmetry

Smallness of $V_{cb}$

Maximal (large) 2-3 leptonic mixing

Universal mass matrices:

$$
\begin{pmatrix}
X & A & A \\
A & B & C \\
A & C & B \\
\end{pmatrix} + \delta m
$$

Quarks, charged leptons: $B \sim C$, $X \ll A \ll B$

Neutrinos: $B \gg C$, $X \sim B$

2-3 symmetry does not contradict mass hierarchy

- Hierarchical mass spectrum
- Small quark mixing
- Degenerate neutrino mass spectrum;
- Large lepton mixing

Still additional symmetries are needed to explain hierarchies/equalities of parameters
Real or accidental?

- Tri-bimaximal mixing
- Small 1-3 mixing
- Koide relation
- Q-L-complementarity
- Maximal 2-3 mixing
Plan

Quark-lepton

Comparing results
Universality
Complementarity
The hope is...