

Translation-invariant star products are studied in the setting of α^* -cohomology. It is explicitly shown that all quantum behaviors and especially the scattering matrix of translation-invariant non-commutative quantum field theories are entirely determined by α^* -cohomology classes of the star products. Moreover it is shown that in each α^* -cohomology class there exists a unique 2-cocycle, the harmonic form, which generates a particular Groenewold-Moyal star product. This leads to an algebraic classification of translation-invariant non-commutative structures and shows that any general translation-invariant non-commutative quantum field theory is physically equivalent to a Groenewold-Moyal non-commutative quantum field theory.