Planck2013 Results: Primordial non-Gaussianity

Mohammad Hossein Namjoo
School of Physics, IPM

Primordial fluctuations

$$\frac{\delta T}{T} \to \Phi(t) \to \Phi_0$$

- Primordial power spectrum

$$\langle \Phi_{k_1} \Phi_{k_2} \rangle = (2\pi)^3 \delta(k_1 + k_2) P_\Phi$$

Primordial Non-Gaussianity

$$\langle \Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\Phi(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_\Phi(k_1,k_2,k_3)$$

- Deviation from Gaussian PDF
- How non-linear are the perturbations
- How strong are the interaction (cubic order action)

$$S = \frac{1}{2}\int \sqrt{g}[R - (\nabla\phi)^2 - 2V(\phi)]$$

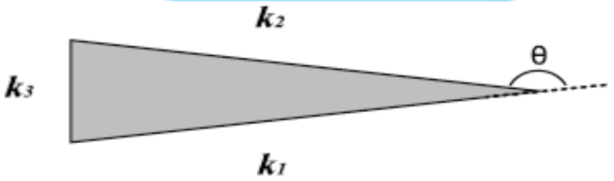
Primordial Non-Gaussianity: Size and shape

$$\langle \Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\Phi(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_\Phi(k_1,k_2,k_3)$$

$$B_\Phi(k_1,k_2,k_3) = f_{\rm NL}F(k_1,k_2,k_3)$$

- Different inflationary models predict different shape and size for non-Gaussianity

Shape of NG



$$
\begin{aligned}
B_{\Phi}^{\text{local}}(k_1,k_2,k_3) &= 2f_{\text{NL}}^{\text{local}}\Big[P_{\Phi}(k_1)P_{\Phi}(k_2)+P_{\Phi}(k_1)P_{\Phi}(k_3) \\
&+ P_{\Phi}(k_2)P_{\Phi}(k_3)\Big] \\
&= 2A^2 f_{\text{NL}}^{\text{local}}\left[\frac{1}{k_1^{4-n_s}k_2^{4-n_s}}+\text{cycl.}\right].
\end{aligned}
$$

Local non-Gaussianity from multi-field models

Curvaton:

$$f_{\rm NL}^{\rm local} = (5/4r_{\rm D}) - 5r_{\rm D}/6 - 5/3$$

Warm inflation

$$f_{\rm NL}^{\rm warm} = -15\ln\left(1 + r_{\rm d}/14\right) - 5/2$$

$$r_{\rm d} = \Gamma/(3H)$$

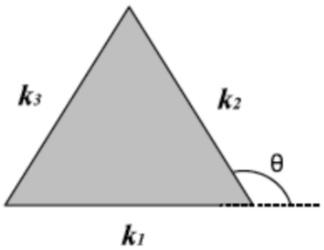
Alternatives to inflation

- Cyclic/Ekpyrotic models

$$|f_{\rm NL}^{\rm local}| > 10$$

Shape of NG

$$B_{\Phi}^{\mathrm{equil}}(k_1,k_2,k_3) = 6A^2 f_{\mathrm{NL}}^{\mathrm{equil}}$$
$$\times \left\{ -\frac{1}{k_1^{4-n_s} k_2^{4-n_s}} - \frac{1}{k_2^{4-n_s} k_3^{4-n_s}} - \frac{1}{k_3^{4-n_s} k_1^{4-n_s}} \right.$$
$$-\frac{2}{(k_1 k_2 k_3)^{2(4-n_s)/3}} + \left[\frac{1}{k_1^{(4-n_s)/3} k_2^{2(4-n_s)/3} k_3^{4-n_s}} \right.$$
$$\left. \left. +(5\ \mathrm{permutations}) \right] \right\} ,$$



(2) Equilateral

DBI

$$\mathcal{L}_{\rm eff} = -\frac{1}{g_s}\sqrt{-g}\left(f(\phi)^{-1}\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} + V(\phi)\right)$$

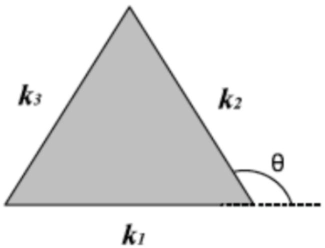
Alishahiha et.al '04

- The sound speed controls the amplitude of non-Gaussianity

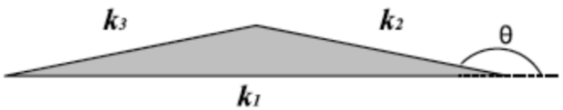
$$f_{NL}^{equil} \propto \frac{1}{c_s^2}$$

Shape of NG

$$B_{\Phi}^{\text{ortho}}(k_1, k_2, k_3) = 6A^2 f_{\text{NL}}^{\text{ortho}}$$
$$\times \left\{ -\frac{3}{k_1^{4-n_s} k_2^{4-n_s}} - \frac{3}{k_2^{4-n_s} k_3^{4-n_s}} - \frac{3}{k_3^{4-n_s} k_1^{4-n_s}} \right.$$
$$- \frac{8}{(k_1 k_2 k_3)^{2(4-n_s)/3}} + \left[\frac{3}{k_1^{(4-n_s)/3} k_2^{2(4-n_s)/3} k_3^{4-n_s}} \right.$$
$$\left. \left. + (5 \text{ perm.}) \right] \right\} .$$



(2) Equilateral



(3) Folded

Orthogonal non-Gaussianity

* Higher derivative models

Galileon model:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(\phi, X) - 2G(\phi, X)\Box\phi]$$

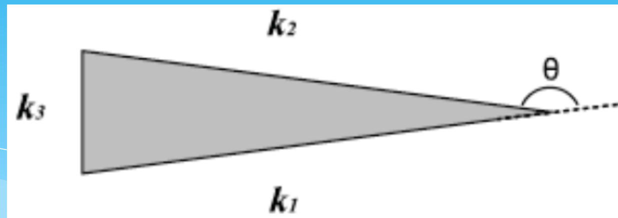
Shape of NG

* There are many other shapes from more complicated models:

  - Non-Bunch-Davis vacuum
  - Feature models
  - …

Consistency relation

Maldacena '03



$$B_\Phi^{\text{single-field}}(k_1 \to 0, k_2, k_3 = k_2) \to \frac{5}{3}(1-n_s)P_\Phi(k_1)P_\Phi(k_2)$$

* Any detection of local type/squeezed limit non-Gaussianity can rule out most single field models of inflation
* Assumptions:
- BD-vacuum
- Attractor phase/ conservation of curvature perturbation at large scales.

MHN, H.Firouzjahi and M.Sasaki '13 / X.chen, H.Firouzjahi, MHN, M.Sasaki '13

Non-primordial bispectrum

- Foreground
- ISW-Lensing

	SMICA	NILC	SEVEM	C-R
KSW	0.81 ± 0.31	0.85 ± 0.32	0.68 ± 0.32	0.75 ± 0.32
Binned	0.91 ± 0.37	1.03 ± 0.37	0.83 ± 0.39	0.80 ± 0.40
Modal	0.77 ± 0.37	0.93 ± 0.37	0.60 ± 0.37	0.68 ± 0.39

- Point sources

	SMICA	NILC	SEVEM	C-R
KSW	7.7 ± 1.5	9.2 ± 1.7	7.6 ± 1.7	1.1 ± 5.1
Binned	7.7 ± 1.6	8.2 ± 1.6	7.5 ± 1.7	0.9 ± 4.8
Modal	10 ± 3	11 ± 3	10 ± 3	0.5 ± 6

Results

$$f_{NL}^{local} = 2.7 \pm 5.8$$

$$f_{NL}^{equil} = -42 \pm 75$$

$$f_{NL}^{ortho} = -25 \pm 39$$

$$\tau_{NL} < 2800$$

Implications for early universe physics

* DBI model

$$c_s^{\mathrm{DBI}} \geq 0.07 \qquad 95\%\ \mathrm{CL}$$

* IR-DBI model

$$V(\phi) = V_0 - \tfrac{1}{2}\beta H^2 \phi^2$$

WMAP: $\beta < 3.7$ Planck: $\beta \leq 0.7 \qquad 95\%\ \mathrm{CL}$

Curvaton

$$f_{\rm NL}^{\rm local} = \frac{5}{4r_{\rm D}} - \frac{5r_{\rm D}}{6} - \frac{5}{3}$$

$$r_{\rm D} \;=\; [3\rho_{\rm curvaton}/(3\rho_{\rm curvaton} + 4\rho_{\rm radiation})]_{\rm D}$$

$$r_{\rm D} \geq 0.15 \qquad 95\%\ \mathrm{CL}$$

Warm inflation

$$f_{\rm NL}^{\rm warm} = -15 \ln(1 + r_{\rm d}/14) - 5/2$$

$$r_{\rm d} = \Gamma/(3H)$$

$$\log_{10} r_{\rm d} \leq 2.6$$

Alternatives to inflation

* Ekpyrotic conversion mechanism

$$f_{\rm NL}^{\rm local} = -(5/12)\,c_1^2 \qquad\qquad 10 \gtrsim c_1 \gtrsim 20$$

* Kinetic conversion ekpyrotic models

$$c_1 \le 4.2 \text{ at } 95\% \text{ CL}$$