Theoretical Setup for Early Universe Cosmology Models

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Introduction and Motivations

- Early Universe usually refers to a cosmos before structures formed.
- Many efforts have gone to finding ways to observationally probe the early Universe era.
- Nonetheless, what we see today is late time cosmology with lots of small and large structures in the sky.

Introduction and Motivations

- Our window to early Universe comes from
  - Background and its fluctuations and perturbations, messengers: mainly CMB photons
  - Foreground: galaxy counts and cluster distribution; scales $\gtrsim 1Mpc$.
- Ability to remove late time effects and extract information about the early Universe.
- Impressive Recent progress, expected to be impressed more in near future....

Cosmological data implies....

Cosmic data imply that background Universe at cosmological scales is

- Homogeneous and isotropic
- big but not so old
- spatially flat.

NOTE: *All the above statements are verified to our current observational precision.*

Cosmological data implies....

Cosmic data indicate that

- there are CMB anisotropies (temperature fluctuations) and
- density perturbations seeding large scale structures are coming from the same source produced CMB temperature anisotropies.

Cosmological data...

- Cosmic data are all distributions,
- in the sky (on a two sphere) and in time (how distant from us they are, parameterized by redshift z).
- Information in distributions is carried in two, three, ..., point correlation functions, as functions of cosmological parameters (parametric plots).

Question to theorists...

Question to theorists and model builders:

Explain the data and produce testable predictions.

We need a framework:

- The main player is gravity:

$$G_{\mu\nu} = M_{\rm pl}^{-2} T_{\mu\nu}\,, \qquad M_{\rm pl}^{-2} = 8\pi G_N \simeq 2.4 \times 10^{18}\ \text{GeV},$$

- matter to gravitate, *cosmic (perfect) fluid*:

$$T^{\mu}_{\ \nu} = diag(-\rho, p, p, p)\,.$$

The framework...

- Almost perfect background homogeneity and isotropy (FLRW cosmology):

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2] \,.$$

  Data clearly prefer $k=0$, flat FLRW.

- Einstein eqs:

$$H^2 = \frac{1}{3M_{\rm pl}^2}\,\rho\,, \qquad \dot{H} = -\frac{1}{2M_{\rm pl}^2}(\rho+p)\,,$$

  where $H = \frac{\dot{a}(t)}{a(t)}$ and $\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}(\rho+3p)$.

The framework...

- To solve the above we need E.o.S: $p = p(\rho)$.
- Horizon and flatness problems are naturally explained with a period of accelerated expansion, *inflation*:

$$\frac{\ddot{a}}{a} \geq 0 \quad \Rightarrow \quad \rho + 3p < 0\,,$$

- which lasts long enough

$$N_e = \int_{inf} H dt \geq N_0$$

Inflationary background

- N_0 depends on details of after inflation dynamics, but it has a lower bound $N_0 \gtrsim 45$. However, we usually take

$$N_0 \gtrsim 60$$

and H to be (almost) constant during inflation:

$$\Delta t = N_e/H \simeq 50 - 100 H^{-1}.$$

Note:

$$\rho \sim \Lambda_{inf}^4 \sim H^2 M_{\rm pl}^2.$$

Inflationary background

- *When did inflation occur? What is H_{inf}?*
- We do not exactly know, but
  - theoretical/particle physics demands prefers reheat temperatures

$$1 - 10\text{TeV} \lesssim T_{reh} \lesssim 10^9\text{GeV}.$$

  - CMB observations (COBE):

$$H \lesssim 10^{-5} M_{\rm pl} \sim 10^{13}\text{GeV}.$$

- The above is favorable to GUTs and resolves heavy relic problem.

The slow-roll

- Enough N_e usually requires slow-roll

$$\epsilon, \eta \ll 1, \quad \epsilon = -\frac{\dot{H}}{H^2} \quad \eta = \frac{\ddot{H}}{2\dot{H}H}.$$

- Slow-roll requires

$$\frac{\rho + p}{\rho} = \frac{2\epsilon}{3} \ll 1, \quad \frac{\dot{\epsilon}}{H\epsilon} = 2\eta - 2\epsilon \ll 1.$$

OR $\quad P \simeq -\rho(1 - \frac{2\epsilon}{3})$.

- During slow-roll spacetime is *almost de Sitter*.

Cosmic perturbations

- *Quantum fluctuations* can never be turned off.
- Cosmic fluid and metric have fluctuations.
- Quantum fluctuations are
  - randomly generated,
  - appear in all wavelength (momenta),
  - almost Gaussian for weakly coupled theories.

Cosmic perturbations

- Quantum fluctuations
  - do not carry energy and hence do not gravitate,
  - can only appear through loop effects (in QFT on flat space).
- In a curved space with horizon quantum fluctuations can become real!
- This is only visible to an observer outside the horizon, not inside.

Cosmic perturbations

- In the case of a black hole this quantum fluctuations appear as Hawking radiation.
- In a cosmological background too, these fluctuations *cross the horizon*, freeze and become classical.
- If inflation never ended we could not see them.

Cosmic perturbations

- Inflation ends, the horizon grows and *superhorizon* modes influence dynamics of the Early Universe.
- They appear as
  - density perturbations seeding large structure,
  - CMB temperature anisotropy.
- When perturbations freeze out *quantum* probabilities of the fluctuations become *statistical* probabilities.

Cosmic perturbations

- Information about correlations of the cosmic data can hence be used to infer information about the inflationary models.
- Fluctuations of an almost freely streaming, almost perfect fluid in the slow-roll approximation is described by
  - how the background density and pressure are varying in time, i.e. ϵ, η,
  - speed of propagation of sound waves c_s^2 in the cosmic fluid.

Cosmic perturbation

- If $\mathcal{R}(k)$ is the comoving curvature perturbations, then the ensemble average of the fluctuations is

$$\langle \mathcal{R}(k)\mathcal{R}(k')\rangle = (2\pi)^3 \delta(k+k') \frac{2\pi^2}{k^3} P_{\mathcal{R}}(k)$$

$$P_{\mathcal{R}}(k) \simeq P_{\mathcal{R}}(k_*) \left(\frac{k}{k_*}\right)^{n_s - 1}$$

- Power spectrum of curvature perturbations $P_{\mathcal{R}}(k)$ is directly related to similar quantity in CMB temperature anisotropy.

Cosmic perturbations

- In terms of parameters of inflationary models

$$P_{\mathfrak{R}}(k_*) = \frac{1}{8\pi^2}\frac{H^2}{\epsilon c_s}, \qquad n_s - 1 = -2\epsilon + \frac{(\epsilon c_s)^{\cdot}}{H\epsilon c_s}.$$

- Power spectrum and tilt of tensor modes in generic simple cosmic fluid models is

$$P_T = \frac{2}{\pi^2}H^2, \qquad n_T = -2\epsilon.$$

- Often reported as tensor-to-scalar ratio r

$$r = \frac{P_{\mathfrak{R}}}{P_T} = 16\epsilon c_s.$$

Cosmic perturbations

- Given the CMB data one can compute three and higher point functions:

$$\langle \mathcal{R}(k_1)\mathcal{R}(k_2)\mathcal{R}(k_3)\rangle = (2\pi)^3\delta(k_1+k_2+k_3)B(k_1,k_2,k_3)\,.$$

- One can also relax the isotropy condition

$$P_{\mathcal{R}}(\vec{k}) = P_{\mathcal{R}}(|k|).$$

- The CMB data are becoming precise enough to probe these.

Inflationary models

- Main questions to a model builder:

  what is the inflationary cosmic fluid made of?

  Can we model the cosmic fluid with a QFT?

- Most commonly scalar field theories with various potentials coupled to Einstein gravity does the job.
- CMB data then restricts the potential.

Inflationary models

- Given the flood of data and progress, one can
  - take a phenomenologic approach and try to design potentials leading to specific observational features on the CMB, or
  - choose a HEP framework and examine inflationary model building in that setup, or
  - extract generic, model independent results.
- All these three lines are actively followed.
- Here is a taste of it.....

Designer Inflationary models

- A single or multi scalar field, with generic kinetic or potential terms, e.g. K-inflation:

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} R + P(\phi, X), \qquad X = -\frac{1}{2}(\partial_\mu \phi)^2 .$$

- Action for scalar field P denotes its pressure and

$$\rho = 2XP,X - P$$

$$\epsilon = \frac{3XP,X}{2XP,X - P} \qquad c_s^2 = \frac{P,X}{P,X + 2XP,XX} .$$

Designer Inflationary models

- The goal here is gaining intuition.....
- Simplest special cases $P = -\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \qquad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,,$$

$$\epsilon = \frac{\dot{\phi}^2}{2\rho} = \frac{\dot{\phi}^2}{2H^2 M_{\rm pl}^2}\,, \qquad \eta = -\frac{\ddot{\phi}}{\dot{\phi}H}\,, \qquad c_s^2 = 1$$

- Chaotic models with $V(\phi) \sim \lambda_n\phi^n$,
- Large field models with $V(\phi) = V_0 + \lambda_n\phi^n$.

Designer Inflationary models

- Two or multifield models, e.g. Hybrid inflation:

$$V(\phi,\chi) = \frac{\lambda}{4}(\chi^2 - \frac{M^2}{\lambda})^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}m^2\phi^2$$

- Anisotropic models, models with background gauge fields.
- One may of course play with various potentials and fields....

Inflationary model building within HEP setups

- Gauge fields and inflation:
- $U(1)$ gauge fields in the background, anisotropic models.
- Non-abelian gauge fields in the background, gauge fields as inflaton: Gauge-flation, chromo-natural model.
- Gauge fields as perturbations, primordial magnetic field generation. *Not successful*.
- See "Gauge Fields and Inflation" arXiv:1212.2921 To appear in Phys.Rept.

Inflationary model building within HEP setups

- Higgs inflation: Use the only observed "fundamental" scalar as inflaton. *Not in a good shape with the data*.
- Inflation in SUSY models:
  - D-term or F-term models.
  - Inflection point inflation
- Generic problem: protect flatness of potential.
- Quantum Gravity motivated models, e.g. loop quantum cosmology, asymptotic safety gravity models, or Hořava-Lifshtz based models.

Inflationary model building within HEP setups

- Inflation within supergravity, e.g. Racetrack model.
- Stringy motivated models:
  - Open string models, e.g. Mobile brane models.
  - Closed string models, e.g. axion monodromy or N-flation.
  - M-flation: Internal D-brane d.o.f. as inflaton.
- Generically plagued with η*-problem* and moduli stabilization problem.

Model Independent results

- Inflation damps out anisotropies.
- Generically we have flat power spectra, how general?
- Vector perturbations are damped.
- Effective field theory of inflation.
- Almost de Sitter-almost conformal invariance.

Concluding Remarks

- It is a very interesting era to do early Universe cosmology.
- *Personal belief*:
  - Quantum Gravity is probably not relevant to inflation.
  - Inflationary model building should be done within well-motivated HEP and particle physics setups, like models involving gauge field theories.

Thank You.