Correlation effects in Fermion Systems
Lecture II

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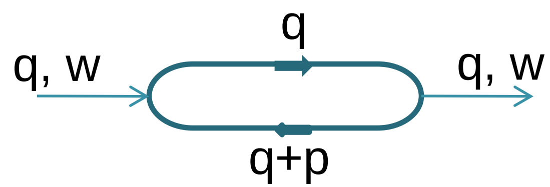
Outlines

1) Correlations in Electron Liquid
   - Introduction
   - Dyson equation and self-energy
   - Effective mass
   - Fermi Liquid & non-Fermi Liquid
   - Luttinger Liquid, Bosonization

2) Hubbard Model
   - Introduction
   - Spin-charge separation
   - Mott insulators
   - Phase diagram

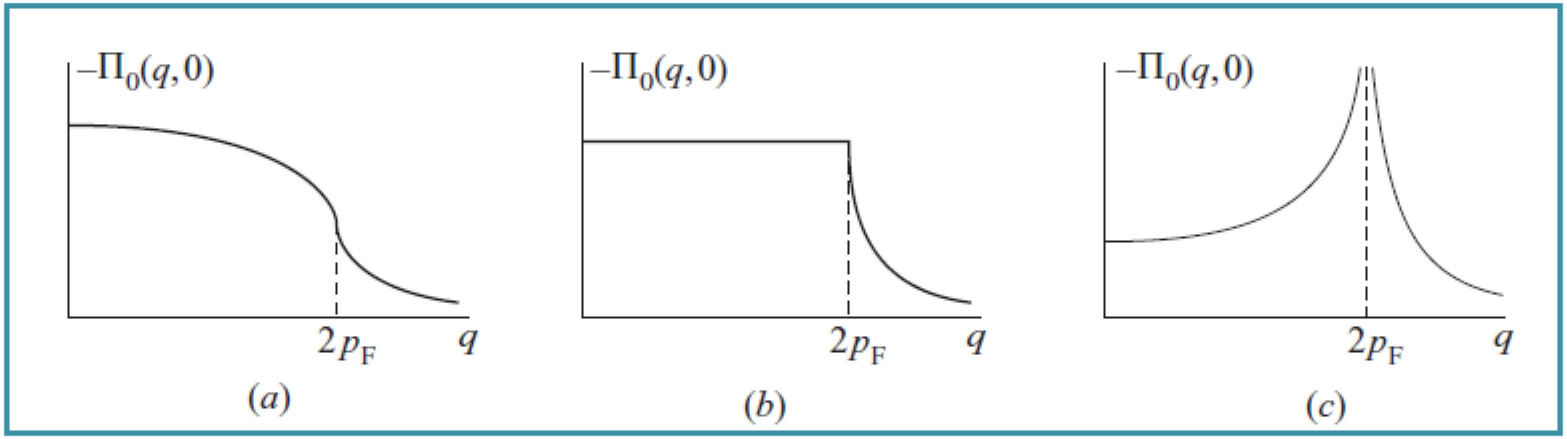
Non-interacting response functions



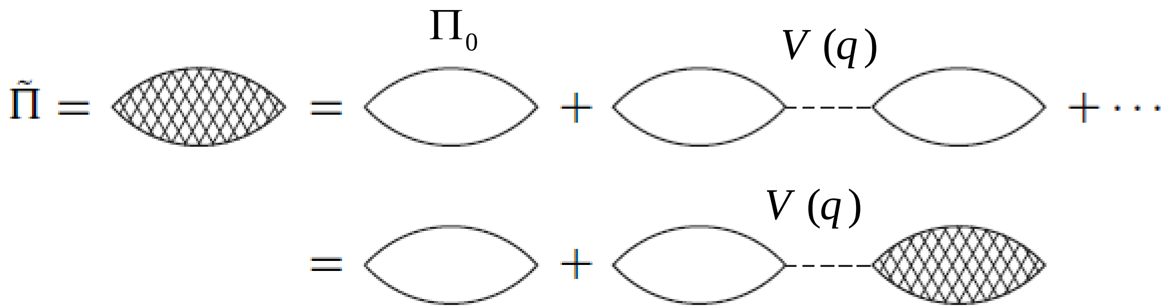
Bubble

$$\Pi_0(\boldsymbol{q},\omega) = \sum_{\alpha\beta} \frac{n_\alpha - n_\beta}{\hbar\omega + \varepsilon_\alpha - \varepsilon_\beta + i\hbar\eta} A_{\alpha\beta} B_{\beta\alpha}$$

$$= -\frac{2i}{(2\pi)^4} \int \frac{d^3 p\, d\omega'}{\Big[\omega+\omega' - \varepsilon(\boldsymbol{p}+\boldsymbol{q}) + i\delta\, \mathrm{sign}\big(\varepsilon(\boldsymbol{p}+\boldsymbol{q}) - \mu\big)\Big]\Big[\omega' - \varepsilon(\boldsymbol{p}) + i\delta\, \mathrm{sign}\big(\varepsilon(\boldsymbol{p}) - \mu\big)\Big]}$$



Minimum theory: RPA



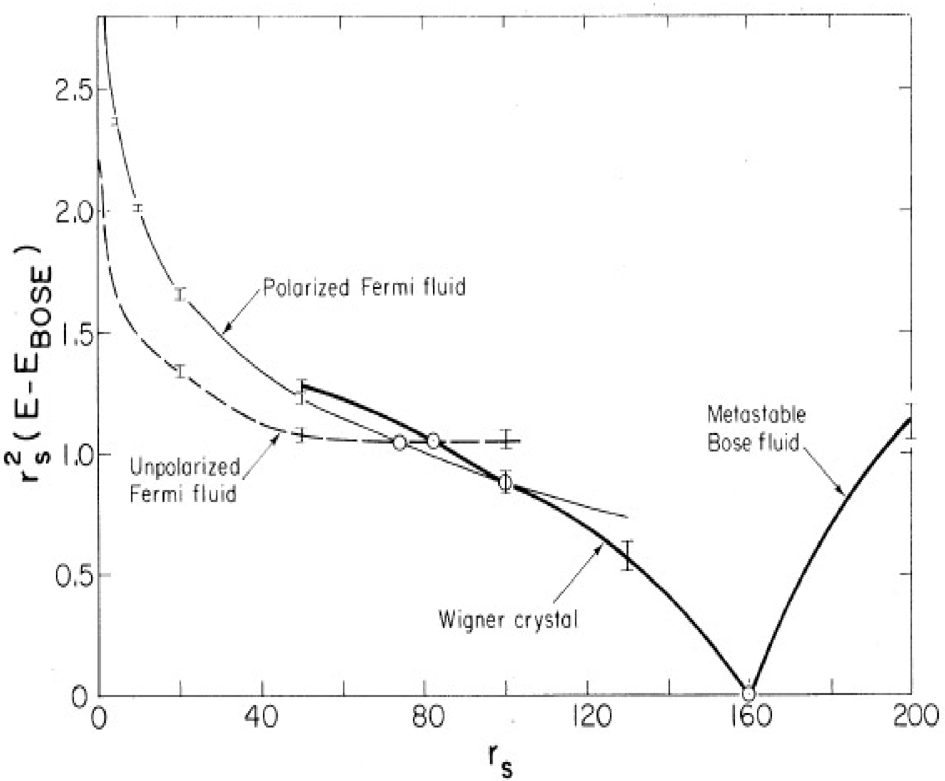
$$\tilde{\Pi} = \Pi_0 + \Pi_0 V \tilde{\Pi} \,.$$

$$\tilde{\Pi} = \frac{\Pi_0}{1 - \Pi_0 V}$$

Effective potential $$v = \frac{V}{1 - \Pi_0 V} = \frac{V}{\epsilon}$$

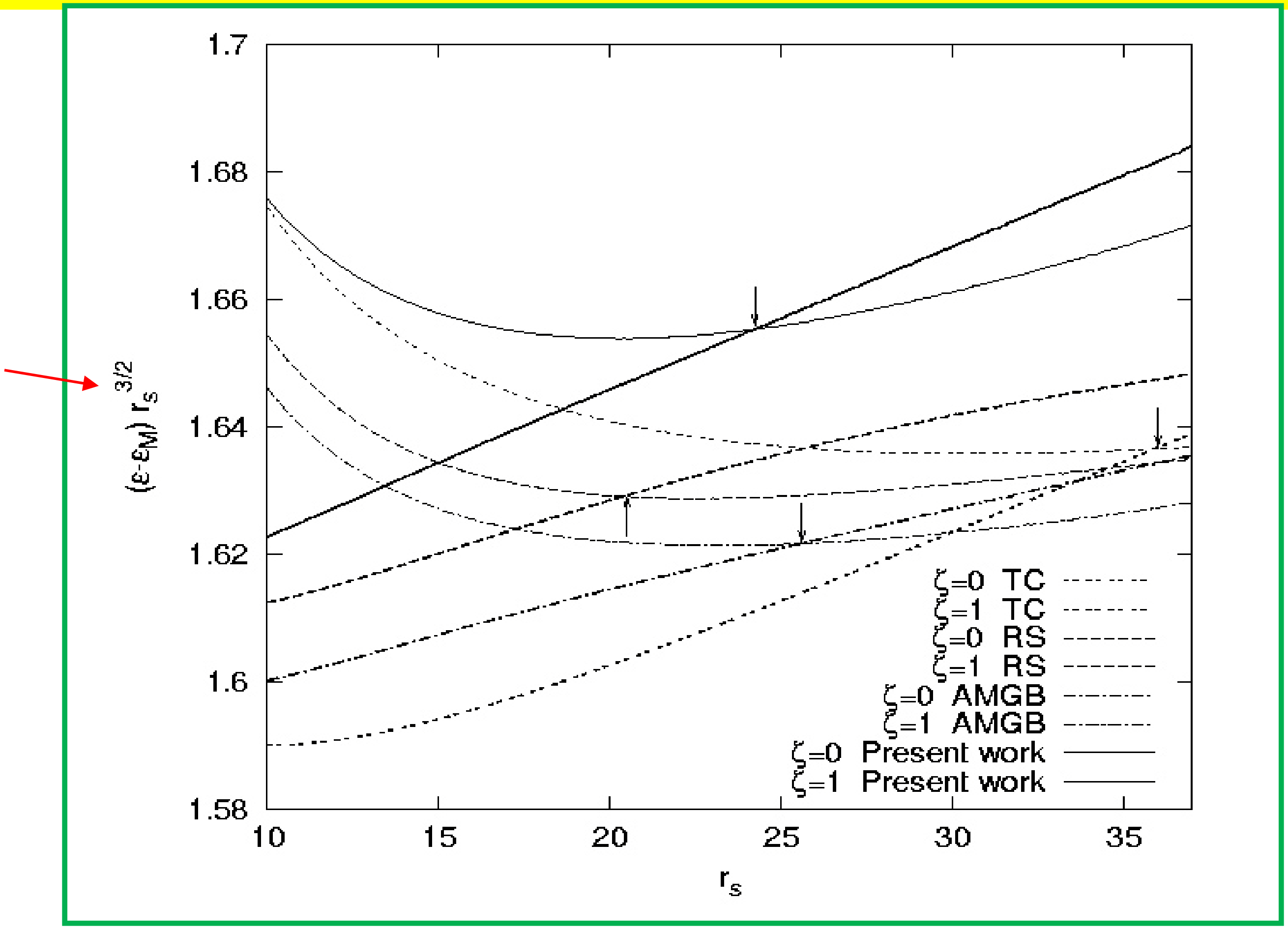
Jellium Model: High density region, Correlation

$$\epsilon(r_s) \simeq \begin{cases} \left(\frac{2.210}{r_s^2} - \frac{0.916}{r_s} + 0.062 \ln r_s - 0.093 + \mathcal{O}(r_s \ln r_s)\right) & Ry \quad , (3D) \\ \left(\frac{1}{r_s^2} - \frac{1.20}{r_s} - 0.17 r_s \ln r_s - (0.38 \pm 0.04) + \mathcal{O}(r_s)\right) & Ry \quad , (2D) \end{cases}$$



Ceperley & Alder, PRL **45**, 566 (1980)

paramagnetic to fully spin-polarized quantum phase transition of a 2D EL



TC: B. Tanatar and D. M. Ceperley, Phys. Rev. B **39**, 5005 (1989)
RS: F. Rapisarda and G. Senatore, Aust. J. Phys. **49**, 161 (1996)
AMGB : C. Attaccalite, *et al*., Phys. Rev. Lett **88**, 256601 (2002)
R. Asgari, B. Davoudi and M. P. Tosi, Solid State Communication 131,1(2004)

Spectral function

$$G(p,\omega) = \int_{-\infty}^{\infty} \frac{A(p,\omega')\, d\omega'}{\omega - \omega' + i\omega'\delta}$$

$$\operatorname{Im} G(p,\omega+\mu) = \begin{cases} -\pi\, A(p,\omega) & \text{for } \omega > 0 \\ \pi\, A(p,\omega) & \text{for } \omega < 0 \end{cases}$$

$$A(p,\omega) = A^*(p,\omega) > 0$$

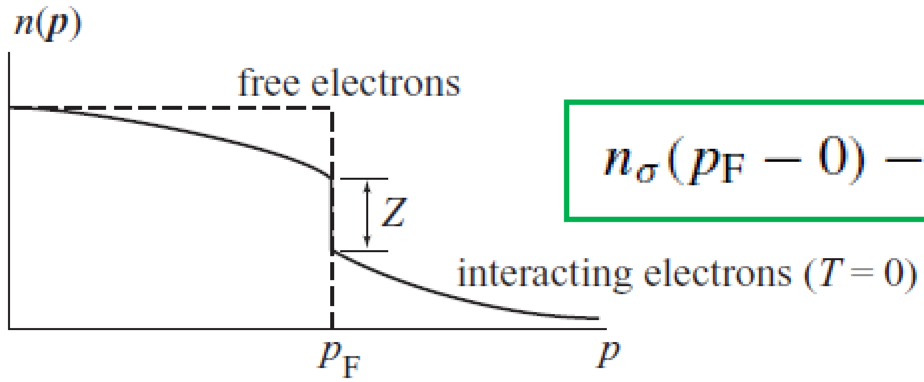
Sum rule $$\int_{-\infty}^{\infty} A(p,\omega)\, d\omega = 1$$

Non-interacting $$A^0(p,\omega) = \delta\big(\omega - [\varepsilon_p - \mu]\big) = \delta(\omega - \xi_p)$$

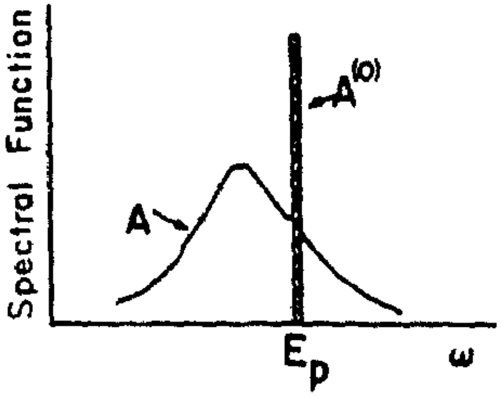
$$A(p,\omega) = \sum_m |(c_p^\dagger)_{m,0}|^2 \delta(\omega - \omega_m^{N+1}) + \sum_m |(c_p)_{m,0}|^2\, \delta(\omega - \omega_m^{N-1})$$

Renormalization constant

$$G(p,\omega) \simeq \frac{Z_p}{\omega - \tilde{\varepsilon}_p + i\gamma}$$



$$n_\sigma(p_F - 0) - n_\sigma(p_F + 0) = Z$$



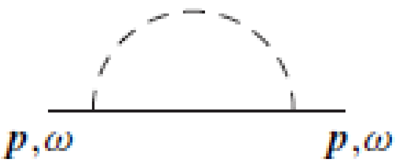
$$A(p,\omega) \simeq Z_p \delta(\omega - \varepsilon_p)$$

Dyson equation

$$G = G_0 + G_0 \Sigma_0 G_0 + G_0 \Sigma_0 G_0 \Sigma_0 G_0 + \cdots$$

Hartree term

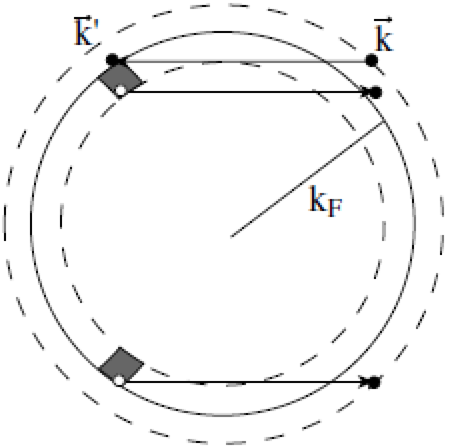
$$\Sigma_0(\boldsymbol{p}, \omega) = \int V(\boldsymbol{q}, \omega') \, G_0(\boldsymbol{p} - \boldsymbol{q}, \omega - \omega') \, \frac{d^3 q \, d\omega'}{(2\pi)^4}$$



$$G(\boldsymbol{p}, \omega) = \frac{G_0}{1 - \Sigma_0 G_0} = \frac{1}{G_0^{-1} - \Sigma_0} = \frac{1}{\omega - \varepsilon_{\boldsymbol{p}} - \Sigma_0(\boldsymbol{p}, \omega) + i\delta_{\boldsymbol{p}}}$$

$$G = G_0 + G_0 \Sigma_0 G ;$$

Fermi Liquid Theory



$$E[\{\mathcal{N}_{\vec{k}\sigma}\}] = E_0 + \sum_{\vec{k}\sigma} \mathcal{E}_{\vec{k}\sigma} \delta\mathcal{N}_{\vec{k}\sigma} + \frac{1}{2} \sum_{\vec{k}\sigma,\vec{k}'\sigma'} f_{\vec{k}\sigma,\vec{k}'\sigma'} \delta\mathcal{N}_{\vec{k}\sigma} \delta\mathcal{N}_{\vec{k}'\sigma'}$$

$$\mathcal{E}_{\vec{k}\sigma} = \left(\frac{\delta E}{\delta\mathcal{N}_{\vec{k}\sigma}} \right)_{\mathcal{N}_{k\sigma} = \mathcal{N}^{(0)}_{k\sigma}} \qquad \mathcal{N}^{(0)}_{\vec{k}\sigma} = \Theta(k_F - k)$$

$$\mathcal{E}_{\vec{k}\sigma} \simeq \mu + \hbar v_F^*(k - k_F) \qquad v_F^* = \frac{1}{\hbar} \left| \frac{\partial \mathcal{E}_{\vec{k}\sigma}}{\partial \vec{k}} \right|_{k=k_F}$$

$$\tilde{\mathcal{E}}_{\vec{k}\sigma} = \frac{\delta E}{\delta\mathcal{N}_{\vec{k}\sigma}} = \mathcal{E}_{\vec{k}\sigma} + \sum_{\vec{k}'\sigma'} f_{\vec{k}\sigma,\vec{k}'\sigma'} \delta\mathcal{N}_{\vec{k}'\sigma'}$$

Fermi Liquid Theory

$$F_\ell^{s,a} = \frac{L^d N^*(0)}{2} \int \frac{d\Omega_d}{\Omega_d} [f_{\uparrow\uparrow}(\cos\theta) \pm f_{\uparrow\downarrow}(\cos\theta)] \begin{cases} P_\ell(\cos\theta), & 3D, \\ \cos\ell\theta, & 2D, \end{cases}$$

$$f_{\uparrow\uparrow}(\cos\theta) \pm f_{\uparrow\downarrow}(\cos\theta) = \frac{2}{L^d N^*(0)} \sum_{\ell=0}^{\infty} F_\ell^{s,a} \begin{cases} (2\ell+1) P_\ell(\cos\theta), & 3D, \\ (2-\delta_{\ell 0}) \cos\ell\theta, & 2D, \end{cases}$$

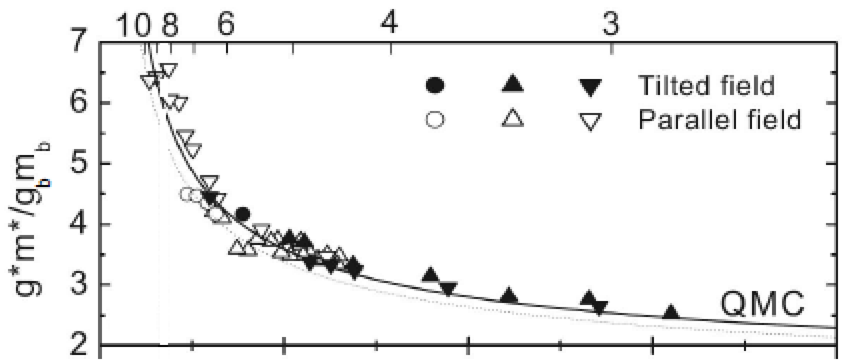
$$\frac{K}{K_0} = \frac{\frac{m^*}{m}}{1+F_0^s} .$$

$$\frac{\chi_S}{\chi_P} = \frac{\frac{m^*}{m}}{1+F_0^a} ,$$

$$\frac{\chi_S}{\chi_P} = \frac{m^*}{m}\frac{g^*}{g}$$

$$\frac{m^*}{m} = 1 + F_1^s ,$$

$$\vec{v}\cdot\vec{k} = \frac{m}{m^*}\vec{v}\cdot\vec{k} + \frac{m}{m^*}\vec{v}\cdot\vec{k}\frac{L^d N^*(0)}{2}\int \frac{d\Omega'_d}{\Omega_d}\cos\theta' \sum_{\sigma'} f_{\sigma\sigma'}(\cos\theta') ,$$



Effective mass

$$A_\sigma(\vec{k},\omega) = -\frac{\hbar}{\pi}\frac{\Im m\,\Sigma_\sigma^{ret}(\vec{k},\omega)}{[\hbar\omega - \varepsilon_{\vec{k}\sigma} - \Re e\,\Sigma_\sigma^{ret}(\vec{k},\omega)]^2 + [\Im m\,\Sigma_\sigma^{ret}(\vec{k},\omega)]^2}.$$

$$\hbar\omega - \varepsilon_{\vec{k}\sigma} - \Re e\,\Sigma_\sigma^{ret}(\vec{k},\omega) \approx \frac{(\hbar\omega - \mathcal{E}_{\vec{k}\sigma})}{Z_{\vec{k}\sigma}},\qquad Z_{\vec{k}\sigma} \equiv \left(1 - \frac{1}{\hbar}\frac{\partial}{\partial\omega}\Re e\,\Sigma_\sigma^{ret}(k,\omega)\Big|_{\hbar\omega=\mathcal{E}_{\vec{k}\sigma}}\right)^{-1}$$

$$A_\sigma(k,\omega) \simeq \frac{Z_{\vec{k}\sigma}}{\pi}\frac{\frac{1}{2\tau_{\vec{k}\sigma}}}{\left(\omega - \frac{\mathcal{E}_{\vec{k}\sigma}}{\hbar}\right)^2 + \left(\frac{1}{2\tau_{\vec{k}\sigma}}\right)^2},$$

$$\mathcal{E}_{k\sigma} \simeq \mu + \frac{\hbar^2 k_F(k-k_F)}{m^*}\qquad \frac{\hbar^2 k_F}{m^*} = \frac{d\mathcal{E}_{k\sigma}}{dk}\Big|_{k=k_F}$$

$$\frac{m^*}{m} = \frac{1}{Z_{k_F\sigma}\left[1 + \frac{m}{\hbar^2 k_F}\frac{\partial}{\partial k}\Re e\,\Sigma_\sigma^{ret}(k,\mu)\Big|_{k=k_F}\right]}$$

Effective mass

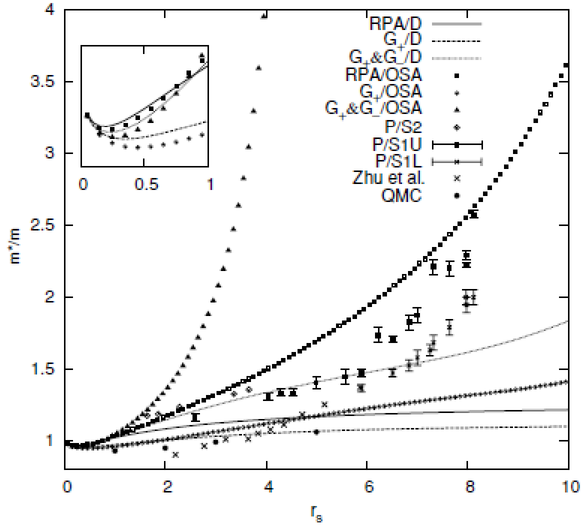
$$\Sigma_\sigma(\vec{k}, i\omega_n) = -\frac{1}{\hbar\beta}\int \frac{d\vec{q}}{(2\pi)^d}\sum_{m=-\infty}^{\infty}\mathcal{G}_\sigma(\vec{k}-\vec{q}, i\omega_n - i\Omega_m)W(q, i\Omega_m),$$

$$W(q,\omega) = v_q + v_n^2(q)\chi_{nn}(q,\omega) + 3v_s^2(q)\chi_{S_z S_z}(q,\omega)$$

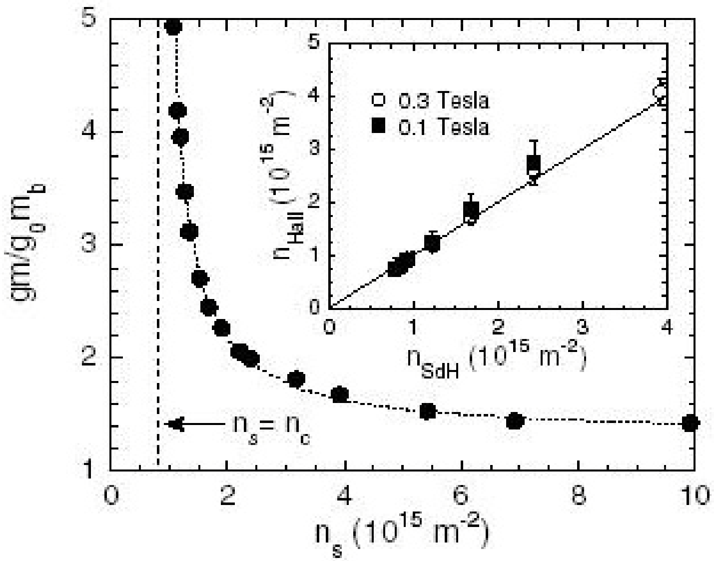
$$V_{\uparrow\uparrow}^{eff}(q) = v_q + \{v_q[1-G_+(q)]\}^2\chi_{nn}(q) + \{v_q G_-(q)]\}^2\chi_{S_z,S_z}(q),$$

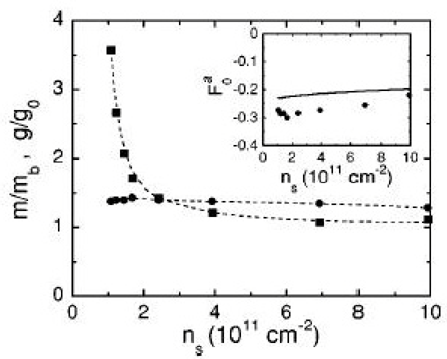
$$V_{\uparrow\downarrow}^{eff}(q) = \{v_q G_T(q)\}^2\frac{1}{4}\chi_{S_+S_-}(q),$$

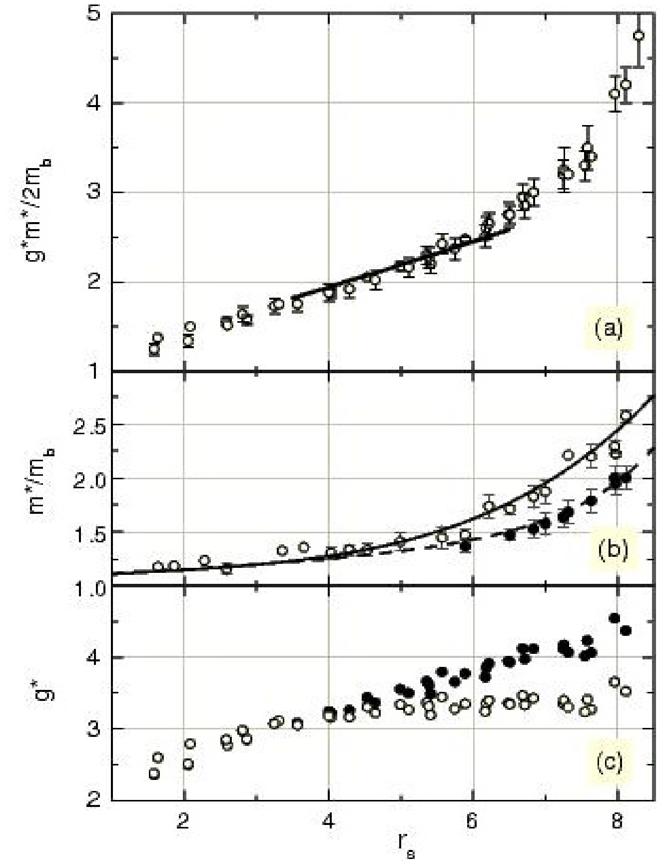
Local field factors



Some of the recent experimental results in Silicon MOSFETs







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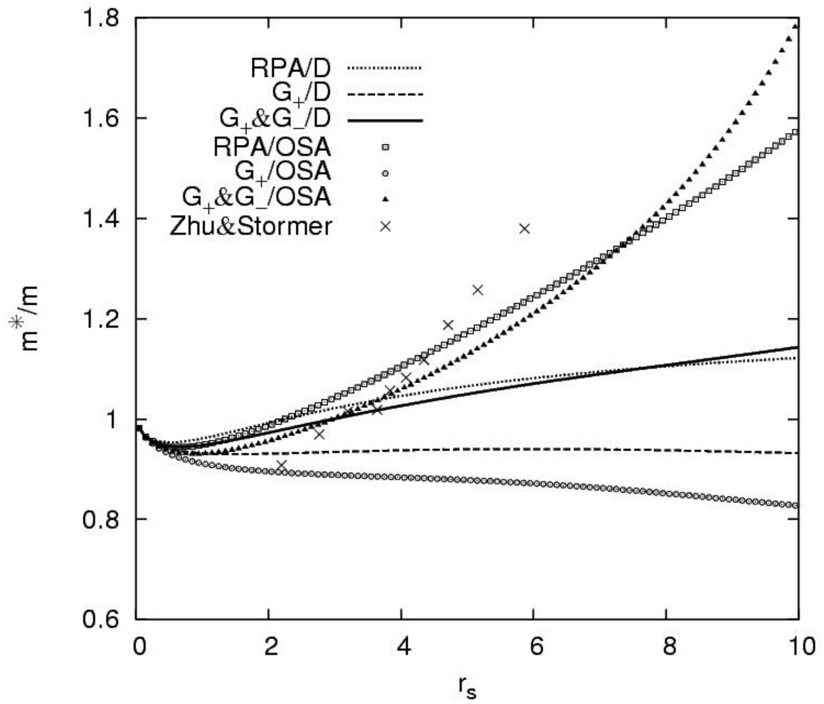
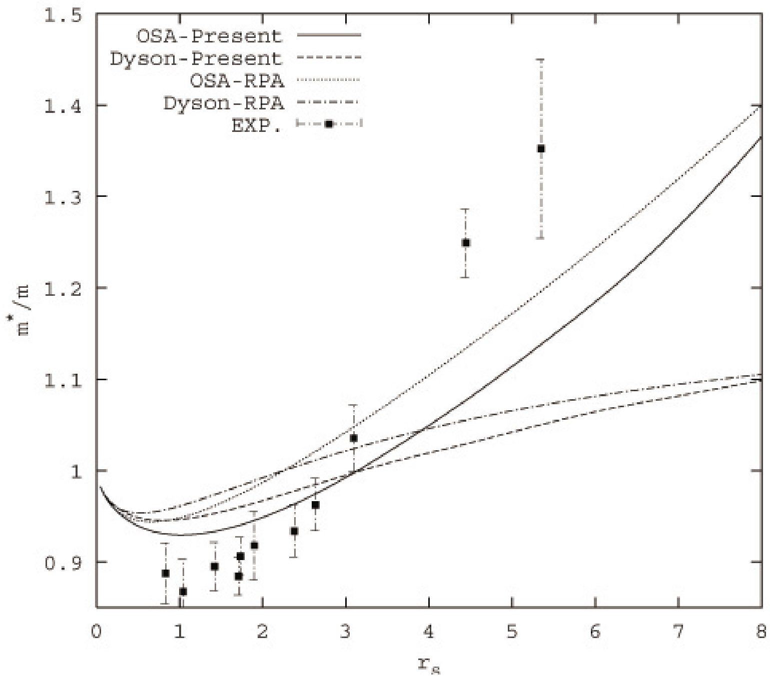
Numerical results: theory versus experiment

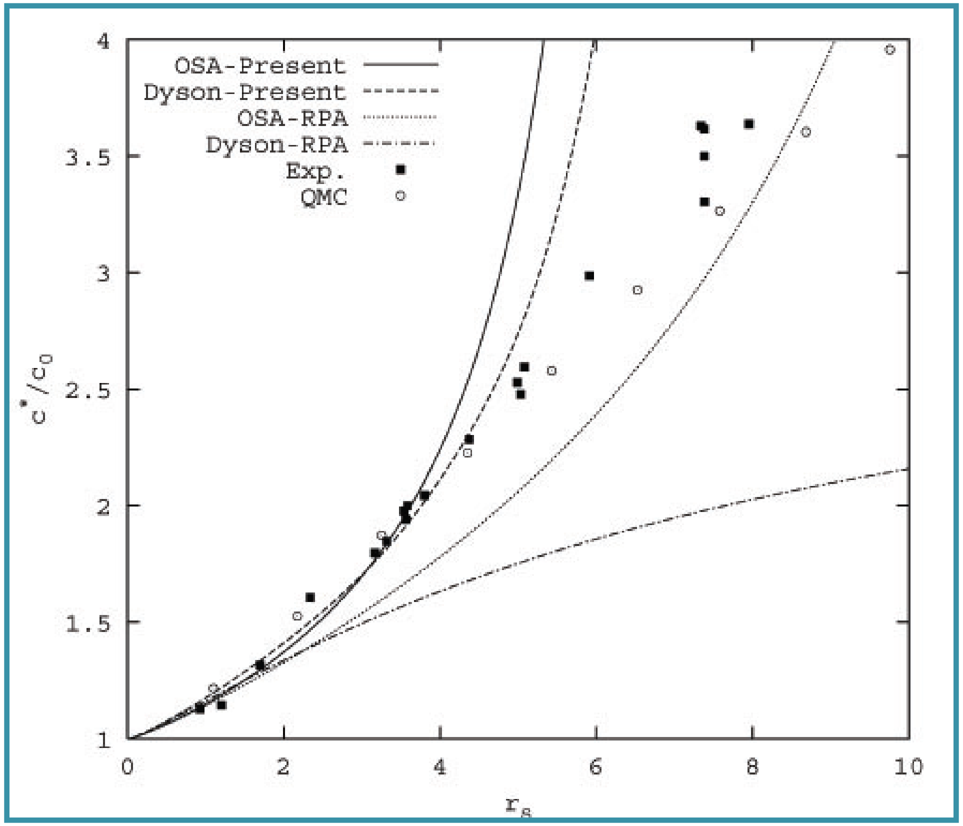




R. Asgari, B. Tanatar Phys. Rev. B 74, 075301 (2006)

R. Asgari, B. Davoudi and B. Tanatar, Solid State Commun.**130,**13(2004**)**
R. Asgari ,B. Davoudi, M. Polini, G. Giuliani,M.P. Tosi and G.Vignale, Phys. Rev. B,**71** 045323 (2005)

Spin Susceptiblity: theory versus experiment



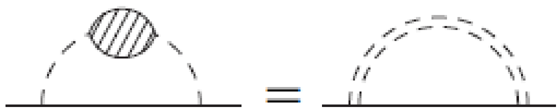
R. Asgari, B. Tanatar Phys. Rev. B 74, 075301 (2006)
R. Asgari, L. Subasi, A. Sabori and B. Tanatar Phys. Rev. B 74, 155319 (2006)

Marginal Fermi Liquid

$$\mathrm{Im}\,\Pi(\boldsymbol{q},\omega) = \begin{cases} -\rho(\varepsilon_\mathrm{F})\,\omega/T & \text{for } \omega \ll T \\ -\rho(\varepsilon_\mathrm{F}) & \text{for } T \ll \omega \ll \omega_\mathrm{c}\,. \end{cases}$$

$$\mathrm{Re}\,\Pi(\boldsymbol{q},\omega) \sim \rho(\varepsilon_\mathrm{F})\ln\left(\frac{\omega}{T}\right)$$

$\Sigma =$

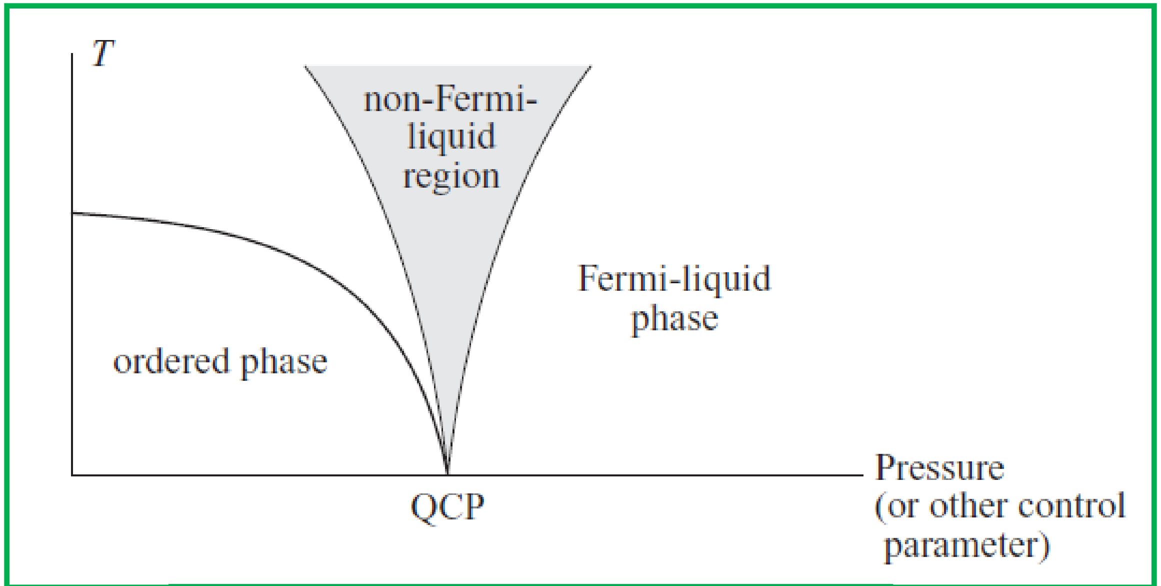


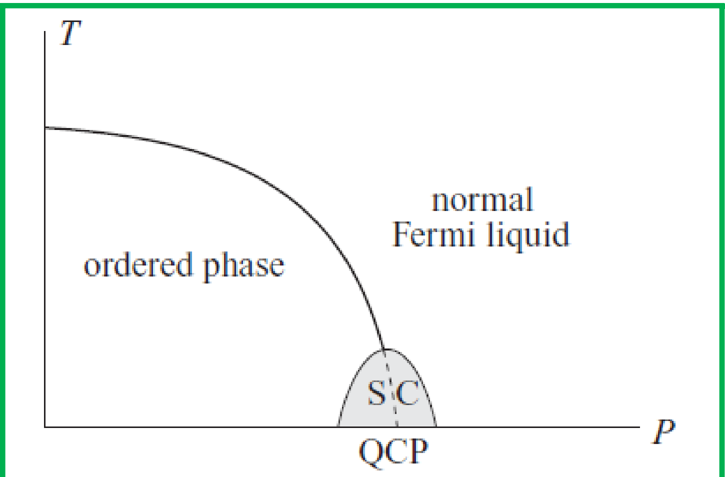
$$\Sigma(\omega) = \lambda\omega\left[\ln\frac{x}{\omega_\mathrm{c}} + i\frac{\pi}{2}x\,\mathrm{sign}(\omega)\right] \qquad x = \max(\omega, T).$$

$$Z = \frac{1}{1-\partial\,\mathrm{Re}\,\Sigma/\partial\omega} \sim \frac{1}{1-\lambda\ln\left(y/\omega_\mathrm{c}\right)} \qquad y = \max[(\varepsilon_p-\mu), T]$$

$$\tau^{-1} \sim (\varepsilon_p - \mu).$$

Non-Fermi Liquid close to a QCP

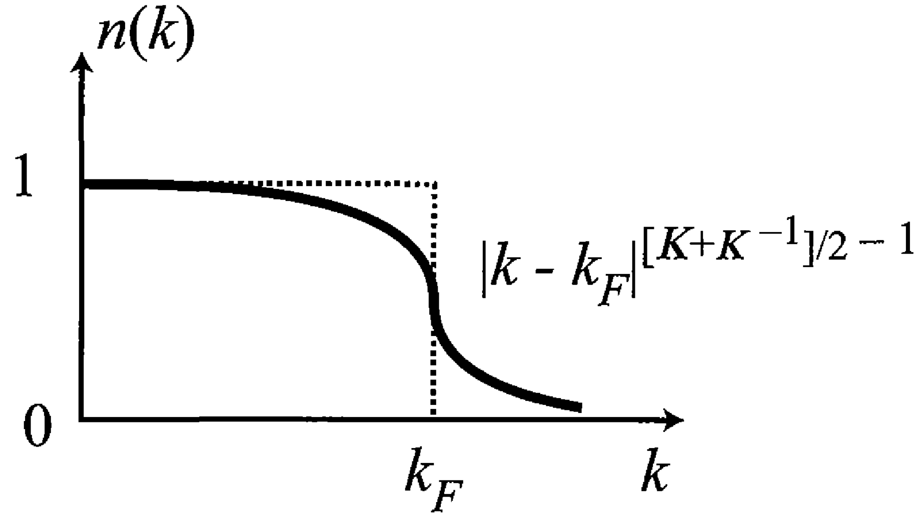




SC – superconducting phase

Luttinger Liquid

$$n(p) - n(p_{\mathrm{F}}) \sim |p - p_{\mathrm{F}}|^{\delta} \operatorname{sign}(p - p_{\mathrm{F}}).$$



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