Textures in Quantum Hall Systems
Past and New Developments

Bahman Roostaei

IUPUI, IPM

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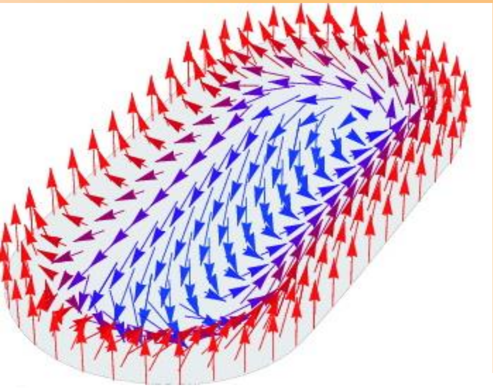
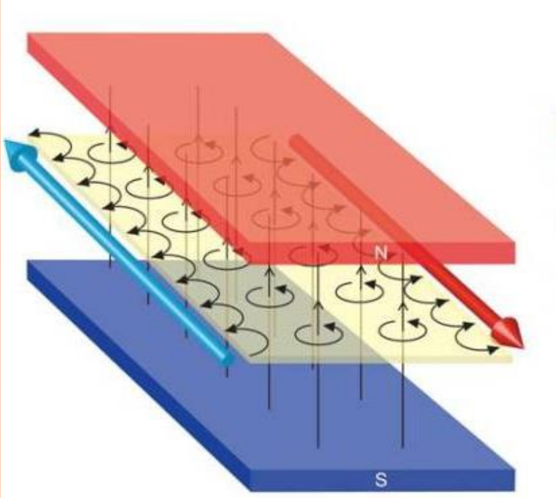
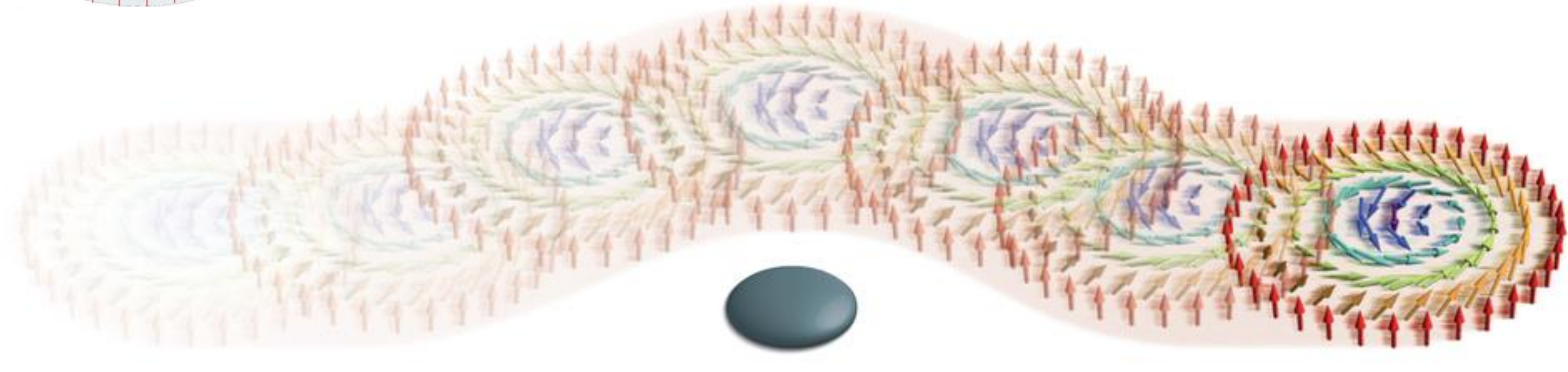






What is Interesting?

- Collective behavior in extreme quantum regime
- Internal degrees of freedom
- Interplay between charge and other degrees of freedom such as spin and …
- How in the world can we detect them ?
- A rich phase diagram taking years to explore!

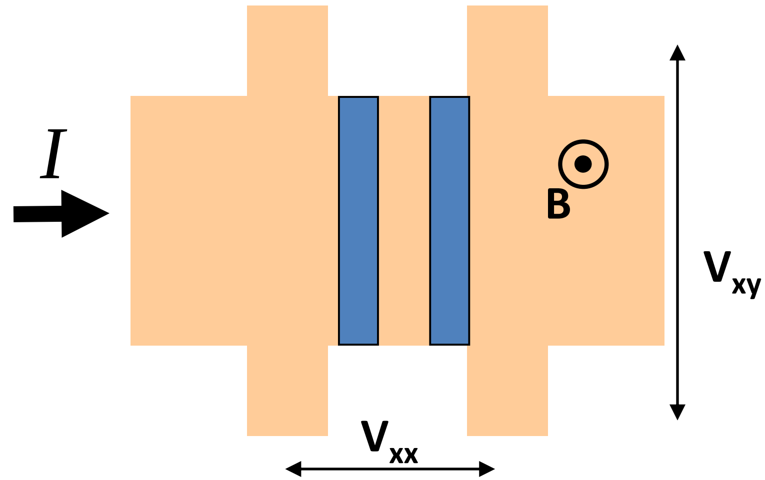
Plan of the talks

- First Day:
  - Introduction to Integer Quantum Hall Phenomena
  - Spin Textures and Detection

- Second Day:
  - Double Quantum Well Systems
  - Interesting Intertwined Textures and Detection

Graphene

Quantum Hall Effect



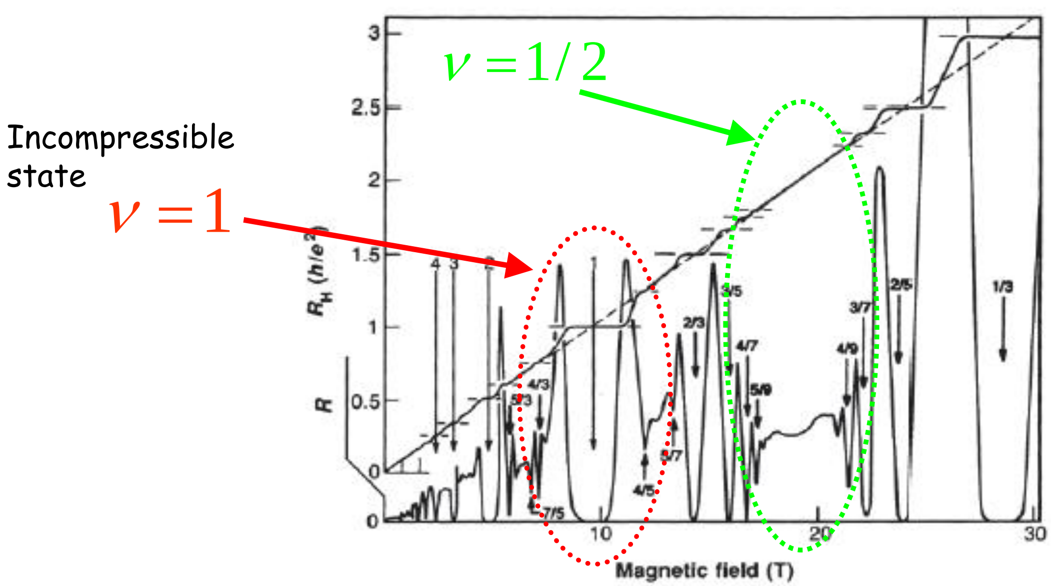
➢**Hall Conductivity is quantized**

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

➢**Longitudinal resistivity is zero (Dissipationless)**

$$\rho_{xx} = 0$$

H.L. StÄormer, PhysicaB 177, 401(1992)

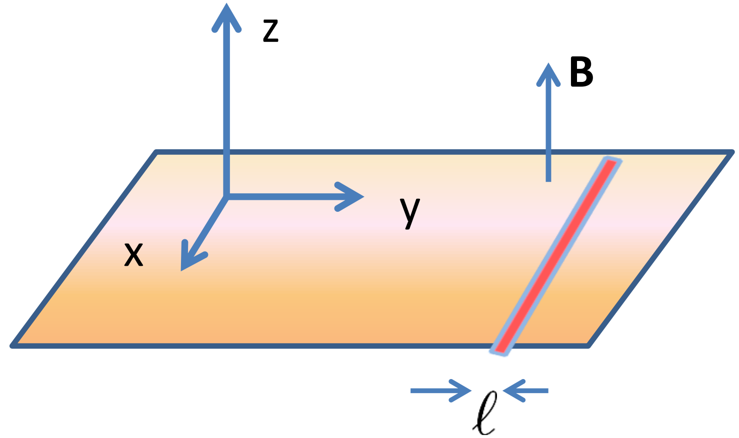


Infinite System (Translationally Symmetric):

$$\rho = \frac{ne}{B} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

No Quantum Hall Effect !

Landau Levels



$$\hat{H} = \frac{1}{2m^*}\left(\mathbf{P} - e\mathbf{A}\right)^2$$

$$\nabla \times \mathbf{A} = B\hat{z}$$

$$\mathbf{A} = \frac{B}{2}(-y, x, 0)$$

Landau Gauge

$$\omega_c = \frac{eB}{m^*}$$

Cyclotron Frequency

$$\hat{H} \approx \frac{\mathbf{P}^2}{2m^*} + \frac{1}{2}\omega_c^2 x^2$$

Harmonic Oscillator in x-direction

$$\ell = \sqrt{\frac{\hbar}{eB}}$$

Magnetic length

Eigenstates:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

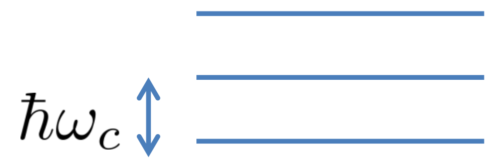
$$\psi_{n,X}(x,y) = H_n(x - X)e^{iXy/\ell^2}$$

$$X = -k\ell^2$$

Translational symmetry in y-direction

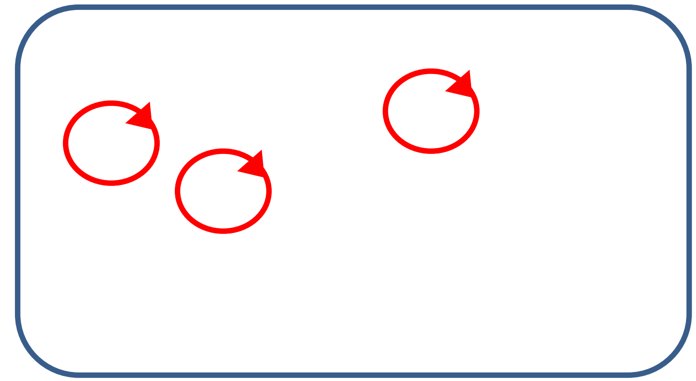
Landau Level Degeneracy

Each Eigenstate is localized in an area: $2\pi\ell^2$



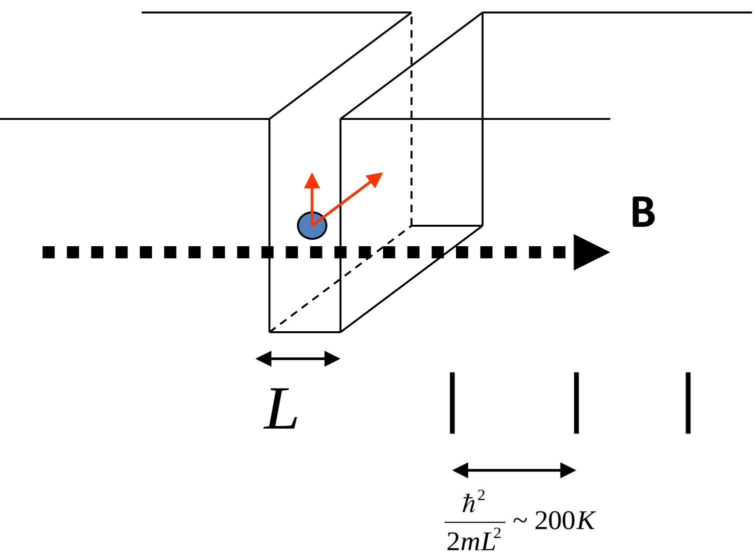
Degeneracy associated with the location of the center of the wavefunction:

$$\frac{S}{2\pi\ell^2} = \frac{\Phi}{\phi_0}$$

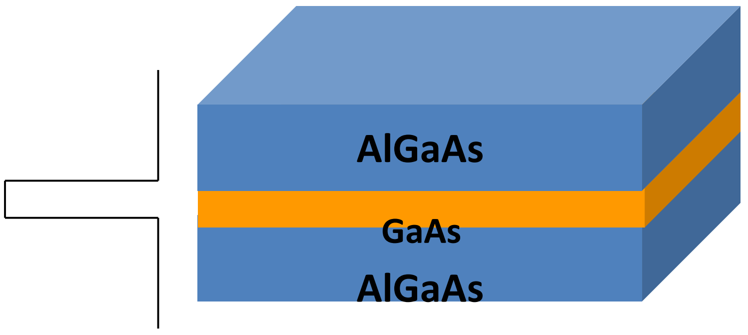


$$\phi_0 = h/e^2$$ **Quantum of Flux**

Quantum Wells

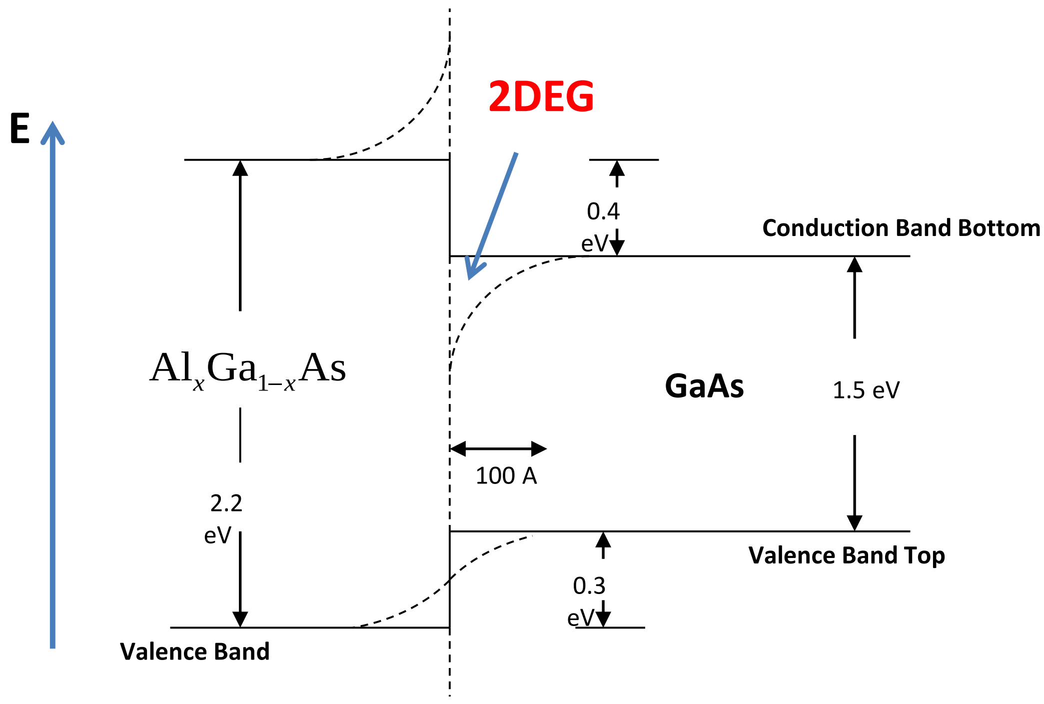


Molecular Beam Epitaxy (MBE)



- **A high mobility electron gas is formed in the lowest subband of the quantum Well at low enough temperatures.**

- **This electron gas is thus two dimensional.**



Quantum Conductance

Total number of electrons

Landau level filling factor:

$$2\pi\ell^2 \times n = \nu = \frac{N}{g}$$

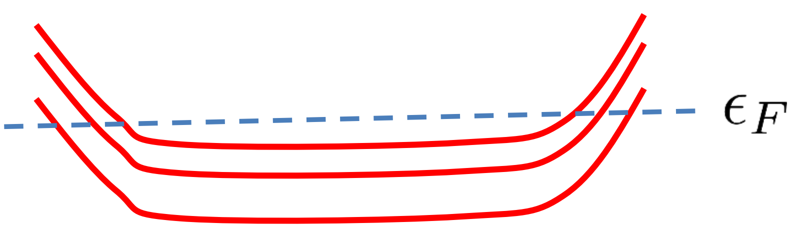
$$g = \frac{\Phi}{\phi_0}$$

At certain values of density Hall conductance is quantized:

$$\rho_{xx} = 0 \qquad \sigma_{xx} = 0$$

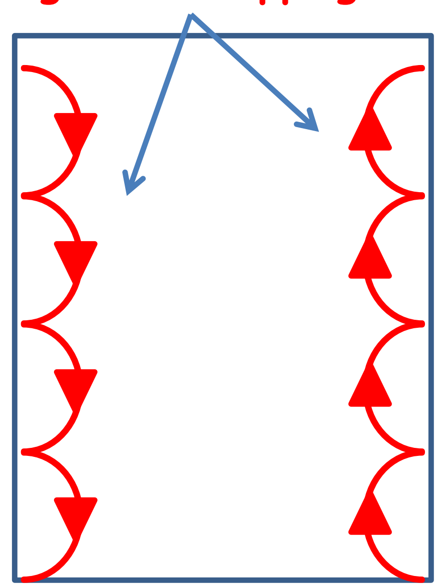
$$\rho_{xy} = \frac{1}{\nu}\frac{h}{e^2} \qquad \sigma_{xy} = -\nu\frac{e^2}{h}$$

Using Landauer-Buttiker formula we can calculate the above values for edge states:



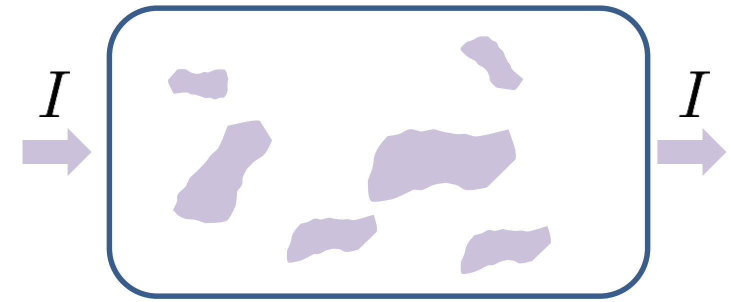
$$I = -\frac{e}{h}\nu\left[\mu_L - \mu_R\right]$$

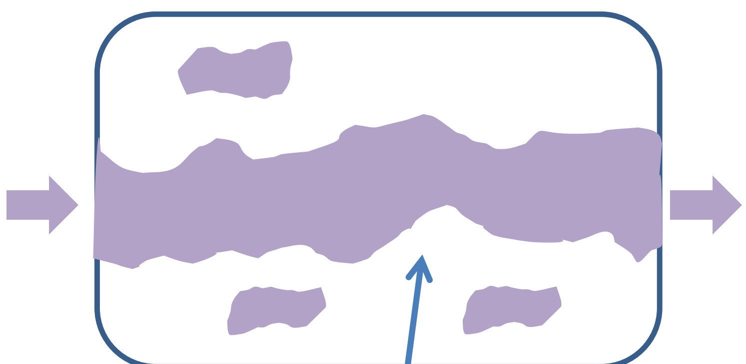
The only conducting channels are at the edges via skipping orbits.



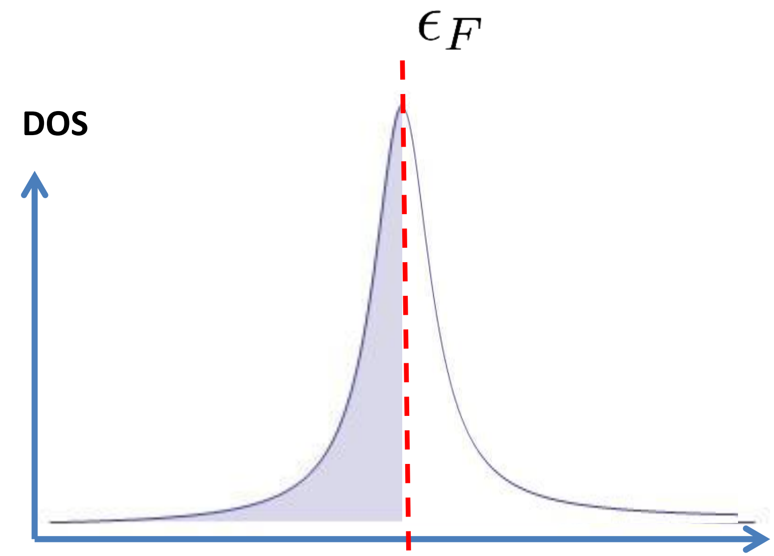
Disorder

Quantum Hall Plateau's are the result of disorder localizing states in the bulk.

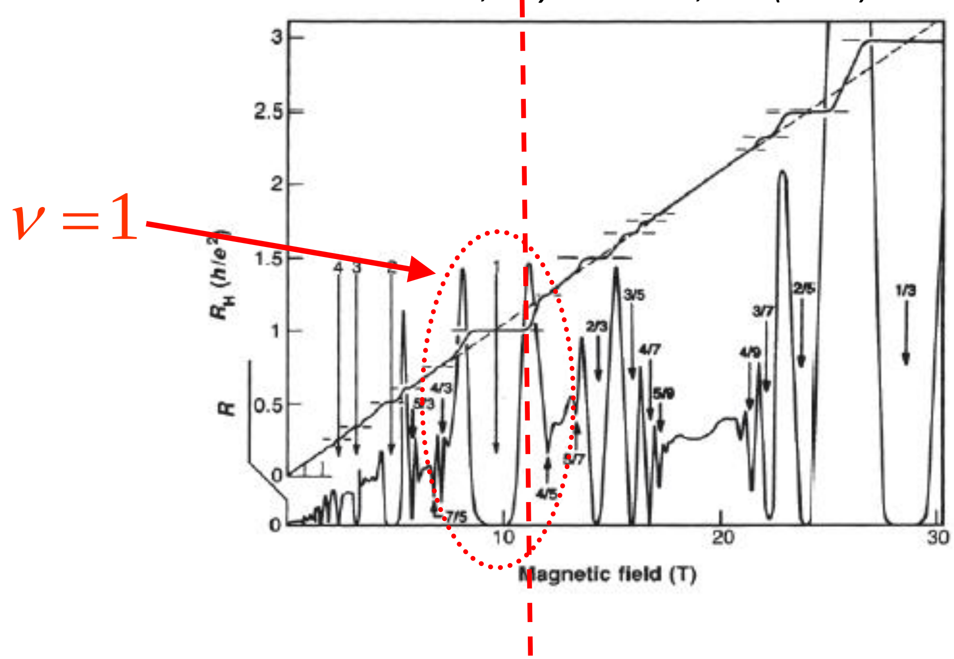




Onset of percolation :
transition to next Hall plateau



H.L. StÄormer, PhysicaB 177, 401(1992)



Quasiparticles with Fractional Charge

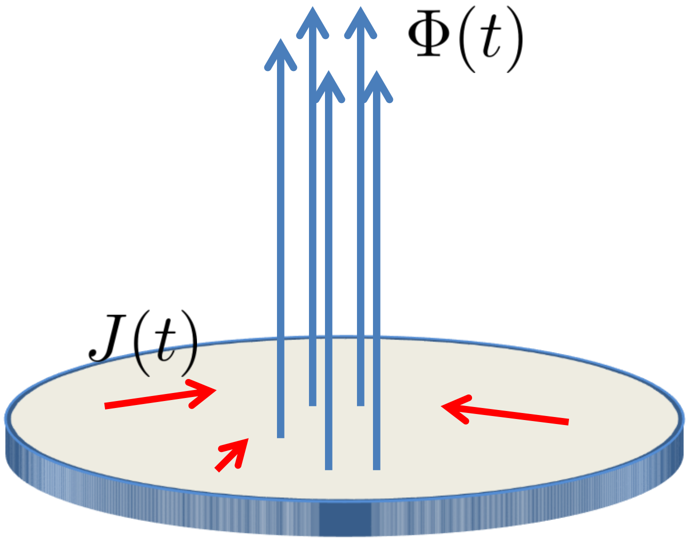
Quantized Hall Conductance implies quantized elementary charged excitations.

a) An adiabatic turning on of a flux moves a certain amount of charge which is proportional to Hall conductance:

$$Q(t) = \sigma_{xy}\Phi(t)$$

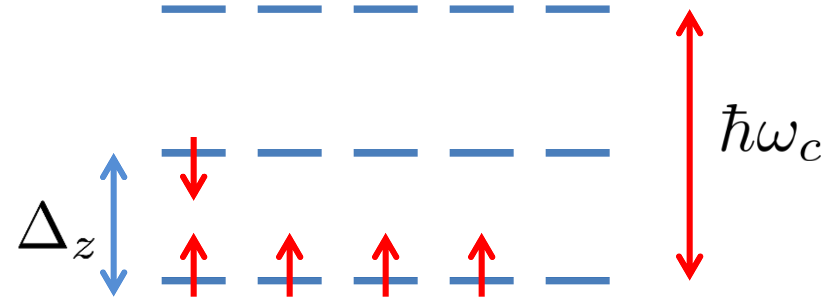
b) At the time when one quantum of flux is completed the system goes back to its ground state:

$$Q(t) = -\nu e$$



Note: Because of zero longitudinal resistance the system is gapped hence adiabatic transformation is possible.

Quantum Hall Ferromagnet



Order of the spin of electrons:

$$\hat{H} = \frac{1}{2m^*}\left(\mathbf{P} - e\mathbf{A}\right)^2 + \Delta_z S_z$$

Zeeman Energy

$$\Delta_z = \eta B$$

$$\eta = 2\mu_B \Rightarrow \Delta_z = \hbar\omega_c$$ vacuum

Possibility of spin fluctuations

$$\eta \approx -0.4\mu_B \Rightarrow \Delta_z \approx 1K$$ GaAs

Does Coulomb interaction still allow a ferromagnetic order in ground state ?

We focus on completely filled Lowest Landau Level (LLL)

The answer is Yes!

Quantum Hall Ferromagnet

a) Ferromagnetic order is an eigenstate of the Hamiltonian.

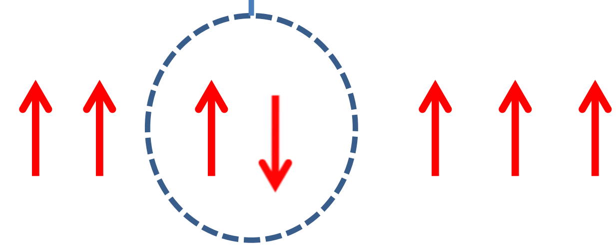
$$z = (x + iy)/\ell$$

$$\Psi(z_1, ..., z_N) = \Pi_{i<j}^{N}(z_i - z_j)\Pi_{i=1}^{N} e^{-|z_i|^2/4\ell^2}$$

b) Coulomb exchange energy can be estimated, using Laughlin wavefunction to be high:

$$\langle V \rangle \approx \sqrt{\frac{\pi}{8}}\frac{e^2}{\epsilon\ell} \approx 200K$$

No kinetic energy for electrons in LLL to compensate (Bad screening)



But this is not enough:

How about collective excitations ?

Dynamics in Lowest Landau Level

Confining electrons to
lowest Landau level

Projection of the operators
to LLL Hilbert space.

$$\bar{z} = z$$
$$\overline{z^*} = \sqrt{2}\frac{\partial}{\partial z}$$

$$\Rightarrow \quad \overline{\rho}_{\mathbf{q}} = \frac{1}{\sqrt{S}}\overline{e^{i\mathbf{q}.\mathbf{r}}} = \frac{1}{\sqrt{S}}e^{-|q|^2/4}\exp\left(-iq\frac{\partial}{\partial z} - \frac{i}{2}q^* z\right)$$

$$\tau_{\mathbf{q}} = \exp\left(-iq\frac{\partial}{\partial z} - \frac{i}{2}q^* z\right)$$

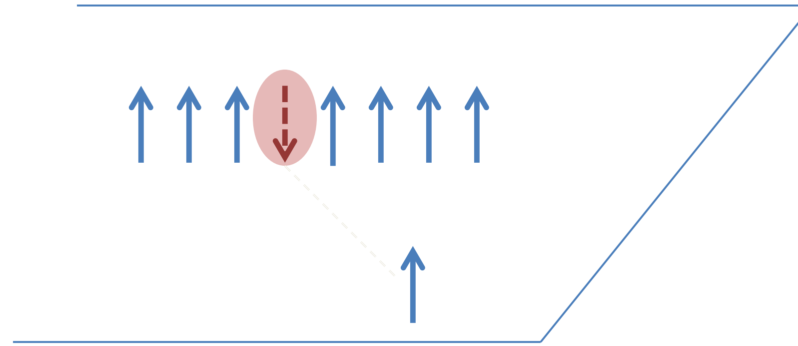
$$\tau_{\mathbf{q}}\tau_{\mathbf{k}} = \tau_{\mathbf{q}+\mathbf{k}}e^{i/2\mathbf{q}\times\mathbf{k}} \qquad [\tau_{\mathbf{q}}, \tau_{\mathbf{k}}] = 2i\tau_{\mathbf{q}+\mathbf{k}}\sin\left(\frac{i}{2}\mathbf{q}\times\mathbf{k}\right)$$

$$\tau_{\mathbf{q}} = \text{Translation along } \ell^2\hat{z}\times\mathbf{q}$$

$$\overline{\rho}_{\mathbf{q}} = \frac{1}{\sqrt{S}}e^{-|q|^2/4}\tau_{\mathbf{q}}$$

$$\boxed{[\overline{\rho}_{\mathbf{q}}, \overline{\rho}_{\mathbf{k}}] \neq 0}$$

Excitations in Lowest Landau Level



There are two types of excitations:

a) Spin waves: b) Quasiparticles

Using spin lowering operator we construct spin wave states.

$$\overline{S_{\mathbf{q}}^{-}} = \sum_{j=1}^{N} \overline{e^{-i\mathbf{q}\cdot\mathbf{R}_j}} S_j^{-} = e^{-|q|^2/4} \sum_{j=1}^{N} \hat{\tau}_{\mathbf{q}}(j) S_j^{-}$$

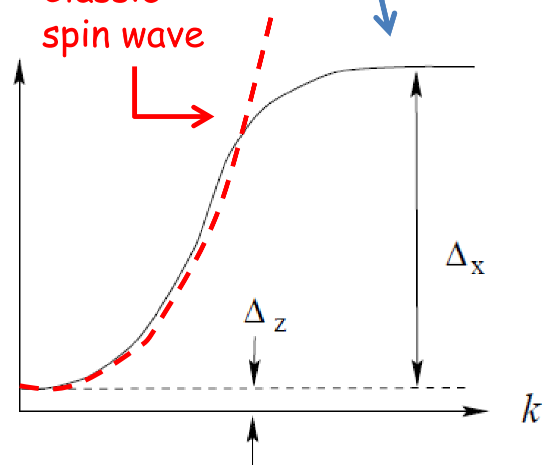
$$\left[\hat{H}, \overline{S_{\mathbf{q}}^{-}}\right] |\Psi_0\rangle = \epsilon_{\mathbf{q}} \overline{S_{\mathbf{q}}^{-}} |\Psi_0\rangle$$

$$\hat{H} = \frac{1}{2} \sum_{k \neq 0} v(k) \overline{\rho}_{-\mathbf{k}} \overline{\rho}_{\mathbf{k}} - \Delta_z \sum_j S_j^z$$

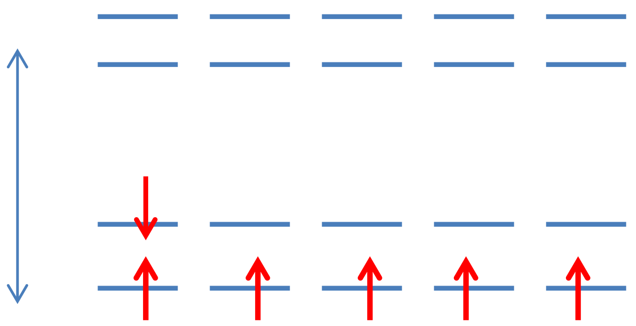
$$\epsilon_{\mathbf{q}} = 2 \sum_{k \neq 0} e^{-|k|^2/2} v(k) \sin^2(\mathbf{q} \times \mathbf{k}/2) + \Delta_z$$

Almost all the exchange energy is spent because of magnetic translation

Classic spin wave



Excitations in Lowest Landau Level



b) Quaisparticles

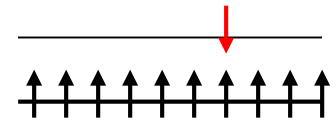
Addition of an electron to the filled Landau level creates a quasiparticle.

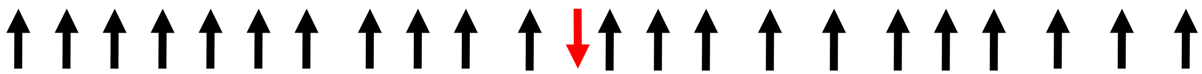
What is the lowest energy configuration?

- Because of the exchange interaction the single spin flip excitations cost a lot of energy.

Excitations in Lowest Landau Level

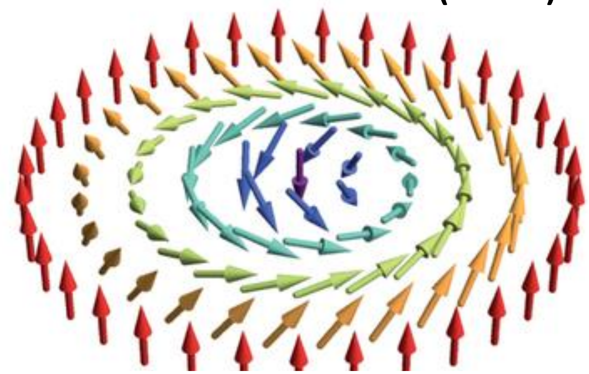
- Cheapest excitation is the collective one.



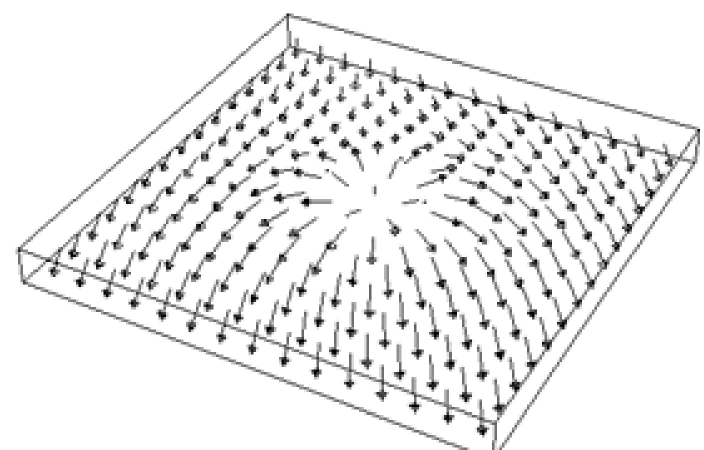


Achim Rosch (Koeln)

Skyrmion



- Their in-plane projection is a vortex (anti-vortex)
- They cost energy to create
- They are spread
- Very stable (Topological)
- Has electrostatic charge



Adding an electron to the system = Adding one Skyrmion.

Kieran Mullen (OU)

Charged Spin Textures

spin configurations in LLL may induce a charge distribution.

It is possible to show that for smooth distributions the induced charge is proportional to the Pontryagin index of the spin field:

PRB 51,5138 (1995)

$$\delta\rho(\mathbf{r}) = -\frac{\nu}{8\pi}\epsilon_{\mu\nu}\mathbf{m}(\mathbf{r})\cdot[\partial_\mu\mathbf{m}(\mathbf{r})\times\partial_\nu\mathbf{m}(\mathbf{r})]$$

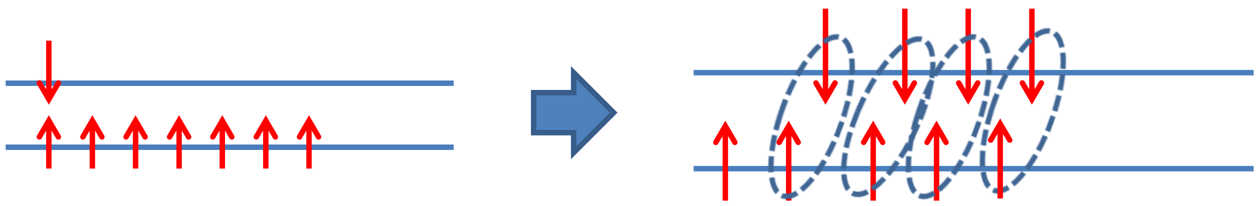
$$\int \delta\rho(\mathbf{r})d^2r = -\nu Q_T$$

Total induced electrostatic charge is proportional to the topological charge!

It is also possible to argue, using Berry phase of the spin that in a quantum Hall system spin textures carry electrostatic charge proportional to topological index of the texture.

Skyrmion Wavefunction

$$|\Psi\rangle = \Pi_m \left[u_m c^\dagger_m + v_m c^\dagger_{m+1} \right] |0\rangle$$

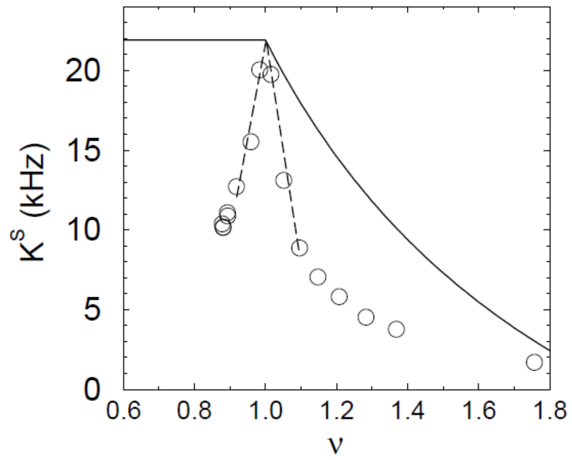


Skyrmions in the Lab

a) Knight Shift

➢Number of reversed spins in a skyrmion is large.

➢This will effect polarization of nuclear spins.



Barret , et. Al. , PRL 74,5112(1995),

Electron spin density at the nuclear site.

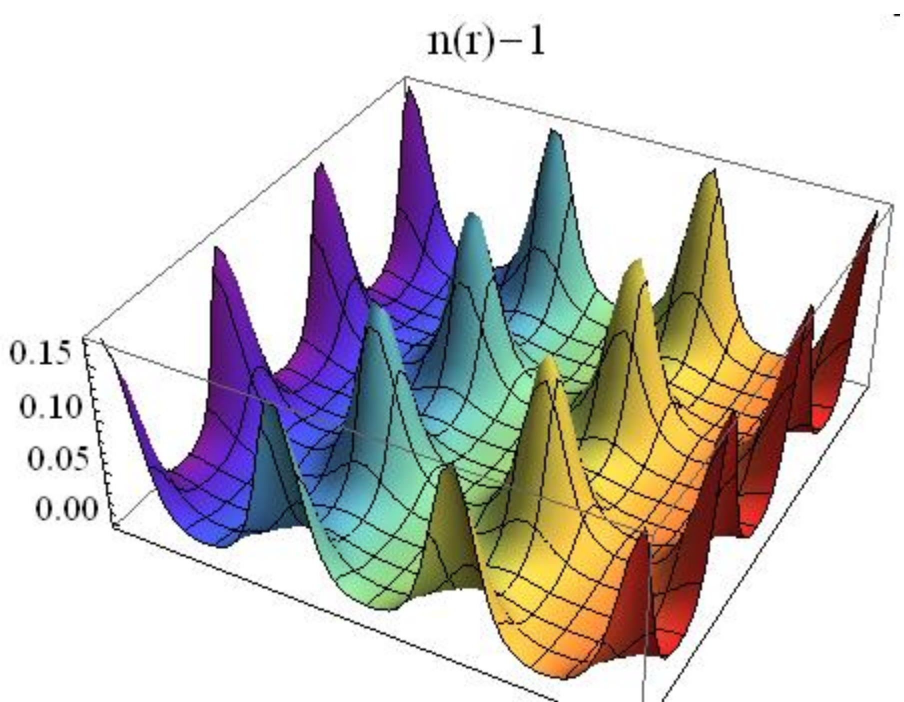
$$H_N = -\Delta_N I_z + \Omega \mathbf{I} \cdot \mathbf{S} \longrightarrow H_N \approx (-\Delta_N + \Omega \langle S_z \rangle) I_z.$$

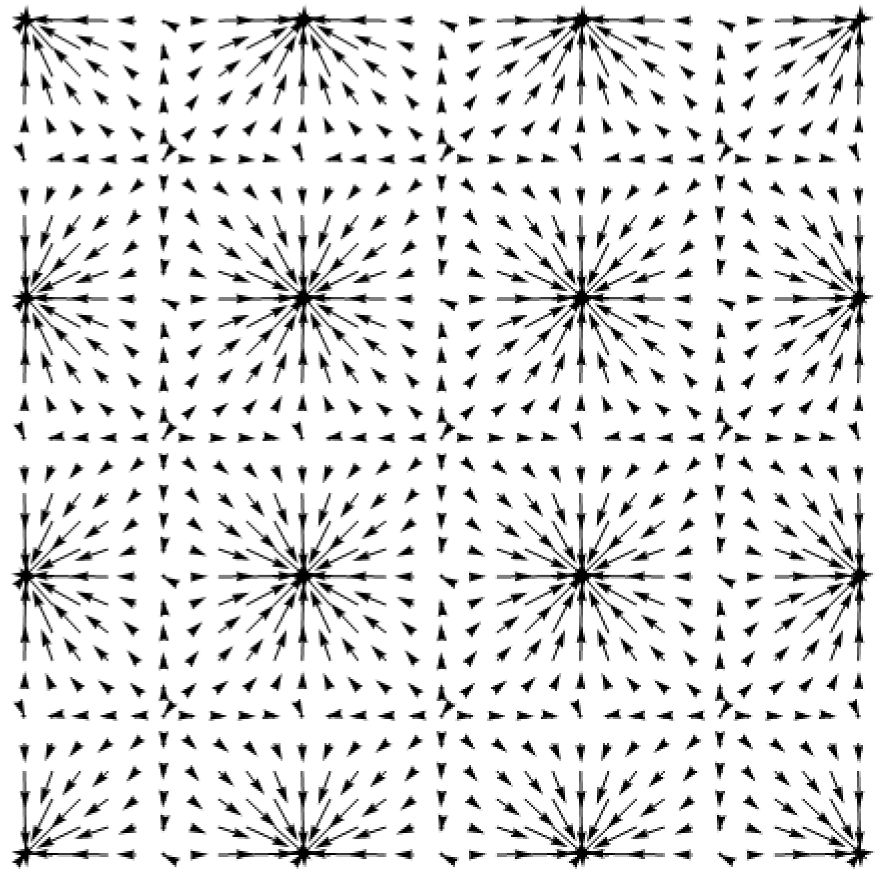
Skyrmions in the Lab

Skyrmion Lattice

- We can create skyrmions by increasing/decreasing density of electrons.

- Skyrmions form a lattice. Confirmed by NMR experiments.





Skyrmions in the Lab

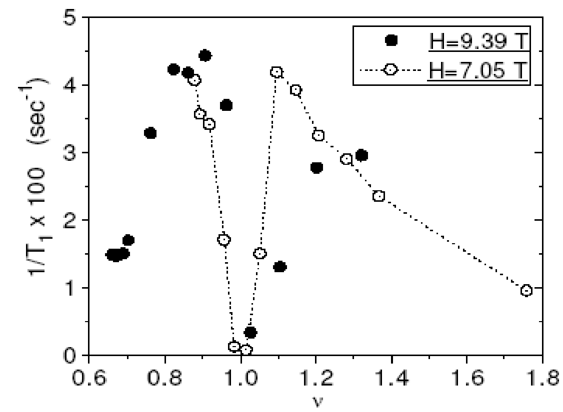
b) NMR – Skyrmion Dynamics

- Single electron spin is not able to exchange angular momentum with nuclear spins.

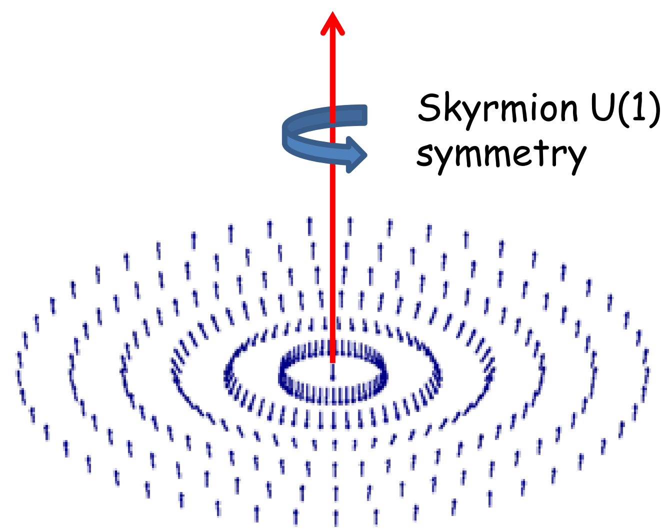
- A density of skyrmions form a lattice because skyrmions have electric charge.

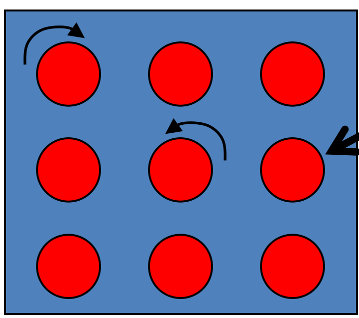
- Long wavelength modes in spin pattern allow nuclear spin relaxation.

- Gapless collective mode associated with broken U(1) symmetry in skyrmion lattice state.



Tycko, et.al.,*Science*, **268**,1460(1995)





$$\hbar\omega_q = \sqrt{UJ}qa$$

Texture Lattice and Hartree-Fock Approximation

- At filling factors away from one, the excited quasiparticles of the system form a lattice.
- Using equation of motion for Green's function : (Cote and MacDonald, 1990) .

Green's function : $$G_{\alpha\beta}(XX',\tau) \equiv -\hat{T}\langle c_{\alpha' X}(\tau) c^{\dagger}_{\beta' X'}(0)\rangle$$

Density : $$\rho_{\alpha\beta}(XX') \equiv \langle c^{\dagger}_{\alpha' X} c_{\beta' X'}\rangle.$$

$$\alpha, \beta = 1,2$$

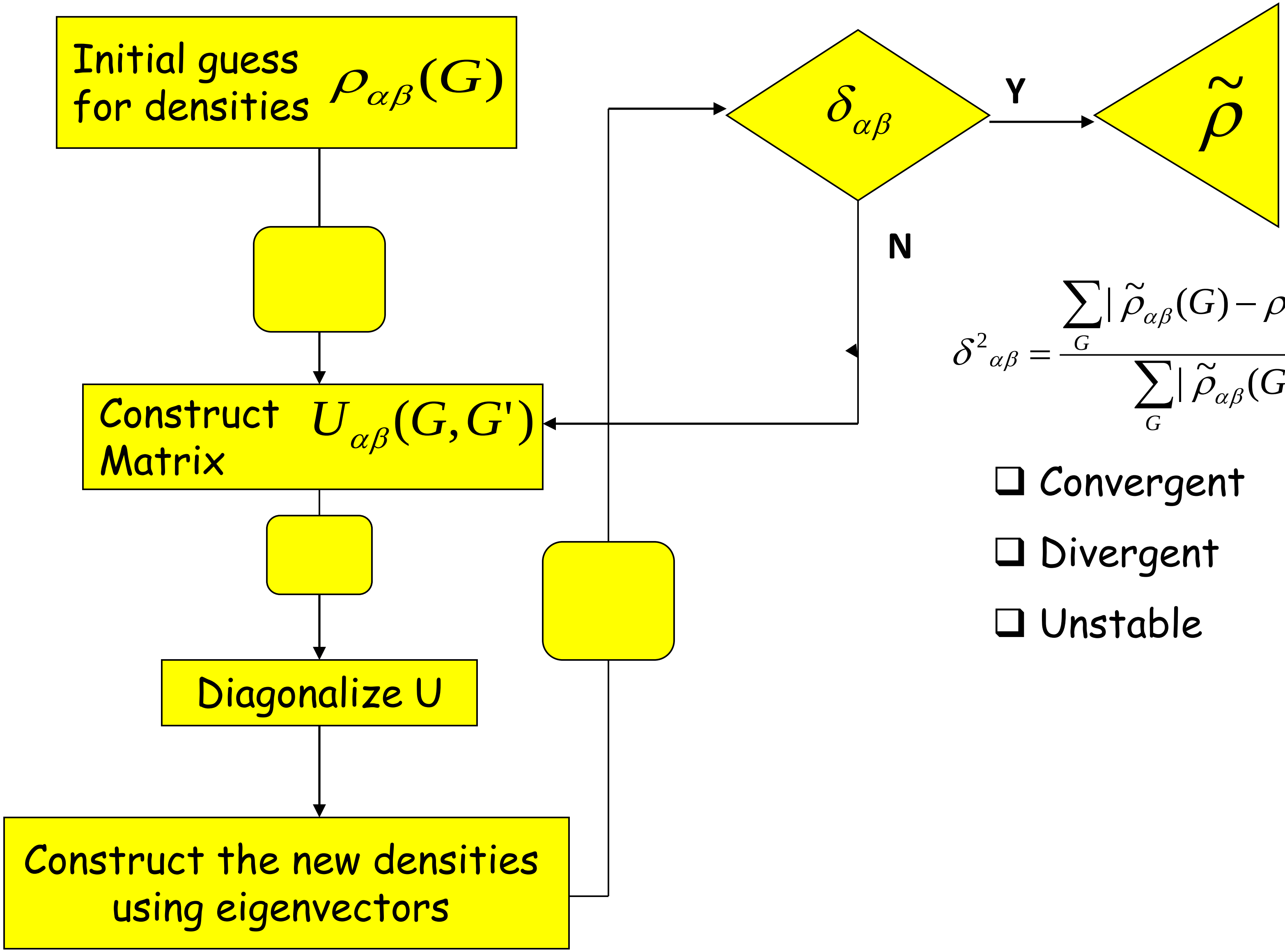
$$G_{\alpha\beta}(\mathbf{q},\tau) = \frac{1}{g}\sum_{XX'} G_{\alpha\beta}(XX',\tau) e^{-iq_x/2(X+X')}\delta_{X',X-q_y\ell^2}$$

$$\frac{\partial G_{\alpha\beta}}{\partial\tau} = -\hat{T}\langle[\hat{H}-\mu\hat{N}, c_{\alpha X}(\tau)]c^{\dagger}_{\beta X'}(0)\rangle - \delta_{\alpha\beta}\delta_{XX'}\delta(\tau).$$

Hartree-Fock

$$\rho_{\alpha\beta}(\mathbf{k}) = G_{\beta\alpha}(\mathbf{k},\tau\to 0^-)$$

$$-i\omega G = U.G - \mathbf{1}$$



$$\delta^2{}_{\alpha\beta} = \frac{\sum_G |\,\tilde{\rho}_{\alpha\beta}(G) - \rho_{\alpha\beta}(G)\,|^2}{\sum_G |\,\tilde{\rho}_{\alpha\beta}(G)\,|^2}$$

- ❑ Convergent
- ❑ Divergent
- ❑ Unstable

$$\tilde{\rho}_{\alpha\beta}(G) = \sum_{\omega_\lambda < \mu} V_\alpha^\lambda(G)\,\overline{V_\beta^\lambda}(G=0)$$

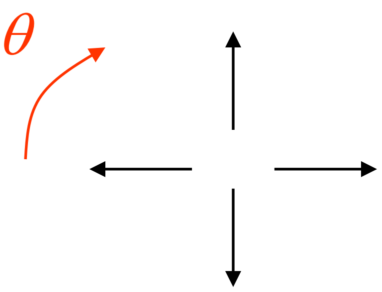
Vortex Symmetry and Densities

- The initial guess for densities must have the vortex symmetry in order to find vortex lattice solutions.

$$S_x + iS_y = \rho_{\uparrow\downarrow}(\vec{r})$$

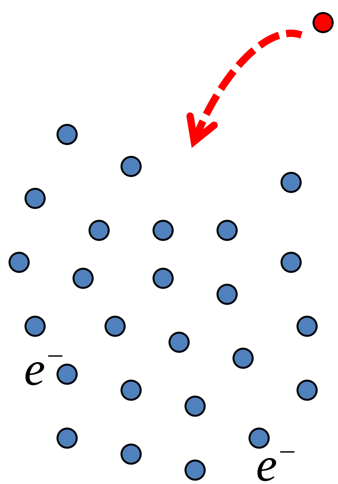
$$\rho_{\uparrow\downarrow}(\Re_\theta \vec{r}) = e^{i\theta}\rho_{\uparrow\downarrow}(\vec{r})$$

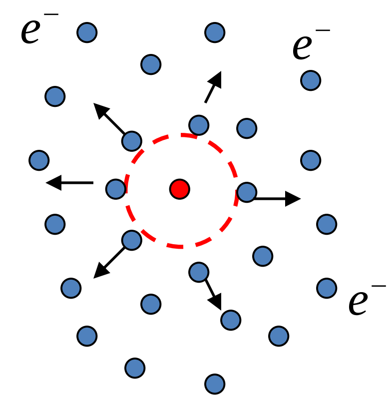
$$S_z = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow}$$



Tunneling into the Electron System

- What happens when an electron tries to tunnel into a Fermi Liquid?



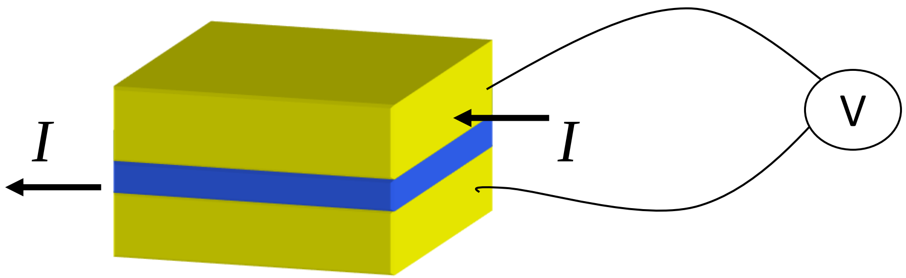


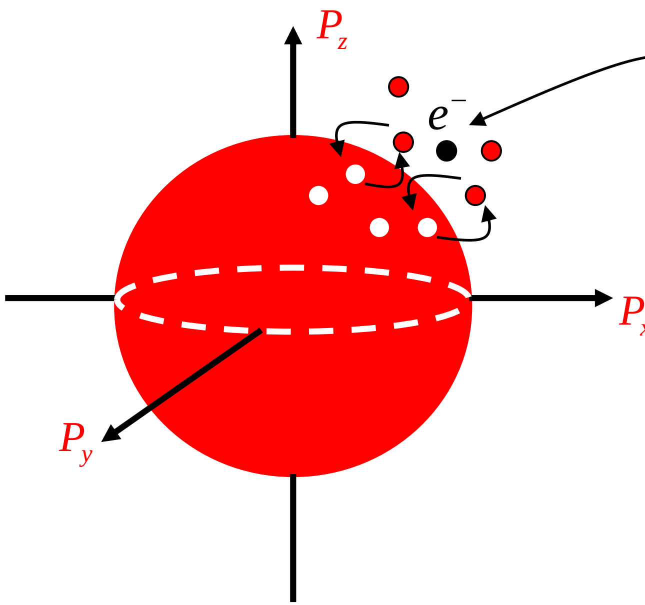
In regular electron gas the electrons become dressed.

The Quantum state of the system does not change drastically.

Tunneling into the Electron System

- We inject electrons into the gas and study the Tunneling Conductance.
- Tunneling into a regular Fermi Gas

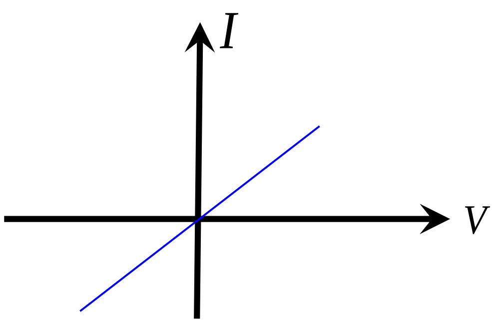


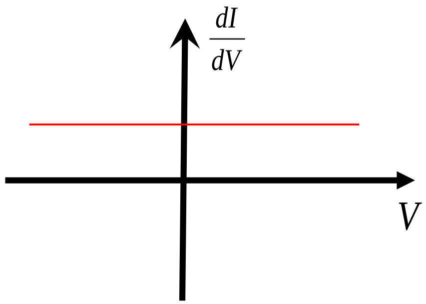


- Electrons are fast and they easily avoid the newcomer.

$$\langle K \rangle >> \langle V_{e-e} \rangle$$

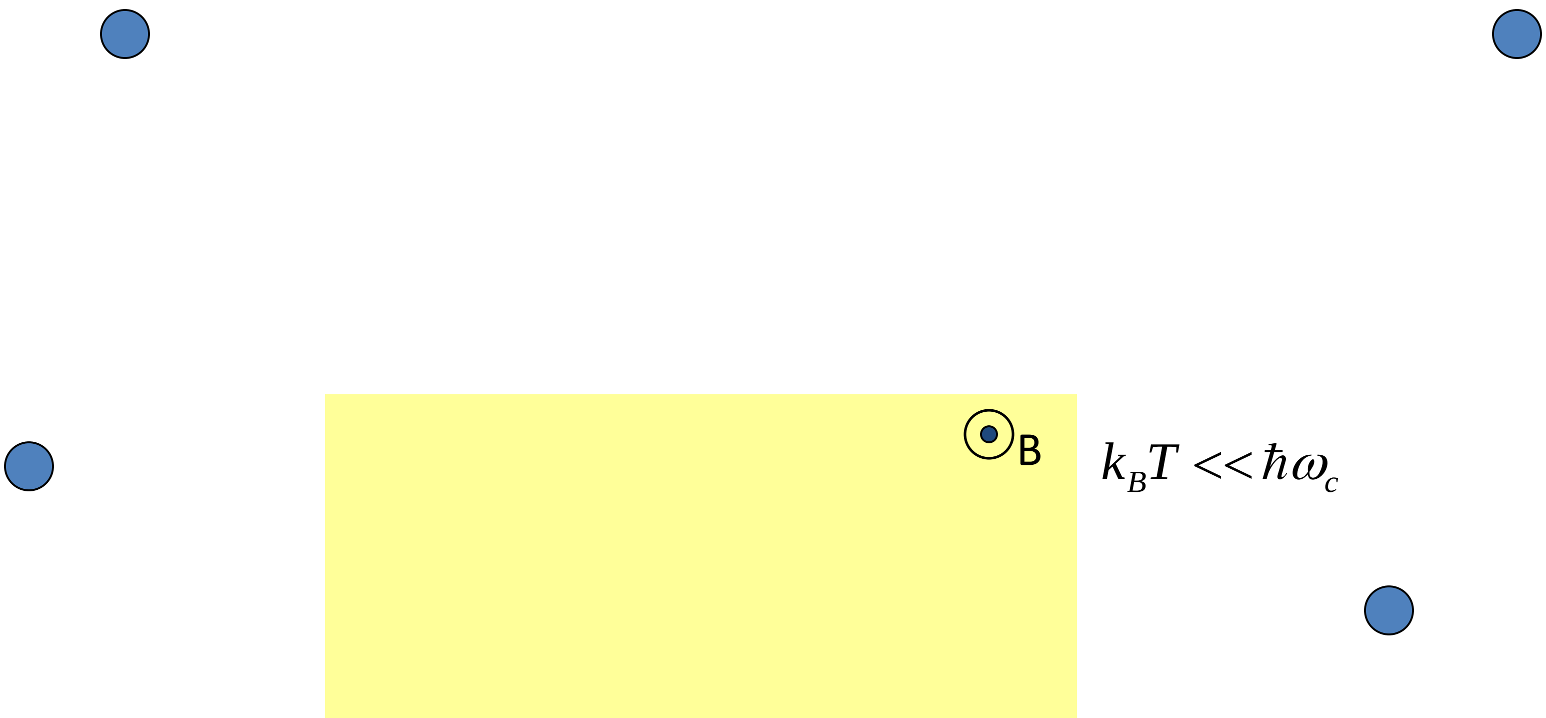
Tunneling Conductance $G = \dfrac{I}{V}$





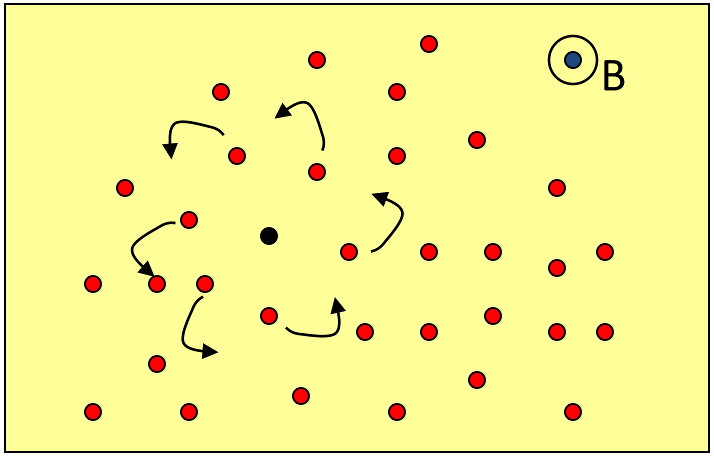
Dynamics in Lowest Landau Level

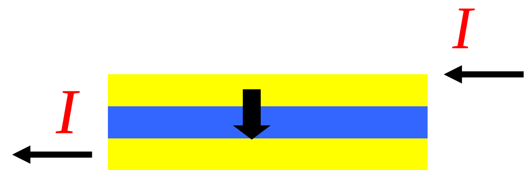
In the lowest Landau level electrons are not able to increase their kinetic energy. ➡ Electrons are not able to escape from each other.



$$k_B T << \hbar\omega_c$$

Tunneling into the Lowest Landau Level





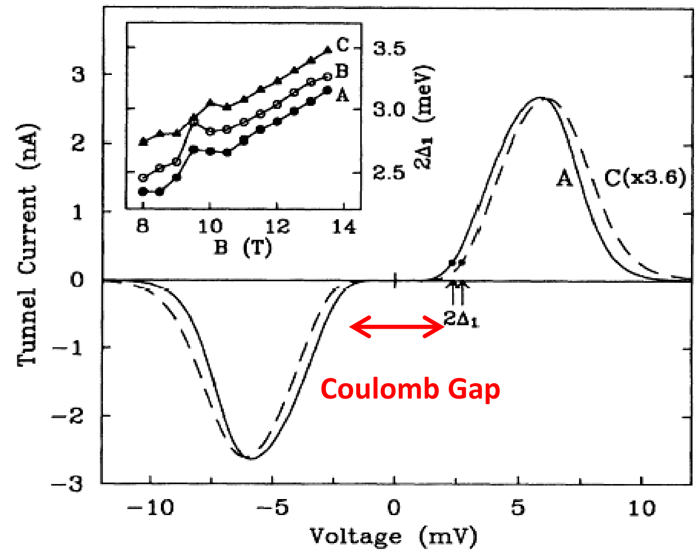


Fig. 1. Tunneling current-voltage characteristics at $B = 8$ T and $T = 0.6$ K for samples A and C. For sample C the current has been multiplied by 3.6 to aid comparison. The onset voltage $2\Delta_1$ is indicated [13]. Inset: magnetic field dependence of $2\Delta_1$ for samples A, B, and C. Tunnel barrier thicknesses are 175, 246, and 340 Å, respectively.

Electrons stay close to each other

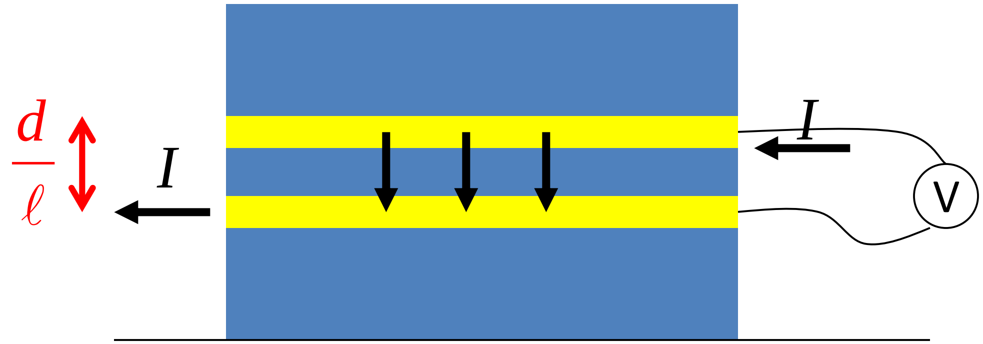
Bad screening

High Coulomb energy

Energy Gap

Tunneling into the Lowest Landau Level

We can vary the layer separation easily.



$$\frac{d}{\ell} = \frac{d}{\sqrt{\frac{\hbar}{eB}}} \propto \sqrt{B}$$

What happens if we bring the layers very close to each other ?

Close enough so that interlayer and interalayer Coulomb forces become of the same order.

$$d \sim \ell$$

Suddenly electrons find it extremely easy to tunnel to the other layer !

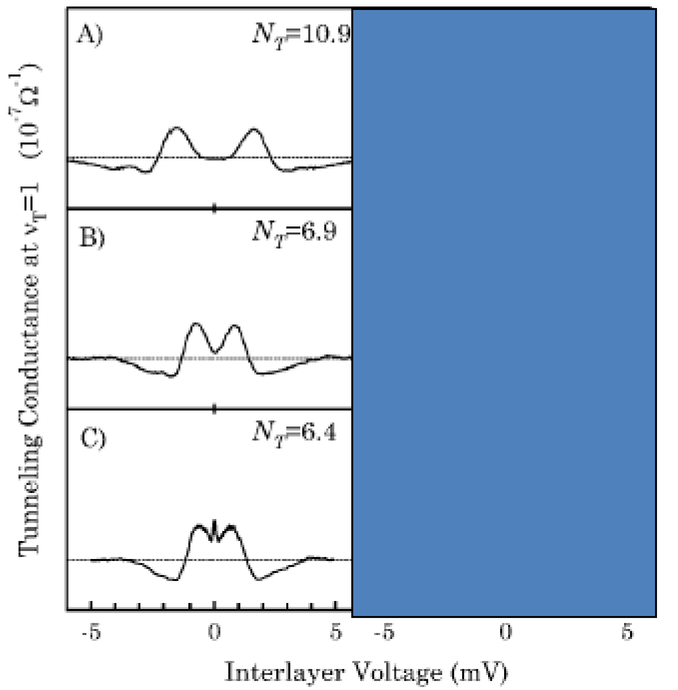


FIG. 1. Tunneling conductance dI/dV vs interlayer voltage V at $\nu_T = 1$ and $T = 40$ mK in a balanced double layer 2D electron system. Each trace corresponds to a different total density N_T (in units of 10^{10} cm^{-2}), and thus a different magnetic field. Trace A, at the highest density, shows a deep suppression of the tunneling near zero bias. By trace D, the lowest density of the four shown, this suppression has been replaced by a tall peak. The vertical scale is the same for all traces.

I.B. Spielman, J.P. Eisenstein, L.N. Pfeiffer, K. W. West, *Phys. Rev. Lett.* 84, 5808 (2000)

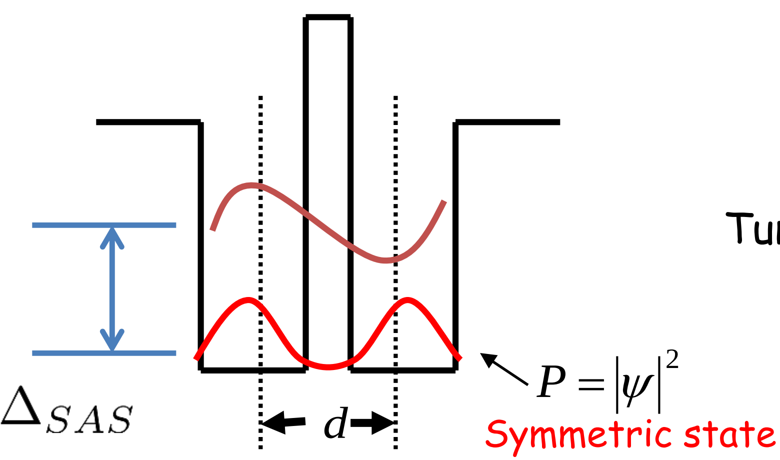
Coherent Layers

Puzzle : How can electrons overcome the Coulomb barrier so easily ?

Quantum mechanics says:

electrons can be in two states at the same time.

$$\frac{1}{\sqrt{2}}\underset{+z}{\left|\uparrow\right\rangle}+\frac{1}{\sqrt{2}}\underset{-z}{\left|\downarrow\right\rangle}=\underset{+x}{\left|\rightarrow\right\rangle}$$



Electrons have acquired a new degree of freedom : Psudo-spin

Tunneling leaves the system in its ground state.

→ No energy is required

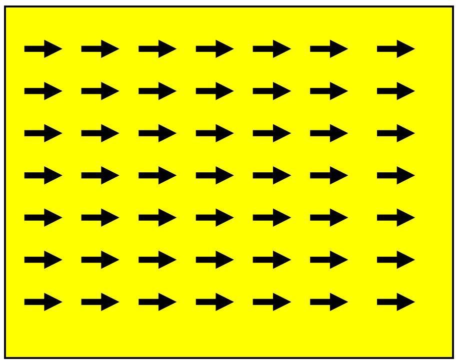
→ Zero bias tunneling peak.

Coherent Layers

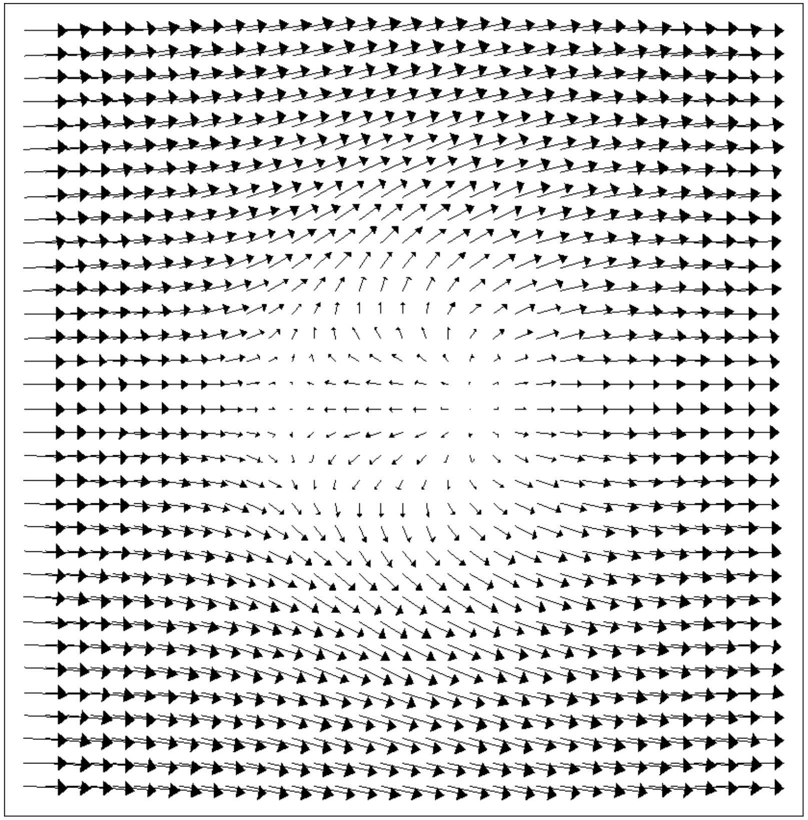
Let's forget the real spin. $\Delta_{SAS} < \Delta_z$

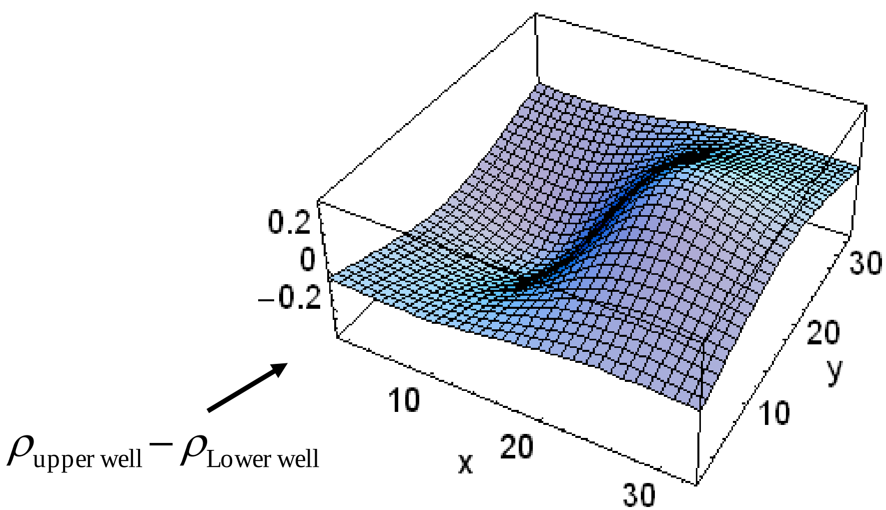
Electrons only have pseudo-spin. → **We expect the same physics as real spin to happen.**

Broken Symmetry Ground State (Pseudo Ferromagnet)



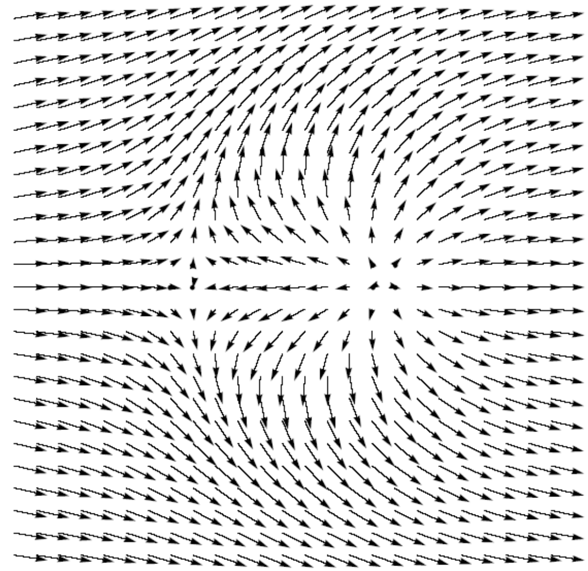
Skyrmion Quasiparticles Or Bimerons





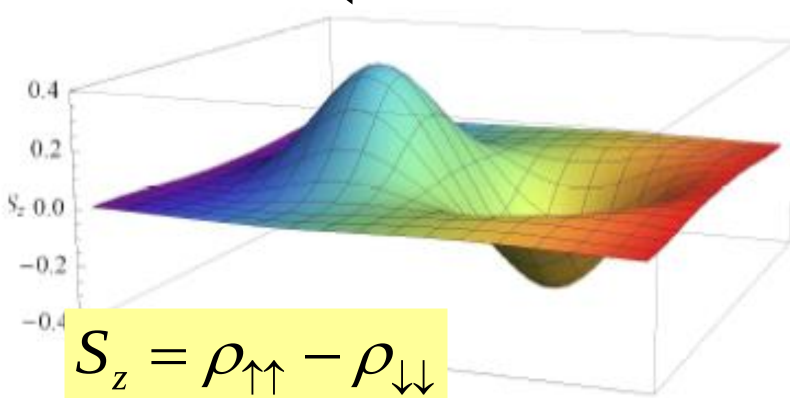
Bimeron Structure

- Bimeron is composed of two bound merons.
- Each meron has charge ±e/2, electric dipole moment and vorticity.
- At large separations exchange energy is low enough to let the meron binding decrease.



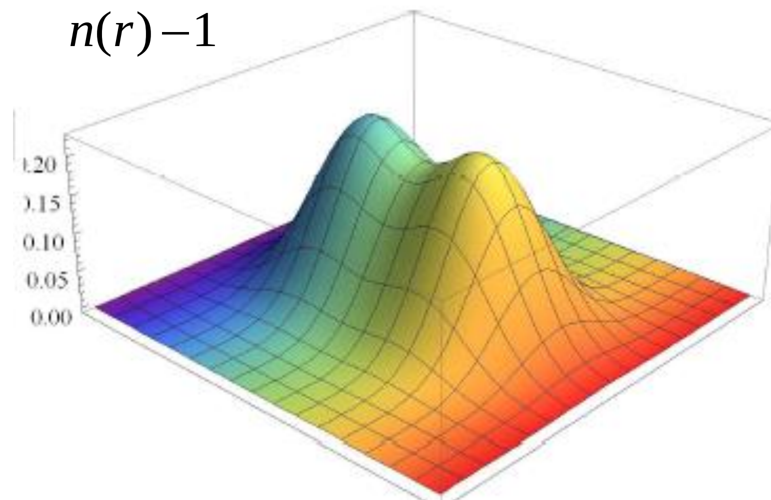
$$\frac{d}{\ell} = 0.8, \nu_T = 1.02$$

S_z



$$S_z = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow}$$

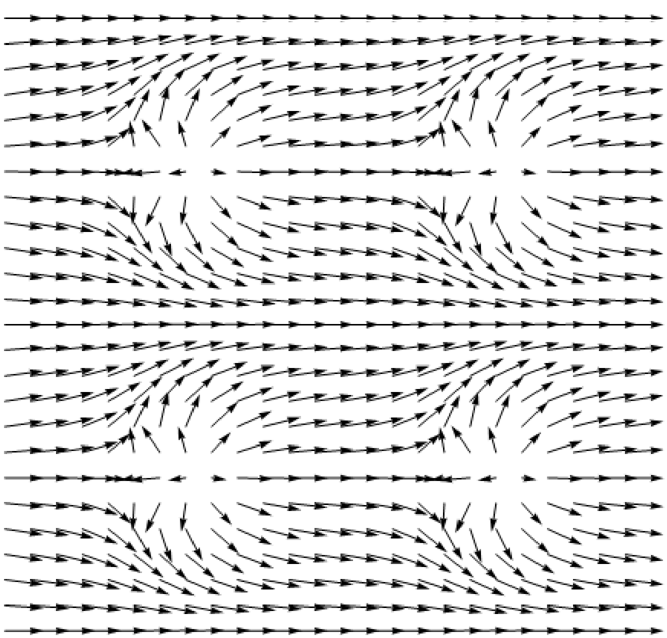
$n(r) - 1$

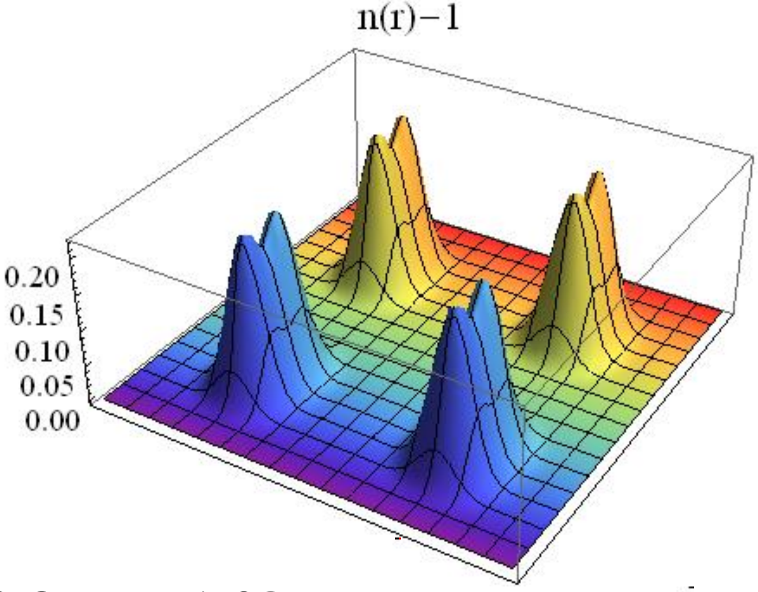


Bimeron Lattice State v.s. Meron Lattice State

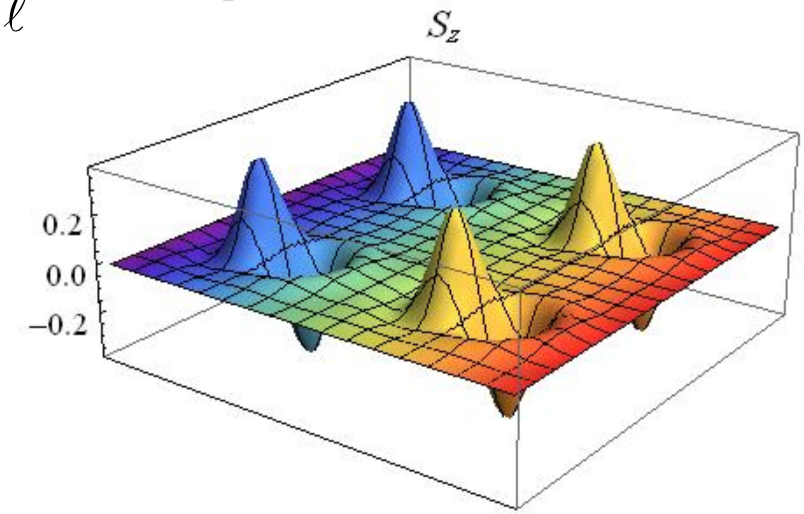
At large separations and high density Bimerons tend to split and rearrange into meron lattice.

Bimeron Lattice

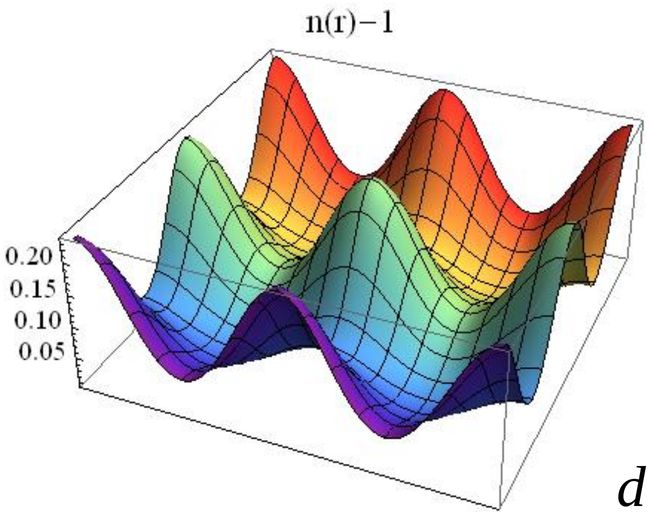




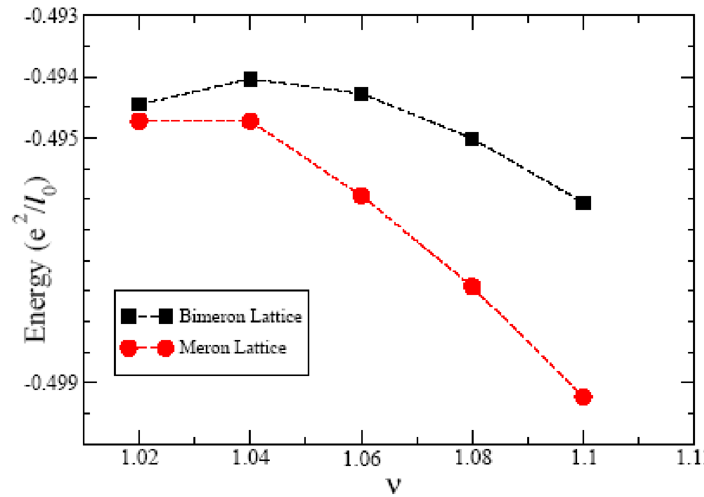
$$\frac{d}{\ell} = 0.8, \nu_T = 1.02$$



Bimeron Lattice State v.s. Meron Lattice State

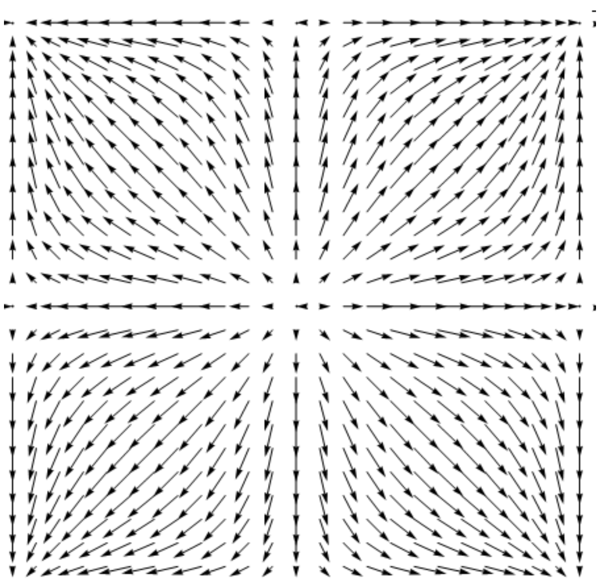


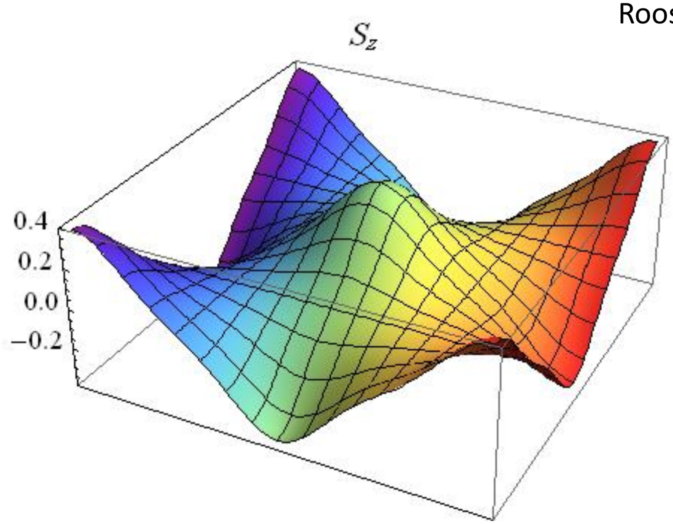
Energy of meron and bimeron lattice per electron v.s. filling factor.



$$\frac{d}{\ell} = 0.8, \nu_T = 1.08$$

Roostaei, *et.al., unpublished*





Coherent Bilayer

- SU(2) symmetry does not exist at d ≠ 0
- U(1) Symmetry breaks.

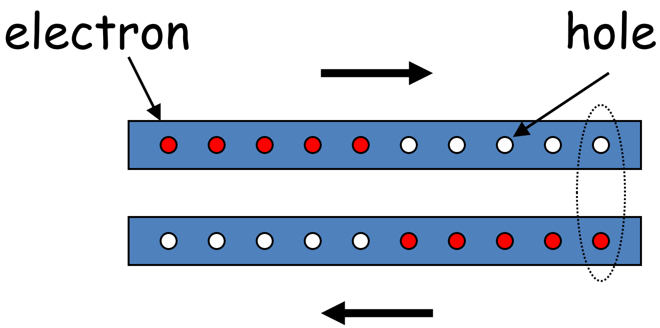
$$\hat{H}_{eff} \approx \int d^2r \left[\frac{1}{2}\rho_s |\nabla\varphi|^2 + \frac{C}{2} m_z^2 - \frac{t}{2\pi\ell^2}\cos\varphi \right]$$

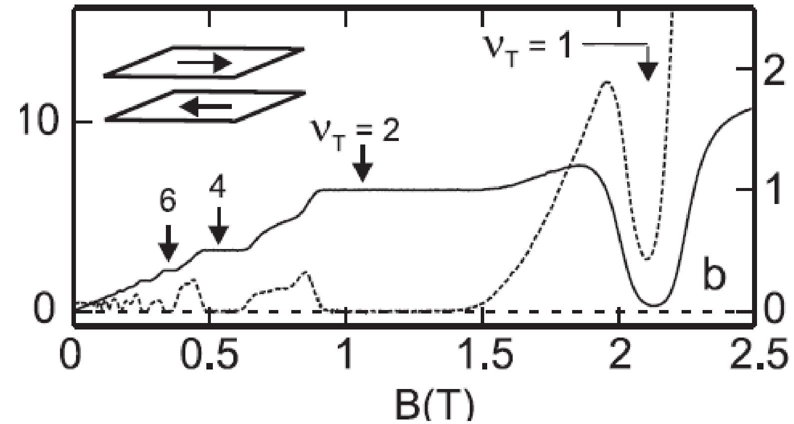
- Is there a Kosterlitz-Thouless Transition ?
  - No Critical Temperature Has been observed!
- The answer is speculated to be the effect of disorder and the charge of merons.

Exciton Superfluidity

At total filling factor one the electrons in one layer can pair with holes in another layer.

$$|\Psi\rangle = \prod_X \left[c^+_{X,\uparrow} + e^{i\varphi} c^+_{X,\downarrow} \right]$$



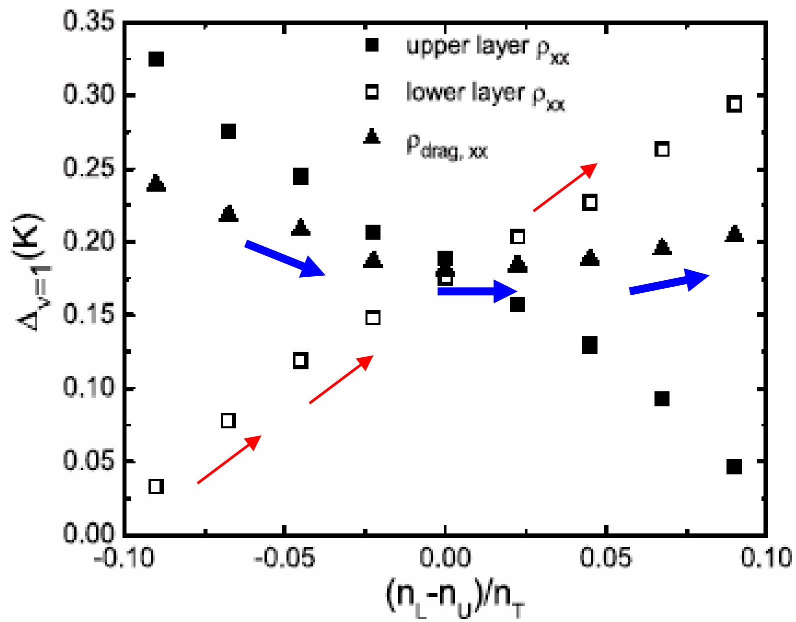


M. Kellogg, J. P. Eisenstein, L. N. Pfeifer, and K.W.West, *Phys. Rev. Lett.* 93, 036801 (2004)

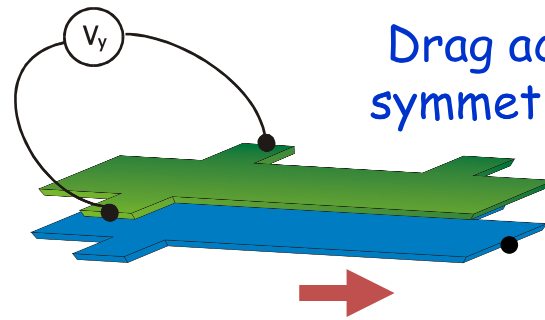
- The coherent state of the bilayer can be interpreted as the condensed state of the exciton gas.
- A counterflow current can couple to this excitonic superfluid.

Puzzle in Excitonic Superfluid : Drag and Drive

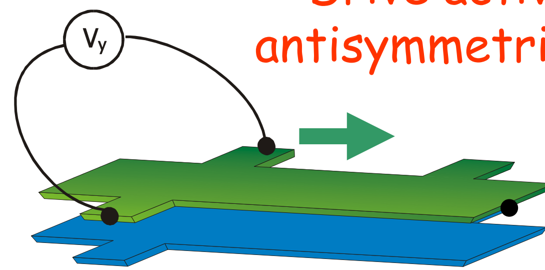
➢ Measured activation energies behave differently with respect to bias for drag and drive layer !



R. Wiersma, et. Al. PRL **93**,266805(2004)

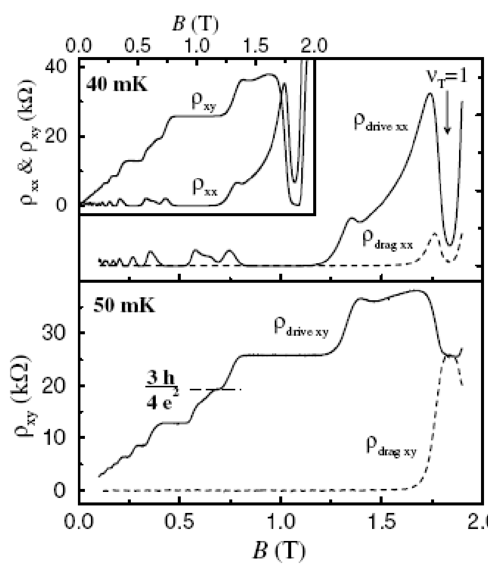


Drag activation: symmetric in bias



Drive activation: antisymmetric in bias

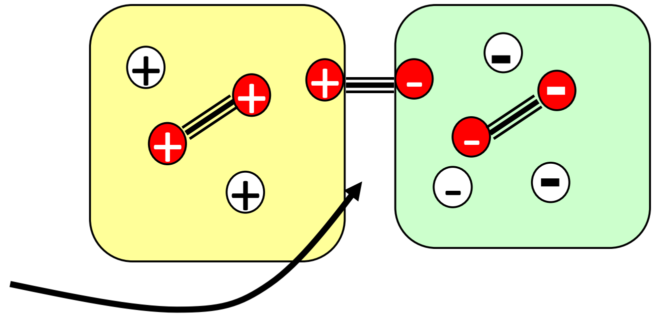
The Hall resistance is still quantized.



R. Wiersma, et. Al,PRL 93,266805(2004)

Effect of Disorder

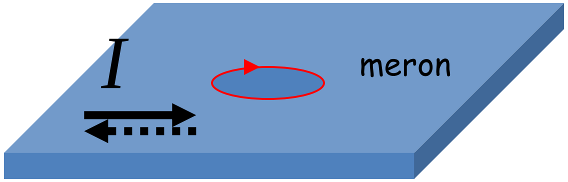
H.A. Fertig,G. Murthy,PRL 95 (2005)

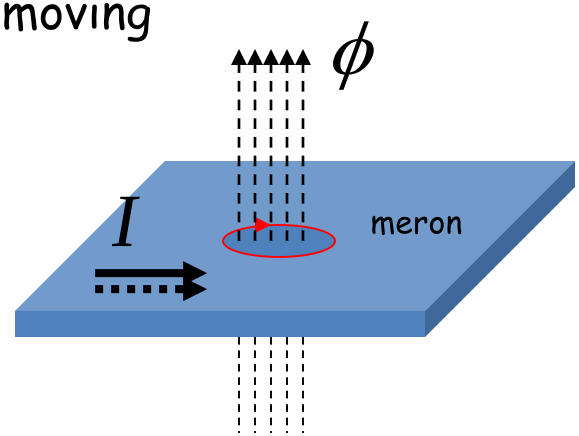


- Dopants form a smooth disorder potential inducing puddles of charged merons and anti-merons.

- There is a barrier for them to hop over an incompressible (ν=1) region from one puddle to the other.

- They are driven by external fields:
  - Their *vorticity* couples to counterflow current(Magnus).
  - Their attached flux is affected by moving charges(Lorentz).





- The driven motion of merons induces a voltage drop.

Drag and Drive Activation Energies

The total force on the meron from an arbitrary current distribution (Roostaei, Fertig, Mullen, Simon, unpublished) :

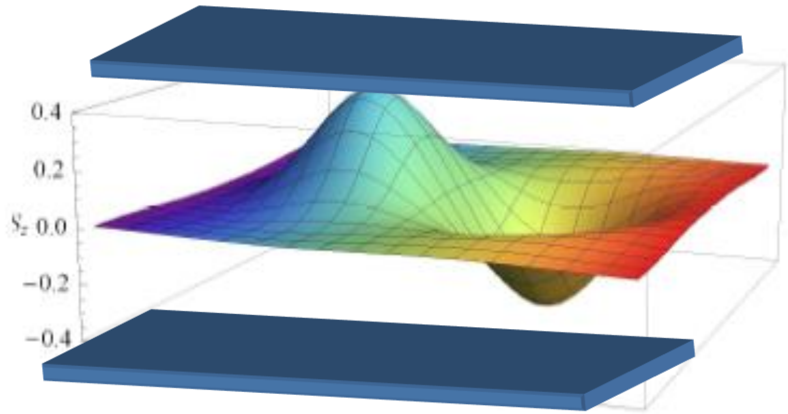
$$\mathbf{F}_{meron} = \frac{\phi_0}{\nu_T}\{[(1-\nu_L)s - q\nu_T]\mathbf{J}_L - [s\nu_L + q\nu_T]\mathbf{J}_U\} \times \hat{z}.$$

➢Direct consequence: For Drive distribution the force on merons with only one type of polarity is nonzero !

❾ Only one type of meron is activated.

$$\Delta \sim \Delta_0 + pV$$

Antisymmetric behavior of activation energy.



➢Merons with opposite vorticity and polarity interact with each other.

➢Secondary merons will induce a much smaller voltage drop in the drag layer.

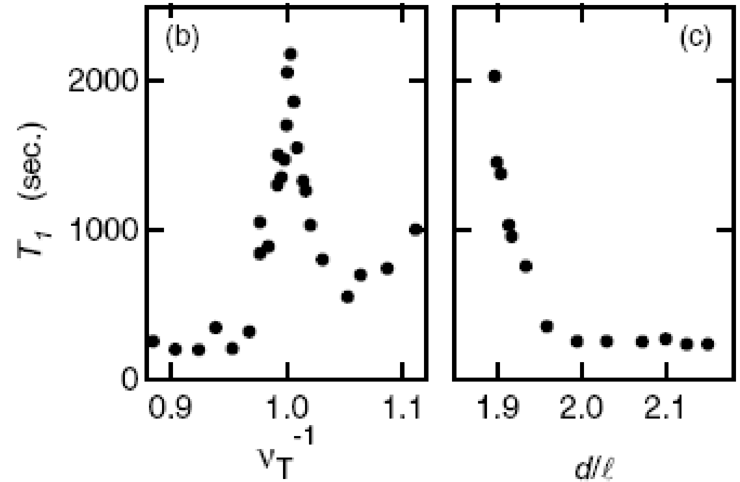
❾ Since the barriers are narrow, the Drag activation energy would be maximum of the two.❾ Symmetric !

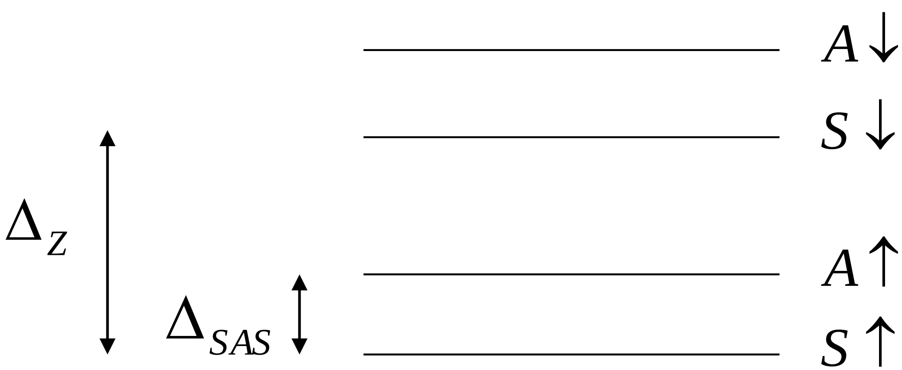
Experiments on spin transitions in Bilayers

I.B. Spielman,et.al.,PRL**94**,076803(2005)

- NMR experiments on bilayer systems reveal effects similar to single layer systems.

- Possibility of involvement of spin in quasiparticles of bilayer systems.





- We hope that some of the charge imbalance at the center of merons can be relaxed by lesser cost of real spin flip.

Textures in Four Component Systems: CP3 Property

Field Theory approach

Ghosh and Rajaraman, *PRB* **63**, 035304 (2000);
Z. F. Ezawa and K. Hasebe , *PRB* **65**, 075311 (2002) .

$$a(X)=\begin{pmatrix} a_1(X) \\ a_2(X) \\ a_3(X) \\ a_4(X) \end{pmatrix}=\begin{pmatrix} \text{upper layer, up spin} \\ \text{lower layer, down spin} \\ \text{upper layer, up spin} \\ \text{lower layer, down spin} \end{pmatrix}$$

Considering ansatz state : $\left|\Psi\right\rangle=\prod_X [\sum_\sigma a_\sigma(X)\hat{c}^{+}_{X,\sigma}]\left|0\right\rangle$

$$\langle H_Z\rangle = g\sum_X \left[|a_1(X)|^2-|a_2(X)|^2+|a_3(X)|^2-|a_4(X)|^2\right]$$

$$\langle H_T\rangle = -t\sum_X \left[a_1(X)a_3^*(X)+a_2(X)a_4^*(X)+c.c.\right]$$

$$\langle H_C\rangle_{direct}=\frac{1}{2}\sum_{X_1,X_2}\left\{D^s+(D^d-D^s)\left[F_u(X_1)F_l(X_2)+F_l(X_1)F_u(X_2)\right]\right\}$$

$$\langle H_C\rangle_{exchange}=\frac{1}{2}\sum_{X_1,X_2}\left[E^s(|G_u|^2+|G_l|^2)+E^d(G_uG_l^*+G_lG_u^*)\right]$$

$$G_u(X_1,X_2)=\sum_{i=1,2} a^i(X_1)a^{*i}(X_2)$$

$$G_l(X_1,X_2)=\sum_{i=3,4} a^i(X_1)a^{*i}(X_2)$$

$$F_u=|a_1(X)|^2+|a_2(X)|^2$$

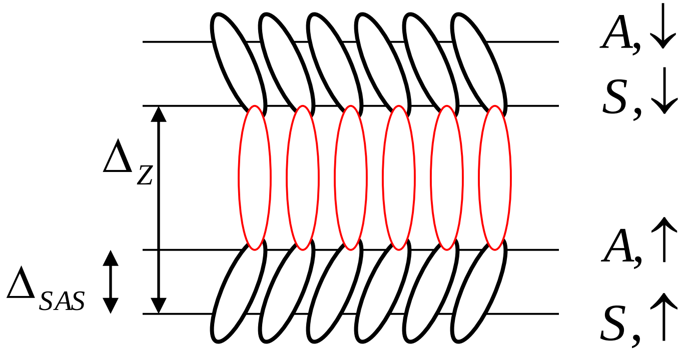
$$F_l=|a_3(X)|^2+|a_4(X)|^2$$

E and D are projected matrix elements of Coulomb potential into LLL .

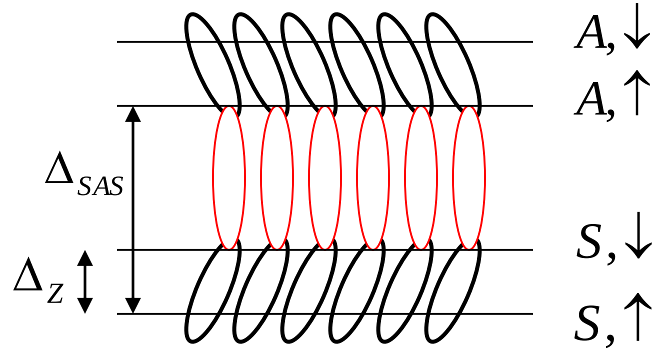
Exchange and tunneling causes the energy to be invariant under only U(1) local gauge transformation : $a_\sigma(X)\rightarrow a_\sigma(X)e^{i\Lambda(X)}$ **CP3 Theory**

Construction of CP3 State

$$\Delta_z > \Delta_{SAS}$$



$$\Delta_{SAS} > \Delta_Z$$

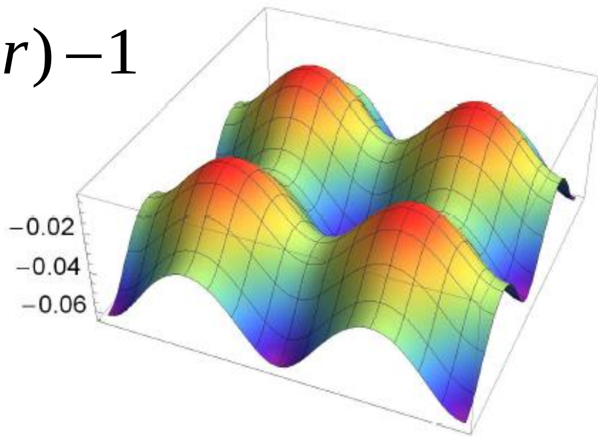


We examine different wavefunctions to minimize the energy using numerical Hartree-Fock approximation.

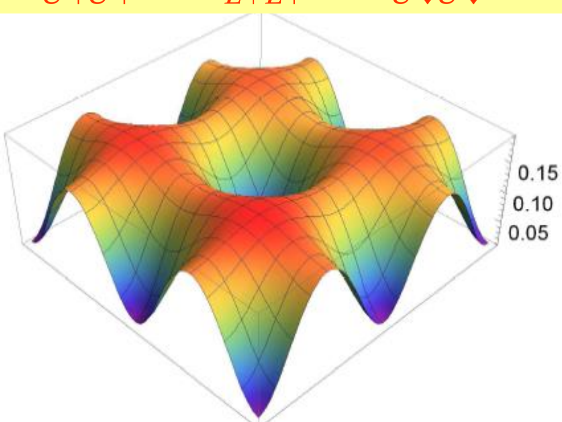
CP3 Skyrmion Lattice State

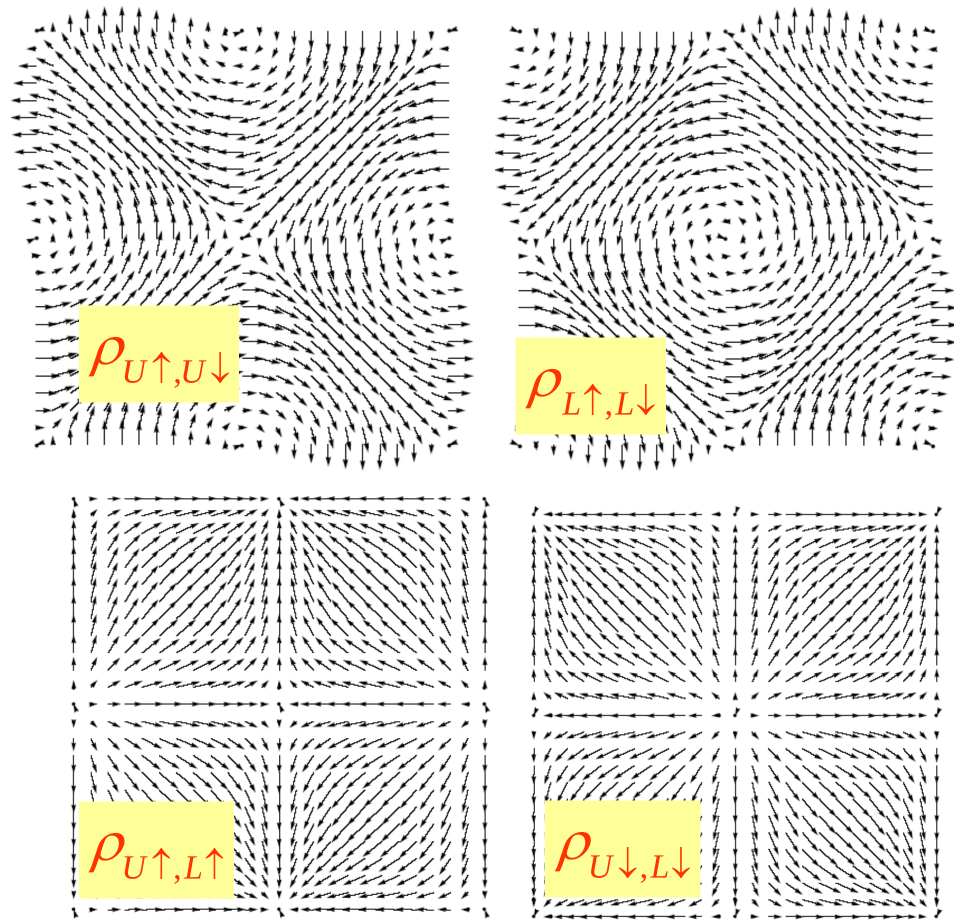
There are spin and pseudo-spin texture lattices in this state at the same time.

$$n(r)-1$$



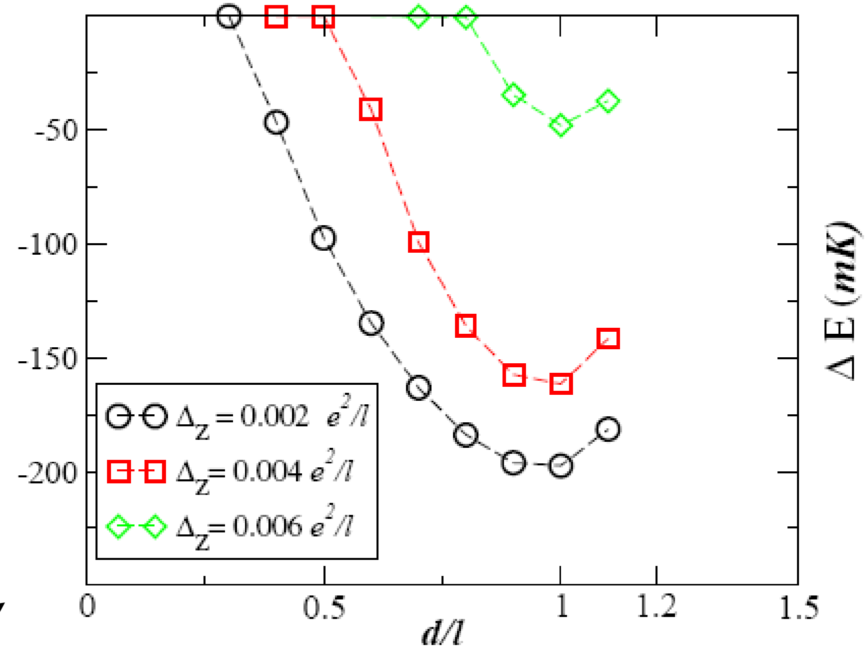
$$S_z = \rho_{U\uparrow U\uparrow} + \rho_{L\uparrow L\uparrow} - \rho_{U\downarrow U\downarrow} - \rho_{L\downarrow L\downarrow}$$



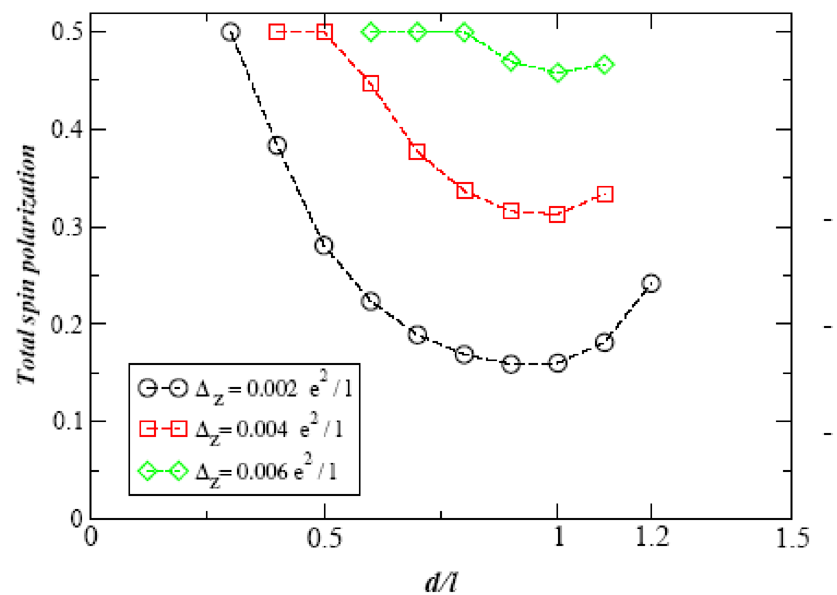


CP3 Lattice Energy and spin polarization

The Hartree-Fock energy of CP3 lattice per electron minus the energy of meron lattice per electron at zero tunneling.



- At large separations the exchange coupling is smaller and it is less costly to flip the spin.



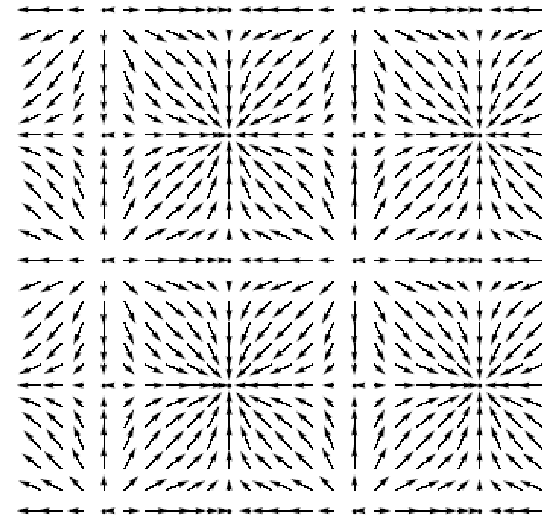
Burassar,Roostaei,Fertig,Cote,Mullen, *PRB* 74 , 195320 (2006)

Other possible spin involvements in bilayer lattice states

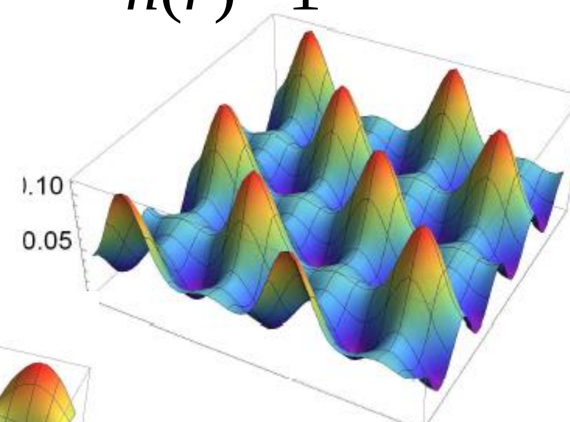
Symmetric Skyrmions: All electrons are in symmetric state but their spin has topological order.

This state exists at high tunneling values and low Zeeman couplings.

$\rho_{S\uparrow,S\downarrow}$

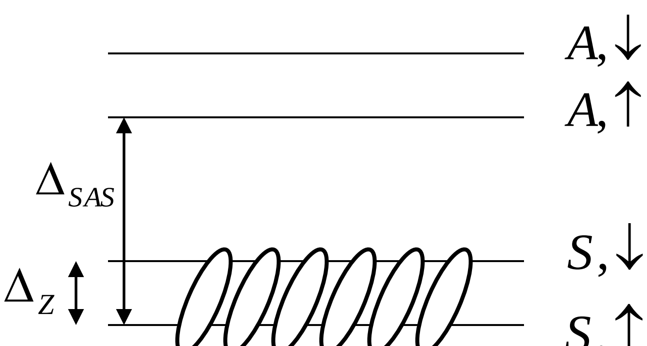


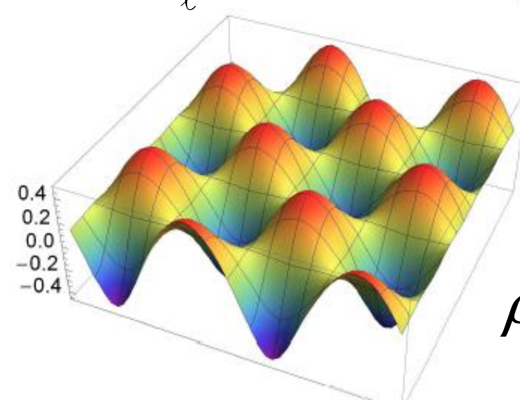
$n(r)-1$



$$\frac{d}{\ell}=0.8, \Delta_{SAS}=0.04(\frac{e^2}{\ell})$$

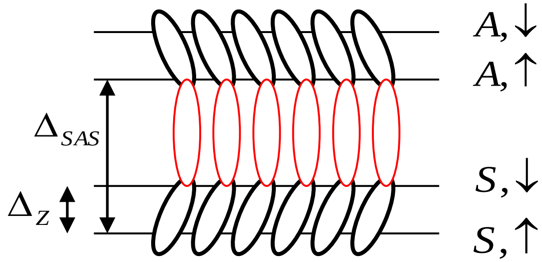
$$\Delta_Z=0.003(\frac{e^2}{\ell})$$



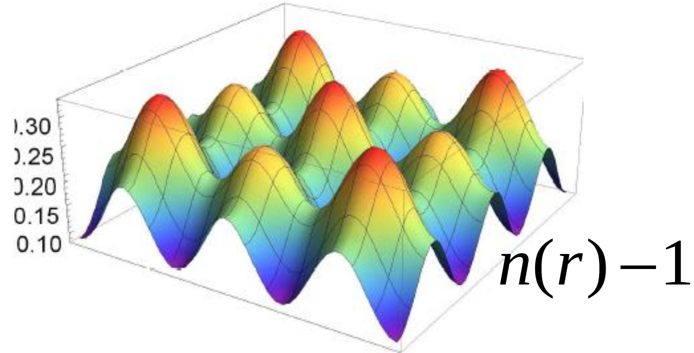


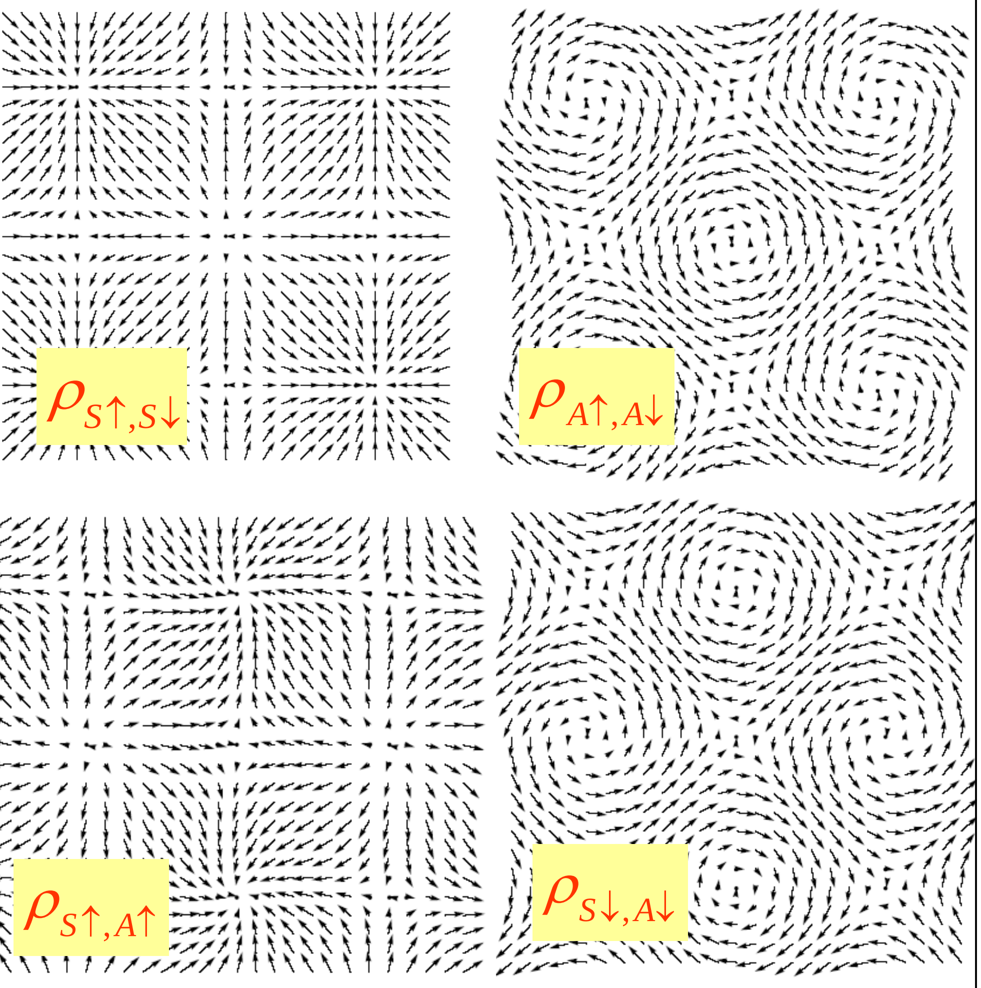
$\rho_{S\uparrow S\uparrow}-\rho_{S\downarrow S\downarrow}$

spin involvements in bilayer lattice states

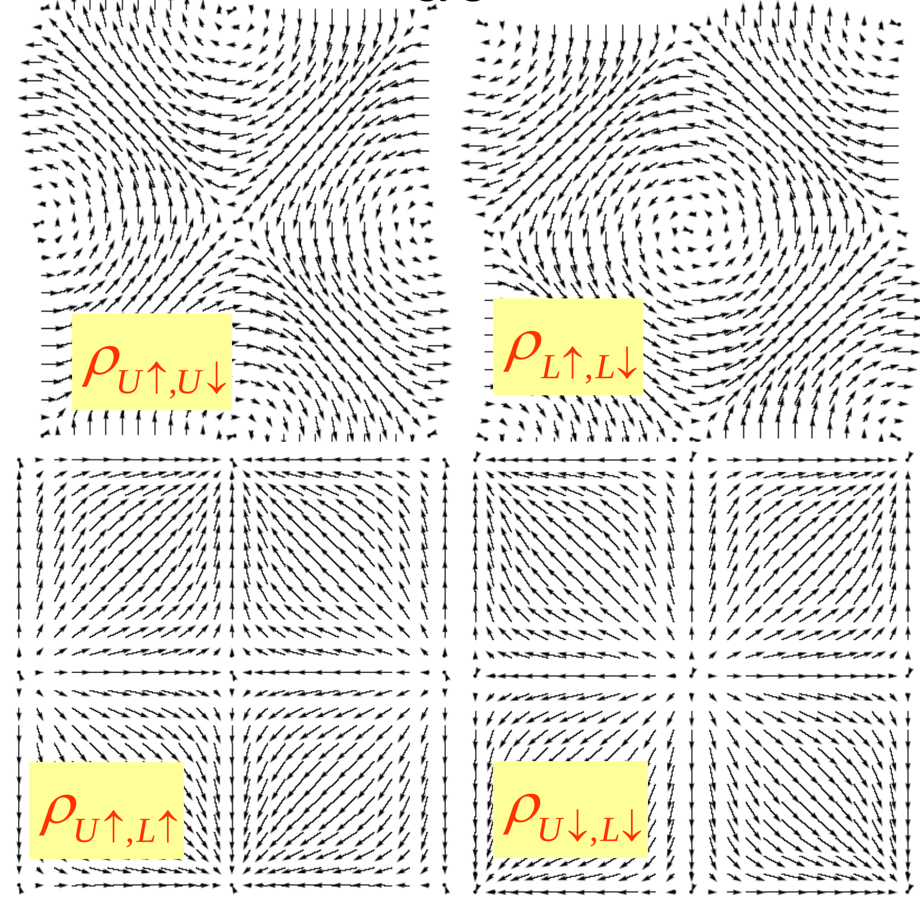


HCP3



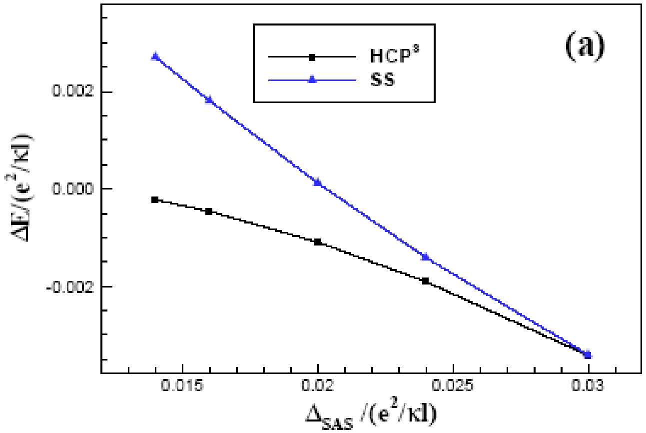


CP3

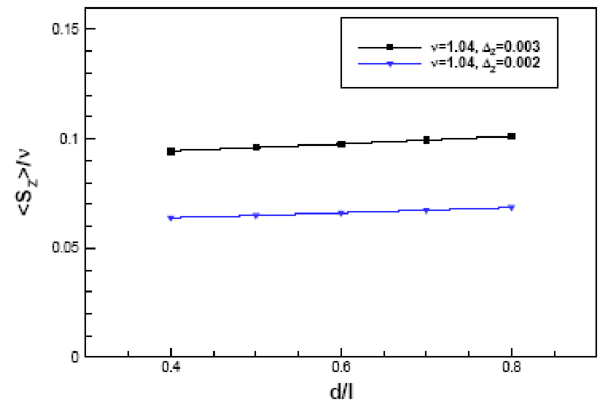


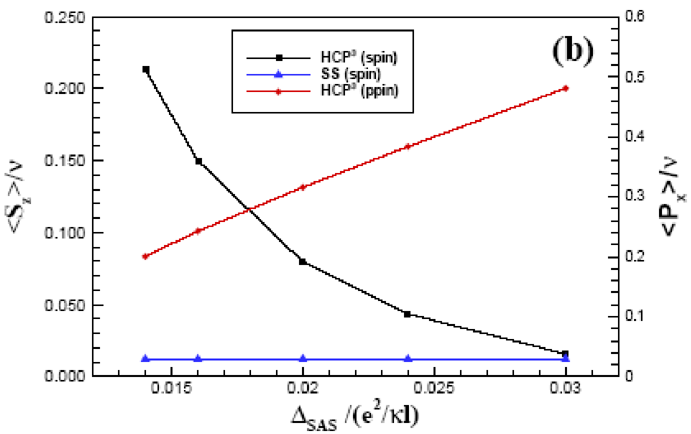
SS and HCP3 Energy and Polarization

$\nu_T = 1.2, \Delta_Z = 0.0015(e^2/\ell)$ and $d/\ell = 0.1$



$\Delta_{SAS} = 0.04(e^2/\ell)$





- The spin polarization behaves very smoothly for SS state.

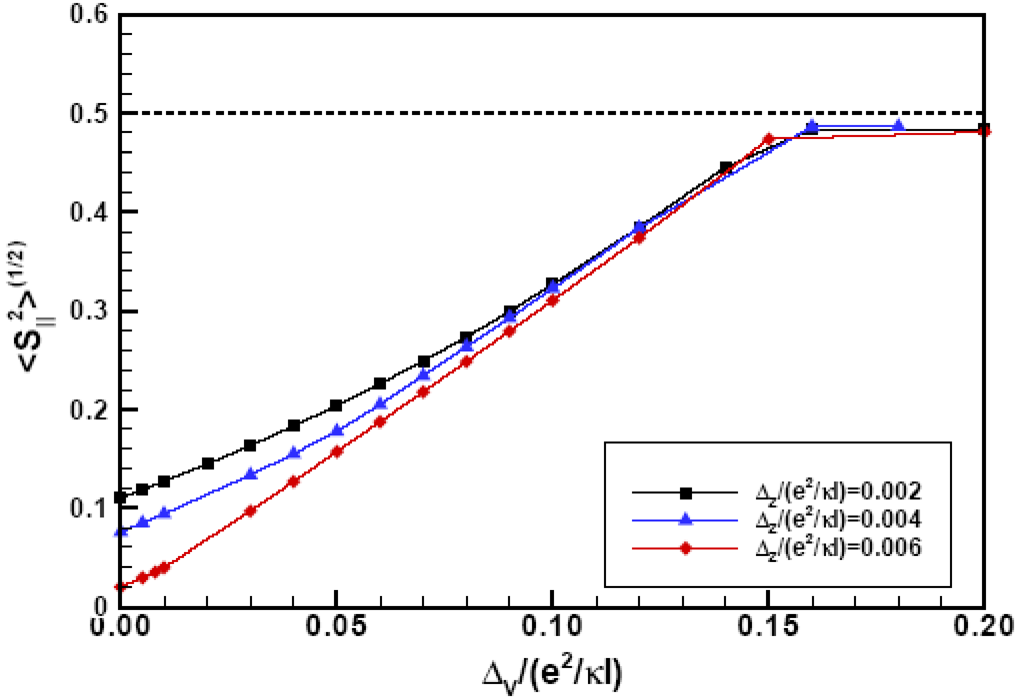
Burassar,Roostaei,Fertig,Cote,Mullen, *PRB* 74 , 195320 (2006)

Spin and Charge can talk : Manipulating spin by interlayer bias.

- Interlayer bias gradually suppresses the CP3 lattice.

- This behavior in principle can be detected in NMR experiments.

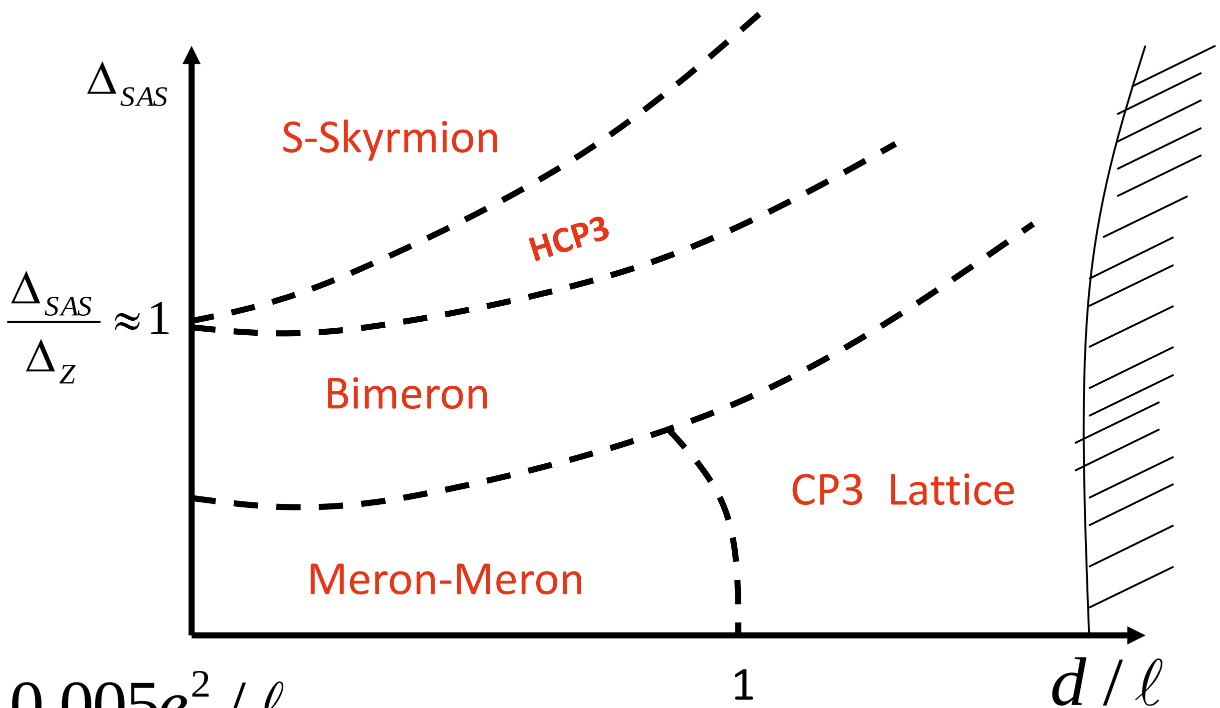
$$T_1^{-1} \propto \left\langle S_\parallel^{\,2} \right\rangle$$



Burassar,Roostaei,Fertig,Cote,Mullen, *PRB* 74 , 195320 (2006)

Summary of Spin-Pseudospin States

- HF equations support the spin-pseudospin excitations at large layer separations and small tunneling.
- These excitations are combination of spin and pseudospin vortex-antivortex lattices.
- The observed signatures of low energy spin excitations at QH state in double layer systems could be explained by taking into account spin-pseudospin textures.



$\Delta_Z = 0.005 e^2/\ell$

Graphene: New Scales

- Dirac electrons are massless so apparently the cyclotron frequency must be infinite!

$$\omega_c = \frac{eB}{m^*}$$ **Cyclotron Frequency**

- On the other hand magnetic length is finite.

$$\ell = \sqrt{\frac{\hbar}{eB}}$$ **Magnetic length**

- There is another scale : The Fermi velocity

$$E_n = sgn(n)\sqrt{|n|}\frac{\sqrt{2}\hbar v_F}{\ell}$$ Graphene Landau Levels

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$ 2DEG Landau Levels

- Coulomb Energy is still smaller than Landau Level splitting.

Zeeman is much smaller.

$$\Delta E \approx 426\sqrt{B[T]}\ \mathrm{K}$$

$$E_C \approx e^2/\epsilon\ell \approx 261\sqrt{B[T]}\ \mathrm{K}$$

$$\Delta_Z \approx 1.34B[T]\ \mathrm{K}$$

Graphene: Questions

Landau levels are almost 4-fold degenarate: Already in the regime of complicated topological textures.

Theoretical works suggest broken SU(4) symmetry ground states.

Field Theory calculations suggest a more robust Skyrmion states.

Spin Skyrmions and valley skyrmions have been detected in higher Landau levels!

There is not still a concrete result on topological texture phase diagram even for monolayer Graphene.