Wiggling Throat of Extremal Black Holes

Ali Seraj

School of Physics
Institute for Research in Fundamental Sciences (IPM),
Tehran, Iran

Recent Trends in String Theory and Related Topics
May 2016, IPM

based on

arXiv:1503.07861 and arXiv:1506.07181

together with

G. Compère, K. Hajian, M.M. Sheikh-Jabbari

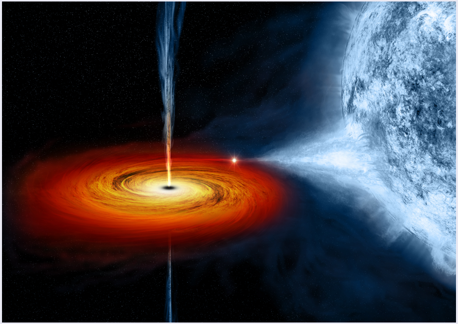
Institute for Research in

Introduction

Motivation

Black holes in gravity

- Attractive nature of gravity
- Gravitational collapse
- Formation of horizon
- Black hole



picture from www.nasa.gov

Black hole Thermodynamics

- Laws of black hole mechanics [*Bardeen et al. (1973)*]
- Black holes are thermodynamic systems [*Hawking(1973)*]
- Black hole entropy [*Bekenstein(1973)*]

Motivation

Microscopic description of black hole entropy

- Boltzmann principle
- The microscopic origin of black hole entropy

Microstates vs. No hair theorem

No hair theorems in GR

- No hair theorems [*Robinson(1975), Mazur(1982), Chrusciel et al. (2012)*]

Microstates vs. No hair theorem

No hair theorems in GR

- No hair theorems [*Robinson(1975)*, *Mazur(1982)*, *Chrusciel et al. (2012)*]

Microstates ?

- Expensive approaches (String theory, Fuzzballs, Loop Q.Gr, etc.)

Microstates vs. No hair theorem

No hair theorems in GR

- No hair theorems [*Robinson(1975)*, *Mazur(1982)*, *Chrusciel et al. (2012)*]

Microstates ?

- Expensive approaches (String theory, Fuzzballs, Loop Q.Gr, etc.)
- Minimal resolution (GR compatible): Surface degrees of freedom

Microstates vs. No hair theorem

No hair theorems in GR

- No hair theorems [*Robinson(1975)*, *Mazur(1982)*, *Chrusciel et al. (2012)*]

Microstates ?

- Expensive approaches (String theory, Fuzzballs, Loop Q.Gr, etc.)
- Minimal resolution (GR compatible): Surface degrees of freedom

- Black hole thermodynamics is associated to the horizon

$$T_H = \frac{\kappa}{2\pi}, \qquad S = \frac{\text{Area of horizon}}{4G_N}$$

Motivation

Surface degrees of freedom

- Horizon as an inner boundary
- Simultaneous breaking of diffeomorphism
- Surface gravitons as Goldstone modes
- How to detect these modes?

Noether's theorems [*E.Noether(1918)*]

Noether's theorem [*E.Noether(1918)*]

Associated with each continuous symmetry of a theory, there exists a *conserved charge* (given suitable boundary conditions)

Noether's theorems [*E.Noether(1918)*]

Noether's theorem [*E.Noether(1918)*]

Associated with each continuous symmetry of a theory, there exists a *conserved charge* (given suitable boundary conditions)

- **Noether's Second theorem.**
  Local symmetries imply Bianchi identities as strong equalities

Using both above theorems imply

global symmetries $\longleftrightarrow$ ***Conserved charge as a volume integral***

local symmetries $\longleftrightarrow$ ***Conserved charge as a surface integral***

Motivation

Microstates at the horizon

Using the charges we can distinguish states at the horizon which were previously gauge equivalent.

Microstates at the horizon

Using the charges we can distinguish states at the horizon which were previously gauge equivalent.
These states have different names in the literature:

- Boundary gravitons [*Brown-Henneaux (1986)*]

Microstates at the horizon

Using the charges we can distinguish states at the horizon which were previously gauge equivalent.
These states have different names in the literature:

- Boundary gravitons [*Brown-Henneaux (1986)*]
- Edge states [*Balachandran et al. (1994),Carlip (2005)*]

Microstates at the horizon

Using the charges we can distinguish states at the horizon which were previously gauge equivalent.
These states have different names in the literature:

- Boundary gravitons [*Brown-Henneaux (1986)*]
- Edge states [*Balachandran et al. (1994),Carlip (2005)*]
- Soft gravitons [*Strominger et al. (2013-2015)*]

Microstates at the horizon

Using the charges we can distinguish states at the horizon which were previously gauge equivalent.
These states have different names in the literature:

- Boundary gravitons [*Brown-Henneaux (1986)*]
- Edge states [*Balachandran et al. (1994),Carlip (2005)*]
- Soft gravitons [*Strominger et al. (2013-2015)*]
- Symplectic symmetries
  [*G.Compère, K.Hajian, A.S. and M. M.Sheikh-Jabbari, JHEP* **1510**, *093 (2015).*]

Microstates at the horizon

Using the charges we can distinguish states at the horizon which were previously gauge equivalent.
These states have different names in the literature:

- Boundary gravitons [*Brown-Henneaux (1986)*]
- Edge states [*Balachandran et al. (1994),Carlip (2005)*]
- Soft gravitons [*Strominger et al. (2013-2015)*]
- Symplectic symmetries
  [*G.Compère, K.Hajian, A.S. and M. M.Sheikh-Jabbari, JHEP* **1510**, *093 (2015).*]
- Adiabatic modes [*Mehrdad Mirbabayi, Marko Simonovi (2016)*]

Extremal black holes
and
Near horizon geometry

Extremal Rotating Black Hole

Extremal black holes

- Vanishing Hawking temperature
- "Minimum energy" for given value of angular momenta $M = M(J_i)$
- Degenerate Killing horizon
- Finite entropy
  $T_H \longrightarrow 0 \quad \Longrightarrow \quad S \longrightarrow S_0$

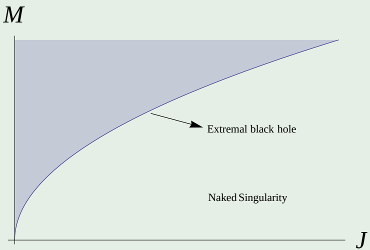


Figure: Parameter space of Kerr black hole

Near Horizon Geometry

Approach

Motivation

Near Horizon Geometry

Approach

- In the extremal case, the near horizon region can be decoupled using the near horizon limit

Near Horizon Geometry

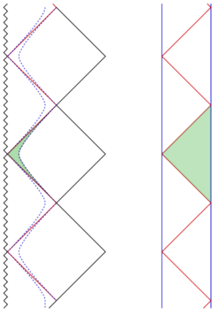
Approach

- In the extremal case, the near horizon region can be decoupled using the near horizon limit
- Enhancement of symmetries in NHEG $SL(2,\mathbb{R}) \times U(1)^N$

Near Horizon Geometry

Approach

- In the extremal case, the near horizon region can be decoupled using the near horizon limit
- Enhancement of symmetries in NHEG $SL(2,\mathbb{R}) \times U(1)^N$

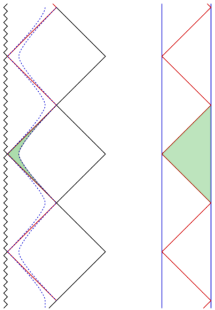


Near horizon limit
picture from: Tom Hartman

Near Horizon Geometry

Approach

- In the extremal case, the near horizon region can be decoupled using the near horizon limit
- Enhancement of symmetries in NHEG $SL(2,\mathbb{R}) \times U(1)^N$



Near horizon limit
picture from: Tom Hartman

$$ds^2 = \Gamma(\theta)\Big[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sum_{i,j=1}^{d-3} \gamma_{ij}(\theta)(d\varphi^i + k^i r dt)(d\varphi^j + k^j r dt)\Big]$$

Dynamics in the near horizon geometry

Kerr/CFT proposal

- A dynamical duality between asymptotic NHEK geometries and a two dimensional chiral CFT [*Strominger et.al.'09*]

Dynamics in the near horizon geometry

Kerr/CFT proposal

- A dynamical duality between asymptotic NHEK geometries and a two dimensional chiral CFT [*Strominger et.al.'09*]
- ***No Dynamics*** in NHEK [*Reall et al. (2009) , Horowitz et al. (2009)*]

Dynamics in the near horizon geometry

Kerr/CFT proposal

- A dynamical duality between asymptotic NHEK geometries and a two dimensional chiral CFT [*Strominger et.al.'09*]
- ***No Dynamics*** in NHEK [*Reall et al. (2009) , Horowitz et al. (2009)*]

Conclusion. The only states penetrating the near horizon geometry are the soft states

Phase Space of Near Horizon Extremal Geometries

Field Configurations

- Start from the background metric

$$\bar{g}_{\mu\nu}: \quad ds^2 = \Gamma(\theta)\Big[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sum_{i,j=1}^{d-3}\gamma_{ij}(\theta)(d\varphi^i + k^i r dt)(d\varphi^j + k^j r dt)\Big]$$

Field Configurations

- Start from the background metric

$$\bar{g}_{\mu\nu}: \quad ds^2 = \Gamma(\theta)\Big[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sum_{i,j=1}^{d-3}\gamma_{ij}(\theta)(d\varphi^i + k^i r dt)(d\varphi^j + k^j r dt)\Big]$$

- ***Soft modes*** are given by

$$g^{(\epsilon)}_{\mu\nu} = \bar{g}_{\mu\nu} + \nabla_{(\mu}\xi_{\nu)}$$

Field Configurations

- Start from the background metric

$$\bar{g}_{\mu\nu}: \quad ds^2 = \Gamma(\theta)\Big[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sum_{i,j=1}^{d-3}\gamma_{ij}(\theta)(d\varphi^i + k^i r dt)(d\varphi^j + k^j r dt)\Big]$$

- ***Soft modes*** are given by

$$g^{(\epsilon)}_{\mu\nu} = \bar{g}_{\mu\nu} + \nabla_{(\mu}\xi_{\nu)}$$

- By some physical arguments, we fix ξ to be

$$\xi[\epsilon(\vec{\varphi})] = \epsilon(\varphi^i)k^i\partial_{\varphi^i} - (k^i\partial_{\varphi^i}\epsilon)\,(\frac{1}{r}\partial_t + r\partial_r)$$

Field Configurations

- Start from the background metric

$$\bar{g}_{\mu\nu}: \quad ds^2 = \Gamma(\theta)\Big[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sum_{i,j=1}^{d-3}\gamma_{ij}(\theta)(d\varphi^i + k^i r dt)(d\varphi^j + k^j r dt)\Big]$$

- ***Soft modes*** are given by

$$g^{(\epsilon)}_{\mu\nu} = \bar{g}_{\mu\nu} + \nabla_{(\mu}\xi_{\nu)}$$

- By some physical arguments, we fix ξ to be

$$\xi[\epsilon(\vec{\varphi})] = \epsilon(\varphi^i)k^i\partial_{\varphi^i} - (k^i\partial_{\varphi^i}\epsilon)\,(\frac{1}{r}\partial_t + r\partial_r)$$

- $g^{(\epsilon)}_{\mu\nu}$ is smooth at poles

Field Configurations

- Generic geometries in the phase space are obtained by ***exponentiating*** the infinitesimal transformations

Field Configurations

- Generic geometries in the phase space are obtained by ***exponentiating*** the infinitesimal transformations

$$\bar{\varphi}^i = \varphi^i + k^i F(\vec{\varphi}), \qquad \bar{r} = r e^{-\Psi(\vec{\varphi})}, \qquad \bar{t} = t - \frac{e^{\Psi(\vec{\varphi})} - 1}{r},$$

where $e^\Psi \equiv 1 + \vec{k} \cdot \vec{\partial} F$. Therefore

$$\bar{g}_{\mu\nu} \to g_{\mu\nu}[\Psi(\varphi^i)]$$

Field Configurations

- Generic geometries in the phase space are obtained by ***exponentiating*** the infinitesimal transformations

$$\bar{\varphi}^i = \varphi^i + k^i F(\vec{\varphi}), \qquad \bar{r} = r e^{-\Psi(\vec{\varphi})}, \qquad \bar{t} = t - \frac{e^{\Psi(\vec{\varphi})} - 1}{r},$$

where $e^{\Psi} \equiv 1 + \vec{k} \cdot \vec{\partial} F$. Therefore

$$\bar{g}_{\mu\nu} \to g_{\mu\nu}[\Psi(\varphi^i)]$$

- ***NHEG phase space:***

$$\boxed{\mathcal{M} = \Big\{ g_{\mu\nu}[\Psi(\varphi^i)], \qquad \forall\ \Psi(\varphi^i) \quad \text{periodic} \Big\}}$$

Covariant phase space description of Gravity

Covariant phase space description of Gravity

- Using the covariant phase space method, one can make $\mathcal{M}$ a **phase space**

$$\mathcal{M} \to \Gamma = (\mathcal{M}, \Omega)$$

Covariant phase space description of Gravity

- Using the covariant phase space method, one can make $\mathcal{M}$ a **phase space**

$$\mathcal{M} \to \Gamma = (\mathcal{M}, \Omega)$$

- **Symplectic structure** Ω is constructed covariantly through

$$\Omega(\delta_1\phi, \delta_2\phi) = \int_\Sigma \boldsymbol{\omega}(\phi, \delta_1\phi, \delta_2\phi)$$

  where $\boldsymbol{\omega}$ is constructed from the second variation of Lagrangian

Covariant phase space description of Gravity

- Using the covariant phase space method, one can make $\mathcal{M}$ a **phase space**

$$\mathcal{M} \to \Gamma = (\mathcal{M}, \Omega)$$

- **Symplectic structure** Ω is constructed covariantly through

$$\Omega(\delta_1\phi, \delta_2\phi) = \int_\Sigma \boldsymbol{\omega}(\phi, \delta_1\phi, \delta_2\phi)$$

  where $\boldsymbol{\omega}$ is constructed from the second variation of Lagrangian

- **Symmetries.** ξ such that $\phi + \mathcal{L}_\xi\phi \in \mathcal{M}, \quad \forall\phi \in \mathcal{M}$
- **Conserved Charges** are defined through

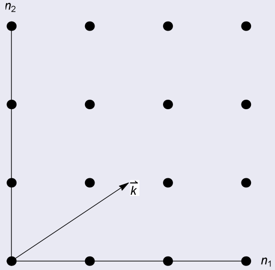
$$\delta Q_\xi = \int_\Sigma \boldsymbol{\omega}(\phi, \delta\phi, \mathcal{L}_\xi\phi)$$

Symmetry Algebra

- Vectors $\xi[\epsilon(\vec{\varphi})]$ are also symmetries of the phase space
- Fourier expansion $\quad\rightarrow\quad \xi = \sum_{\vec{n}} c_{\vec{n}}\, \xi_{\vec{n}}$

$$\boxed{[\xi_{\vec{n}}, \xi_{\vec{m}}] = i\vec{k}\cdot(\vec{n}-\vec{m})\, \xi_{\vec{n}+\vec{m}}}$$

- $d = 4 \rightarrow$ Witt algebra
- $d = 5 \rightarrow$ See figure
- Infinitely many Virasoro sub algebras
- $\vec{k} \in \mathbb{Q}^n$ vs. $k \in \mathbb{Q}^{*n}$
- $U(1)$ factors



Algebra of Charges: The "NHEG Algebra"

- There is a well defined ***symplectic structure*** on this phase space

Algebra of Charges: The "NHEG Algebra"

- There is a well defined ***symplectic structure*** on this phase space
- Algebra of charges is a ***central extension*** of the algebra of symmetry vectors

Algebra of Charges: The “NHEG Algebra”

- There is a well defined *symplectic structure* on this phase space
- Algebra of charges is a ***central extension*** of the algebra of symmetry vectors
- Quantized Algebra after $\{\quad\} \to \frac{1}{i\hbar}[\quad]\,,\, H_{\vec{n}} \to \hbar\, L_{\vec{n}},$

$$\boxed{[L_{\vec{m}}, L_{\vec{n}}] = \vec{k}\cdot(\vec{m}-\vec{n})L_{\vec{m}+\vec{n}} + \frac{S}{2\pi}(\vec{k}\cdot\vec{m})^3\delta_{\vec{m}+\vec{n},0}}$$

Algebra of Charges: The "NHEG Algebra"

- There is a well defined *symplectic structure* on this phase space
- Algebra of charges is a ***central extension*** of the algebra of symmetry vectors
- Quantized Algebra after $\{\quad\} \to \frac{1}{i\hbar}[\quad]\,, H_{\vec{n}} \to \hbar\, L_{\vec{n}}$,

$$\boxed{[L_{\vec{m}}, L_{\vec{n}}] = \vec{k}\cdot(\vec{m}-\vec{n})L_{\vec{m}+\vec{n}} + \frac{S}{2\pi}(\vec{k}\cdot\vec{m})^3\delta_{\vec{m}+\vec{n},0}}$$

- In $d=4$ dimensions, it is a chiral Virasoro algebra
- In $d\geq 5$ we have an extended Virasoso algebra

Charges

- Charge $H_{\vec{n}}[\Psi]$: Generator of $\xi_{\vec{n}}$ over the field configuration $g_{\mu\nu}[\Psi(\varphi^i)]$

$$\boxed{H_{\vec{n}} = \oint \boldsymbol{\epsilon}\, T[\Psi] e^{-i\vec{n}\cdot\vec{\varphi}}}$$

- Charges are Fourier modes of a **Liouville-type stress tensor**

$$\boxed{T[\Psi] = \frac{1}{16\pi G}\Big((\Psi')^2 - 2\Psi'' + 2e^{2\Psi}\Big)}$$

- Ψ is a “primary field of *weight one*” $\delta_\epsilon \Psi = \epsilon\, \Psi' + \epsilon'$,
- $T[\Psi]$ is a “quasi-primary of *weight two*” $\delta_\epsilon T = \epsilon\, T' + 2\epsilon'\, T - \frac{1}{8\pi G}\epsilon'''$

- However $\Psi = \Psi(\varphi^i)$, $\Psi' = \vec{k}\cdot\vec{\partial}\Psi$

Summary and Outlook

Summary

- We constructed the classical phase space of extermal black holes
- We obtrained the symmetry algebra (The NHEG algebra)
- We obtained the exact form of charges on the phase space

Outlook

- Look for a field theory with the same charges (it should be much similar to Liouville theory)
- Is there a notion of modular invariance here?
- Look for a Cardy like formula for counting of states? Is the black hole entropy reproduced?

Thank you for your attention

Details

- Infinitesimal phase space transformation

$$\chi[\epsilon(\vec{\varphi})] = \epsilon(\varphi^i)k^i\partial_{\varphi^i} - (k^i\partial_{\varphi^i}\epsilon)\,(\frac{b}{r}\partial_t + r\partial_r) \tag{1}$$

- Phase space field configurations

$$ds^2 = \Gamma(\theta)\Big[-(\boldsymbol{\sigma} - d\Psi)^2 + \Big(\frac{dr}{r} - d\Psi\Big)^2 + d\theta^2 + \gamma_{ij}(d\tilde{\varphi}^i + k^i\boldsymbol{\sigma})(d\tilde{\varphi}^j + k^j\boldsymbol{\sigma})\Big] \tag{2}$$

where $\tau = t + \frac{1}{r}$ and

$$\boldsymbol{\sigma} = e^{-\Psi} r d\tau + \frac{dr}{r}, \qquad \tilde{\varphi}^i = \varphi^i + k^i(F - \Psi)\,.$$