Abdus Salam International Center for Theoretical Physics - Trieste

Aspects of N=2 Gauge Theories from String Theory

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Based on work with

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Plan

- Introduction
- The Refinement of the Topological String
  - Generelised F-terms and amplitudes
  - Emergence of topological properties
- Mass deformations of N=4 in String Theory
  - Freely-acting orbifolds and adjoint masses
  - Recovering Nekrasov from the string
- On the Dual Gravitational Backgrounds
  - From the pure N=4 to its deformations
- Conclusions and Outlook

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Introduction & Motivations

- Seiberg-Witten theory : toy model for QCD
  - Explicitly solvable => prepotential of SYM
  - Including gravity and Ω-deformation

Seiberg, Witten (94')

Ferrara, Harvey, Strominger, Vafa (95')

Losev, Nekrasov, Shatashvili (98')

- String theory interpretation
  - Gravitational couplings, BPS amplitudes
  - String dualities

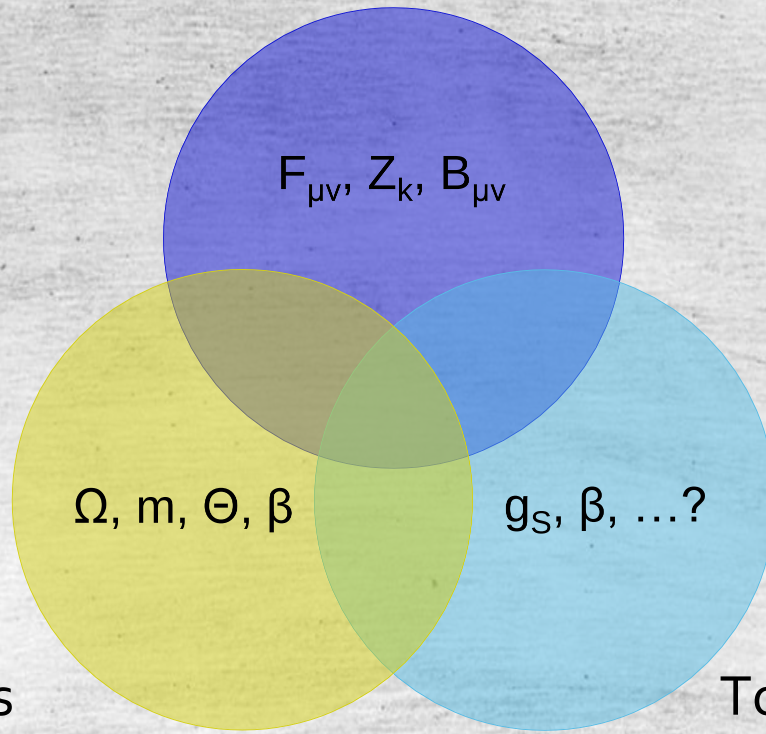
Bershadsky, Cecotti, Ooguri, Vafa (93')

Antoniadis, Gava, Narain, Taylor (93')

- Topological string theory
  - Counting indices (BPS states, wall-crossing, etc.)
  - Matrix models
  - Non-perturbative physics

Klemm, Labastida, Llatas, Mariño, Ooguri, Vafa, Witten, etc.

String amplitudes, BPS indices



N=2 gauge theories

Topological String Theory

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The Ω-deformed N=2 gauge theory

Nekrasov, Okounkov (04')

- Dimensionally reduce N=1 gauge theory in 6D

$$\mathcal{N}=1,\ 6D \xrightarrow[T^2\times\mathbb{R}^4]{\text{Dimensional Reduction}} \mathcal{N}=2,\ 4D\ \text{SYM}$$

+ R-symmetry rotation

$$ds^2 = Adzd\bar{z} + g_{IJ}\left(dx^I + V^I dz + \bar{V}^I d\bar{z}\right)\left(dx^J + V^J dz + \bar{V}^J d\bar{z}\right)$$

$$V^I \equiv \Omega^I{}_J x^J$$

$$\Omega^{IJ} = \begin{pmatrix} 0 & \epsilon_1 & 0 & 0 \\ -\epsilon_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \epsilon_2 \\ 0 & 0 & -\epsilon_2 & 0 \end{pmatrix}$$

$$\epsilon_\pm = \frac{\epsilon_1 \pm \epsilon_2}{2}$$

$$g_s^2 = -\epsilon_1\epsilon_2$$

$$\beta = -\frac{\epsilon_1}{\epsilon_2}$$

The underlying twisted N=2 gauge theory

Nekrasov (02')

- Using localisation, calculate the partition function (perturbative AND non-perturbative) of this theory

$$Z_{\text{Nek}} = \mathrm{Tr}(-)^F e^{-2\epsilon_- J_-^3 - 2\epsilon_+ (J_+^3 + J_R^3)} e^{-\beta H}$$

- The Ω-background regularises the instanton moduli space divergence $V_\Omega = \frac{1}{\epsilon_1 \epsilon_2} \xrightarrow[\epsilon_{1,2} \to 0]{} \infty$

- Leading term in the Ω-background expansion gives the SW prepotential

- Higher order terms : TST partition function

$$\sum_{g=0}^{\infty} g_s^{2g-2} F_g \Big|_{\text{field theory}} = \log Z_{\text{Nek}}(\epsilon_+ = 0, \epsilon_- = g_s)$$

TST partition function as higher-derivative F-terms

- This computes ½ - BPS F-terms in the effective action involving the N=2 supergravity multiplet :

$$W^{ij}_{\mu\nu} = \frac{1}{2}\epsilon^{ij}T_{\mu\nu} - R_{\mu\nu\lambda\rho}\theta^i\sigma^{\lambda\rho}\theta^j + \cdots$$

Internal $SU(2)_R$

SUSY completion

$$\int d^4x d^4\theta F_g\left(t_a\right)\left(W^2\right)^g = \int d^4x F_g\left(t_a\right)\left(T^2\right)^{g-1} R^2 + \cdots$$

- The coupling depends *only* on holomorphic vector multiplets

Toward a Refinement of the Topological String

1. Is there a one-parameter extension of the TST partition function capturing the second parameter (ε_+) of the Ω-background?
2. What is the corresponding coupling in the effective action?

Generalised F-terms

- The coupling can be realised through the insertion of the 'chiral projection' of anti-chiral superfields :

$$\mathcal{I}_{g,n} = \int d^4x \int d^4\theta\, \tilde{\mathcal{F}}_{g,n}(X)\, (W^{ij}_{\mu\nu} W^{\mu\nu}_{ij})^g \Upsilon^n$$

$$\sim \int d^4x\, \mathcal{F}_{g,n}(X, X^\dagger)\, R^2_{(-)} [F^G_{(-)}]^{2g-2} [F_{(+)}]^{2n}$$

$$\Upsilon := \Pi \frac{h(\hat{X}^I, (\hat{X}^I)^\dagger)}{(X^0)^2}$$

Vector superfield

$$\Pi := (\epsilon_{ij} \bar{D}^i \bar{\sigma}_{\mu\nu} \bar{D}^j)^2$$

- Possible mixing between chiral and anti-chiral superfields in a supersymmetric fashion

Proposal

Antoniadis, Florakis, Hohenegger, Narain, AZA (13')

- Identify the anti-chiral superfield as the $\bar{T}$ -vector multiplet in Heterotic on K3 x T^2
- T : Kähler modulus of T^2

General strategy

$$F_{g,n} = \left\langle R_-^2 \, V_{G,-}^{2g-2} \, V_{\bar{T},-}^{2n} \right\rangle_{1-\text{loop,Het}}$$

- Generating function trick

$$\mathcal{F}(\epsilon_-, \epsilon_+) = \sum_{g,n} \frac{\epsilon_-^{2g} \epsilon_+^{2n}}{g!^2 n!^2} F_{g,n} = \left\langle e^{-S_{\text{def}}(\epsilon_-, \epsilon_+)} \right\rangle_{1-\text{loop,Het}}$$

- Exactness of the sigma-model: Gaussian deformation
- Can be interpreted as a background

Structure of the deformation

$$S_{\text{def}}^{\text{bos}} = \tilde{\epsilon}_- \int d^2z \left(Z^1 \bar{\partial} Z^2 + \bar{Z}^2 \bar{\partial} \bar{Z}^1 \right) + \check{\epsilon}_+ \int d^2z \left(Z^1 \partial \bar{Z}^2 + Z^2 \partial \bar{Z}^1 \right)$$

$\epsilon_- \langle \partial X \rangle$

SU(2)$_-$ current

$\epsilon_+ \langle \bar{\partial} X \rangle$

SU(2)$_+$ current

$$S_{\text{ferm}}^{\text{def}} = \check{\epsilon}_+ \int d^2z (\chi^4 \chi^5 + \text{c.c.})$$

Effective R-symmetry current

- This is the structure of the N=2 gauge theory partition function in the Ω-background
- Here we work on a FLAT background

Field theory limit

$$\mathcal{F}(\epsilon_-, \epsilon_+) \;\sim\; (\epsilon_-^2 - \epsilon_+^2) \int_0^\infty \frac{dt}{t} \; \frac{-2\cos(2\epsilon_+ t)}{\sin(\epsilon_- - \epsilon_+)t \;\sin(\epsilon_- + \epsilon_+)t} \; e^{-\mu t}$$

- The would-be massless BPS states have mass

$$\mu \sim \sqrt{(T-\bar{T})(U-\bar{U}) - \tfrac{1}{2}(\vec{Y}-\bar{\vec{Y}})^2}\; \bar{P} = a_2 - \bar{U} a_1$$

- The leading singularity behaviour : $\mu^{2-2g-2n}$
- It perfectly matches the perturbative part of Nekrasov's partition function
- Not symmetric under $\varepsilon_+ \leftrightarrow \varepsilon_-$
- Only even powers in ε_+, ε_- (required by Lorentz invariance)

The dual type I theory

- In D=10 : Heterotic $\xleftrightarrow[\text{SO(32)}]{\text{S-duality}}$ Type I

Polchinski, Witten (98')

- In D=4 (K3 x T² compactifications): weakly coupled regimes on both sides

Antoniadis, Partouche, Taylor (98')

- Mapping of universal multiplets

Heterotic	Type I
$S = \alpha + i\, e^{-2\phi_4}$	$S = \alpha + i\, G^{\frac{1}{4}} V^{\frac{1}{2}} e^{-\phi_4}$
$T = B_{45} + i\, G^{\frac{1}{2}}$	$S' = B_{45} + i\, G^{\frac{1}{4}} V^{-\frac{1}{2}} e^{-\phi_4}$
$U = (G_{45} + i\, G^{\frac{1}{2}})/G_{44}$	$U = (G_{45} + i\, G^{\frac{1}{2}})/G_{44}$

Yang-Mills From Open Strings

- Classical action $\longrightarrow$ tree-level (disk)
- Gauge theory : $\alpha' \longrightarrow 0$ limit
- Calculate all disk diagrams with open string states and the take field theory limit

Ω-Deformed ADHM Action

$$\mathcal{S}_{\mathrm{ADHM}} = -\mathrm{Tr}\Big\{[\chi^\dagger, a_{\alpha\dot\beta}]\Big([\chi, a^{\dot\beta\alpha}] + \epsilon_-(a\tau_3)^{\dot\beta\alpha}\Big) - \chi^\dagger\,\bar\omega_{\dot\alpha}\left(\omega^{\dot\alpha}\chi - \tilde a\,\omega^{\dot\alpha}\right) - (\chi\bar\omega_{\dot\alpha} - \bar\omega_{\dot\alpha}\,\tilde a)\,\omega^{\dot\alpha}\,\chi^\dagger$$
$$+\epsilon_+\left[\chi^\dagger, a_{\alpha\dot\beta}\right](\tau_3 a)^{\dot\beta\alpha} - \epsilon_+\,\bar\omega_{\dot\alpha}\,(\tau_3)^{\dot\alpha}{}_{\dot\beta}\,\chi^\dagger\,\omega^{\dot\beta}\Big\}$$

- Deformed action used by Nekrasov to derive the instanton partition function (localisation)

Nekrasov (02')

- Action is Q-exact, where Q is the susy preserved by D5/D9 system

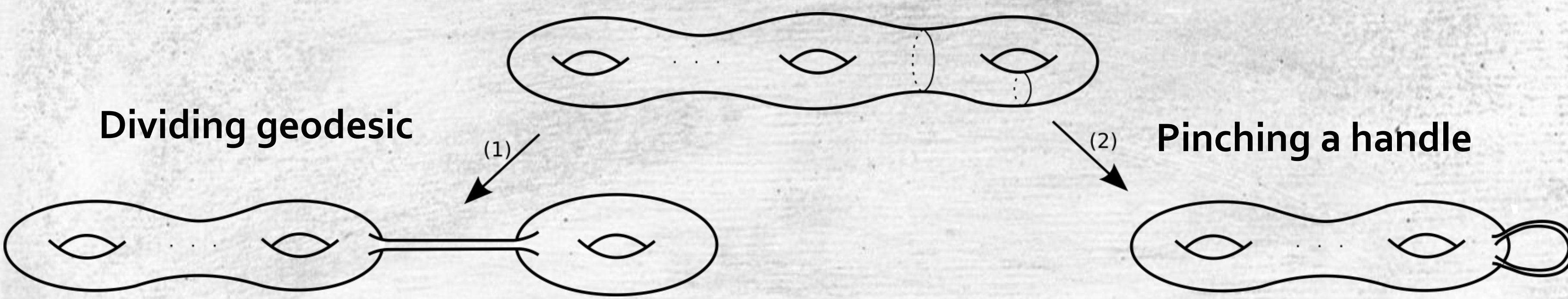
- Nekrasov partition function : holomorphic in the parameters (vevs, Ω-background)

Holomorphic Anomaly Equations

- BCOV holomorphic anomaly : recursion relation satisfied by the TST partition function as powerful tool for non-perturbative calculations

Bershadsky, Ceccotti, Ooguri, Vafa (93')

- Quantifies the non-decoupling of unphysical (Q-exact) states in TST
- Contributions from boundary of moduli space



Holomorphic Anomaly Equations

- The recursion relations

$$\partial_{\bar{i}} F_g = \frac{1}{2} C_{\bar{i}}^{\ jk} \left(\sum_{g'} D_j F_{g'} D_k F_{g-g'} + D_j D_k F_{g-1} \right)$$

3-point function

Bershadsky, Ceccotti, Ooguri, Vafa (93')

- Breaking of holomorphicity of F_g (predicted from SUGRA)

- Integrate the equations with appropriate boundary conditions (field theory limit, conifold, etc.)

Holomorphicity Properties of the Refined Couplings

- Explicit breaking of holomorphicity
  - Related to the compactness of the CY
- Non-compact CY for
  - R-symmetry current
  - Decoupling of hypers
  - Define generating function refined topological invariants
- Use the generic CY compactification + appropriate limit

Holomorphicity Properties of the Refined Couplings

- Type II dual coupling

$$\hat{\phi} = \oint dz\rho(z) \oint d\bar{z}\tilde{\rho}(\bar{z})\,\bar{\phi}$$

anti-chiral

hol. 3-form

$$F_{g,n} = \int_{\mathcal{M}_{g,n}} \left\langle \prod_{k=1}^{3g-3+n} |\mu_k \cdot G^-|^2 \left(\int \bar{\phi}\right)^n \left(\hat{\phi}\right)^n \right\rangle_{\text{twist}}$$

Antoniadis, Hohenegger, Narain, Taylor (10')

- Generic case : differential equation with not just boundary terms

Antoniadis, Florakis, Hohenegger, Narain, AZA (15')

- Required limit : $\mathcal{D}_{\bar{\imath}} F_{g,n} = 0\,, \quad \mathcal{D}_\star F_{g,n,\bar{\imath}} = (n-1)\Psi_{g,n}^{(\star\star\bar{i})}\,.$

- The equation becomes a standard recursion

$$D_{\bar{i}} F_g = \frac{1}{2} C_{\bar{i}}^{\ jk} \left(\sum_{g',n'} D_j F_{g',n'} D_k F_{g-g',n-n'} + D_j D_k F_{g-1,n} \right)$$

Holomorphicity Properties of the Refined Couplings

- Explicit test in the weak-coupling limit
  - Heterotic amplitude
  - Large volume limit

$$C_{\bar{i}}{}^{j} = C_{\bar{i}\bar{j}\bar{S}} e^{2K} G^{S\bar{S}} G^{j\bar{j}}$$

- Recursion relations

$$D_{\bar{i}} F_{g,n} = 2\pi i\, C_{\bar{i}}{}^{j} D_j F_{g-1,n}$$

- Coincides with the weak-coupling limit of the exact (Type II) equation

Challenges for refinement

- How to realise these constraints at the CFT level
- CY should experience a severe transition
  - Local U(1) invariance should become global
  - Which one?
- Potential role of contact geometry in the underlying supergravity
  - Reeb vector plays a special role
- Is decompactification enough? If yes, in which direction?

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Why N=2*?

- Interpolating between N=4 and N=2
- Simplest deformation of N=4
  - Vanishing beta-function
- Typical example in the context of AGT
  - Liouville theory on a one-punctured torus
- Connection to the (2,0) theory
  - M2/M5 brane configurations

Intuition from branes

- N Dp-branes ⟶ N=4 SU(N) gauge theory
  - Massless excitations of open strings ⟶ N=4 vector multiplet

Green, Gutperle (00')

- Tilt the branes in some internal direction
  - Break N=4 ⟶ N=2
  - Give mass to the N=2 hypermultiplet (~ angle)
- Example: NS5-branes on a circle and TN geometry
- Branes at angles ↔ magnetic fluxes

I. Florakis, AZA (15')

Consider freely-acting orbifolds of N=4 compactifications as a string theory realisation of N=2*

The class of models

I. Florakis, AZA (15')

- Z_2 freely-acting orbifolds of heterotic on T^6
  - Symmetric or asymmetric
  - Can be done in any string theory (we have explicit examples in type II and type I as well)
- Notation
  - 1,2,3: internal directions, 4,5: space-time
- Action of the orbifold: $g = e^{i\pi Q_L}\delta$

$$e^{i\pi Q_L} : Z_L^2 \to e^{i\pi} Z_L^2 \quad , \quad Z_L^3 \to e^{-i\pi} Z_L^3$$

$$\delta : Z^1 \to Z^1 + i\pi\sqrt{\frac{T_2}{U_2}}$$

Spectrum of the theory

- Partition function:

$$Z = \frac{1}{2^2\,\eta^{12}\,\bar\eta^{24}} \sum_{\substack{a,b=0,1\\ H,G=0,1}} (-1)^{a+b+ab}\,\vartheta\left[{}^{a}_{b}\right]^2 \vartheta\left[{}^{a+H}_{b+G}\right]\vartheta\left[{}^{a-H}_{b-G}\right]\, Z_{4,4}\left[{}^{H}_{G}\right]\,\Gamma_{2,18}\left[{}^{H}_{G}\right](T,U)$$

Twisted lattice partition function

Shifted Narain lattice (T^2 + gauge bundle)

- SO(4) character decomposition:

$$O_4 = \frac{1}{2\eta^2}(\vartheta_3^2 + \vartheta_4^2)\ , \qquad V_4 = \frac{1}{2\eta^2}(\vartheta_3^2 - \vartheta_4^2)\ ,$$

$$S_4 = \frac{1}{2\eta^2}(\vartheta_2^2 - \vartheta_1^2)\ , \qquad C_4 = \frac{1}{2\eta^2}(\vartheta_2^2 + \vartheta_1^2)\ .$$

Spectrum of the theory

- Partition function:

$$Z = \frac{1}{2^2\,\eta^{12}\,\bar\eta^{24}} \sum_{\substack{a,b=0,1\\ H,G=0,1}} (-1)^{a+b+ab}\,\vartheta\left[{a \atop b}\right]^2 \vartheta\left[{a+H \atop b+G}\right]\vartheta\left[{a-H \atop b-G}\right] Z_{6,22}\left[{H \atop G}\right](T,U)$$

- SO(4) character decomposition:

$$O_4 = \frac{1}{2\eta^2}(\vartheta_3^2 + \vartheta_4^2)\ , \qquad V_4 = \frac{1}{2\eta^2}(\vartheta_3^2 - \vartheta_4^2)\,,$$

$$S_4 = \frac{1}{2\eta^2}(\vartheta_2^2 - \vartheta_1^2)\ , \qquad C_4 = \frac{1}{2\eta^2}(\vartheta_2^2 + \vartheta_1^2)\,.$$

Spectrum of the theory

- Partition function:

$$\begin{aligned}\eta^4\bar{\eta}^4\, Z =& (V_4O_4 - S_4S_4)\, Z_{6,22}\begin{bmatrix}0\\+\end{bmatrix} + (O_4V_4 - C_4C_4)\, Z_{6,22}\begin{bmatrix}0\\-\end{bmatrix}\\ &\xcancel{+(V_4C_4 - S_4V_4)\, Z_{6,22}\begin{bmatrix}1\\\mp\end{bmatrix} + (O_4S_4 - C_4O_4)\, Z_{6,22}\begin{bmatrix}1\\\pm\end{bmatrix}}\,.\end{aligned}$$

N=2 vector multiplet

Parity under the orbifold action

N=2 hypermultiplet (massive)

- SO(4) character decomposition:

$$O_4 = \frac{1}{2\eta^2}(\vartheta_3^2 + \vartheta_4^2)\,, \qquad V_4 = \frac{1}{2\eta^2}(\vartheta_3^2 - \vartheta_4^2)\,,$$

$$S_4 = \frac{1}{2\eta^2}(\vartheta_2^2 - \vartheta_1^2)\,, \qquad C_4 = \frac{1}{2\eta^2}(\vartheta_2^2 + \vartheta_1^2)\,.$$

Matching N=2*

- For simplicity, I focus on the SU(2) case
  - Generalises to exceptional groups straightforwardly
  - Enhancement from T^2 only
  - Wilson lines break the gauge group to U(1) factors
- Enhancement point: $TU - \frac{1}{2}W^2 = -1$
  - Not equivalent to T=U point!
- $W^{\pm}$ states: $m_1=n^1=0$, $m_2=n^2=\pm 1$, $m_{\rm vec}^2 = \frac{|TU - \frac{1}{2}W^2 + 1|^2}{T_2 U_2 - \frac{1}{2}W_2^2}$
- Adjoint hypermultiplet: $m_{\rm h}^2 = \frac{|1 + TU - \frac{1}{2}W^2 + U|^2}{T_2 U_2 - \frac{1}{2}W_2^2}$

Ω-deformed N=2* partition function

- For concreteness, I focus on an asymmetric orbifold of heterotic
- Topological amplitude in this background

$$\mathcal{F}(\epsilon)=\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}\,\frac{G_{\rm bos}(\epsilon)}{4\bar{\Delta}}\sum_{\substack{H,G=0,1\\(H,G)\neq(0,0)}}\ \sum_{\substack{\gamma,\delta=0,1\\(\gamma,\delta)\neq(1,1)}}(-1)^{G(\gamma+1)}\,\bar{\vartheta}\big[{}^{\gamma+H}_{\delta+G}\big]^4\,\Gamma_{2,18}\big[{}^{H}_{G}\big](T,U,W)$$

$$\Delta=\eta^{24}$$

$$G_{\rm bos}(\epsilon)=\frac{(2\pi i\epsilon)^2\bar{\eta}^6}{\bar{\vartheta}_1(\tilde{\epsilon})^2}\,e^{-\frac{\pi\tilde{\epsilon}^2}{\tau_2}}$$

$$\tilde{\epsilon}=\frac{\tau_2 P_L}{\sqrt{T_2U_2-W_2^2/2}}\,\epsilon$$

Ω-deformed N=2* partition function

$$\mathcal{F}(\epsilon) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\,\frac{G_{\rm bos}(\epsilon)}{4\bar{\Delta}} \sum_{\substack{H,G=0,1\\(H,G)\neq(0,0)}}\ \sum_{\substack{\gamma,\delta=0,1\\(\gamma,\delta)\neq(1,1)}} (-1)^{G(\gamma+1)}\,\bar\vartheta[{}^{\gamma+H}_{\delta+G}]^4\,\Gamma_{2,18}[{}^H_G](T,U,W)$$

- Field theory limit at the SU(2) point:

$$\frac{1}{(2\pi\epsilon)^2}\,\mathcal{F}(\epsilon)\big|_{\rm F.T.} = \sum_{k=0,\pm1}\left[\gamma_\hbar(k\mu) - \gamma_\hbar(k\mu + m_{\rm h})\right]$$

- Perfect matching with the gauge theory result

Barnes double gamma function

BPS vector mass

$$\hbar = 2i\pi\epsilon$$

Star remarks

- Mass deformation implemented directly in the background
- It works also in type I and type II
- Analyticity is guaranteed
- $N=4,2$ limits are crystal clear
- Equivalently: the mass deformation is topological
  - It satisfies the standard BCOV holomorphic anomaly equation
  - This property is obvious by construction
  - Not clear from the geometric point of view

N=2* vertex and instantons

- Scherk-Schwarz ↔ Freely-acting orbifolds
- Works for symmetric or asymmetric
- Example

$$e^{i\pi Q_L} : Z_L^2 \to e^{i\pi} Z_L^2 \quad , \quad Z_L^3 \to e^{-i\pi} Z_L^3$$

$$\delta : Z^1 \to Z^1 + i\pi R_1 \sqrt{\frac{T_2}{U_2}}$$

- Field redefinition

$$\hat{Z}^{2,3} = e^{\mp \frac{Z^1}{R_1}\sqrt{\frac{U_2}{T_2}}} Z^{2,3}$$

N=2* vertex and instantons

- Scherk-Schwarz ↔ Freely-acting orbifolds
- More generally

$$\delta S = i \int \mathrm{d}^2 z F^a_{IJ} \left[\hat{Z}^I \overleftrightarrow{\partial} \hat{Z}^J \tilde{J}^a + J^a \, \hat{Z}^I \overleftrightarrow{\bar{\partial}} \hat{Z}^J \right]$$

can be re-absorbed through

$$Z^I = \left(e^{F^a X^a} \right)^I{}_J \, \hat{Z}^J$$

- This leads to a freely-acting orbifold with a flat sigma-model (modulo a quadratic term...)

N=2* vertex and instantons

- Vertex operator for the mass deformation

  - Linear part

$$V_{X^k}(z) = (-)^k \frac{m}{4\pi}\left[\left(\bar{Z}^k(z) - \bar{Z}^k(\bar{z})\right)\partial X(z)\bar{\partial}Z^k(\bar{z}) - \left(\bar{Z}^k(z) - \bar{Z}^k(\bar{z})\right)\partial Z^k(z)\bar{\partial}X(\bar{z})\right.$$
$$\left. -\bar{\psi}^k\chi(z)\bar{\partial}Z^k(\bar{z}) - \partial Z^k(z)\bar{\psi}^k\chi(z) + \partial X(z)\bar{\psi}^k\psi^k(\bar{z}) + \bar{\psi}^k\psi^k(z)\bar{\partial}X(\bar{z})\right] ,$$
$$V_{\bar{X}^k}(z) = (-)^k \frac{\bar{m}}{4\pi}\left[\left(\bar{Z}^k(z) - \bar{Z}^k(\bar{z})\right)\partial \bar{X}(z)\bar{\partial}Z^k(\bar{z}) - \left(\bar{Z}^k(z) - \bar{Z}^k(\bar{z})\right)\partial Z^k(z)\bar{\partial}\bar{X}(\bar{z})\right.$$
$$\left. -\bar{\psi}^k\bar{\chi}(z)\bar{\partial}Z^k(\bar{z}) - \partial Z^k(z)\bar{\psi}^k\bar{\chi}(z) + \partial \bar{X}(z)\bar{\psi}^k\psi^k(\bar{z}) + \bar{\psi}^k\psi^k(z)\bar{\partial}\bar{X}(\bar{z})\right] .$$

  - Quadratic part

$$V_{XX} = \frac{m^2}{8\pi}\sum_{k=1,2}\left[\psi^k\chi(z)\bar{\psi}^k\chi(\bar{z}) + \bar{\psi}^k\chi(z)\psi^k\chi(\bar{z})\right] ,$$
$$V_{X\bar{X}} = \frac{|m|^2}{8\pi}\sum_{k=1,2}\left[\psi^k\chi(z)\bar{\psi}^k\bar{\chi}(\bar{z}) + \bar{\psi}^k\chi(z)\psi^k\bar{\chi}(\bar{z}) + \psi^k\bar{\chi}(z)\bar{\psi}^k\chi(\bar{z}) + \bar{\psi}^k\bar{\chi}(z)\psi^k\chi(\bar{z})\right] .$$

N=2* ADHM recovered

- Calculate all tree-level amplitudes with the mass vertices
- Result : agreement with the perturbative and non-perturbative actions!
- For instance, bosonic Yang-Mills scalars

$$\mathcal{L}_m \ni -\tfrac{1}{2} \sum_{k=1,2} |m|^2 |\phi_k|^2 - m\bar{\chi}\left([\phi_1, \bar{\phi}_1] - [\phi_2, \bar{\phi}_2]\right) - \bar{m}\chi\left([\phi_1, \bar{\phi}_1] - [\phi_2, \bar{\phi}_2]\right) .$$

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Emergence of AdS from branes

Ferrari (12')

- Probe dual geometry by probe branes in some brane configuration

- D3/D(-1) system: D-instantons see the emergent geometry from the D3's (near-horizon limit)

$$\int \mathrm{d}\Psi_3 \mathrm{d}\Psi_{3/-1} \mathrm{d}\Psi_{-1}\, \mathrm{e}^{-S_{\mathrm{D3}} - S_{\mathrm{D3/D(-1)}} - S_{\mathrm{D(-1)}}} = \int \mathrm{dZ}\, \mathrm{e}^{-S_{\mathrm{eff}}(\mathrm{Z})}$$

- The resulting effective action exactly encodes the $AdS_5 \times S^5$ geometry!

Emergence of AdS from branes

- Large N limit is sometimes crucial to perform the path integral
- The integration over the D3-D3 moduli is possible when quantum corrections vanish or can be neglected
  - Ex: when conformal invariance is unbroken
  - Should hold in more general cases
- Generalisation: implement gauge theory deformation and study gravity dual
  - Mass deformation
  - Ω-deformation

Towards the dual of N=2*

$$\int d\Psi_3 d\Psi_{3/-1} d\Psi_{-1}\, e^{-S_{D3}-S_{D3/D(-1)}-S_{D(-1)}} = \int dZ\, e^{-S_{\text{eff}}(Z)}$$

- Do this for N=2* and read dual background
- Involves calculating a chiral correlation function (integration over D3 d.o.f)

Nekrasov, Pestun (12')

$$R_N(t,q,x) = \frac{1}{\theta(t,q)} \sum_{k=0}^{N} a_k \left(x - m \frac{\partial}{\partial \log t} \right)^k \theta(t,q) = 0$$

Hol. gauge coupling

(coord. on elliptic curve)

Vacuum configuration

Chiral observables of N=2*

Plan

- Introduction
- The Refinement of the Topological String
  - Generelised F-terms and amplitudes
  - Emergence of topological properties
- Mass deformations of N=4 in String Theory
  - Freely-acting orbifolds and adjoint masses
  - Recovering Nekrasov from the string
- On the Dual Gravitational Backgrounds
  - From the pure N=4 to its deformations
- Conclusions and Outlook

Concluding remarks

- N=2 is an interesting playground for string theory and gauge theory
  - Learn new aspects on both sides
- Beyond the topological string paradigm
  - Refinement
  - Mass deformation
  - Non-commutativity
- Worldsheet realisation of N=2* and its universality
- Promising candidate for a worldsheet realisation of the refined topological string

Concluding remarks

- Refinement and input from supergravity?
  - Translate ST conditions into WS ones
  - Understand the resulting geometry
- Holographic application through D-instanton probes?
  - Gravitational duals of gauge theory deformations
  - Beyond the conformal case
- Understand these deformations from the topological string point of view: the worldsheet is crucial!

Aspects of N=2 Gauge Theories from String Theory

Thank You!

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