$pp$-Waves and AdS-Plane Waves in Null Aether Theory

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Outline

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Null Aether Theory (NAT)

- NAT is described by

\[
I = \frac{1}{16\pi G} \int d^D x \sqrt{-g} [R - 2\Lambda - K_{\mu\nu}^{\alpha\beta} \nabla_\alpha u^\mu \nabla_\beta u^\nu + \lambda (u^\mu u_\mu + \epsilon)],
\]

\[
K^{\mu\nu}_{\alpha\beta} = c_1 g^{\mu\nu} g_{\alpha\beta} + c_2 \delta^\mu_\alpha \delta^\nu_\beta + c_3 \delta^\mu_\beta \delta^\nu_\alpha - c_4 u^\mu u^\nu g_{\alpha\beta}.
\]
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\[ K^{\mu\nu}_{\alpha\beta} = c_1 g^{\mu\nu} g_{\alpha\beta} + c_2 \delta^\mu_\alpha \delta^\nu_\beta + c_3 \delta^\mu_\beta \delta^\nu_\alpha - c_4 v^\mu v^\nu g_{\alpha\beta}. \]
Null Aether Theory (NAT)

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\]

\[
K_{\mu\nu}^{\alpha\beta} = c_1 g^{\mu\nu} g_{\alpha\beta} + c_2 \delta_{\alpha}^{\mu} \delta_{\beta}^{\nu} + c_3 \delta_{\beta}^{\mu} \delta_{\alpha}^{\nu} - c_4 v^\mu v^\nu g_{\alpha\beta}.
\]

- The aether field has the fixed-norm constraint

\[
\text{[sign} = (-, +)] \quad v^\mu v_\mu = -\varepsilon, \quad (\varepsilon = 0, \pm 1)
\]

- \(\varepsilon = +1 \Rightarrow \text{Einstein-Aether theory.} \quad \text{[Jacobson & Mattingly (2001)]}\)
The eqns. of motion are

\[
G_{\mu\nu} + \Lambda g_{\mu\nu} = \nabla_\alpha \left[ J^\alpha_{(\mu \nu)} - J_{(\mu} J^{\alpha \nu)} + J_{(\mu\nu)} v^\alpha \right] \\
+ c_1 (\nabla_\mu v_\alpha \nabla_\nu v^\alpha - \nabla_\alpha v_\mu \nabla^\alpha v_\nu) \\
+ c_4 \dot{v}_\mu \dot{v}_\nu + \lambda v_\mu v_\nu - \frac{1}{2} L g_{\mu\nu},
\]

\[
c_4 \dot{v}^\alpha \nabla_\mu v_\alpha + \nabla_\alpha J^\alpha_{\mu} + \lambda v_\mu = 0,
\]

where \( \dot{v}^\mu \equiv v^\alpha \nabla_\alpha v^\mu \) and

\[
J_{\mu \nu} \equiv K^{\mu\alpha}_{\nu\beta} \nabla_\alpha v^\beta, \\
L \equiv J^\mu_{\nu} \nabla_\mu v^\nu.
\]

From now on, \( \varepsilon = 0 \Rightarrow \text{NAT.} \)
**pp-Wave Spacetimes**

- *pp*-waves (plane-fronted waves with parallel rays) are defined by

\[
\nabla_\mu l_\nu = 0, \quad l_\mu l^\mu = 0.
\]

which immediately implies that

\[
\begin{align*}
l^\mu \nabla_\mu l_\nu &= 0, \\
\nabla_\mu l_\nu + \nabla_\nu l_\mu &= 0, \\
\nabla_\mu l_\nu - \nabla_\nu l_\mu &= 0.
\end{align*}
\]

\( (\text{covariantly const. null vector field}) \)

\( (\text{geodesic with } l^\mu = \frac{dx^\mu}{dv}) \)

\( (\text{automatically a Killing vector}) \)

\( (l_\mu = \partial_\mu u) \)
Consider Kerr-Schild class of $pp$-waves:

$$g_{\mu\nu} = \eta_{\mu\nu} + 2V(x)l_{\mu}l_{\nu},$$

In the coord. sys. $x^{\mu} = (u, v, x^{i})$,

$$ds^2 = 2dudv + 2V(u, x^{i})du^2 + dx_{i}dx^{i}.$$

For such metrics,

$$R_{\mu\nu} = -(\nabla_{\perp}^2 V)l_{\mu}l_{\nu} \Rightarrow R = 0,$$

$$\nabla_{\perp}^2 \equiv \partial_{i}\partial^{i}$$
Plane Waves

- Plane waves are **subclass** of *pp*-waves for which

\[ V(u, x^i) = h_{ij}(u)x^i x^j, \]

\[ R_{\mu\nu} = -2\text{Tr}(h)l_\mu l_\nu, \]

- In Einstein gravity,

\[ R_{\mu\nu} = 0 \implies \text{Tr}(h) = 0 \]

\[ \text{in } D = 4, \]

\[ x^i = (x, y) \]

\[ ds^2 = 2du dv + 2[h_{11}(u)(x^2 - y^2) + 2h_{12}(u)xy]du^2 + dx^2 + dy^2 \]
pp-Waves in NAT

- *pp*-Waves constitute exact solutions to NAT.

- To show this, take the null aether field as

  \[ v^\mu = \phi(x) l^\mu, \quad l_\mu l^\mu = 0 \]

  and assume that

  \[ \nabla_\mu l_\nu = 0, \quad l^\mu \partial_\mu \phi = 0. \]

  so that

  \[ l^\mu \nabla_\mu l_\nu = 0, \quad l^\mu \nabla_\nu l_\mu = 0, \quad \dot{v}_\mu = 0, \]

  (scalar spin–0 aether field)
Then the field eqns. of NAT become

\[ G_{\mu\nu} + \Lambda g_{\mu\nu} = -c_3 \left[ \nabla_\alpha \phi \nabla^\alpha \phi - \frac{\lambda}{c_1} \phi^2 \right] l_\mu l_\nu, \]

\[ (c_1 \Box \phi + \lambda \phi) l_\mu = 0, \]

\[ \Box \phi - m^2 \phi = 0 \]

\[ m^2 \equiv -\frac{\lambda}{c_1} \geq 0 \]

\[ T_{\mu\nu} = \varepsilon l_\mu l_\nu \]

\[ \varepsilon \geq 0 \Rightarrow c_3 \leq 0 \& \frac{\lambda}{c_1} \leq 0 \]

**null dust**
**pp-Waves in NAT**

- For *pp*-waves spacetimes,

\[
\nabla_\bot^2 V = c_3 \left[ \partial_i \phi \partial^i \phi + m^2 \phi^2 \right],
\]

\[
\nabla_\bot^2 \phi - m^2 \phi = 0, \quad \Lambda = 0.
\]

(with the ansatz)

\[
V(u, x^i) = V_0(u, x^i) + \alpha \phi(u, x^i)^2,
\]

(they decouple)

\[
\nabla_\bot^2 V_0 = 0 \quad \text{for} \quad \alpha = \frac{c_3}{2}.
\]

- Thus, *pp*-waves are solutions if the Laplace eqn. for \(V_0\), and the Klein-Gordon eqn. for \(\phi\) are satisfied.
Plane Waves in NAT

- Plane waves \( V(u, x^i) = h_{ij}(u)x^ix^j \) can also be constructed:

  - when \( c_3 = 0 \implies V = V_0 : \)
    \[
    \nabla^2 V = 0 \implies \text{Tr}(h) = 0 \quad \& \quad \nabla^2 \phi - m^2 \phi = 0
    \]
    \[
    ds^2 = 2du dv + 2[h_{11}(u)(x^2 - y^2) + 2h_{12}(u)xy]du^2 + dx^2 + dy^2 \quad (D = 4)
    \]

  - when \( c_3 \neq 0 \), but \( V_0 = t_{ij}(u)x^ix^j : \)
    \[
    V = V_0 + \frac{c_3}{2} \phi^2,
    \]
    \[
    \nabla^2 V_0 = 0 \implies \text{Tr}(t) = 0, \quad \text{zero}
    \]
    \[
    [h_{jk}(h_{ij} - t_{ij}) - (h_{ki} - t_{ki})(h_{kj} - t_{kj})]x^ix^j - m^2[(h_{ij} - t_{ij})x^ix^j] = 0. \quad \text{zero}
    \]
**Kerr-Schild-Kundt (KSK) Class of Metrics**

- KSK class of metrics are defined by [Gürses, Şişman, Tekin (2012)]

\[
g_{\mu\nu} = \bar{g}_{\mu\nu} + 2Vl_{\mu}l_{\nu},
\]

(Generalized KS form)

\[
\begin{align*}
l_{\mu}l^{\mu} &= 0, \\
\nabla_{\mu}l_{\nu} &= \frac{1}{2}(l_{\mu}\xi_{\nu} + l_{\nu}\xi_{\mu}), \\
l_{\mu}\xi^{\mu} &= 0, \\
l^{\mu}\partial_{\mu}V &= 0.
\end{align*}
\]
### Kerr-Schild-Kundt (KSK) Class of Metrics

- **KSK class of metrics** are defined by [Gürses, Şişman, Tekin (2012)]

\[ g_{\mu \nu} = \bar{g}_{\mu \nu} + 2V l_\mu l_\nu, \]

\[ (\text{generalized KS form}) \]

\[ l_\mu l^\mu = 0, \quad \nabla_\mu l_\nu = \frac{1}{2}(l_\mu \xi_\nu + l_\nu \xi_\mu), \]

\[ l_\mu \xi^\mu = 0, \quad l^\mu \partial_\mu V = 0. \]

\[ \bar{R}_{\mu \alpha \nu \beta} = K(\bar{g}_{\mu \nu} \bar{g}_{\alpha \beta} - \bar{g}_{\mu \beta} \bar{g}_{\nu \alpha}) \]

\[ K = \frac{\bar{R}}{D(D-1)} = \text{const.} \]

- **Kundt class**

\[ K > 0 \quad dS \]

\[ K = 0 \quad M \]

\[ K < 0 \quad \text{AdS} \]
Again assume that

but with

so that

\( \nabla_\mu l_\nu = \frac{1}{2}(l_\mu \xi_\nu + l_\nu \xi_\mu), \quad l_\mu \xi_\mu = 0, \quad l_\mu \partial_\mu \phi = 0, \)

\( l^\mu \nabla_\mu l_\nu = 0, \quad l^\mu \nabla_\nu l_\mu = 0, \quad \nabla_\mu l^\mu = 0, \quad \dot{v}_\mu = 0, \)
• Then the field eqns. of NAT become

\[ G_{\mu\nu} + \Lambda g_{\mu\nu} = \left[ -c_3 \nabla_\alpha \phi \nabla^\alpha \phi + (c_1 - c_3) \phi \Box \phi - 2c_3 \phi \xi^\alpha \partial_\alpha \phi \\
+ \left( \lambda - \frac{c_1 + c_3}{4} \xi_\alpha \xi^\alpha \right) \phi^2 \right] l_\mu l_\nu - (c_1 + c_3) \phi^2 R_{\mu\alpha\nu\beta} l^\alpha l^\beta, \]

\[ [c_1 (\Box \phi + \xi^\alpha \partial_\alpha \phi) + \lambda \phi] l_\mu + (c_1 + c_3) \phi R_{\mu\nu} l^\nu = 0, \]

• For the KSK metrics, we have

\[ G_{\mu\nu} = -\frac{(D - 1)(D - 2)}{2} K \bar{g}_{\mu\nu} - \rho l_\mu l_\nu, \]

\[ \rho \equiv \Box V + 2\xi^\alpha \partial_\alpha V + \left[ \frac{1}{2} \xi_\alpha \xi^\alpha + (D + 1)(D - 2) K \right] V, \]
Then we obtain

\[ \Lambda = \frac{(D - 1)(D - 2)}{2} K, \]

\[ \Box V + 2\xi^\alpha \partial_\alpha V + \left[ \frac{1}{2} \xi_\alpha \xi^\alpha + 2(D - 2)K \right] V = c_3 \left[ \overrightarrow{\nabla}_\alpha \phi \overrightarrow{\nabla}^\alpha \phi - \frac{\lambda}{c_1} \phi^2 \right] + (c_1 + c_3) \phi \xi^\alpha \partial_\alpha \phi + \frac{c_1 + c_3}{c_1} \left\{ [c_1(D - 2) - c_3(D - 1)] K + \frac{c_1}{4} \xi_\alpha \xi^\alpha \right\} \phi^2, \]

\[ c_1(\Box \phi + \xi^\alpha \partial_\alpha \phi) + [\lambda + (c_1 + c_3)(D - 1)K] \phi = 0, \]

\[ (\Box + \xi^\alpha \partial_\alpha) \phi - m^2 \phi = 0, \]

\[ m^2 \equiv -\frac{1}{c_1} [\lambda + (c_1 + c_3)(D - 1)K] \]

\( \text{For } K = 0 \text{ and } \xi^\mu = 0, \quad (\text{we recover the pp-wave case!}) \)
• Let us assume the ansatz

\[ V(x) = V_0(x) + \alpha \phi(x)^2, \]

• There are two possible choices for \( \alpha \):

\[
\Box V_0 + 2\xi^\alpha \partial_\alpha V_0 + \left[ \frac{1}{2} \xi_{\alpha} \xi^\alpha + 2(D - 2)K \right] V_0 \\
= c_1 \left\{ \phi \xi^\alpha \partial_\alpha \phi + \left[ (D - 2)K + \frac{1}{4} \xi_\alpha \xi^\alpha \right] \phi^2 \right\},
\]

\[
\Box V_0 + 2\xi^\alpha \partial_\alpha V_0 + \left[ \frac{1}{2} \xi_{\alpha} \xi^\alpha + 2(D - 2)K \right] V_0 \\
= -c_1 \left[ \nabla_\alpha \phi \nabla^\alpha \phi + m^2 \phi^2 \right].
\]

\[
\text{for } \alpha = \frac{c_3}{2}
\]

\[
\text{for } \alpha = \frac{c_1 + c_3}{2}
\]

• Thus, exact wave solutions propagating in nonflat backgrounds can be constructed in NAT.
Now assume that the background is AdS; i.e.,

$$d\bar{s}^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = \frac{\ell^2}{z^2} (2du dv + dx^i dx^i + dz^2),$$

Then taking $l_\mu = \delta_\mu^u$, one can show that

$$l^\mu = g^{\mu\nu} l_\nu = \bar{g}^{\mu\nu} l_\nu = \frac{z^2}{\ell^2} \delta_\nu^u \implies l^\alpha \partial_\alpha V = \frac{z^2}{\ell^2} \frac{\partial V}{\partial v} = 0 \quad \& \quad l^\alpha \partial_\alpha \phi = \frac{z^2}{\ell^2} \frac{\partial \phi}{\partial v} = 0,$$

$$\nabla_\mu l_\nu = \tilde{\nabla}_\mu l_\nu = \frac{1}{z} (l_\mu \delta_\nu^z + l_\nu \delta_\mu^z),$$

$$\begin{aligned}
\xi_\mu &= \frac{2}{z} \delta_\mu^z, \\
\xi^\mu &= \frac{2z}{\ell^2} \delta_\mu^z,
\end{aligned}
\implies \xi_\mu \xi^\mu = \frac{4}{\ell^2},$$

$z = 0$ represents the AdS boundary.
Therefore, AdS-plane waves can be constructed as follows:

\[ ds^2 = [\bar{g}_{\mu\nu} + 2V(u, x^i, z)l_\mu l_\nu]dx^\mu dx^\nu = \bar{ds}^2 + 2V(u, x^i, z)du^2, \]

\[ V(u, x^i, z) = V_0(u, x^i, z) + \alpha \phi(u, x^i, z)^2 \]

\[ z^2 \hat{\phi}^2 + (4 - D)z \partial_z \phi - m^2 \ell^2 \phi = 0, \]

\[ z^2 \hat{\phi}^2 (6 - D)z \partial_z V_0 + 2(3 - D)V_0 = c_1[2z\phi \partial_z \phi + (3 - D)\phi^2], \]  
\[ (\text{for } \alpha = \frac{c_3}{2}) \]

\[ z^2 \hat{\phi}^2 (6 - D)z \partial_z V_0 + 2(3 - D)V_0 = -c_1[z^2(\hat{\phi})^2 + m^2 \ell^2 \phi^2], \]  
\[ (\text{for } \alpha = \frac{c_1 + c_3}{2}) \]

where \( \hat{\phi}^2 \equiv \partial_i \partial^i + \partial_z^2 \) and \( (\hat{\phi})^2 \equiv \partial_i \phi \partial^i \phi + (\partial_z \phi)^2 \).
AdS-Plane Waves in 3D

• The field eqns. can be solved exactly in 3D because

\[ x^\mu = (u, v, z) \quad \Rightarrow \quad V_0 = V_0(u, z) \quad \& \quad \phi = \phi(u, z) \]

• The solution of the aether eqn. is

\[ z^2 \partial_z^2 \phi + z \partial_z \phi - m^2 \ell^2 \phi = 0 \]

\[ \phi(u, z) = a_1(u) z^{m\ell} + a_2(u) z^{-m\ell} \quad (\text{for } m \neq 0) \]

\[ \phi(u, z) = a_1(u) + a_2(u) \ln z \quad (\text{for } m = 0) \]

where

\[ m^2 \equiv -\frac{1}{c_1} \left[ \lambda - \frac{2(c_1 + c_3)}{\ell^2} \right] \]

\[ (a_{1,2} \text{ are arbitrary funcs.}) \]
And the Einstein-Aether eqns. become

\[ z^2 \partial_z^2 V_0 + 3z \partial_z V_0 = E_1(u) z^{2m \ell} + E_2(u) z^{-2m \ell}, \]

\[
\begin{align*}
E_1(u) &\equiv 2c_1 m \ell a_1(u)^2, \\
E_2(u) &\equiv -2c_1 m \ell a_2(u)^2, \\
\end{align*}
\]

\[
\begin{align*}
E_1(u) &\equiv -2c_1 m^2 \ell^2 a_1(u)^2, \\
E_2(u) &\equiv -2c_1 m^2 \ell^2 a_2(u)^2, \\
\end{align*}
\]

for \( \alpha = \frac{c_3}{2} \),

for \( \alpha = \frac{c_1 + c_3}{2} \).

\[
V_0(u, z) = b_1(u) + b_2(u) z^{-2} + \frac{1}{4m \ell} \left[ \frac{E_1(u)}{m \ell + 1} z^{2m \ell} + \frac{E_2(u)}{m \ell - 1} z^{-2m \ell} \right],
\]

when \( m \ell \pm 1 \neq 0 \). (can be absorbed into AdS)
AdS-Plane Waves in 3D

- When \( m\ell + 1 = 0 \),

\[
V_0(u, z) = b_1(u) + b_2(u)z^{-2} - \frac{E_1(u)}{2} z^{-2} \ln z + \frac{E_2(u)}{8} z^2,
\]

- When \( m\ell - 1 = 0 \),

\[
V_0(u, z) = b_1(u) + b_2(u)z^{-2} + \frac{E_1(u)}{8} z^2 - \frac{E_2(u)}{2} z^{-2} \ln z.
\]
AdS-Plane Waves in 3D

- When \( m = 0 \),

\[
z^2 \partial_z^2 V_0 + 3z \partial_z V_0 = E_1(u) + E_2(u) \ln z,
\]

\[
\begin{align*}
E_1(u) &\equiv 2c_1a_1(u)a_2(u), \\
E_2(u) &\equiv 2c_1a_2(u)^2,
\end{align*}
\]

for \( \alpha = \frac{c_3}{2} \),

\[
\begin{align*}
E_1(u) &\equiv -c_1a_2(u)^2, \\
E_2(u) &\equiv 0,
\end{align*}
\]

for \( \alpha = \frac{c_1 + c_3}{2} \).

\[
V_0(u, z) = b_1(u) + b_2(u)z^{-2} + \frac{E_1(u)}{2} \ln z + \frac{E_2(u)}{4} \ln z(\ln z - 1).
\]
AdS-Plane Waves in 3D

• For a valid behavior as $z \to 0$,

\[-1 < m\ell < 1 \Rightarrow 0 < m < \sqrt{|\Lambda|},\]

\[(c_1 + 2c_3)|\Lambda| < \lambda < 2(c_1 + c_3)|\Lambda| \quad \text{if } c_1 > 0,
\]

\[2(c_1 + c_3)|\Lambda| < \lambda < (c_1 + 2c_3)|\Lambda| \quad \text{if } c_1 < 0.
\]

• Thus, the AdS-plane wave solution is

\[ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{\ell^2}{z^2}(2dudv + dz^2) + 2V(u, z)du^2,
\]

\[V(u, z) = V_0(u, z) + \alpha\phi(u, z)^2\]
AdS-Plane Waves in Higher D>3

- In dimensions D>3, generic solution is not possible!

- However, solutions can be obtained if we assume the wave is **homogeneous** along the transverse coords.:

  \[ x^\mu = (u, v, x^i, z) \Rightarrow V_0 = V_0(u, z) \ & \ \phi = \phi(u, z) \]

- In this case,

  \[ z^2 \partial_z^2 \phi + (4 - D)z \partial_z \phi - m^2 \ell^2 \phi = 0 \ \Rightarrow \ \phi(u, z) = a_1(u) z^{r_+} + a_2(u) z^{r_-} \]

  where

  \[ r_{\pm} = \frac{1}{2} \left[ D - 3 \pm \sqrt{(D - 3)^2 + 4m^2 \ell^2} \right]. \]
And

\[ z^2 \partial_z^2 V_0 + (6 - D)z \partial_z V_0 + 2(3 - D)V_0 = E_1(u)z^{2r_+} + E_2(u)z^{2r_-}, \]

\[
\begin{align*}
E_1(u) & \equiv c_1(2r_+ + 3 - D)a_1(u)^2, \\
E_2(u) & \equiv c_1(2r_- + 3 - D)a_2(u)^2,
\end{align*}
\]

for \( \alpha = \frac{c_3}{2} \),

\[
\begin{align*}
E_1(u) & \equiv -c_1(r_+^2 + m^2 \ell^2)a_1(u)^2, \\
E_2(u) & \equiv -c_1(r_-^2 + m^2 \ell^2)a_2(u)^2,
\end{align*}
\]

for \( \alpha = \frac{c_1 + c_3}{2} \).

whose general solution is

\[ V_0(u, z) = b_1(u)z^{D-3} + b_2(u)z^{-2} + \frac{E_1(u)}{d_+} z^{2r_+} + \frac{E_2(u)}{d_-} z^{2r_-}, \]

\[
\begin{align*}
d_+ & \equiv 4r_+^2 + 2(5 - D)r_+ + 2(3 - D) \neq 0, \\
d_- & \equiv 4r_-^2 + 2(5 - D)r_- + 2(3 - D) \neq 0.
\end{align*}
\]
• On the other hand, when $d_+ = 0$,

\[ V_0(u, z) = b_1(u)z^{D-3} + b_2(u)z^{-2} + \frac{E_1(u)}{4r_+ + 5 - D} z^{2r_+ \ln z} + \frac{E_2(u)}{d_-} z^{2r_-}, \]

or, when $d_- = 0$,

\[ V_0(u, z) = b_1(u)z^{D-3} + b_2(u)z^{-2} + \frac{E_1(u)}{d_+} z^{2r_+} + \frac{E_2(u)}{4r_- + 5 - D} z^{2r_- \ln z}. \]

• All these mean that

\[ r_- > -1 \implies m < \sqrt{\frac{2|\Lambda|}{D - 1}} \implies m < 10^{-42} \text{ GeV} \]

\[ [D = 4 \& |\Lambda| < 10^{-52} \text{ m}^{-2} \approx 10^{-84} (\text{GeV})^2] \]
Then the solution is

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{\ell^2}{z^2} (2dudv + dx_1 dx_2 + dz^2) + 2V(u, z) du^2, \]

\[ V(u, z) = V_0(u, z) + \alpha \phi(u, z)^2 \]

( exact plane wave propagating in D−dim. AdS background in NAT )
We constructed exact plane wave solutions in NAT.

These are important in that they are exact solutions.

Waves propagating in dS backgrounds can also be constructed! (In progress)