From Basu-Harvey to Nahm equation and M2 to D2 via 3-Lie bialgebra

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1. BLG model

2. 3-Lie bialgebra

3. BLG model on 3-Lie bialgebra
    - From Basu-Harvey to Nahm equation via 3-Lie bialgebra
    - From M2 to D2

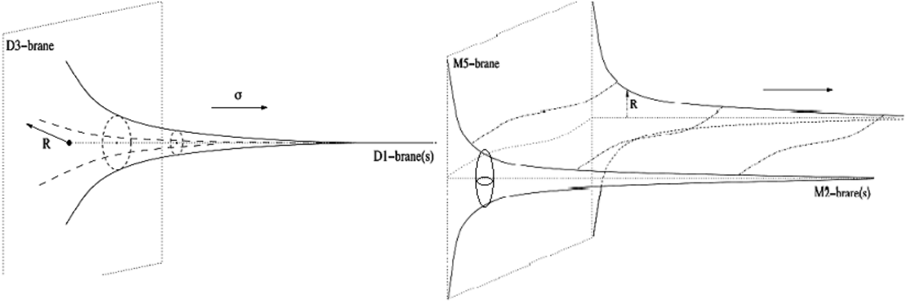
Bagger-Lambert-Gustavsson model

Basu-Harvey equation

$$\frac{dX^i}{dx^2} = \frac{1}{2}\epsilon^i_{jkl}[X^j, X^k, X^l], \quad (1)$$

Nahm-equation

$$\partial_\sigma X^i = \frac{i}{2}\epsilon^i_{jk}[X^j, X^k]. \quad (2)$$



A. Basu, J. A. Harvey, Nucl. Phys. B713 (2005) 136- W. Nahm, Phys. Lett. B90 (1980)

Bagger-Lambert-Gustavsson model

Bagger- Lambert- Gostavsson developed (BLG model)

1. a SCFT involving Chern-simon theory coupled to Matter
2. world volume theory for 2 coincident M2-brane with 16 supersymmetric

supersymmetric transformations

$$\begin{aligned} \delta X^I_a &= i\bar{\epsilon}\Gamma^I\Psi_a, \\ \delta\Psi_a &= D_\mu X^I_a\Gamma^\mu\Gamma_I\epsilon - \frac{1}{2}X^I_bX^J_cX^K_d f^{bcd}{}_a\Gamma_{IJK}\epsilon, \\ \delta(\hat{A}_\mu)^a_b &= i\bar{\epsilon}\Gamma_\mu\Gamma_I X^I_c\Psi_d f^{cda}{}_b, \end{aligned} \tag{3}$$

J. Bagger, N. Lambert, Phys. Rev. D75 (2007) 045020.
J. Bagger and N. Lambert, Phys. Rev. D77 (2008) 065008.
J. Bagger, N. Lambert, JHEP 02 (2008) 105.
A. Gustavsson, Nucl. Phys. B811 (2009) 66.

BLG Model

equations of motion

$$\Gamma^{\mu}D_{\mu}\Psi_{a}+\frac{1}{2}\Gamma_{IJ}X_{c}^{I}X_{d}^{J}\Psi_{b}f^{cdb}{}_{a}=0, \tag{4}$$

$$D^{2}X_{a}^{I}-\frac{i}{2}\tilde{\Psi}_{c}\Gamma^{I}_{J}X_{d}^{J}\Psi_{b}f^{cdb}{}_{a}+\frac{1}{2}f^{bcd}{}_{a}f^{efg}{}_{d}X_{b}^{J}X_{c}^{K}X_{e}^{I}X_{F}^{J}X_{g}^{K}=0, \tag{5}$$

$$(\hat{F}_{\mu\nu})_{a}^{b}+\epsilon_{\mu\nu\lambda}(X_{c}^{J}D^{\lambda}X_{d}^{J}+\frac{i}{2}\bar{\Psi}_{c}\Gamma^{\lambda}\Psi_{d})f^{cdb}{}_{a}=0, \tag{6}$$

BLG Lagrangian

$$\begin{aligned} L=&-\frac{1}{2}D_{\mu}X^{A(I)}D^{\mu}X_{A}^{(I)}+\frac{i}{2}\bar{\psi}^{A}\Gamma^{\mu}D_{\mu}\psi_{A}+\frac{i}{4}f_{ABCD}\bar{\psi}^{B}\Gamma^{IJ}X^{C(I)}X^{D(J)}\psi^{A}\\ &-\frac{1}{12}f_{ABCD}f_{EFG}{}^{D}X^{A(I)}X^{B(J)}X^{C(K)}X^{E(I)}X^{F(J)}X^{G(K)}\\ &+\frac{1}{2}\epsilon^{\mu\nu\lambda}[f_{ABCD}A_{\mu}{}^{AB}\partial_{\nu}A_{\lambda}{}^{CD}+\frac{2}{3}f_{AEF}{}^{G}f_{BCDG}A_{\mu}{}^{AB}A_{\nu}{}^{CD}A_{\lambda}{}^{EF}]\\ &I=1,...,8,\mu,\nu=0,1,2. \end{aligned} \tag{7}$$

Lie bialgebras[1] are algebraic structure of $N=(2,2)$ and $N=(4,4)$ may be $N=(8,8)$ supersymmtric WZW models[2].

Definition[a] of 3-Lie bialgebra

[a] arXiv:1604.04475

3-Lie algebra $\mathcal{A}$ with the co commutator map $\delta : \mathcal{A} \rightarrow \mathcal{A}\otimes\mathcal{A}\otimes\mathcal{A}$

1. δ is a 1-cocycle of $\mathcal{A}$ with value in $\otimes^3\mathcal{A}$, i.e:

$$\delta([T_a, T_b, T_c]) = ad^{(3)}{}_{T_b\otimes T_c}\delta(T_a) - ad^{(3)}{}_{T_a\otimes T_c}\delta(T_b) + ad^{(3)}{}_{T_a\otimes T_b}\delta(T_c), \quad (8)$$

$$ad^{(3)}{}_{T_b\otimes T_c} = ad_{T_b\otimes T_c}\otimes 1\otimes 1 + 1\otimes ad_{T_b\otimes T_c}\otimes 1 + 1\otimes 1\otimes ad_{T_b\otimes T_c}, \quad (9)$$

2. the dual map ${}^t\delta : \otimes^3\mathcal{A}^* \rightarrow \mathcal{A}^*$ is a 3-Lie bracket on $\mathcal{A}^*$ with

$$(\tilde{T}^a\otimes\tilde{T}^b\otimes\tilde{T}^c, \delta(T_d)) = ({}^t\delta(\tilde{T}^a\otimes\tilde{T}^b\otimes\tilde{T}^c), T_d) = ([\tilde{T}^a, \tilde{T}^b, \tilde{T}^c], T_d), \quad (10)$$

[1] Y. K. Schwarzbach, Lecture notes in physics 038, Springer-Verlag (2004)107.
[2] M.Aali-Javanangrouh, A. Rezaei-Aghdam, Arxiv:1402.5600v1].

Definition Manin triple

Triple of 3-Lie algebras $(\mathcal{D}, \mathcal{A}, \mathcal{A}^*)$

1. $\mathcal{A}$ and $\mathcal{A}^*$ are 3-Lie subalgebras of $\mathcal{D}$
2. $\mathcal{D} = \mathcal{A} \oplus \mathcal{A}^*$ as a vector space
3. $\mathcal{A}$ and $\mathcal{A}^*$ are isotropic i.e. $(T^a, \tilde{T}_b) = \delta^a_b, (T^a, T^b) = (\tilde{T}_a, \tilde{T}_b) = 0$. By using

$$\delta(T^a) = \tilde{f}_{bcd}{}^a T^b \otimes T^c \otimes T^d \tag{11}$$

$$\begin{aligned}
f_{aef}{}^g f_{bcdg} &- f_{bef}{}^g f_{acdg} + f_{cef}{}^g f_{abdg} - f_{def}{}^g f_{abcg} = 0, && (12)\\
\tilde{f}^{aef}{}_g \tilde{f}^{bcdg} &- \tilde{f}^{bef}{}_g \tilde{f}^{acdg} + \tilde{f}^{cef}{}_g \tilde{f}^{abdg} - \tilde{f}^{def}{}_g \tilde{f}^{abcg} = 0, && (13)\\
f_{abc}{}^g \tilde{f}^{def}{}_g &= f_{gbc}{}^f \tilde{f}^{deg}{}_a + f_{gbc}{}^e \tilde{f}^{dfg}{}_a - f_{gbc}{}^d \tilde{f}^{efg}{}_a - f_{gac}{}^f \tilde{f}^{deg}{}_b \\
&+ f_{gac}{}^e \tilde{f}^{dfg}{}_b - f_{gac}{}^d \tilde{f}^{efg}{}_b + f_{gab}{}^f \tilde{f}^{deg}{}_c - f_{gab}{}^e \tilde{f}^{dfg}{}_c \\
&+ f_{gab}{}^d \tilde{f}^{efg}{}_c. && (14)
\end{aligned}$$

Proposition

$(\mathcal{A}_{\mathcal{G}}, \mathcal{A}^*_{\mathcal{G}^*})$ is a 3-Lie bialgebra and the structure constants ${f^{abc}}_d$ and ${\tilde{f}_{abc}}^d$ satisfy mix fundamental identity if and only if $(\mathcal{G}, \mathcal{G}^*)$ is Lie bialgebra.

$$[T_A, T_B, T_D] = F^D_{ABC} T_D \tag{15}$$

An Example [3]:

$$[T_-, T_a, T_b] = 0, \quad [T_+, T_i, T_j] = {f_{ij}}^k T_k, \quad [T_i, T_j, T_k] = f_{ijk} T_-, \tag{16}$$

$$[\tilde{T}^-, \tilde{T}^a, \tilde{T}^b] = 0, \quad [\tilde{T}^+, \tilde{T}^i, \tilde{T}^j] = {\tilde{f}^{ij}}_k \tilde{T}^k, \quad [\tilde{T}^i, \tilde{T}^j, \tilde{T}^k] = \tilde{f}^{ijk} T^-, \tag{17}$$

$$\begin{aligned} {f^{+ij}}_k &= {f^{ij}}_k, & {f^{ijk}}_- = f^{ijk}, && {f^{-ab}}_c = 0, && {f^{abc}}_+ = 0, \\ {\tilde{f}_{+ij}}^k &= {\tilde{f}_{ij}}^k, & {\tilde{f}_{ijk}}^- = \tilde{f}_{ijk}, && {\tilde{f}_{-ab}}^c = 0, && {\tilde{f}_{abc}}^+ = 0, \end{aligned} \tag{18}$$

$$-{f^{ij}}_k {\tilde{f}_{lm}}^k + {f^{ik}}_l {\tilde{f}_{km}}^j - {f^{jk}}_m {\tilde{f}_{lk}}^i - {f^{jk}}_l {\tilde{f}_{km}}^i + {f^{ik}}_m {\tilde{f}_{lk}}^j = 0. \tag{19}$$

[3] P-M Ho, Y. Imamura, Y. Matsuo, JHEP 07 (2008) 003, arXiv:0805.1202

Supersymmetric boundary conditions

Euler-Lagrange equations of a Lagrangian field theory

$$\int_{\mathcal{M}} d^m x \, \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \Phi} \delta \Phi \right) \,, \tag{20}$$

If this term goes to zero the action will be invariant. Different properties :

1. non-compact
2. With baundary condition.

Boundary break translation symmetry and so the number of supersymmetry will be change. Global supersymmetry variation of the action

$$\delta_{susy} S = \int d^m x \, \partial_\mu \mathcal{K}^\mu \,, \tag{21}$$

A action with maximal supersymmetry at the boundary

$$\mathcal{K}^n = 0. \tag{22}$$

Supersymmetric boundary conditions

component of the supercurrent normal to the boundary

$$J^n|_{\delta\mathcal{M}} = \frac{\delta\mathcal{L}}{\delta\partial_n\Phi}\delta\Phi\Big|_{\partial\mathcal{M}} - \mathcal{K}^n|_{\partial\mathcal{M}} \ . \tag{23}$$

Therefore boundary conditions for which:

$$J^n|_{\partial\mathcal{M}} = 0 \tag{24}$$

D. Gaiotto, E. Witten,J. Statist. Phys. 135(2009) 789, arXiv:0804.2902.

D. S. Berman, M. J.Perry, E. Sezgin, D. C. Thompson,JHEP 1004:025,2010.

From Basu-Harvey to Nahm equation

supercurrent for BLG model on Manin triple

$$J^{\mu} = -\bar{\epsilon} D_{\nu} X^I_A \Gamma^{\nu} \Gamma^I \Gamma^{\mu} \Psi^A - \frac{1}{6} \bar{\epsilon} X^I_A X^J_B X^K_C F^{ABCD} \Gamma^{IJK} \Gamma^{\mu} \Psi_D \ . \tag{25}$$

$$0 = \left(-\bar{\epsilon} D_{\nu} X^I_A \Gamma^{\nu} \Gamma^I \Gamma^2 \Psi^A - \frac{1}{6} \bar{\epsilon} X^I_A X^J_B X^K_C F^{ABCD} \Gamma^{IJK} \Gamma^2 \Psi_d \right) |_{\partial \mathcal{M}} \, . \tag{26}$$

$$SO(1,10) \to SO(1,2) \times SO(8) \tag{27}$$

$$SO(1,10) \to SO(1,1) \times SO(4) \times SO(4) \tag{28}$$

$$X^V = \{X^3, X^4, X^5, X^6\} \tag{29}$$

$$Y^P = \{X^7, X^8, X^9, X^{10}\} \tag{30}$$

$$
\begin{aligned}
0 \;=\;& -\bar{\epsilon} D_{\hat{\nu}} X_A^V \Gamma^{\hat{\nu}} \Gamma^V \tilde{\Psi}^A \\
& -\bar{\epsilon} D_{\hat{\nu}} Y_A^P \Gamma^{\hat{\nu}} \Gamma^P \tilde{\Psi}^A \\
& -\bar{\epsilon} \left(D_2 Y_A^P \Gamma^2 \Gamma^P \delta^{DA} + \frac{1}{6} Y_A^P Y_B^Q Y_C^R F^{ABCD} \Gamma^{PQR} \right) \tilde{\Psi}_D \\
& -\bar{\epsilon} \left(D_2 X_A^V \Gamma^2 \Gamma^V \delta^{DA} + \frac{1}{6} X_A^V X_B^U X_C^W F^{ABCD} \Gamma^{VUW} \right) \tilde{\Psi}_D \\
& -\bar{\epsilon} \left(\frac{1}{2} X_A^V X_B^U Y_C^P F^{ABCD} \Gamma^{VUP} \right) \tilde{\Psi}_D \\
& -\bar{\epsilon} \left(\frac{1}{2} X_A^V Y_B^P Y_C^Q F^{ABCD} \Gamma^{APQ} \right) \tilde{\Psi}_D
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
&-\bar{\epsilon} D_{\hat{\nu}} X_A^V \Gamma^{\hat{\nu}} \Gamma^V \tilde{\Psi}^A = 0 \\
&-\bar{\epsilon} D_{\hat{\nu}} Y_A^P \Gamma^{\hat{\nu}} \Gamma^P \tilde{\Psi}^A = 0 \\
&-\bar{\epsilon} \left(D_2 Y_A^P \Gamma^2 \Gamma^P \delta^{DA} + \frac{1}{6} Y_A^P Y_B^Q Y_C^R F^{ABCD} \Gamma^{PQR} \right) \tilde{\Psi}_D = 0 \\
&-\bar{\epsilon} \left(D_2 X_A^V \Gamma^2 \Gamma^V \delta^{DA} + \frac{1}{6} X_A^V X_B^U X_C^W F^{ABCD} \Gamma^{VUW} \right) \tilde{\Psi}_D = 0 \\
&-\bar{\epsilon} \left(\frac{1}{2} X_A^V X_B^U Y_C^P F^{ABCD} \Gamma^{VUP} \right) \tilde{\Psi}_D = 0 \\
&-\bar{\epsilon} \left(\frac{1}{2} X_A^V Y_B^P Y_C^Q F^{ABCD} \Gamma^{APQ} \right) \tilde{\Psi}_D = 0
\end{aligned}
\tag{32}
$$

From Basu-Harvey to Nahm equation

Dirichlet conditions

$$D_{\hat{\mu}} Y^P = 0 \tag{33}$$

and the simplest solution

$$Y^P = 0 \tag{34}$$

$$0 = \bar{\epsilon} D_2 Y^P \Gamma^2 \Gamma^P \tilde{\Psi} \,, \tag{35}$$

$$0 = \bar{\epsilon} D_{\hat{\nu}} X^V \Gamma^{\hat{\nu}} \Gamma^V \tilde{\Psi} \,, \tag{36}$$

$$0 = \bar{\epsilon} \left(D_2 X_A^V \delta^{AD} \Gamma^2 \Gamma^V + \frac{1}{6} F^{ABCD} X_A^V X_B^U X_C^W \Gamma^{VUW} \right) \tilde{\Psi}_D \,. \tag{37}$$

$$\Gamma^V = \frac{1}{6} \epsilon^{VUWZ} \Gamma^{UWZ} \Gamma^{3456} \tag{38}$$

$$0 = D_2 X_A^V + \frac{1}{6} \epsilon^{VUWZ} X_B^U X_C^W X_D^Z F^{BCD}{}_A, \tag{39}$$

Basu-Harvey type equations by considering scalar fields X^V.

Proposition

$(\mathcal{A}_{\mathcal{G}}, \mathcal{A}^*_{\mathcal{G}^*})$ is a 3-Lie bialgebra and the structure constants $f^{abc}{}_d$ and $\tilde{f}_{abc}{}^d$ satisfy mix fundamental identity if and only if $(\mathcal{G}, \mathcal{G}^*)$ is Lie bialgebra.

$$[T_A, T_B, T_D] = F^D_{ABC} T_D \tag{40}$$

An Example [4]:

$$[T_-, T_a, T_b] = 0, \quad [T_+, T_i, T_j] = f_{ij}{}^k T_k, \quad [T_i, T_j, T_k] = f_{ijk} T_-, \tag{41}$$

$$[\tilde{T}^-, \tilde{T}^a, \tilde{T}^b] = 0, \quad [\tilde{T}^+, \tilde{T}^i, \tilde{T}^j] = \tilde{f}^{ij}{}_k \tilde{T}^k, \quad [\tilde{T}^i, \tilde{T}^j, \tilde{T}^k] = \tilde{f}^{ijk} T^-, \tag{42}$$

$$\begin{aligned} f^{+ij}{}_k &= f^{ij}{}_k, \quad f^{ijk}{}_- = f^{ijk}, \quad f^{-ab}{}_c = 0, \quad f^{abc}{}_+ = 0, \\ \tilde{f}_{+ij}{}^k &= \tilde{f}_{ij}{}^k, \quad \tilde{f}_{ijk}{}^- = \tilde{f}_{ijk}, \quad \tilde{f}_{-ab}{}^c = 0, \quad \tilde{f}_{abc}{}^+ = 0, \end{aligned} \tag{43}$$

$$-f^{ij}{}_k \tilde{f}_{lm}{}^k + f^{ik}{}_l \tilde{f}_{km}{}^j - f^{jk}{}_m \tilde{f}_{lk}{}^i - f^{jk}{}_l \tilde{f}_{km}{}^i + f^{ik}{}_m \tilde{f}_{lk}{}^j = 0. \tag{44}$$

[4] P-M Ho, Y. Imamura, Y. Matsuo, JHEP 07 (2008) 003, arXiv:0805.1202

$$\begin{aligned} \partial_\sigma X^I_A &= \frac{1}{2}\epsilon^I_{JK} X^J_B X^K_C F^{BC}_A \\ \partial_\sigma X^I_i &= \frac{1}{2}\epsilon^I_{JK} X^J_j X^K_k f^{jk}_i + \frac{1}{2}\epsilon^I_{JK} X^{J\tilde{j}} X^K_k \tilde{f}^k_{\tilde{i}\tilde{j}}. \end{aligned} \tag{45}$$

BPS bound

$$\begin{aligned} E &\geq \frac{1}{6}\epsilon^{JIKL} Tr(\partial_s X^{(I)}, [X^{(J)}, X^{(K)}, X^{(L)}]) \\ &\geq \frac{1}{6}\epsilon^{JIKL} Tr(\partial_s X^{(I)}_+ T^+ + \partial_s X^{(I)}_- T^- + \partial_s X^{(I)}_i T^i + \partial_s X^{(I)}_{\tilde{+}} T^{\tilde{+}} + \partial_s X^{(I)}_{\tilde{-}} T^{\tilde{-}} \\ &\quad + \partial_s X^{(I)}_{\tilde{i}} T^{\tilde{i}}, X^{(J)}_+ X^{(K)}_i X^{(L)}_j [T^+, T^i, T^j] + X^{(J)}_+ X^{(K)}_i X^{(L)}_{\tilde{j}} [T^+, T^i, T^{\tilde{j}}] \\ &\quad + X^{(J)}_{\tilde{+}} X^{(K)}_{\tilde{i}} X^{(L)}_{\tilde{j}} [T^{\tilde{+}}, T^{\tilde{i}}, T^{\tilde{j}}] + X^{(J)}_{\tilde{+}} X^{(K)}_{\tilde{i}} X^{(L)}_j [T^{\tilde{+}}, T^{\tilde{i}}, T^j]) \\ &\geq \frac{1}{2g^2_{YM}} \int d\sigma \epsilon_{IJK} \partial_s X^I [X^J, X^K]. \end{aligned} \tag{46}$$

M2 to D2 and vice versa

equations of motion of BLG mode

$$\Gamma^{\mu} D_{\mu} \Psi_{A} + \frac{1}{2} \Gamma_{IJ} X^{I}_{C} X^{J}_{D} \Psi_{B} F^{CDB}{}_{A} = 0,$$

$$D^{2} X^{I}_{A} - \frac{i}{2} \tilde{\Psi}_{C} \Gamma^{I}_{J} X^{J}_{D} \Psi_{B} F^{CDB}{}_{A} + \frac{1}{2} F^{BCD}{}_{A} F^{EFG}{}_{D} X^{J}_{B} X^{K}_{C} X^{I}_{E} X^{J}_{F} X^{K}_{G} = 0,$$

$$\partial^{2} X^{I}_{+} = 0, \quad (47)$$

$$\Gamma^{\mu} \partial_{\mu} \Psi_{+} = 0, \quad (48)$$

$$\partial^{2} X^{I}_{\tilde{+}} = 0, \quad (49)$$

$$\Gamma^{\mu} \partial_{\mu} \Psi_{\tilde{+}} = 0, \quad (50)$$

$$\begin{aligned} L = & - \frac{1}{2} D_{\mu} X^{A(I)} D^{\mu} X^{(I)}_{A} + \frac{i}{2} \bar{\psi}^{A} \Gamma^{\mu} D_{\mu} \psi_{A} + \frac{i}{4} F_{ABCD} \bar{\psi}^{B} \Gamma^{IJ} X^{C(I)} X^{D(J)} \psi^{A} \\ & - \frac{1}{12} F_{ABCD} F_{EFG}{}^{D} X^{A(I)} X^{B(J)} X^{C(K)} X^{E(I)} X^{F(J)} X^{G(K)} \\ & + \frac{1}{2} \epsilon^{\mu\nu\lambda} [F_{ABCD} A_{\mu}{}^{AB} \partial_{\nu} A_{\lambda}{}^{CD} + \frac{2}{3} F_{AEF}{}^{G} F_{BCDG} A_{\mu}{}^{AB} A_{\nu}{}^{CD} A_{\lambda}{}^{EF}] \end{aligned} \quad (51)$$

$$\epsilon^{\mu\nu\lambda}F^{ABCD}A_{\mu AB}\partial_\nu A_{\lambda CD} = \epsilon^{\mu\nu\lambda}\tilde{f}_{ij}{}^{k}A^{ij}_{\mu}(\partial_\nu A^{+}_{\lambda k} - \partial_\lambda A^{+}_{\nu k}) + \epsilon^{\mu\nu\lambda}f^{ij}{}_{k}A_{\mu ij}(\partial_\nu A^{k\tilde{+}}_{\lambda} - \partial_\lambda A^{k\tilde{+}}_{\nu}) - 2\epsilon^{\mu\nu\lambda}\tilde{f}_{ik}{}^{j}A^{i}_{\mu j}(\partial_\nu A^{k+}_{\lambda} - \partial_\lambda A^{k+}_{\nu}) - 2\epsilon^{\mu\nu\lambda}f^{jk}{}_{i}A^{i}_{\mu j}(\partial_\nu A^{\tilde{+}}_{\lambda k} - \partial_\lambda A^{\tilde{+}}_{\nu k}), \tag{52}$$

and

$$\epsilon^{\mu\nu\lambda}F^{AEF}_{G}F^{BCDG}A_{\mu AB}A_{\nu CD}A_{\lambda EF} = \epsilon^{\mu\nu\lambda}\tilde{f}_{ij}{}^{k}A^{ij}_{\lambda}[A^{+}_{\mu l}, A^{\tilde{+}}_{\nu m}] + \epsilon^{\mu\nu\lambda}f^{ij}{}_{k}A_{\lambda ij}[A^{\tilde{+}l}_{\mu}, A^{+\tilde{m}}_{\nu}] + \epsilon^{\mu\nu\lambda}\tilde{f}_{ik}{}^{j}A^{i}_{\lambda j}[A^{l+}_{\mu}, A^{m+}_{\nu}] + \epsilon^{\mu\nu\lambda}f^{ik}{}_{j}A^{j}_{\lambda i}[A^{\tilde{+}}_{\mu l}, A^{\tilde{+}}_{\nu m}] \tag{53}$$

$$A^{+}_{\mu l} = A_{\mu l},\ A^{+l}_{\mu} = A^{l}_{\mu}, \tag{54}$$

$$A^{\tilde{+}}_{\mu l} = A'_{\mu l},\ A^{\tilde{+}l}_{\mu} = A'^{l}_{\mu}, \tag{55}$$

$$f^{ij}{}_{k}A_{\mu ij} \equiv C_{\mu k},\ \tilde{f}_{ij}{}^{k}A^{ij}_{\mu} \equiv \tilde{C}^{k}_{\mu} \tag{56}$$

General form of the BLG Lagrangian on the especial 3-Lie bialgebra

$$L=-2g_{YM}^2 C_\mu^a C_a^\mu - 2g_{YM} C_{\mu a} D^\mu X^{(8)a} + 2\, f^{\mu\nu\lambda}\, C_{\mu a} F_{\nu\lambda}^a + 2\, f^{\mu\nu\lambda}\, \tilde{C}_\mu^a B_{\nu\lambda a} + ... \quad (57)$$

Integration of $C_{\mu k}$ and $\tilde{C}_\mu^k$:

$$L = \frac{1}{2} F_{\nu\lambda k} F^{\nu\lambda k} + \frac{1}{4} B_{\nu\lambda k} B^{\nu\lambda k} + ..., \quad (58)$$

that

$$B_{\nu\lambda k} = \partial_\nu A'_{\lambda k} - \partial_\lambda A'_{\nu k} - [A'_{\nu k}, A'_{\lambda k}], \quad (59)$$

$$F_{\nu\lambda}^k = \partial_\nu A_\lambda^k - \partial_\lambda A_\nu^k - [A_\nu^k, A_\lambda^k]. \quad (60)$$

is a result of expanding Dirac Born Infeld action[5].

[5] D. Tong, February 2012.

Dirac-Born-Infeld action

$$S = \int d^{p+1}\zeta \sqrt{-det(G_{\mu\nu}h_{AB} + 2\pi\alpha' F_{\mu\nu AB} + B_{\mu\nu AB})}, \tag{61}$$

1. DBI action gives equation of motion proportional to beta-function gained from sigma-model and conformal gauge action.
2. Supersymmetric WZW model is a result of defining sigma model on Lie group and its algebraic structure is Lie bialgrbra.

Relation between D-brane and WZW model

$$S_{WZW} = \int d^3x \epsilon^{\alpha\beta\gamma} L_\mu{}^I L_\nu{}^J L_\lambda^K \partial_\alpha X^\mu \partial_\beta X^\nu \partial_\gamma X^\lambda Tr([T_I, T_J], T_K) \quad (62)$$

$$X^\mu = X^{\mu A} T_A \quad (63)$$

$$S_{WZW-like} = \int d^3x \epsilon^{\alpha\beta\gamma} L_\mu{}^L L_\nu{}^M L_\lambda^N \partial_\alpha X^{I\mu} \partial_\beta X^{J\nu} \partial_\gamma X^{K\lambda} Tr([T_I T_L, T_J T_M], T_K T_N)$$

$$S_{WZW-like} = \int d^2x \, \{ \frac{1}{6} \epsilon^{\beta\gamma} B_{\nu\mu}{}^Q \partial_\beta X^{J\nu} \partial_\gamma X^{I\mu} Tr(T_J T_I T_Q)$$
$$\frac{1}{6} \epsilon^{\alpha\gamma} B_{\mu\lambda}{}^Q \partial_\alpha X^{I\mu} \partial_\gamma X^{K\lambda} Tr(T_I T_K T_Q) \} + \frac{1}{6} \epsilon^{\alpha\gamma} B_{\nu\lambda}{}^Q \partial_\alpha X^{J\nu} \partial_\gamma X^{K\lambda} Tr(T_J T_K T_Q) + \ldots \quad (64)$$

$$B_{\nu\mu}{}^Q = L_\nu{}^L L_\lambda{}^N f_{NL}{}^P x^J f_{PJ}{}^Q \quad (65)$$

$, L_\mu^L X^{I\mu} T_I T_L|_{boundry} = x^L T_L|_{boundry}$

Using the concept of 3-Lie bialgebra; we construct BLG model on the Manin triple $\mathcal{D}$ of the especial 3-Lie bialgebra and shown that: Nahm equation can be obtained from Basu-Harvey equation and vice versa. arXiv:1604.05181
One can construct M2-brane from a D2-brane and vice versa. arXiv:1604.05183
Similar works have been done for BL model with $N = 6$ supersymmetry and multiple membrane which detail are arXiv:1604.05890

Thank You