Reflections on Black Mirrors

A one-to-one correspondence between moving mirrors and black holes.

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arXiv:gr-qc/1507.03489.

Late Time Planck Distribution

$$T = \frac{\hbar c^3}{8\pi k G M} \rightarrow \frac{\kappa}{2\pi}$$

$$T = \frac{\hbar \kappa}{2\pi c k} \rightarrow \frac{\kappa}{2\pi}$$

Planck Distribution

$$\overline{n} = \frac{1}{e^{\frac{\omega}{T}} - 1}$$

$$T = \frac{\kappa}{2\pi}$$

Correspondences

- Hawking ⇔ Davies-Fulling
  - Thermal late times
- Unruh ⇔ Carlitz-Willey
  - Thermal all times (exactly analytic)
- Black Hole ⇔ Moving Mirror
  - All times. (even the non-thermal early times)

Thermal Equilibrium:
Unruh Effect vs Thermal Mirror

Unruh world-line: Black
Thermal mirror: Green

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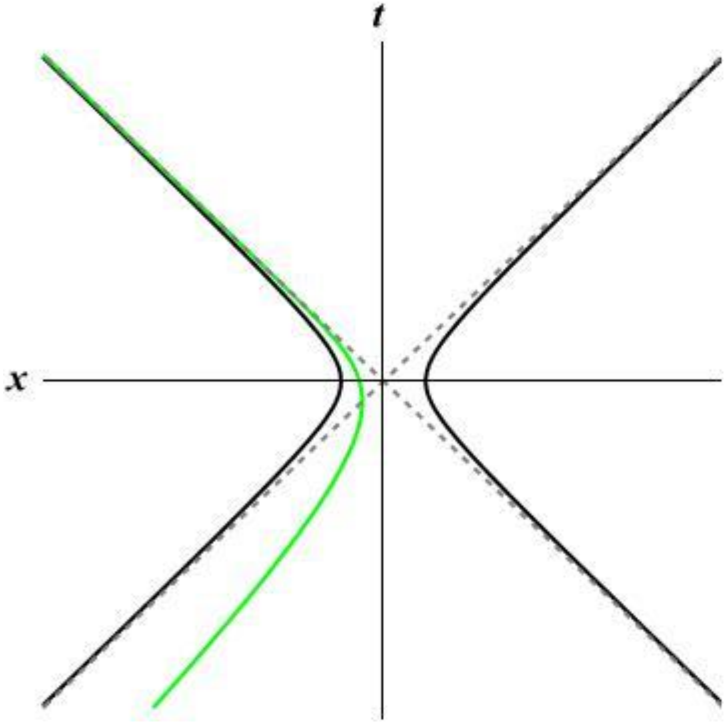
Unruh Effect vs Thermal Mirror



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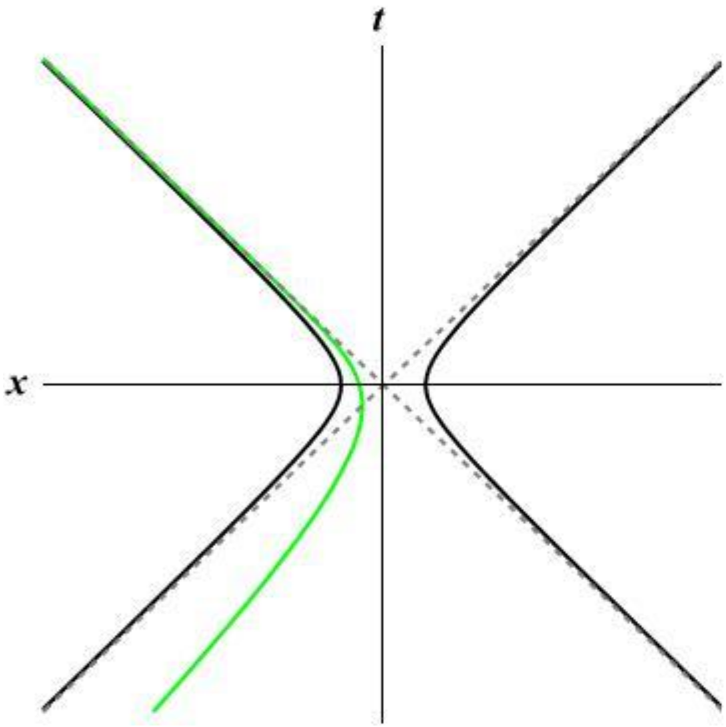

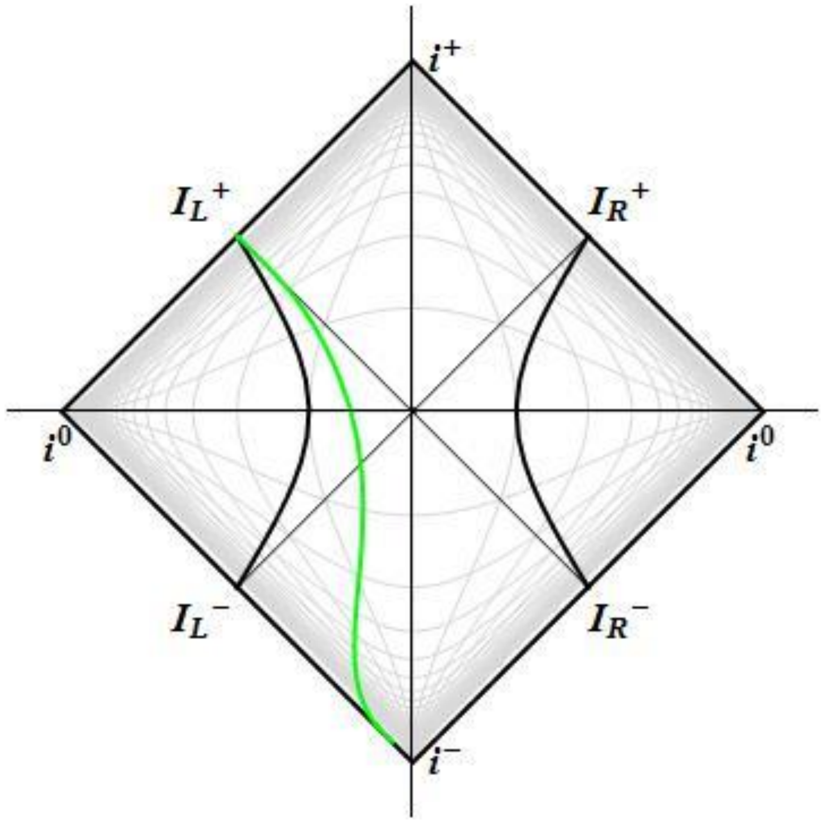

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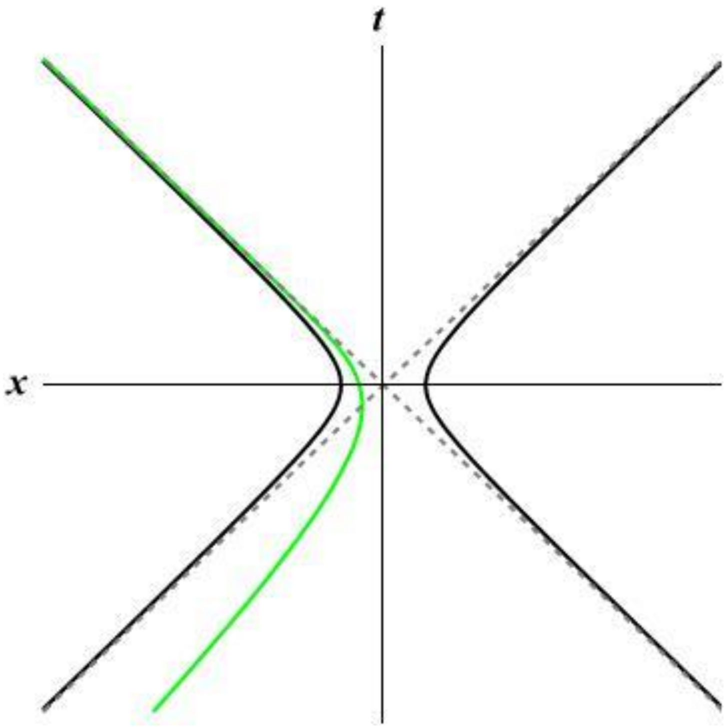
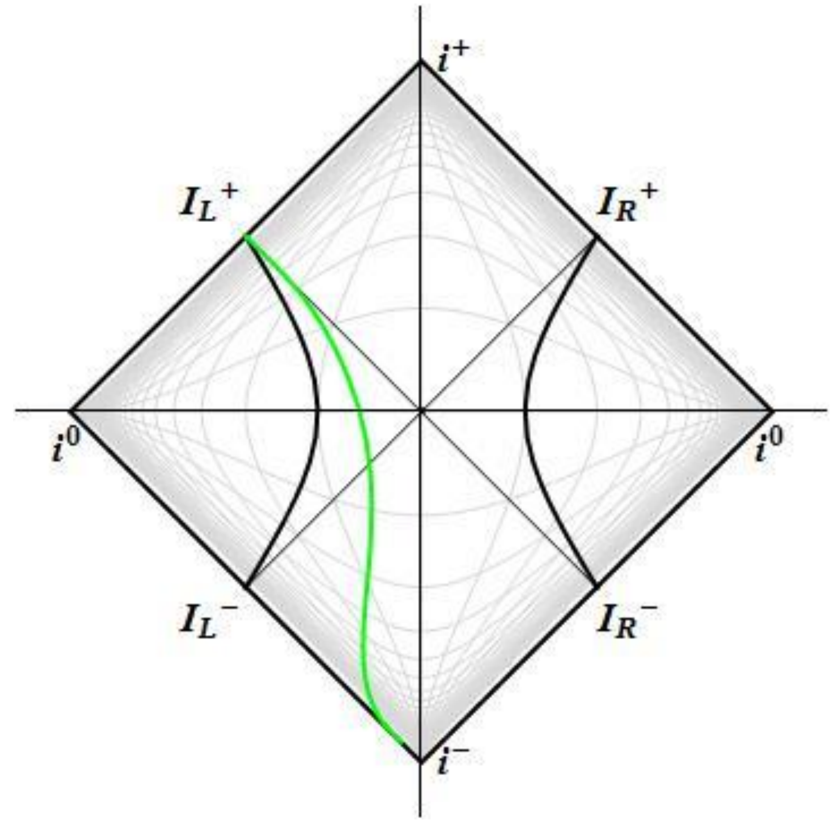


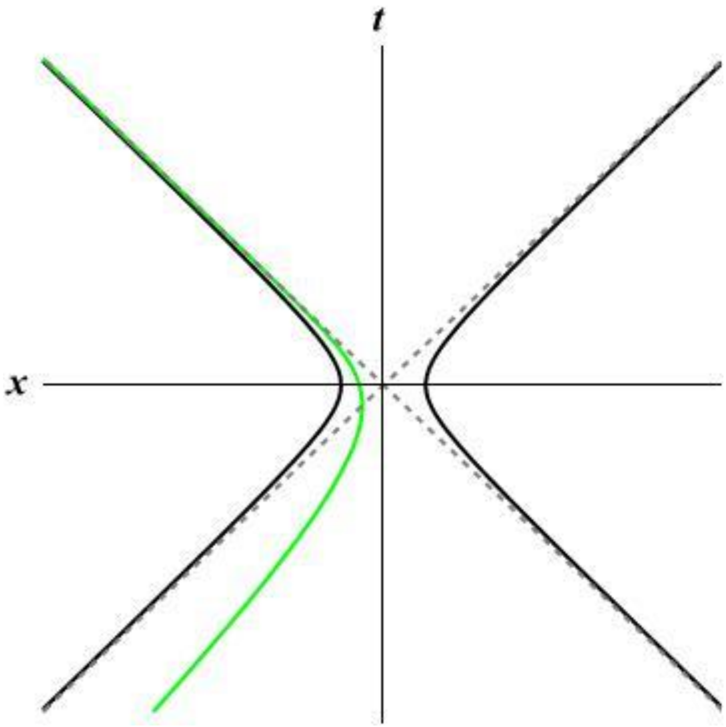


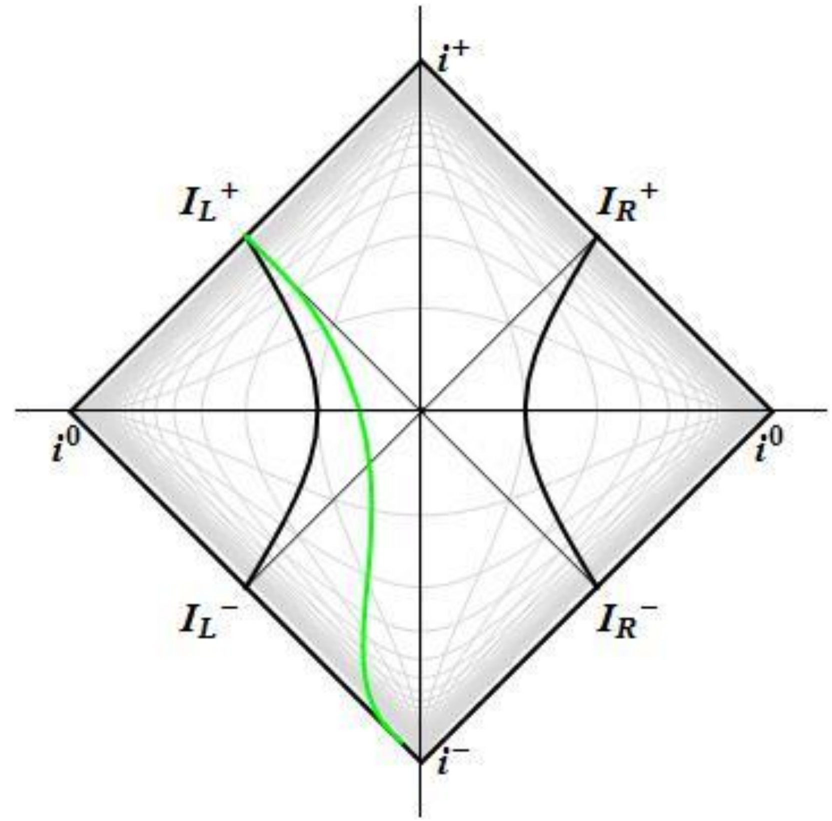
$$z_U^2 = \kappa^{-2} + t^2$$

Unruh world-line: Black

Thermal mirror: Green

Thermal Equilibrium: Unruh Effect vs Thermal Mirror





Unruh world-line: Black $z_U^2 = \kappa^{-2} + t^2$

Thermal mirror: Green $z_M = -t - \kappa^{-1} W(e^{-2\kappa t})$

Product Log

$$xe^{x} = 1$$

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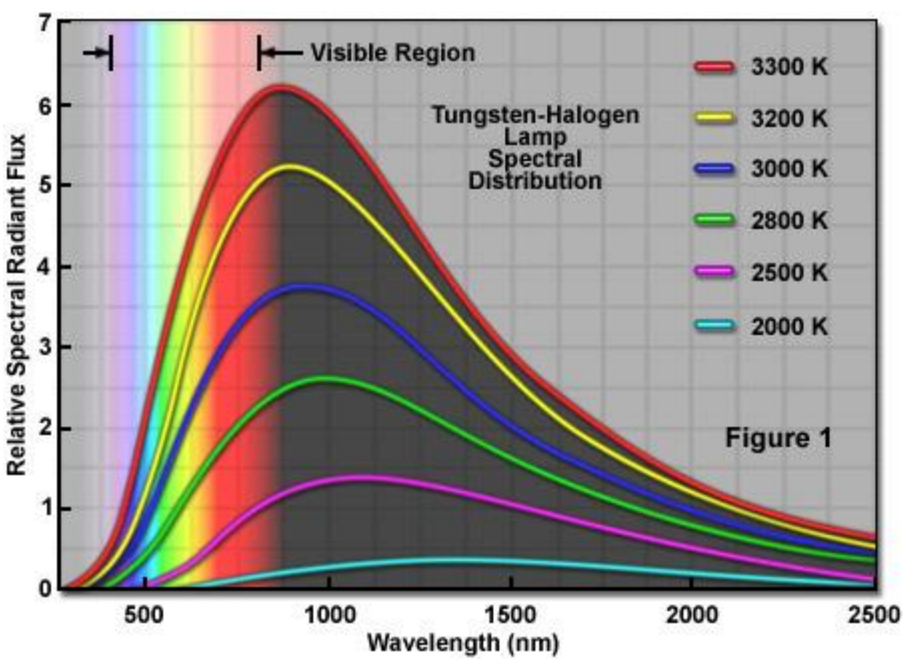
$$xe^x = 1$$

$$x = W(1) = 0.567$$

$$E(\omega)d\omega = \frac{V\hbar}{\pi^2 c^3}\frac{\omega^3}{e^{\beta\hbar\omega}-1}d\omega$$

$$\frac{d}{d\omega}E(\omega) = 0$$

$$\omega_{\max} = \frac{3 + W(-3e^{-3})}{\beta\hbar}$$



$$\omega_{\max} = \zeta\frac{k_B T}{\hbar}$$

$$\zeta = 2.822$$

Thermal Accelerations

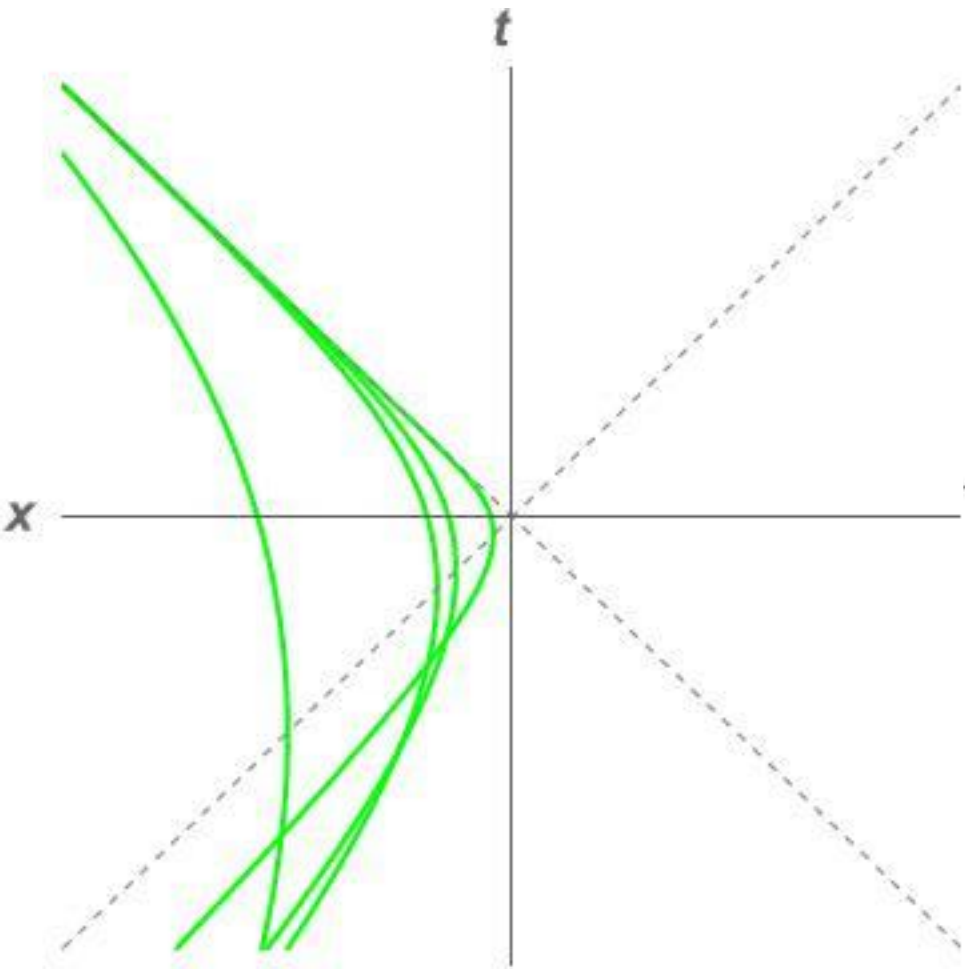
$$|\alpha_M| = \frac{\kappa}{2\sqrt{W(e^{-2\kappa t})}} \qquad |\alpha_U| = \kappa$$

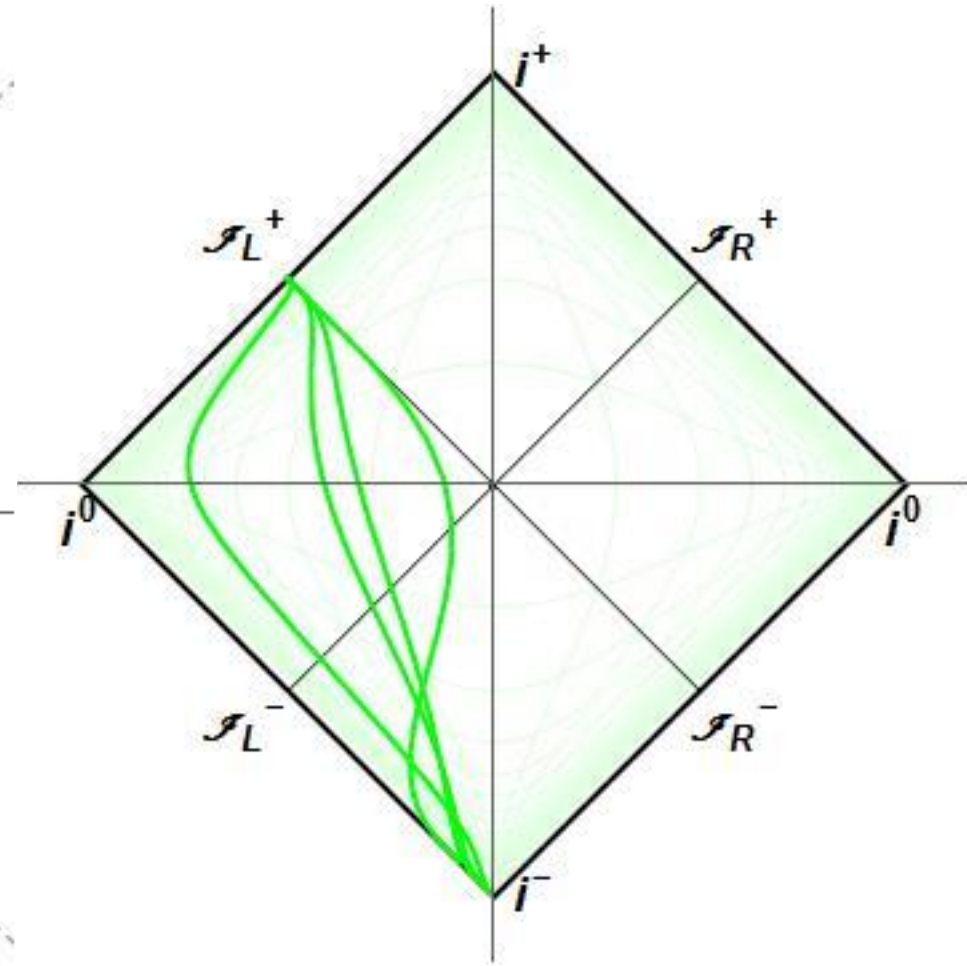
The Unruh effect and the thermal mirror trajectory give off constant energy flux at all times.

Both radiate particles thermally.

$$\frac{1}{e^{2\pi\omega/\kappa} - 1} \qquad T = \frac{\kappa}{2\pi}$$

Eternal Thermal





Carlitz R.D. and R.S. Willey,
“Reflections on Moving Mirrors”. Phys. Rev. D **36**, 2327, 1987

Localization

$$j\,\epsilon \;\le \omega \le (j+1)\epsilon$$

$$\omega_j = (j+1/2)\epsilon$$

$$\frac{(2\,\pi\,n\;-\pi)}{\epsilon} \le u \le \frac{2\,\pi\,n+\pi}{\epsilon}$$

$$u_n = 2\pi n/\epsilon$$

$$\beta_{jn,\omega'} = \frac{1}{\sqrt{\epsilon}} \int_{j\epsilon}^{(j+1)\epsilon} d\omega e^{2\pi i\omega n/\epsilon} \beta_{\omega\omega'}$$

$$\langle in|N_{jn}^{out}|in\rangle = \int_0^\infty d\omega' |\beta_{jn,\omega'}|^2$$

Hawking, S.W.
"Particle Creation by Black Holes", Comm. Math. Phys. 43, 1975.

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Hawking, S.W.
“Particle Creation by Black Holes”, Comm. Math. Phys. 43, 1975.

Time Independent Particle Creation

$$\langle N_{jn} \rangle = \frac{T}{\epsilon} \ln\left(\frac{e^{(j+1)\epsilon/T} - 1}{e^{j\epsilon/T} - 1} \right) - 1$$

Good, M.R.R., Anderson, P.R., and Evans, C.R.
"Time Dependence of Particle Creation from Accelerating Mirrors", arXiv:gr-qc/1303.6756
Physical Review D, 88, 025023, July 2013.

Choices of Packets

$$\epsilon = 1$$

Giddings, Steven B.
"Hawking Radiation, the Stefan-Boltzmann Law, and unitarization",
arXiv:gr-qc/1511.08221 December 2015

Choices of Packets

$$\epsilon = 1$$

$$\epsilon = \pi^{-1} = 0.32$$

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Choices of Packets

$$\epsilon = 1$$

$$\epsilon = \pi^{-1} = 0.32$$

$$\epsilon = T \ln \phi = \frac{\kappa}{2\pi} \ln \frac{1+\sqrt{5}}{2} = 0.0765$$

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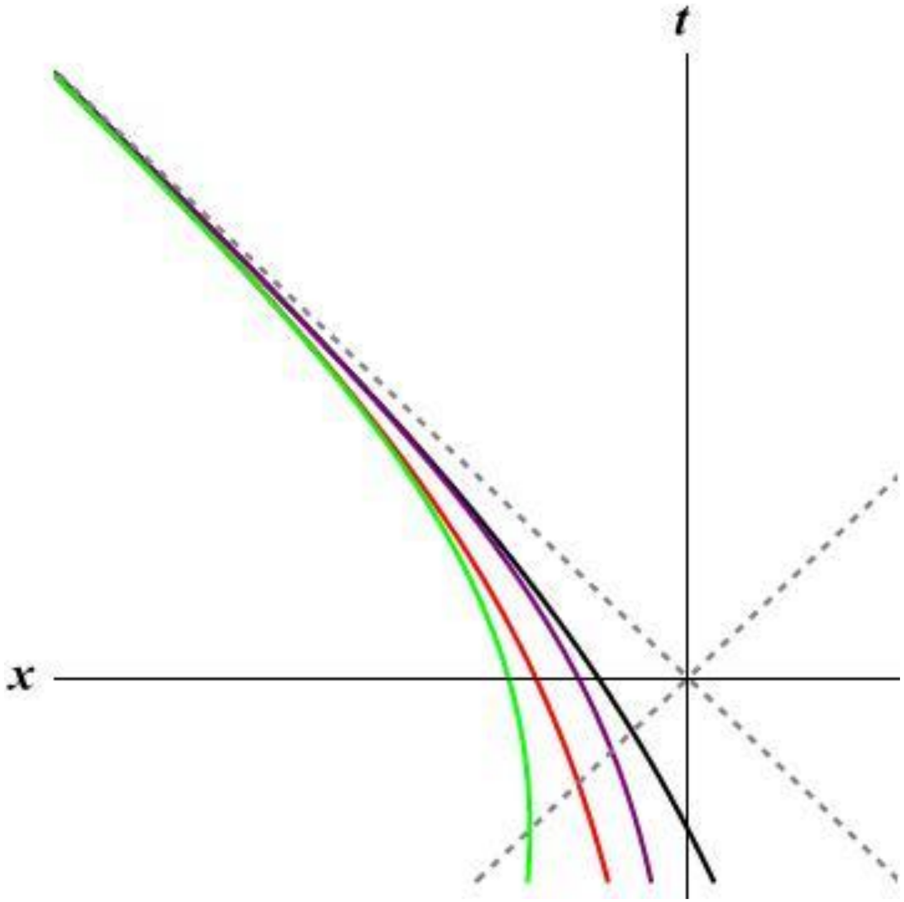
“Hawking Radiation, the Stefan-Boltzmann Law, and unitarization”,
arXiv:gr-qc/1511.08221 December 2015

Evolution to Equilibrium



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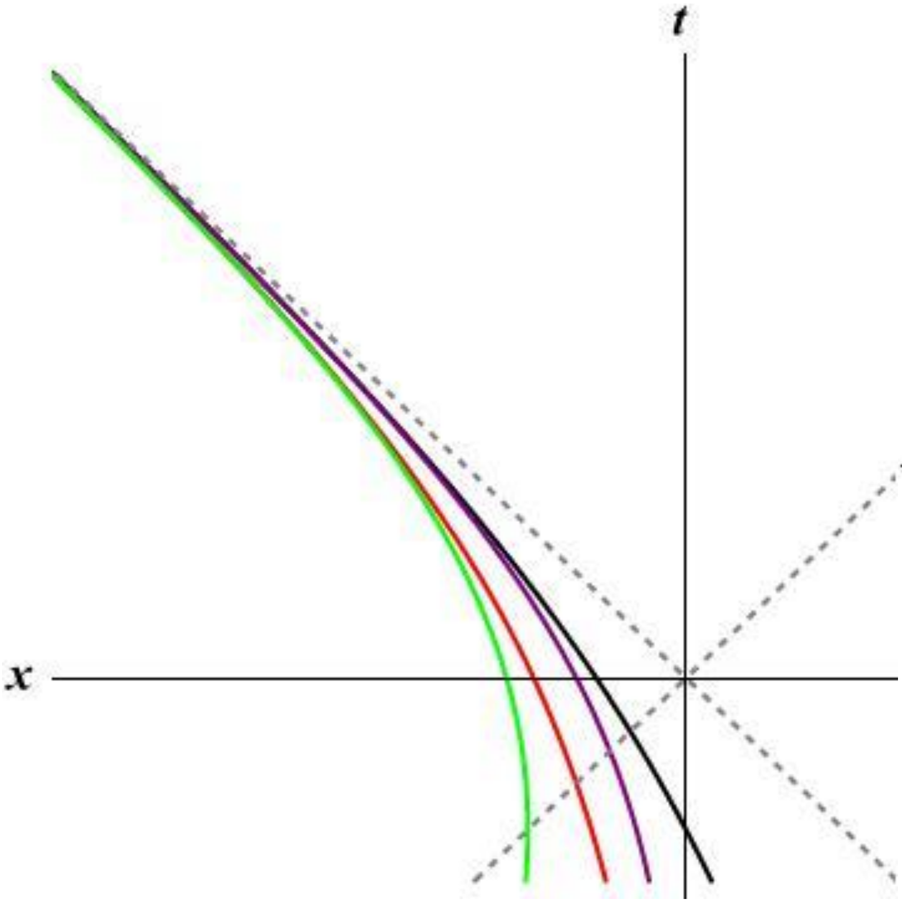
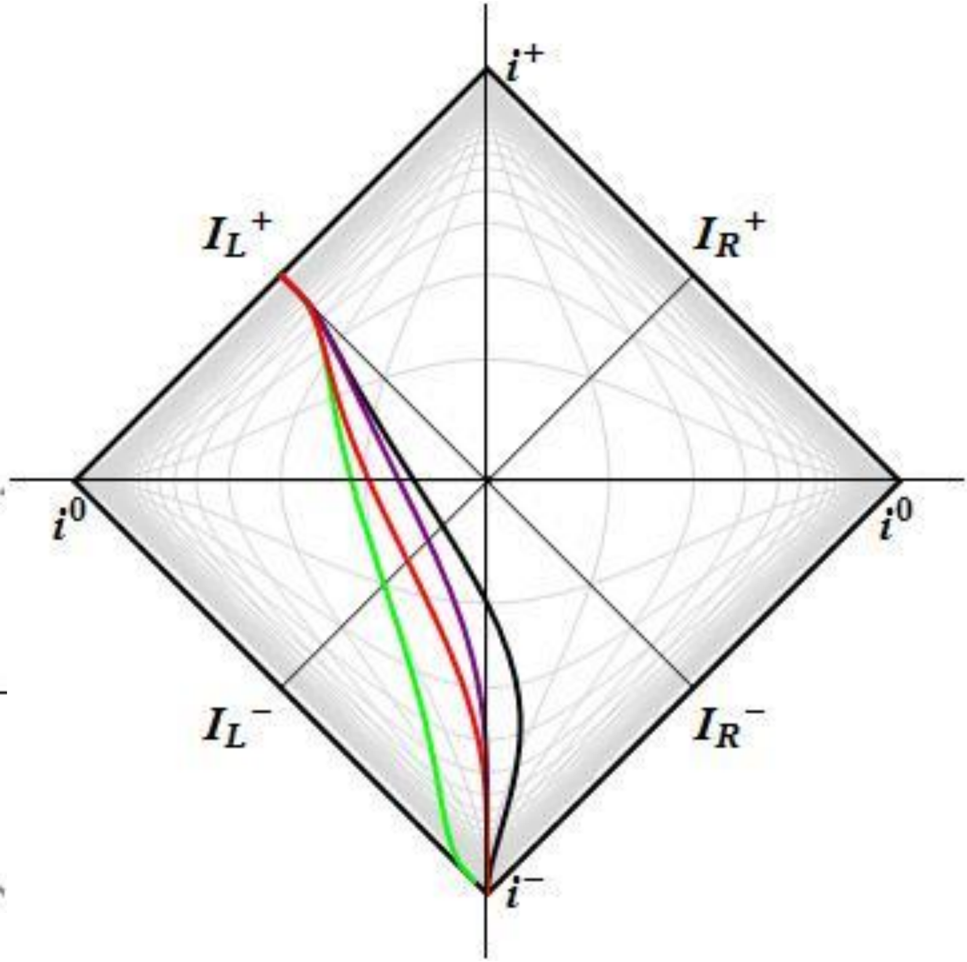
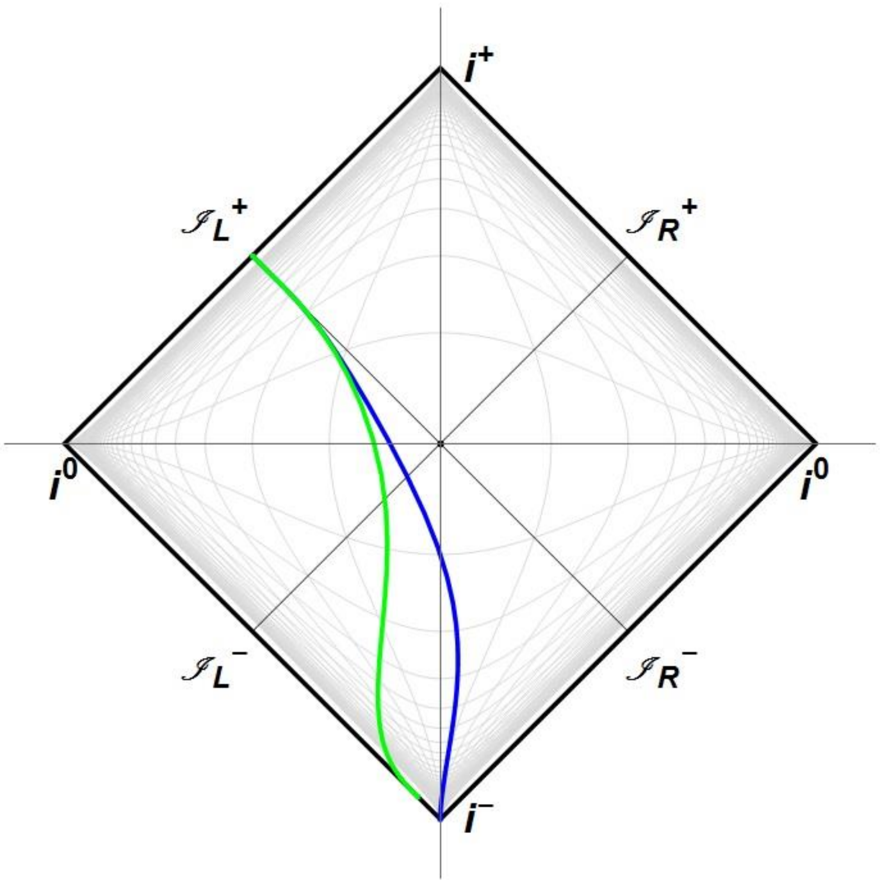
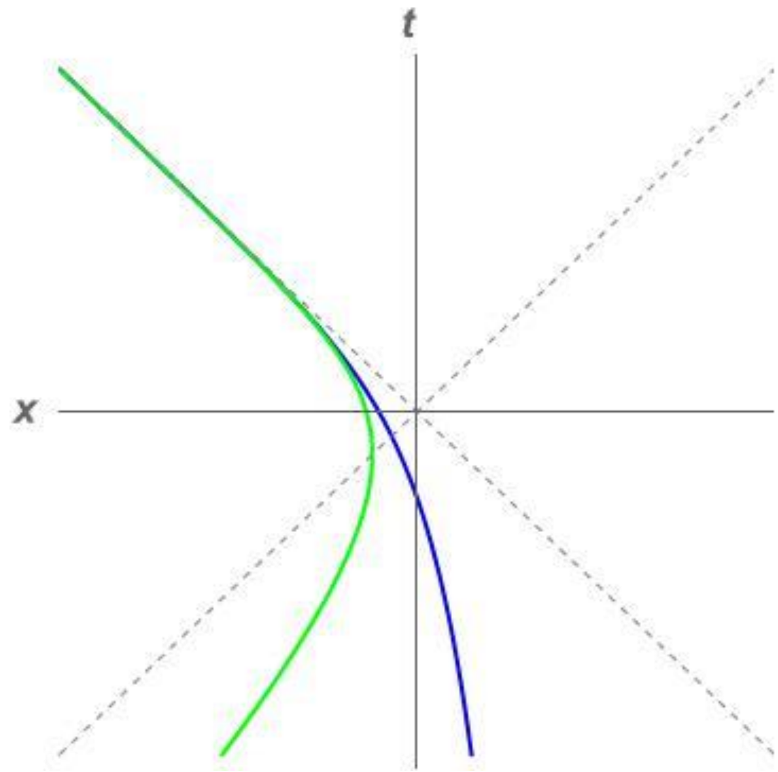




$$z(t) = -t - \frac{W[2e^{-2\kappa t}]}{2\kappa}$$





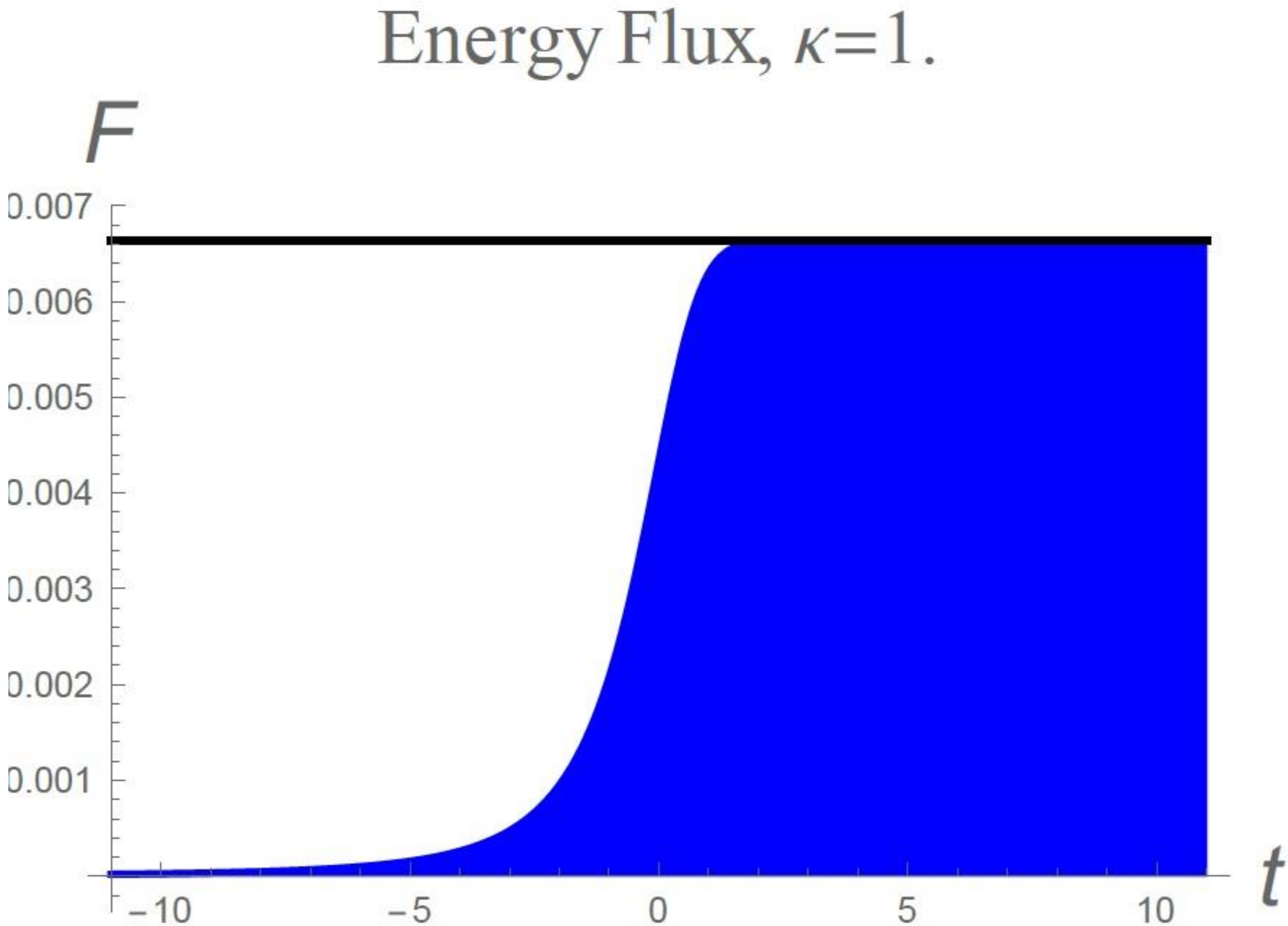
Good, M.R.R., Anderson, P.R., and Evans, C.R.
"Black Hole- Moving Mirror" , 1507.03489 & 1507.05048, 2015

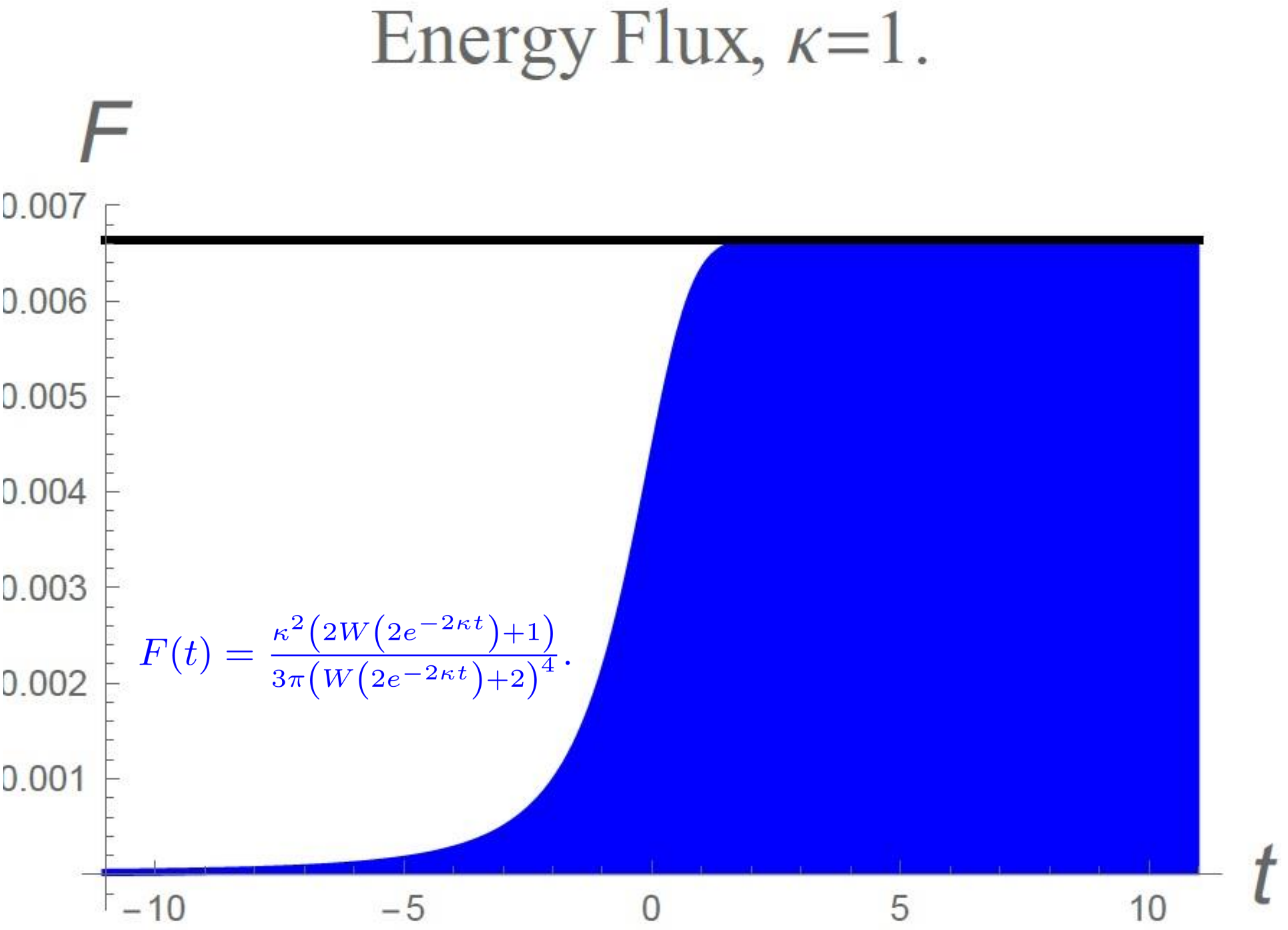
Coordinate Dependent Collapse

- Various horizon moving mirror solutions ⇔ various black hole collapse scenarios.
  - Non-stationary black hole mass function.
  - Different coordinates for geometry matching.

$$u_{\rm out}(u_{\rm in}) = u_{\rm in} - 4M \ln \frac{|v_0 - 4M - u_{\rm in}|}{4M}$$

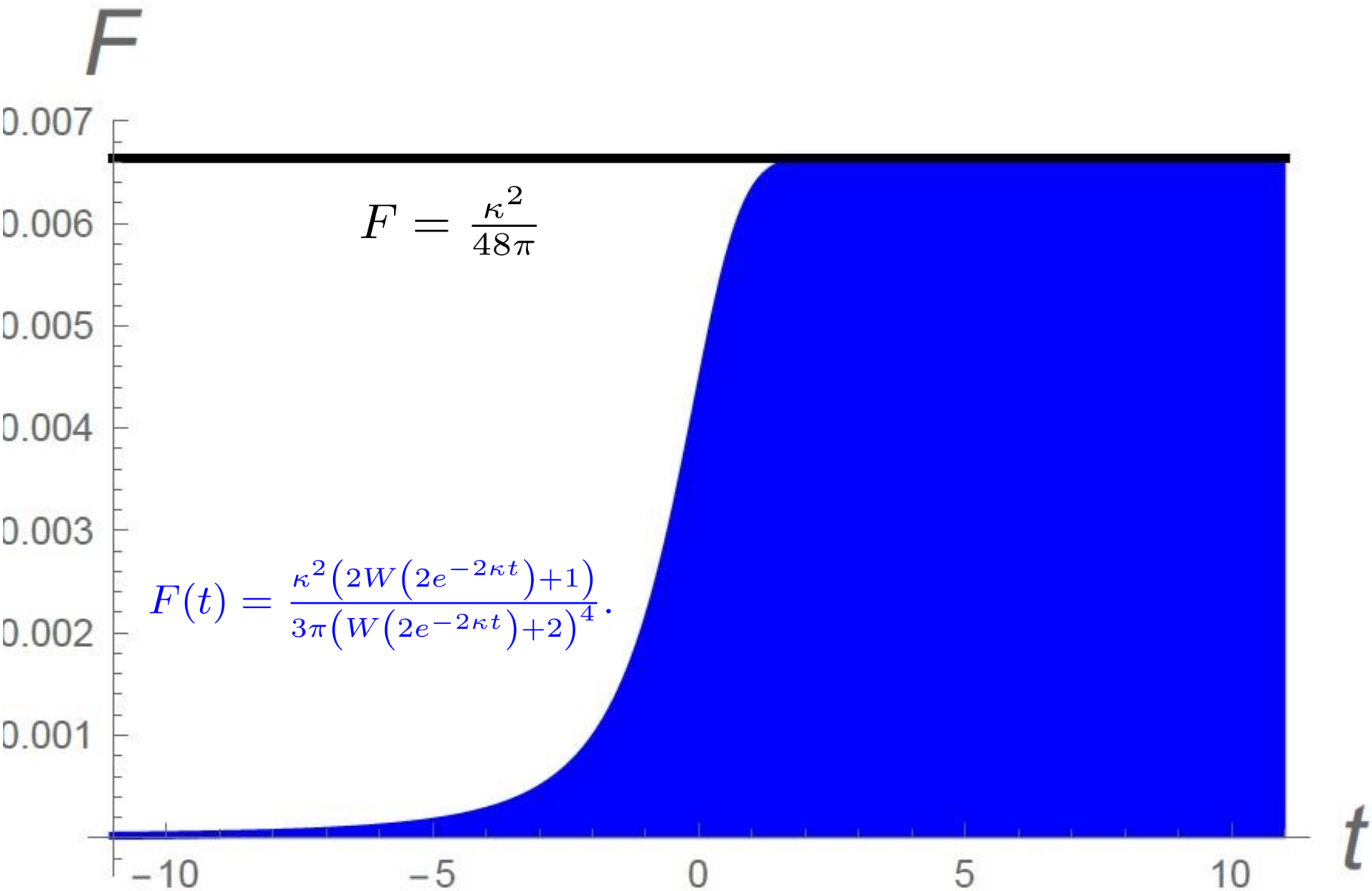
$$r^* = r + 2M \ln \left| \frac{r}{2M} - 1 \right|$$


Energy Flux, κ=1.
F
0.007
0.006
0.005
0.004
0.003
0.002
0.001
t
−10
−5
0
5
10



Energy Flux, κ=1.
F
0.007
0.006
0.005
0.004
0.003
0.002
0.001
$F(t) = \frac{\kappa^2\left(2W\left(2e^{-2\kappa t}\right)+1\right)}{3\pi\left(W\left(2e^{-2\kappa t}\right)+2\right)^4}.$
−10
−5
0
5
10
t

Energy Flux, $\kappa=1$.



Non-eternally thermal Bogolubov Coefficients

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$$\beta_{\omega\omega'} = \frac{1}{4\pi\sqrt{\omega\omega'}}\frac{2\omega'}{\kappa}\Gamma\left(\frac{i\omega}{\kappa}+1\right)\left(-\frac{i(\omega+\omega')}{\kappa}\right)^{-1-\frac{i\omega}{\kappa}}$$

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Late Times

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Late Times $\qquad \omega' \gg \omega$

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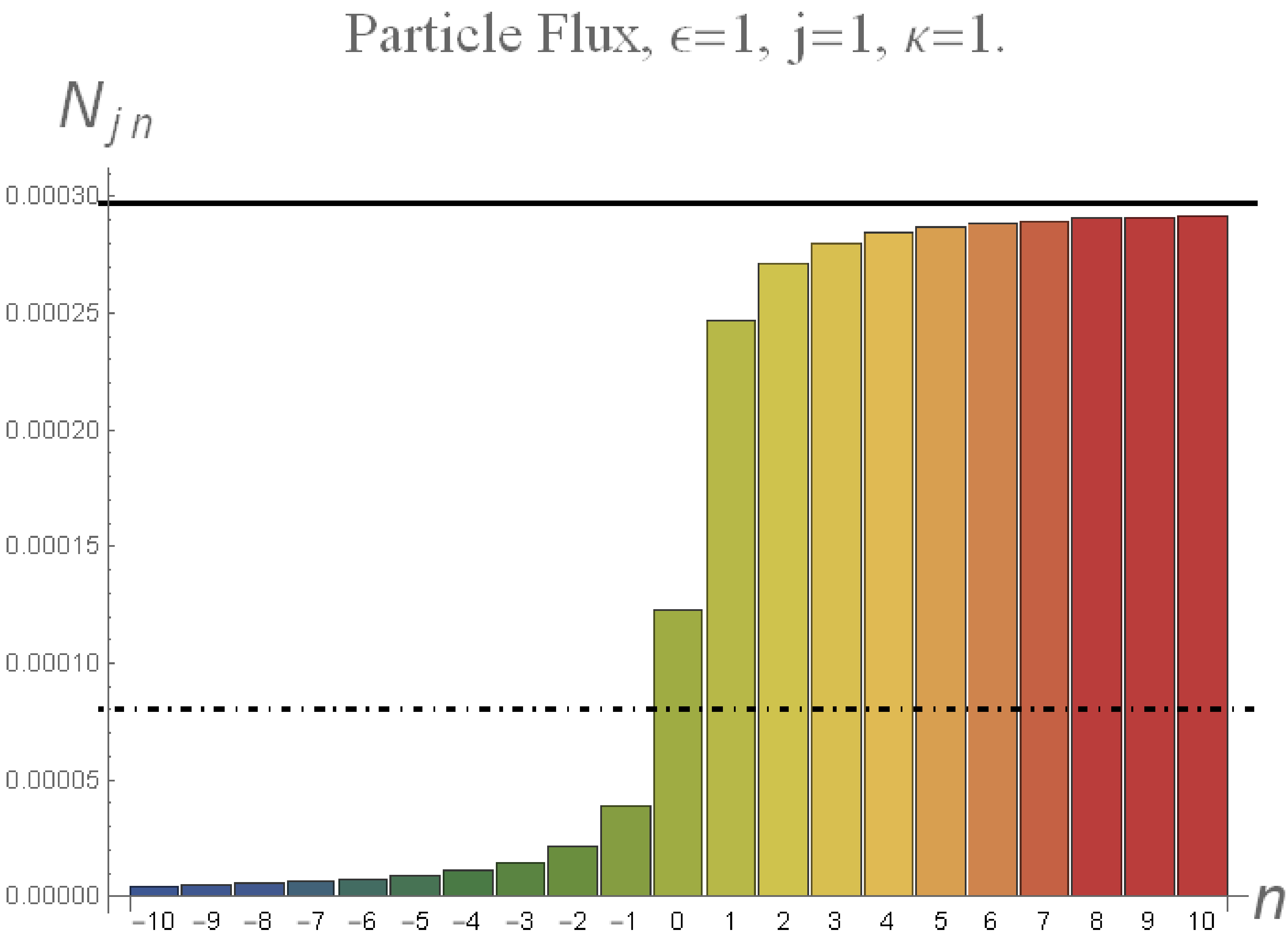
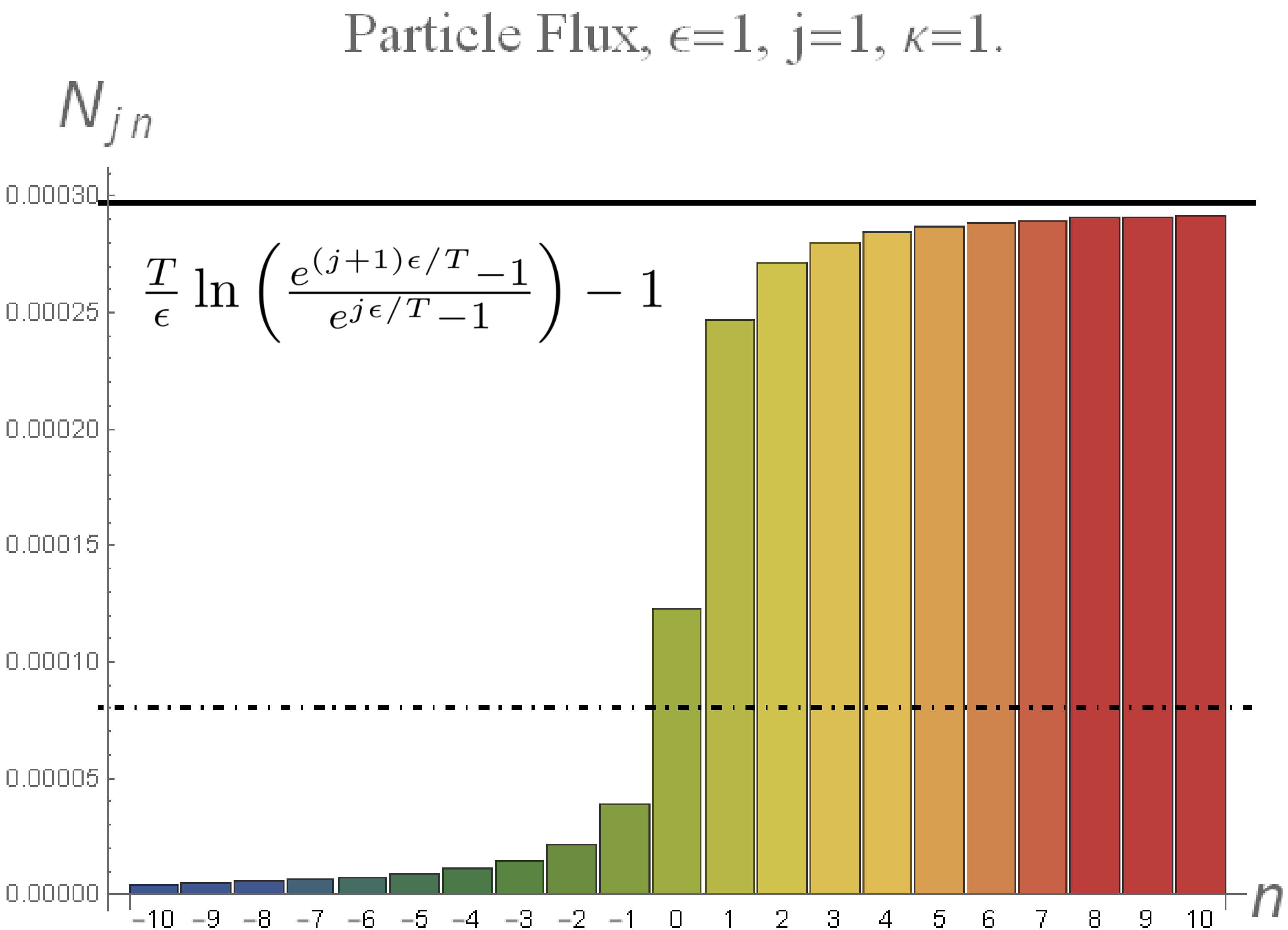
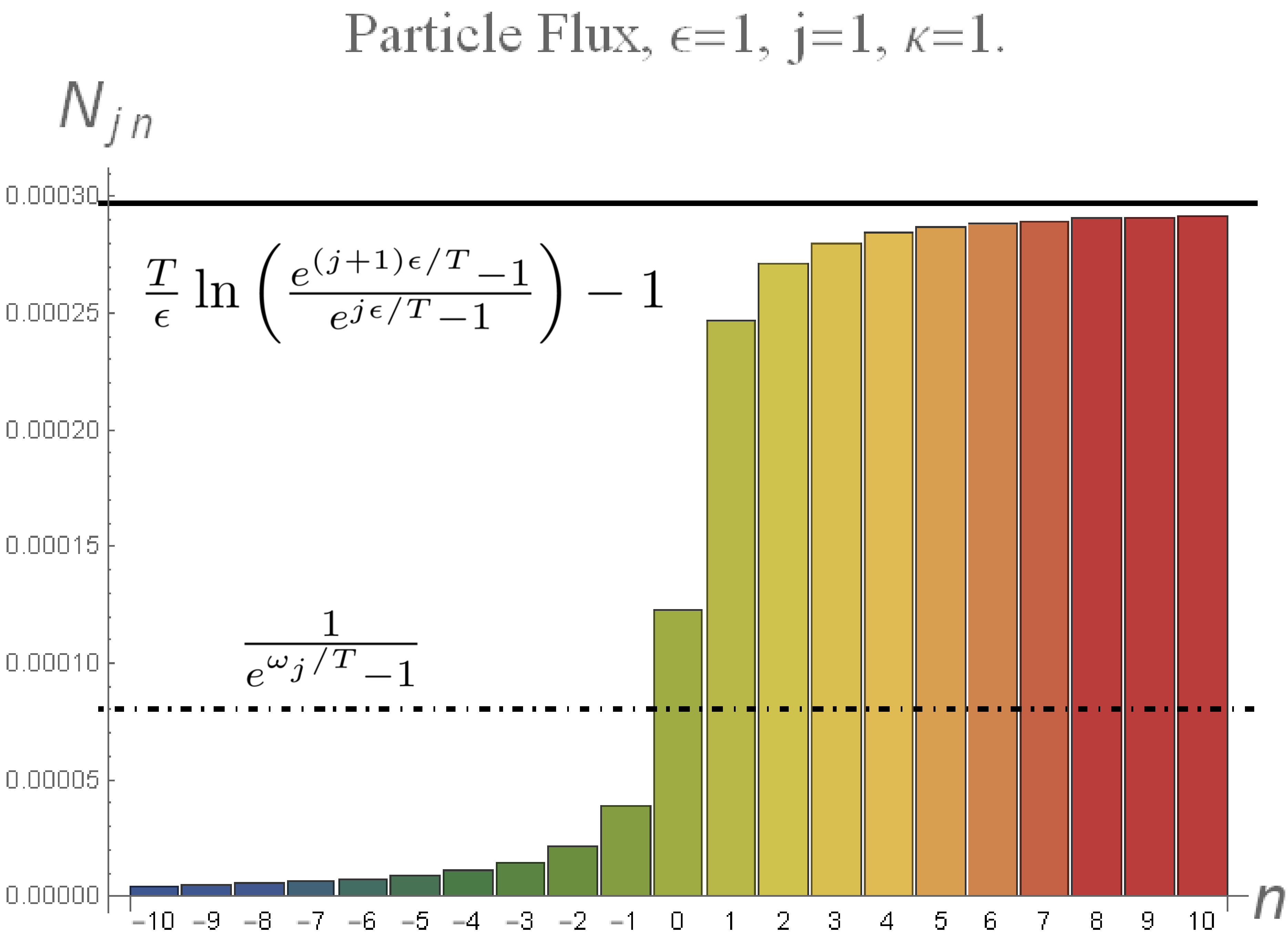
$$T = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}$$



Particle Flux, ϵ=1, j=1, κ=1.
N_{jn}
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
0.00000
n
−10
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7
8
9
10



Particle Flux, ϵ=1, j=1, κ=1.
N_{jn}
$\frac{T}{\epsilon}\ln\left(\frac{e^{(j+1)\epsilon/T}-1}{e^{j\epsilon/T}-1}\right)-1$
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
0.00000
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
n



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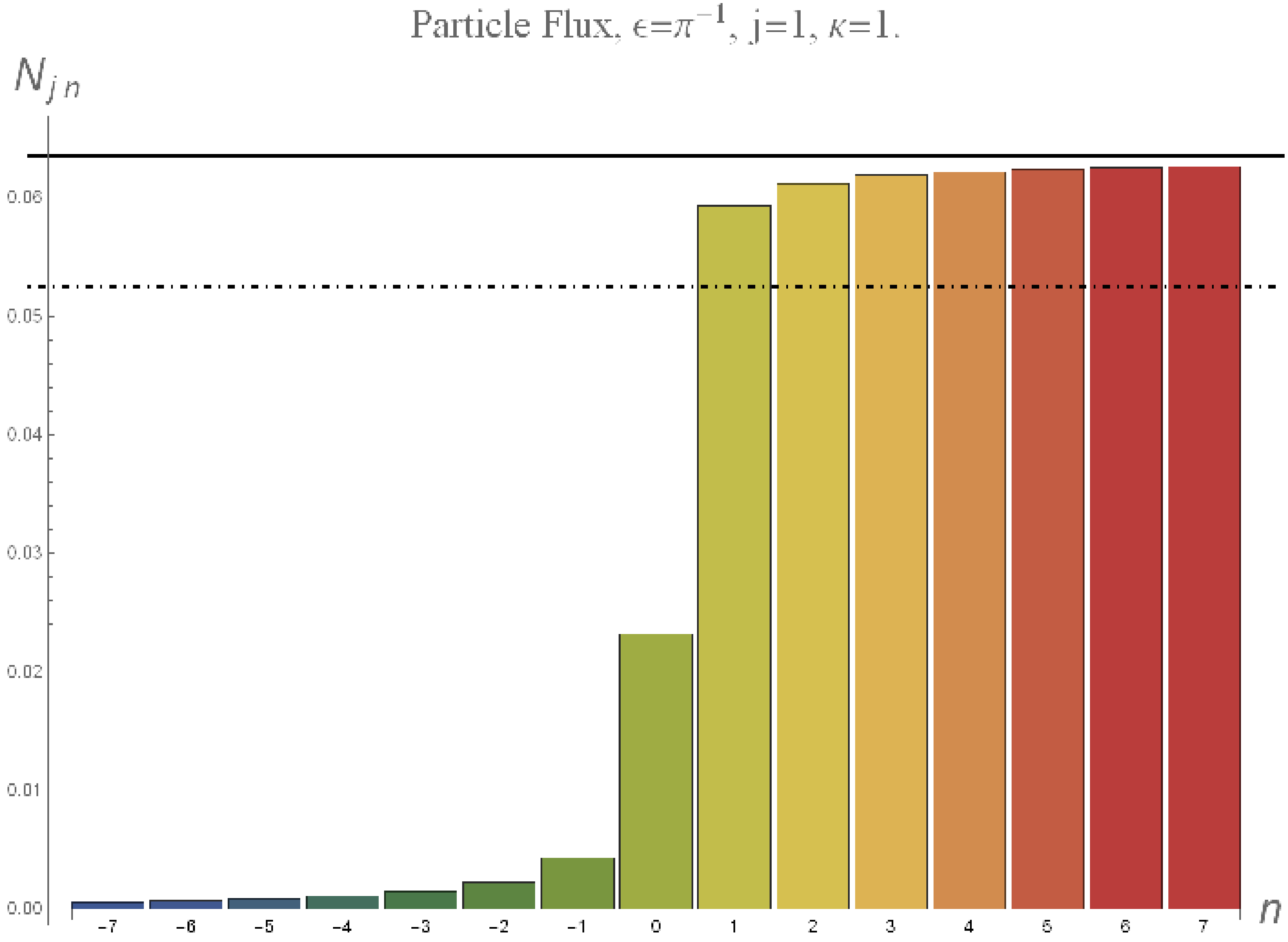
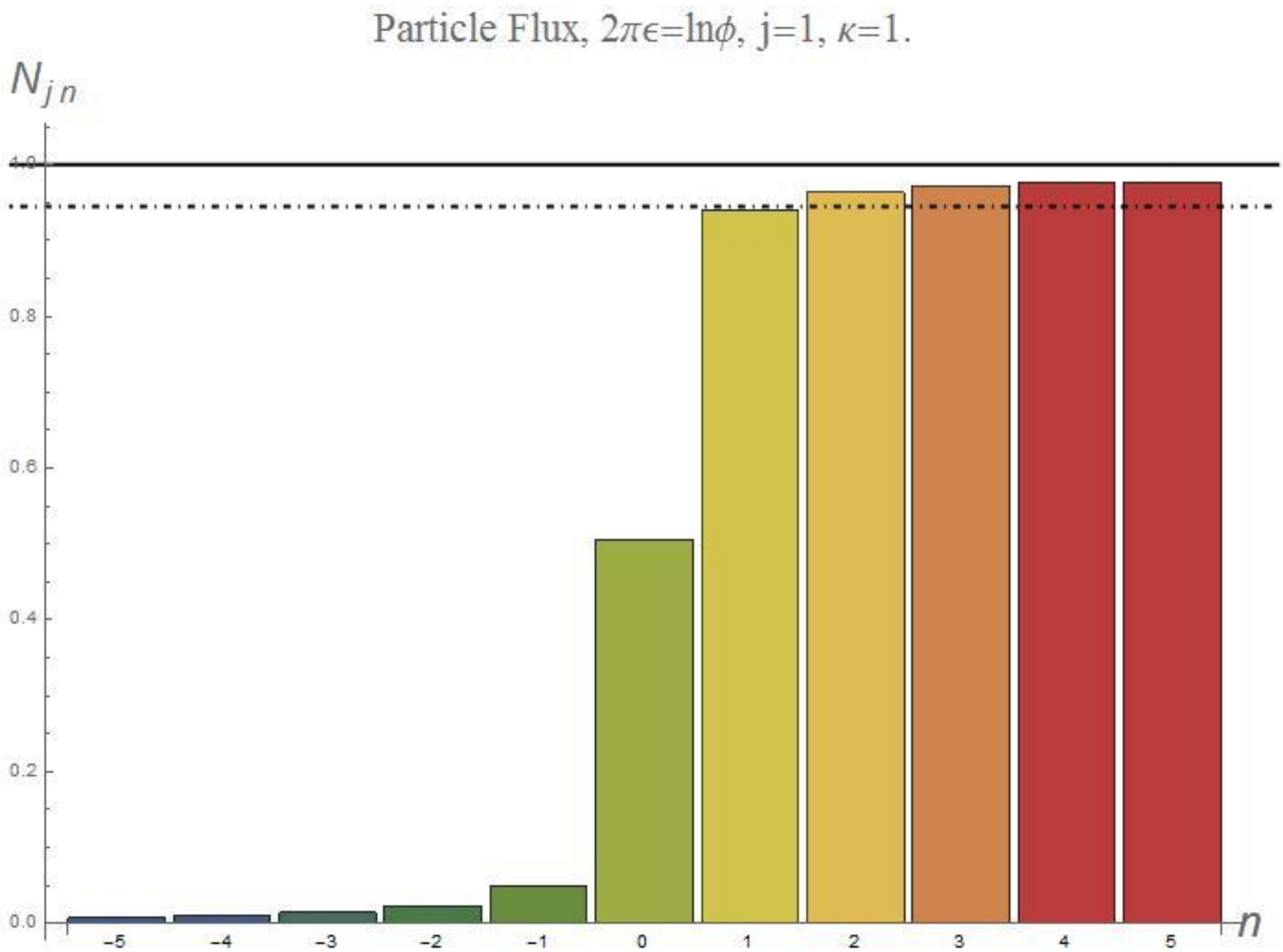
N_{jn}
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
0.00000
$\frac{T}{\epsilon}\ln\left(\frac{e^{(j+1)\epsilon/T}-1}{e^{j\epsilon/T}-1}\right)-1$
$\frac{1}{e^{\omega_j/T}-1}$
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
n



Particle Flux, ϵ=π^−1, j=1, κ=1.
N_jn
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
n


Particle Flux, 2πϵ=lnϕ, j=1, κ=1.
N_{j n}
1.0
0.8
0.6
0.4
0.2
0.0
-5
-4
-3
-2
-1
0
1
2
3
4
5
n

Energy Conservation

- As it stands, the black hole radiates forever.
- However, the emission energy must be finite.

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- However, the emission energy must be finite.

$$E_T \leq M$$

Information Loss

- The mirror must not accelerate forever.
- The mirror must be asymptotically inertial.
- The mirror must emit negative energy flux.

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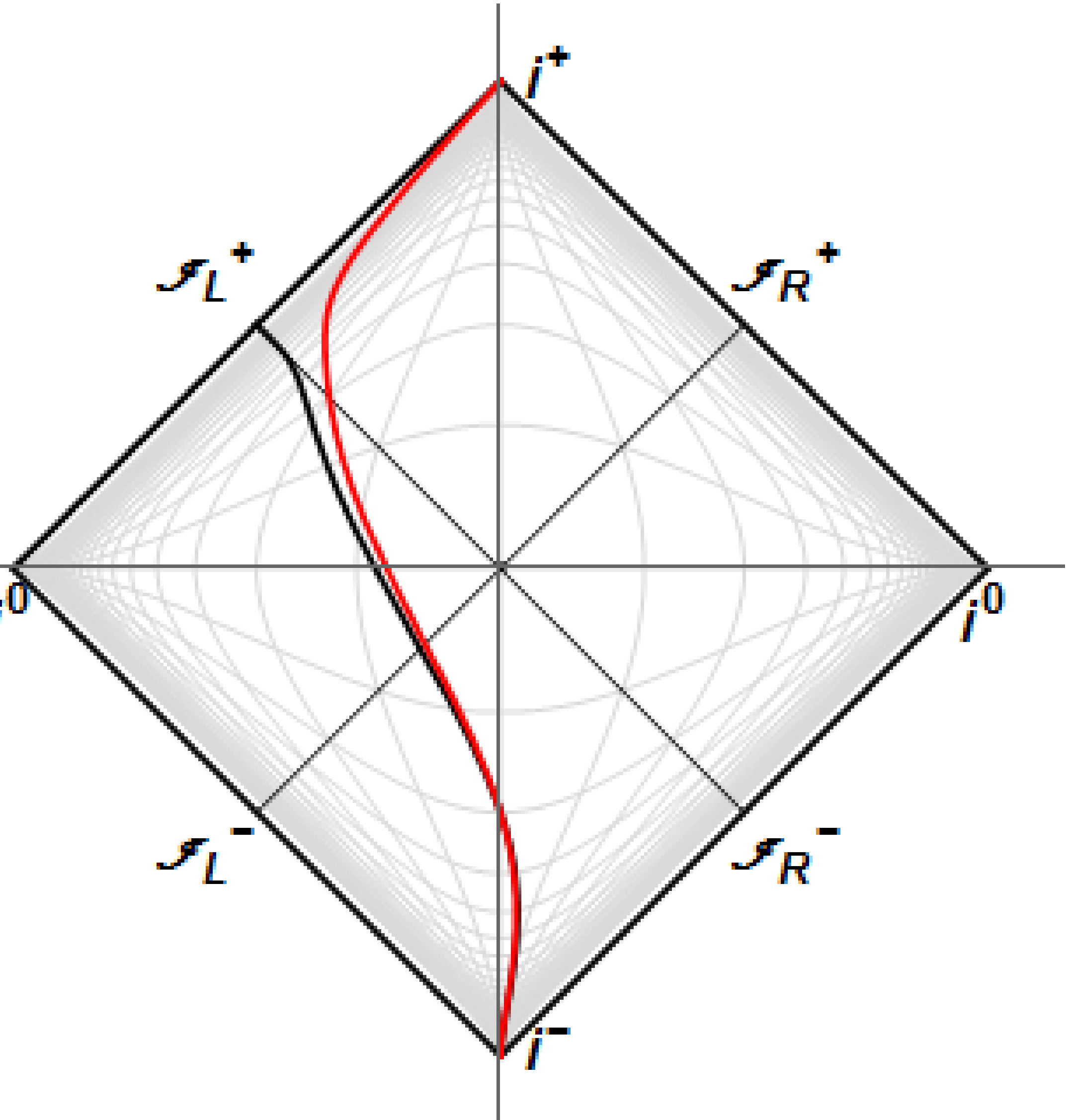
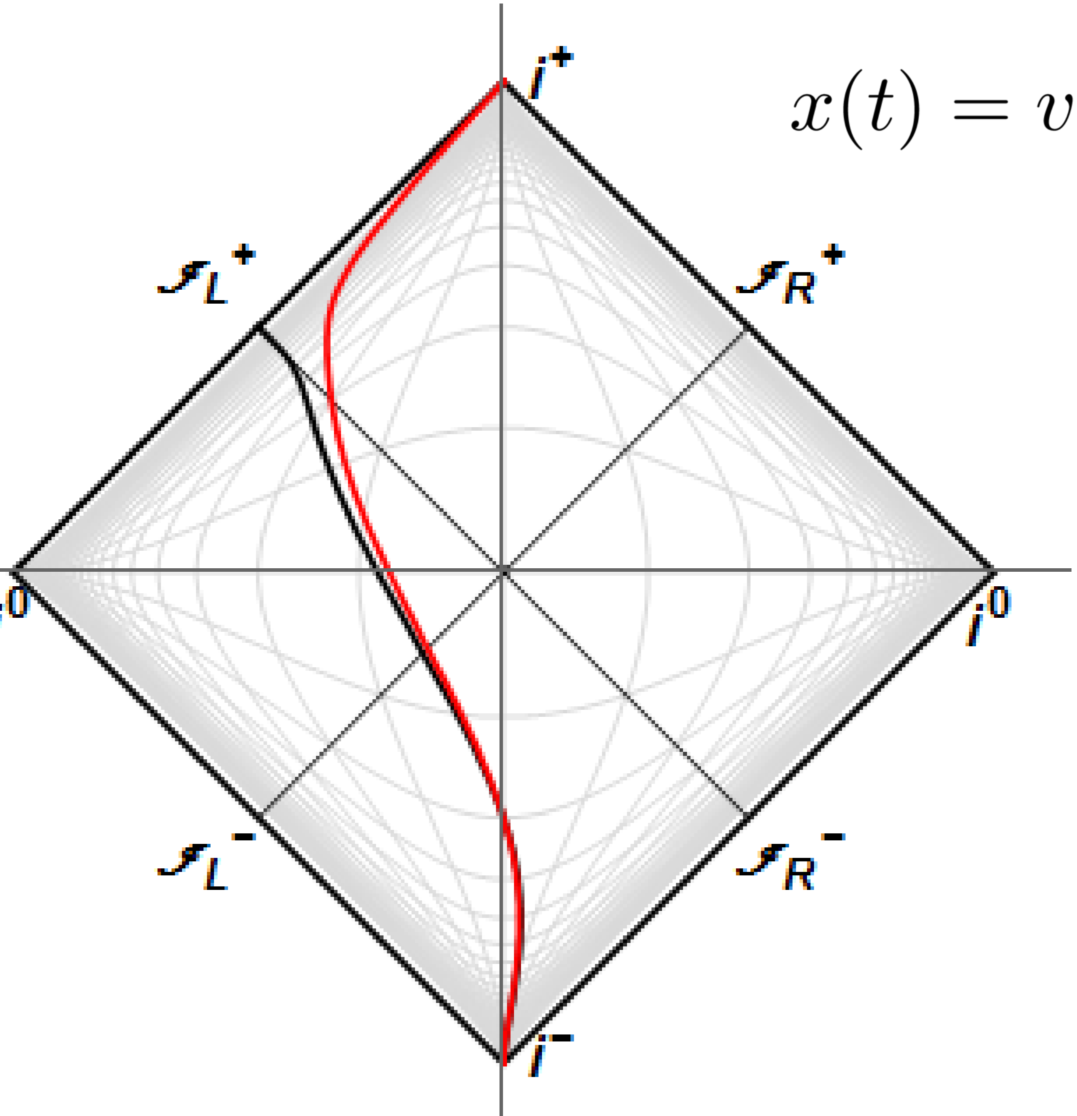
$$12\pi F(u) = [\eta'(u)]^2 - \eta''(u)$$

Trajectory Functions

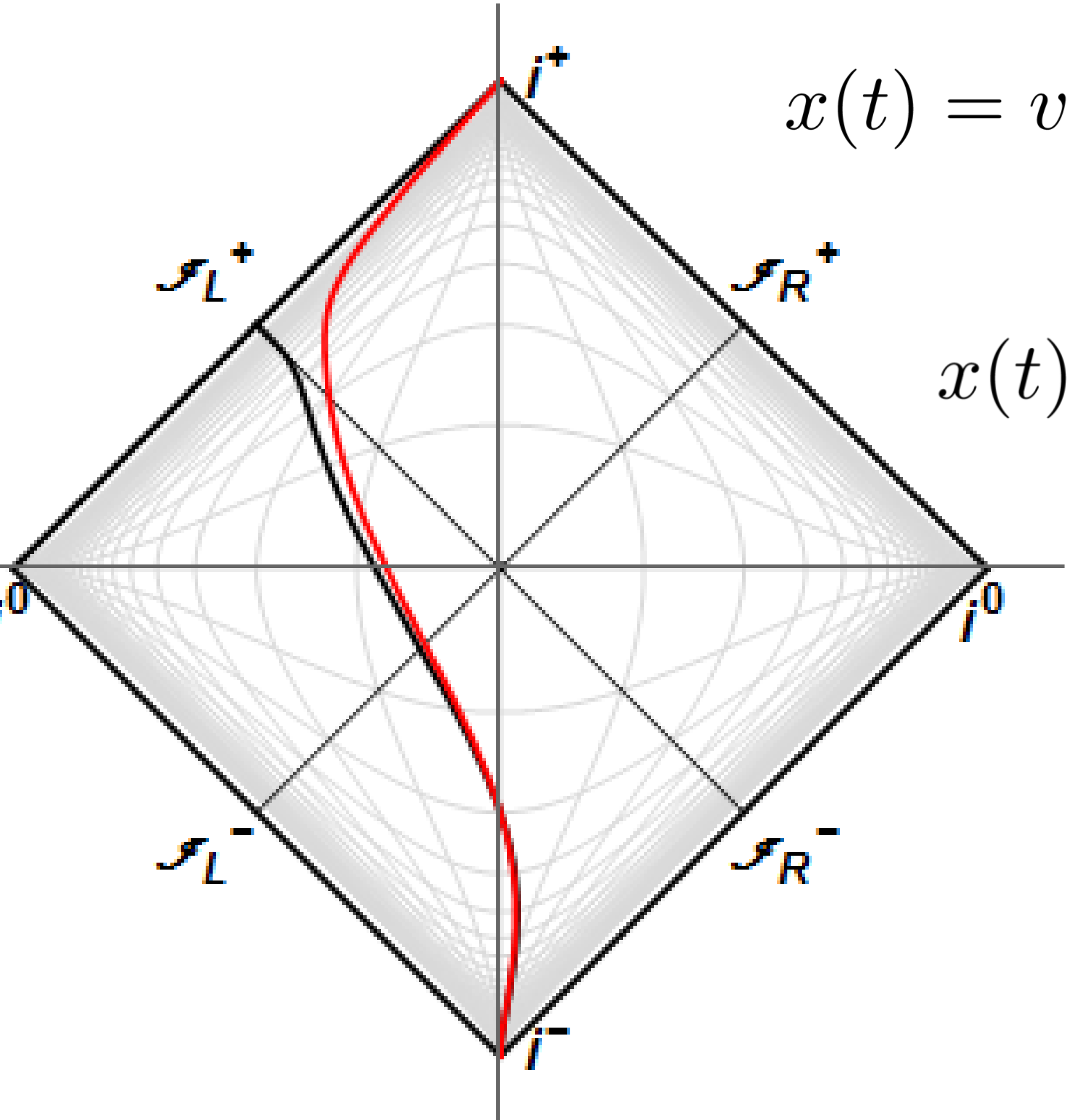


Trajectory Functions



$$x(t) = v_H - t - 2MW\left[2e^{\frac{v_H - t}{2M}}\right]$$

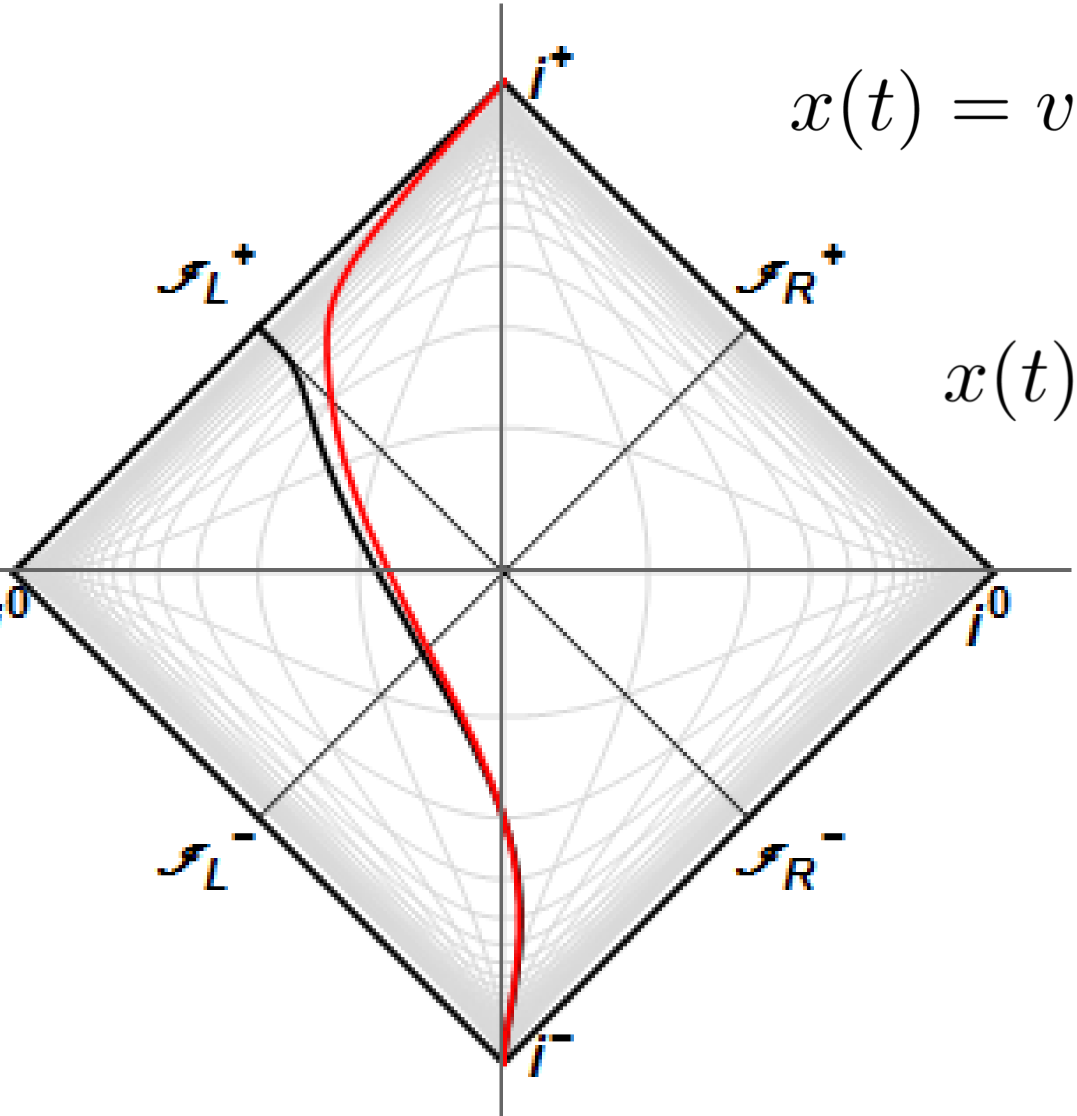
Trajectory Functions



$$x(t) = v_H - t - 2MW\left[2e^{\frac{v_H - t}{2M}}\right]$$

$$x(t) = -t - 2MW\left[2e^{-\frac{t}{2M}}\right]$$

Trajectory Functions



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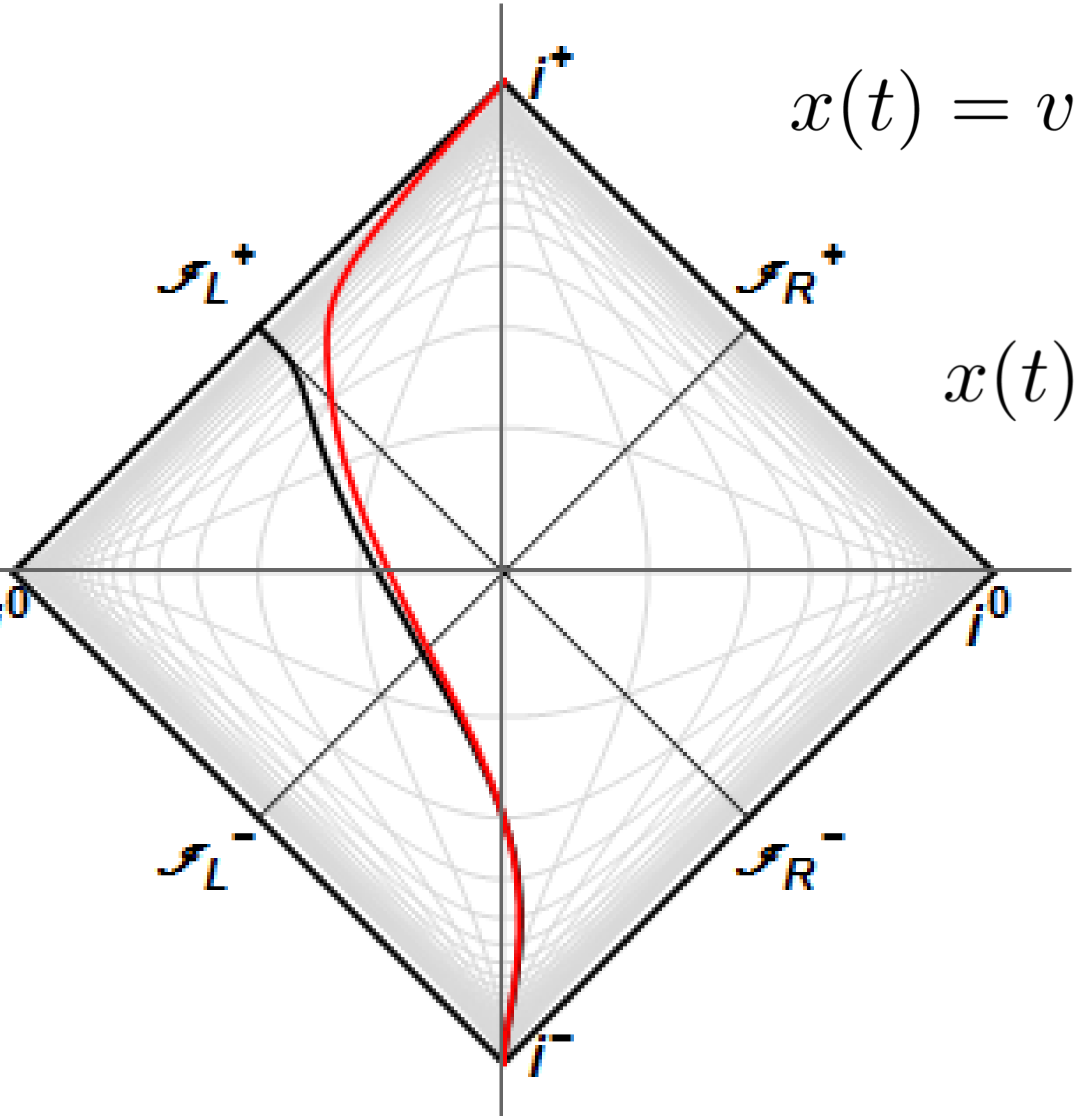
$$x(t) = -t - 2MW\left[2e^{-\frac{t}{2M}}\right]$$

$$z(t) = \xi x(t)$$

Trajectory Functions



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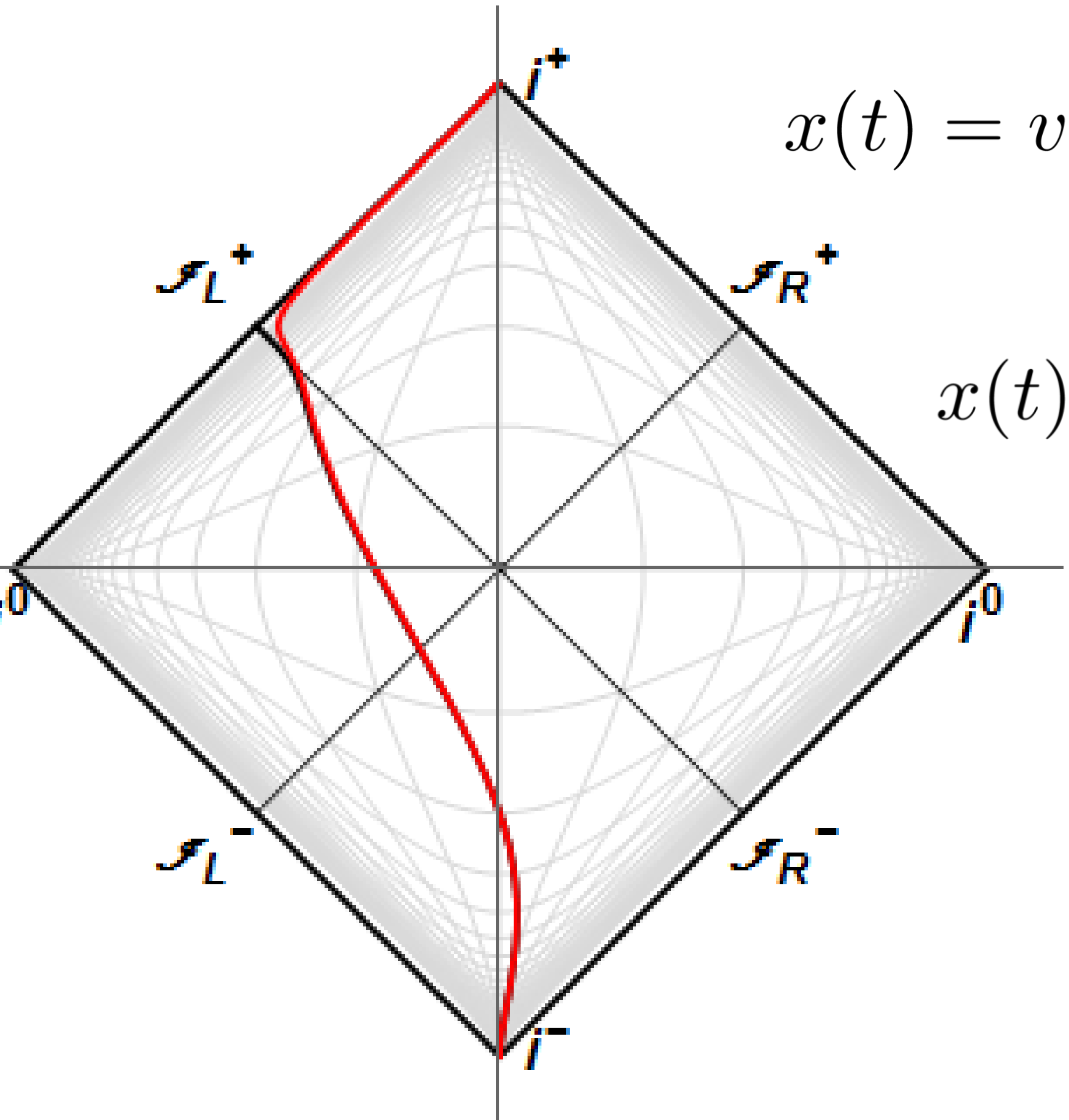
$$\xi = 0.9$$

Trajectory Functions



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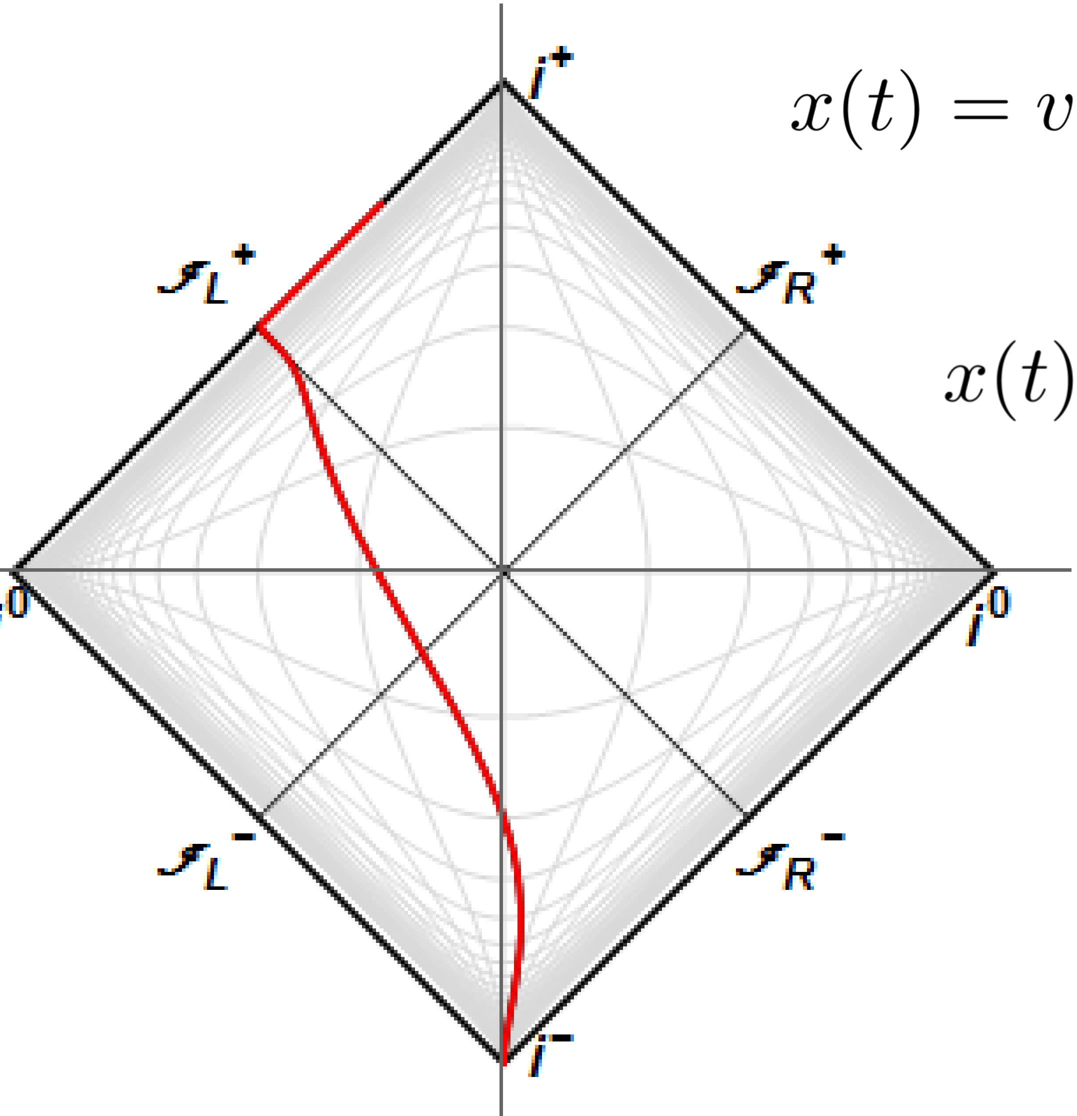
$$z(t) = \xi x(t)$$

$$\xi = 0.9$$
$$\xi = 0.99$$

Trajectory Functions



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$$\xi = 0.99$$
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Ray Tracing Functions

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$$u = v - 4M \ln \frac{v_H - v}{4M}$$

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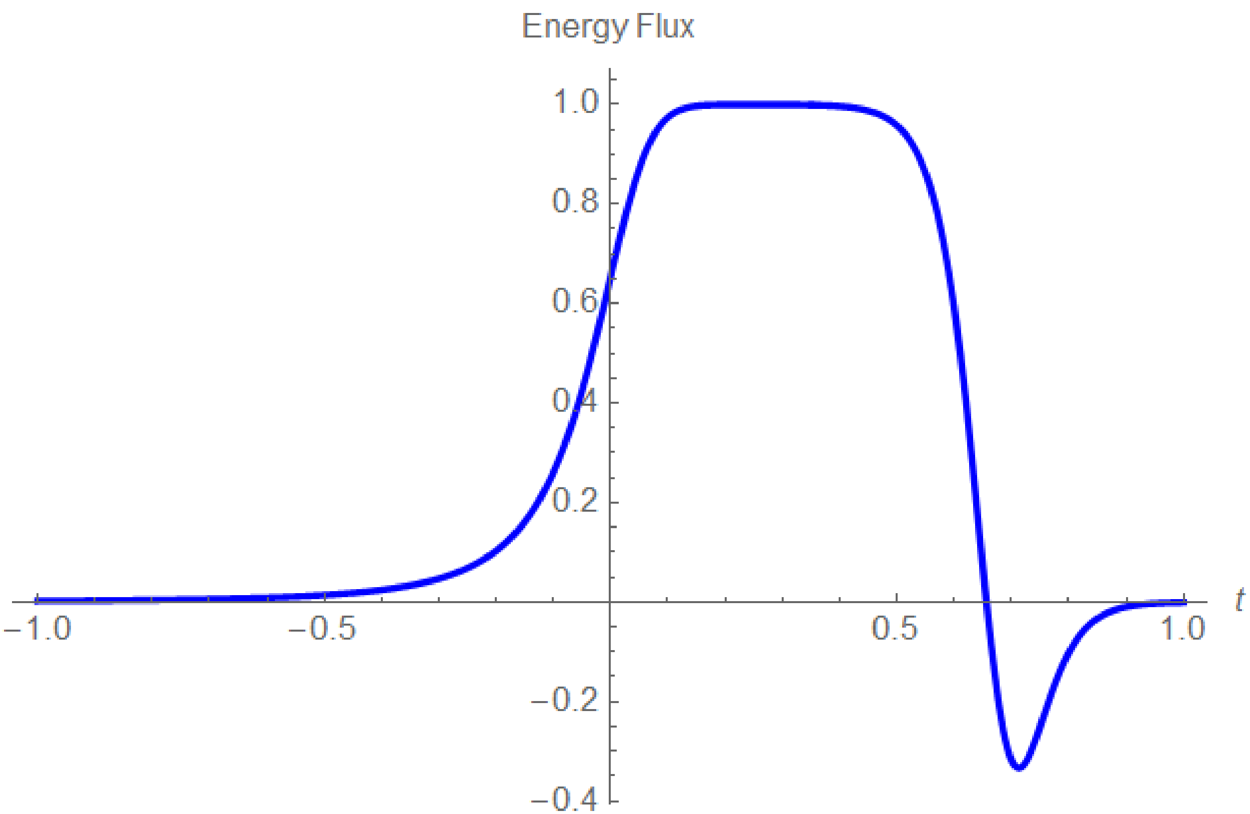
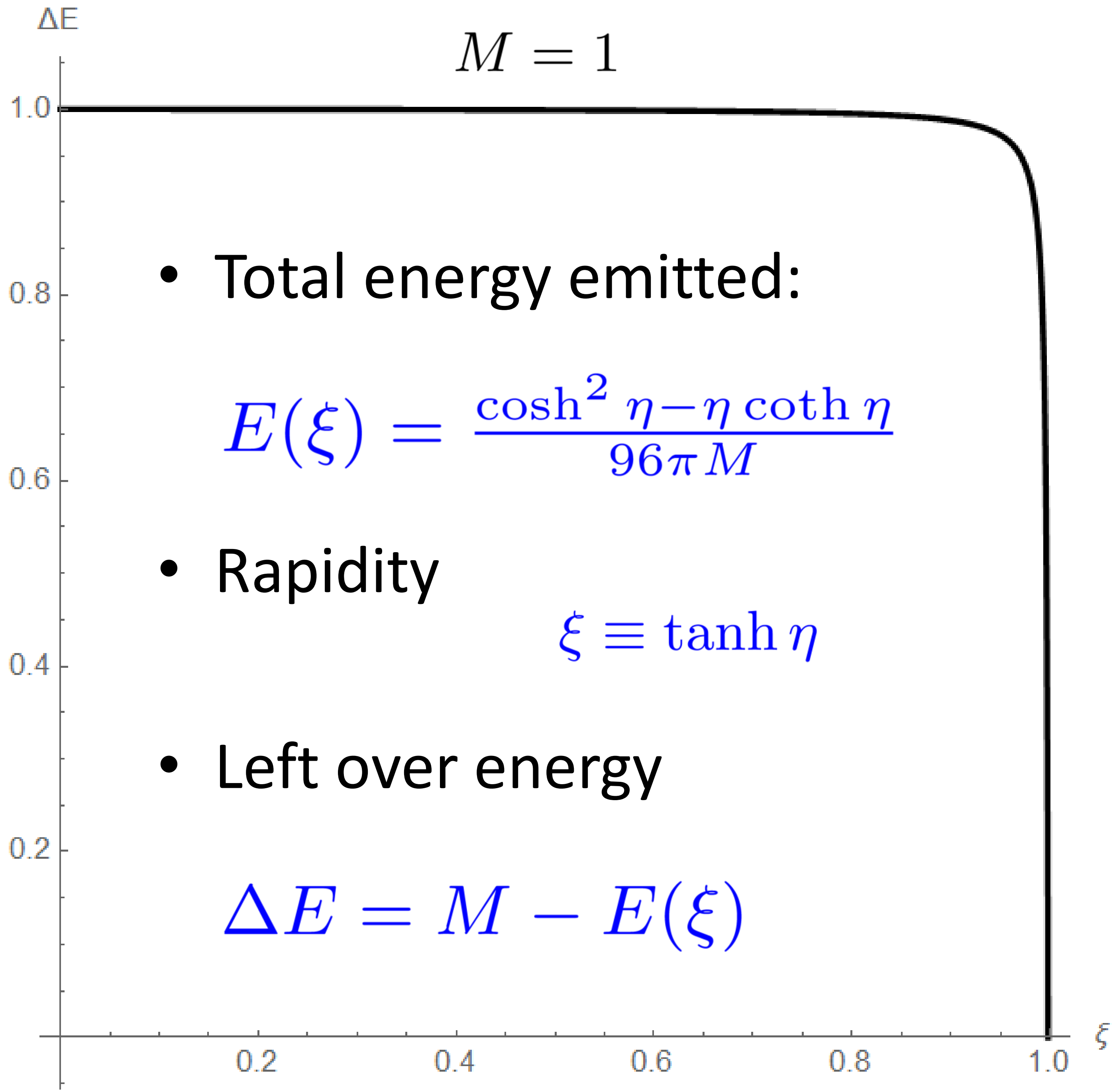
$$u = \frac{1+\xi}{1-\xi} v + 4M\xi W \left[\frac{2e^{\frac{v_H - v}{2M(1-\xi)}}}{1-\xi} \right] - \frac{2\xi}{1-\xi} v_H$$

Energy Flux

$$F = \frac{\kappa^2 \xi W\left(2e^{-2\kappa t}\right)\left(\xi^2 + 2W\left(2e^{-2\kappa t}\right)^2 + W\left(2e^{-2\kappa t}\right) - 1\right)}{3\pi\left(-\xi + W\left(2e^{-2\kappa t}\right)+1\right)^2\left(\xi + W\left(2e^{-2\kappa t}\right)+1\right)^4}$$



$$\xi = 1 - 0.1^7$$
$$= 0.9999999$$



- Total energy emitted:

$$E(\xi) = \frac{\cosh^2\eta - \eta\coth\eta}{96\pi M}$$

- Rapidity

$$\xi \equiv \tanh\eta$$

- Left over energy

$$\Delta E = M - E(\xi)$$

Conclusions

- An exact one-to-one analytic correspondence exists.
- High-frequency limit gives thermal particle creation.
- Early-time particle creation is monotonic.
- Finite energy emission requires negative energy flux.
- Effects of acceleration $\rightarrow$ Effects of gravitation.

$$e^{\epsilon/T} = \phi \quad \rightarrow \quad \langle N_{j=1} \rangle = 1$$

$$E_{st} = \int_{-\infty}^{\infty} \langle T_{uu} \rangle \, du$$

$$E_{st} = \int_{-\infty}^{\infty} \langle T_{uu} \rangle \, du$$

$$E_{qs} = \int_{0}^{\infty} \omega \langle N_{\omega} \rangle \, d\omega$$

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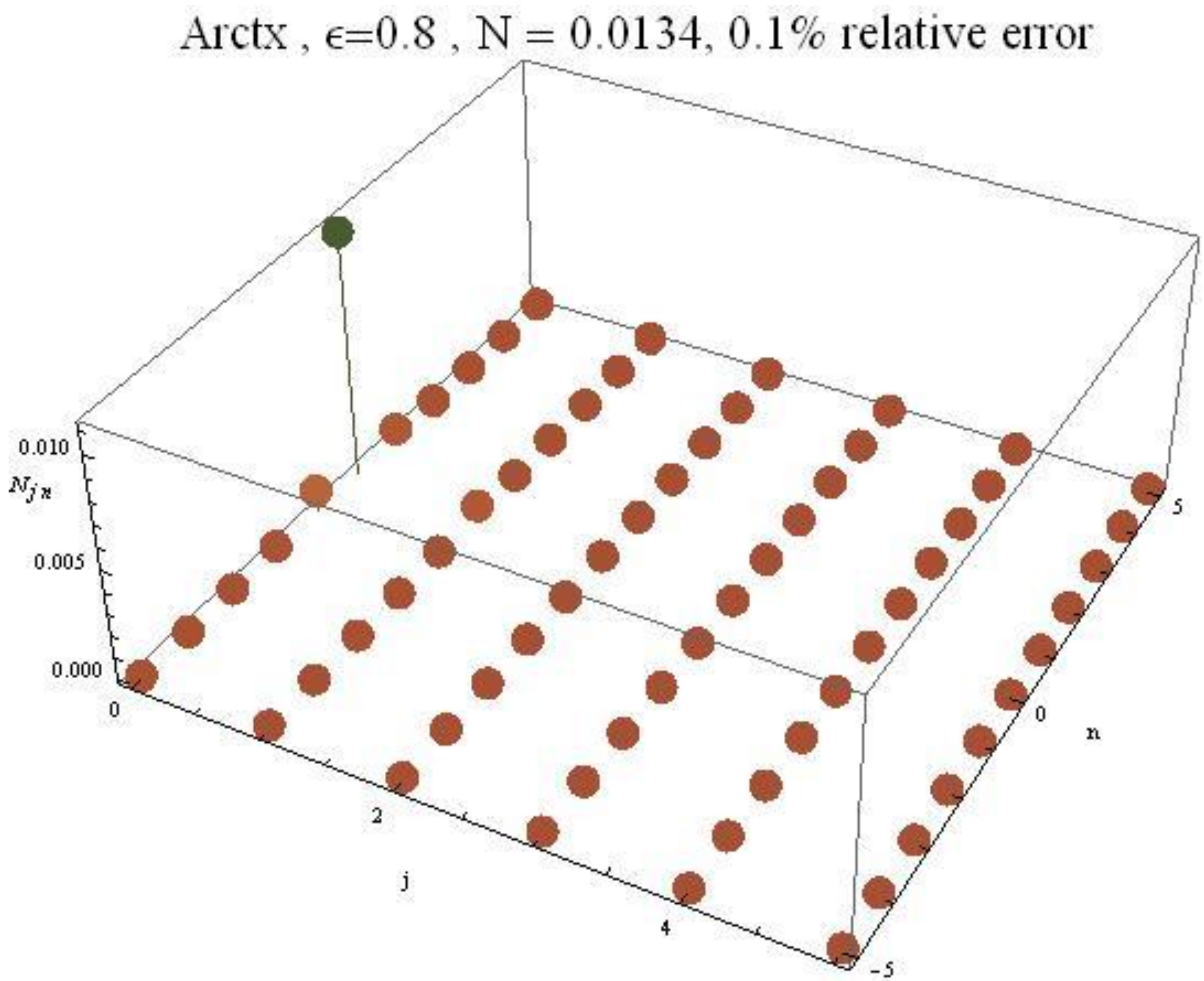
$$E_{qs} = \int_{0}^{\infty} \omega \langle N_{\omega} \rangle \, d\omega$$

$$E_{st} = E_{qs}$$

Spectral Resolution Using Packets



Wavepacket Spectral Resolution

$$\Delta t \Delta \omega \geq \frac{1}{2}$$

$$\Delta T \Delta \epsilon \geq 2\pi$$

insights

- Effects of acceleration will yield insights into the effects of gravitation.
- Existence of trajectories with spectral evolution.
- Moving mirrors provide a simple laboratory to construct the mathematical machinery needed to understand the time evolution of black hole evaporation.

Negative Energy Flux

Constraints on negative energy fluxes

Ford - Physical Review D, 1991

Negative Energy Seen By Accelerated Observers

Ford and Roman - Phys. Rev. D, 2013

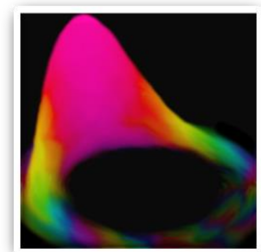
Energy flux correlations and moving mirrors

Ford and Roman - Phys.Rev. D, 2004

Finite Particle Count via Packets

$$\langle in|N_\omega^{out}|in\rangle = \frac{1}{e^{2\pi\omega_1/\kappa}-1}\delta(\omega_1-\omega_2)$$

$$\beta_{jn,\omega'} = \frac{1}{\sqrt{\epsilon}}\int_{j\epsilon}^{(j+1)\epsilon} d\omega e^{2\pi i\omega n/\epsilon}\beta_{\omega\omega'}$$



$$\langle in|N_{jn}^{out}|in\rangle = \int_0^\infty d\omega'|\beta_{jn,\omega'}|^2 \quad = \frac{1}{e^{2\pi\omega_j/\kappa}-1}$$

$$T = \frac{\kappa}{2\pi}$$

$$\langle T_{uu}\rangle = \frac{\kappa^2}{48\pi}$$

W Lambert (Product Log)

$$Y = Xe^X$$

$$X = W(Y)$$

$$W(z) = \sum_{n=1}^{\infty}(-n)^{n-1}\frac{z^n}{n!}$$

$$3^t = 8t$$

$$-\frac{\ln 3}{8} = (-t\ln 3)e^{(-t\ln 3)}$$

$$t = -\frac{W(-\frac{\ln 3}{8})}{\ln 3}$$

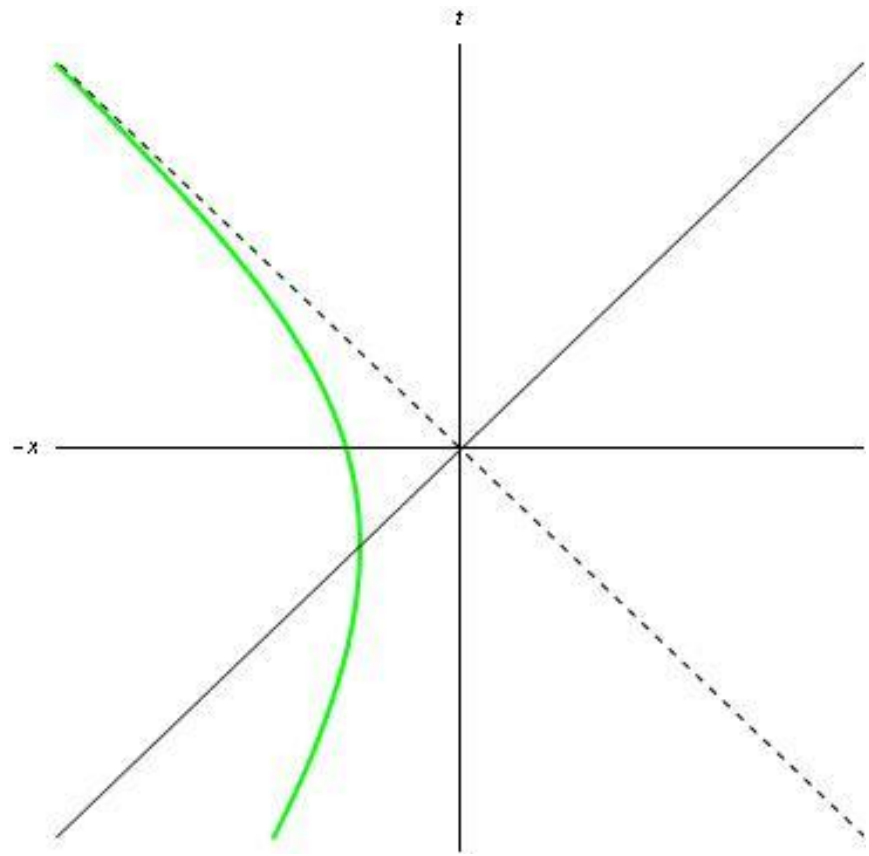
$$t \approx 0.146891$$

We use this function for the transcendental inversion necessary to obtain z(t).

Constant Energy Flux Trajectory

$$z = -t - \frac{1}{\kappa} W(e^{-2\kappa t})$$

$$\alpha = \gamma^3 a = \frac{-\kappa}{2\sqrt{W(e^{-2\kappa t})}}$$



Features:

Future Horizon.

Infinite total energy production.

Infinite total particle production.

Time dependent acceleration.

$$x(0) = -\frac{\Omega}{\kappa}$$

$$e^{-\Omega} = \Omega$$

$$x(-\tfrac{1}{2\kappa}) = -\tfrac{1}{2\kappa}$$

$$\lim_{t\to\pm\infty} \alpha(t) = \left\{ \begin{matrix} -\infty \\ 0 \end{matrix} \right.$$

$$\lim_{t\to\pm\infty} v(t) = \left\{ \begin{matrix} -1 \\ 1 \end{matrix} \right.$$

$$p(u) = 2t_u - u \qquad u = t - z(t)$$

$$f(v) = 2t_v - v \qquad v = t + z(t)$$

Exactly Solvable Trajectory

$$z(t) = -\frac{1}{2\kappa}\sinh^{-1}(e^{\kappa t})$$

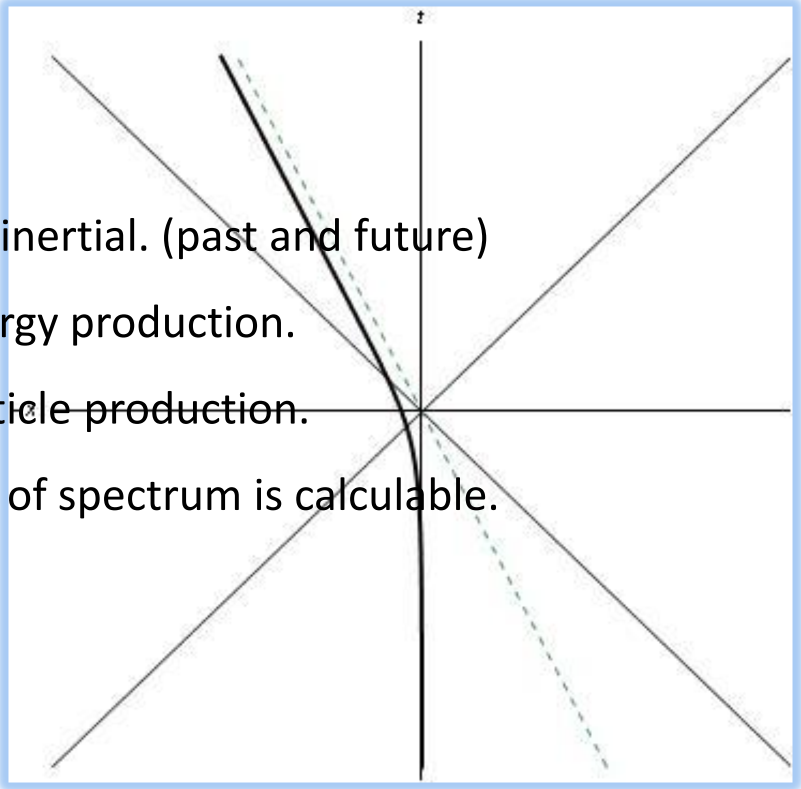
Features:

Asymptotically inertial. (past and future)

Finite total energy production.

Finite total particle production.

Time evolution of spectrum is calculable.



$$E = \int \langle T_{uu}\rangle \, du = \int \omega \langle N_\omega\rangle \, d\omega$$

$$|\beta_{\omega'\omega}|^2 = \frac{1}{16\pi\kappa}\frac{\omega\omega'(\omega+\omega')^{-1}}{(\frac{\omega'}{4\kappa}+\frac{3\omega}{4\kappa})(\frac{\omega}{4\kappa}+\frac{3\omega'}{4\kappa})}\frac{\sinh(\frac{\pi\omega}{4\kappa}+\frac{3\pi\omega'}{4\kappa})}{\sinh(\frac{\pi\omega}{\kappa}+\frac{\pi\omega'}{\kappa})\sinh(\frac{\pi\omega'}{4\kappa}+\frac{3\pi\omega}{4\kappa})}$$