

# Numerical studies of inverse magnetic catalysis in holographic QCD

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# Outline of the talk

Work in progress with Alfonso Ballon-Bayona (ICTP South America, Brazil), Jon Shock (University of Cape Town, South Africa), Dimitrios Zoakos (Universitat Internacional de Catalunya, Spain)

[Chiral Phase Transitions in Holographic QCD at Finite Temperature, Density, Magnetic Field and Quark Mass](#)

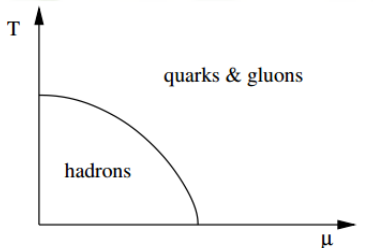
Previous results: Preiss, Rebhan and Schmitt, *Inverse magnetic catalysis in dense holographic matter*, JHEP1103, 033 (2011),

- ▶ **Motivation:** Understanding QCD phase transitions/phase diagram (dense matter in magnetic fields)
- ▶ Why (strong) [magnetic fields](#)?
- ▶ **Magnetic Catalysis:** Chiral symmetry breaking and magnetic fields
- ▶ **Introduction** to gauge/gravity duality and the Sakai-Sugimoto model of holographic QCD.
- ▶ Chiral symmetry breaking in holographic QCD at finite temperature, density and magnetic field.
- ▶ (Preliminary) [results](#) of the numerical analysis

# Understanding QCD phase transitions/phase diagram: Dense matter

- ▶ at **small**  $T, \mu$ , we observe hadrons
- ▶ at **large**  $T, \mu$ , quarks and gluons are weakly coupled (asymptotic freedom)
- ▶ Chiral and deconfinement phase transitions:
  - ▶ Theory: extremely challenging (strong coupling, strong interactions, sign problem at  $\mu \neq 0$ )
  - ▶ Experiment: Two laboratories, **Heavy ion collisions** and **compact stars/magnetars**

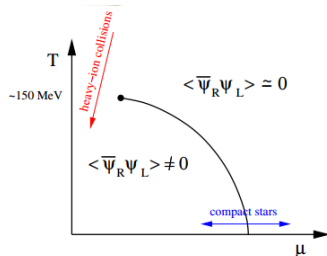
Naive phase diagram:



# Understanding QCD phase transitions/phase diagram: (strong) magnetic fields

Two "experimental laboratories" w/ strong magnetic fields

- ▶ large  $T \gtrsim T_c$ , small  $\mu \ll T$ : (non-central) relativistic heavy-ion collisions w/ magnetic field  $\perp$  to collision plane  $\sim 10^{18} G \approx (140 MeV)^2$ .
- ▶ large  $\mu \sim 400 MeV$ , small  $T \ll \mu$ : magnetars  $\gtrsim 10^{15} G$  (surface) to  $\sim 10^{18} G$  (interior).
- ▶  $\Lambda_{QCD} \sim (200 MeV)^2 \sim 2 \approx 10^{18} G \Rightarrow$  effect on QCD phase transitions ?



# QCD at finite $T$ , $\mu$ , $B$ : Chiral symmetry (breaking), $\chi$ SB

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{L+R}$$

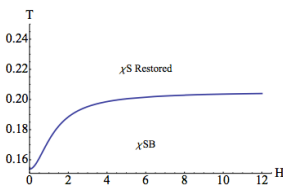
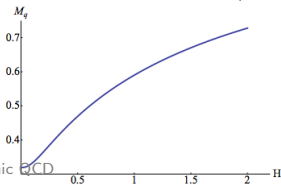
- ▶ Quark masses  $m_q$  break chiral symmetry explicitly
- ▶ Chiral condensate  $\langle \bar{\psi}_L \psi_R \rangle$  breaks  $\chi$ S spontaneously.
- ▶ Real world:  $m_q \neq 0$ , crossover at  $\mu = 0$ , first order transition at  $\mu > 0$ ?
- ▶ Theoretically: Use  $B$  as another tuneable parameter that may provide clues to understand phase transitions

# Magnetic Catalysis of Chiral Symmetry Breaking ( $\mu = 0$ )

Review: V. Filev, R. Rashkov, Magnetic Catalysis of Chiral Symmetry Breaking: A holographic perspective (2010).

- ▶ Universal effect in many field theoretical contexts/models:  
The presence of an external magnetic field  $B$  leads to an effective **dimensional reduction** of the dynamics from  $D \rightarrow D - 2$ .
- ▶ Landau quantization  $E(k_{\perp}, n) = \sqrt{k_{\perp}^2 + 2|q|Bn + M_q(B)^2}$ .
- ▶ Typically lowest Landau level (LLL) dominates (higher LL decouple for sufficiently large magnetic fields).
- ▶ Result: Dim. reduction generically enhances particle - anti-particle pairing/chiral condensate  $\langle \bar{\psi}_L \psi_R \rangle$ , responsible for  $\chi$ SB.

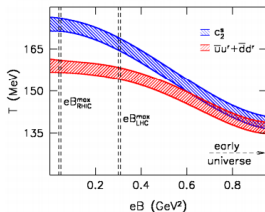
E.g., in holography: Magnetic field enhances  $M_q$  and  $T_c$  (Johnson, Kundu JHEP 0812, 053 (2008))



# Inverse Magnetic Catalysis

Recent result:

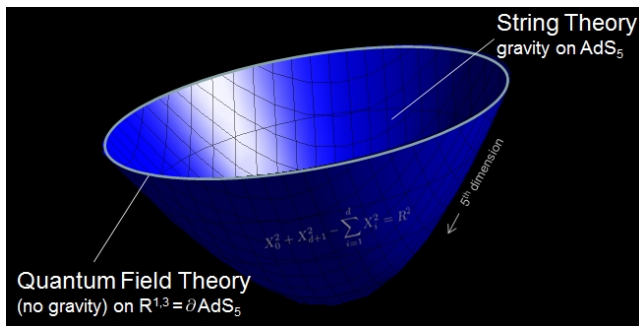
- ▶ Inverse magnetic catalysis: lattice results indicate decrease of  $T_c$  with  $B$  Bali et al. 1111.4956 [hep-lat]  
(Also, e.g. NJL model calculations)



- ▶ Underlying physics not well understood!
- ▶ In holographic QCD (at finite  $T$ ,  $B$ ,  $\mu$ ), IMC can be observed and (partially) understood.

# What is holography?

- ▶ Holography refers to a **duality** between a string theory (in the bulk) and a field theory (on the boundary) .
- ▶ Compare to **hologram**: 3D information stored in 2D picture.
- ▶ Original example: **Maldacena duality** (conjecture).  $\mathcal{N} = 4$  SYM in  $3 + 1$  dim. is dual to type IIB strings on  $AdS_5$ .
- ▶ Strong-weak coupling duality.



# What is holography? AdS/CFT

- ▶ AdS/CFT: Classical (**weakly-coupled**) theory of **gravity** e.g. IIB SUGRA on  $AdS_5 \times S^5 \Leftrightarrow$  **Strongly-coupled field theory** e.g.  $\mathcal{N} = 4$  SU(N) SYM.
- ▶ Allows us to compute strongly-coupled field theory properties from gravity
- ▶ AdS/CFT Dictionary: Field theory  $\Leftrightarrow$  Gravity
  - ▶ Operators  $\mathcal{O}$ , e.g.  $T^{\mu\nu}, J^\mu$  (global sym.)  $\Leftrightarrow$  Fields  $\varphi$ , e.g.  $g_{\mu\nu}, A_\mu$
  - ▶ Expectation values  $\langle \mathcal{O} \rangle \Leftrightarrow \lim_{r \rightarrow \infty} \varphi = J + \frac{\langle \mathcal{O} \rangle}{r} + \dots$
  - ▶ Equilibrium state  $\Leftrightarrow$  Solutions to EOMs

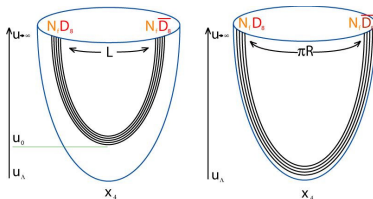
# Towards (large $N_c$ ) QCD

- ▶ Original Maldacena duality:  $\mathcal{N} = 4$  SYM, highly symmetric.
- ▶ Break some (or all) of SUSY and conformal sym. e.g., introduce non-zero temperature.
- ▶ Add flavor degrees of freedom  $\implies$  Fundamental matter, quarks, mesons.  
Probe limit:  $N_f \ll N_c$  flavor branes, no backreaction on geometry, “quenched” approx. [Karch, Katz hep-th/0205236]
- ▶ Semi-realistic models of chiral symmetry breaking, deconfinement transition: e.g., Sakai-Sugimoto, [hep-th/0412141]

# Sakai-Sugimoto model of holographic QCD

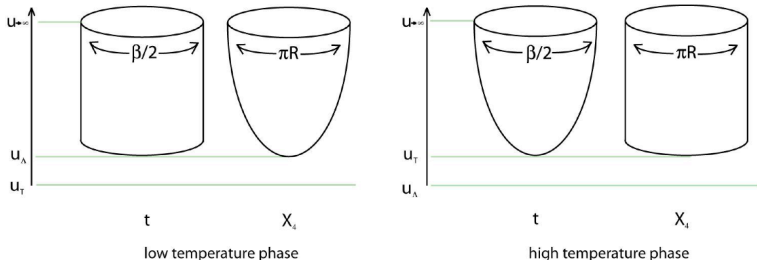
## Introduction [Aharony, Sonnenschein, Yankielowicz hep-th/0604161]

- ▶ String theory dual to a field theory with **chiral symmetry breaking** at  $T = 0$ .
- ▶ IIA string theory on circle  $x^4$ ,  $N_c$  D4-branes filling 01234-directions,  $N_f$  D8-branes at  $x_4 = 0$ ,  $N_f$   $\overline{D8}$ -branes at  $x_4 = L$ ,  $\frac{du}{dx_4} = 0$  at  $u = u_0$ , U-shaped embedding  $x_4(u)$ .
- ▶ Global chiral symmetry visible as gauge theory on  $D8 - \overline{D8}$ -branes.
- ▶ Massless spectrum of 3+1  $U(N_c)$  gauge theory coupled to  $N_f$  massless fermions.



# Sakai-Sugimoto model of holographic QCD

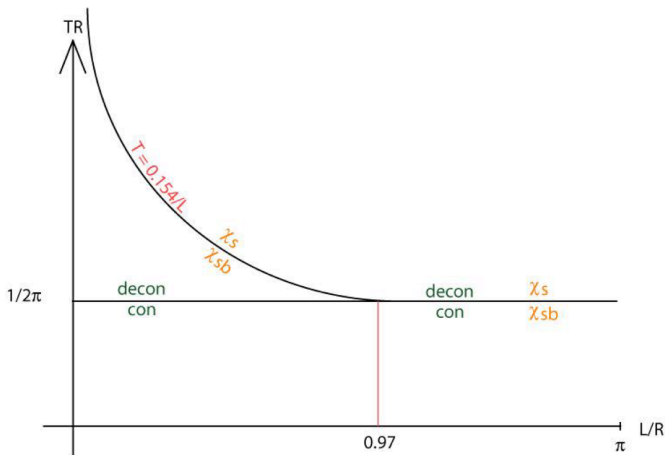
Finite temp.: confinement/deconfinement transition



# Sakai-Sugimoto model of holographic QCD

## Phase diagram (T-L)

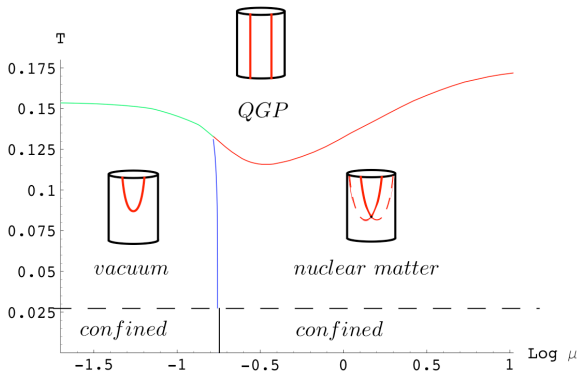
O. Aharony, J. Sonnenschein and S. Yankielowicz, [hep-th/0604161].



# Sakai-Sugimoto model of holographic QCD

## Phase diagram ( $T-\mu$ )

O. Bergman, G. Lifschytz and M. Lippert, arXiv:0708.0326 [hep-th].



# Deconfined phase of Sakai-Sugimoto model with external fields

Specific cases studied by: Bergman, Lifshitz, Lippert (2007, 2008, 2009); Johnson, Kundu (2008)

- ▶ induced metric on  $D8$  branes:

$$ds_{D8}^2 = \left( \frac{U}{R_{D4}} \right)^{3/2} [f(U) d\tau^2 + d\vec{x}^2] \\ + \left( \frac{R_{D4}}{U} \right)^{3/2} \left\{ \left[ f(U)^{-1} + \left( \frac{U}{R_{D4}} \right)^3 X_4'^2 \right] dU^2 + U^2 d\Omega_4^2 \right\}$$

- ▶ study DBI + CS actions and resulting e.o.m.s of flavor  $D8$ -brane embeddings and world volume fields (details omitted for brevity):

$$S_{DBI} = T_8 V_4 \int d^4x \int_{u_T}^{\infty} e^{-\Phi} \sqrt{\det(g_8 + 2\pi\alpha' F)}$$

$$S_{CS} = T_8 \int_{D8} C_3 (2\pi\alpha' F)^3 = \frac{N_c}{24\pi^2} \int d^4x \int_{u_T}^{\infty} du \epsilon_{\mu\nu\rho\sigma} A_\mu F_{\nu\rho} F_{\sigma\mu}$$

# Deconfined phase of Sakai-Sugimoto model with external fields

- ▶  $\mu = 0, B \neq 0$ : The critical temperature increases with  $B$ , S-S-model exhibits magnetic catalysis.
- ▶ **General case**: non-vanishing field strength components  $F_{u0}, F_{u3}, F_{12} = B$ .
- ▶ Dimensionless fields ( $t = TR, \ell = L/R$ ):  
 $a_{0,3} = \frac{2\pi\alpha'}{R} A_{0,3}, b = 2\pi\alpha' B, u = \frac{U}{R}, x_4 = \frac{X_4}{R}.$
- ▶ **Asympt. values**:  $a_0(\infty) = \mu, a_3(\infty) = j, x_4(\infty) = \ell/2.$
- ▶ Resulting (integrated) e.o.m.s:  $f(u) = 1 - \frac{u_T^3}{u^3}$ 
  - ▶  $\frac{\sqrt{u^5 + b^2 u^2} a'_0}{1 + u^3 f x_4'^2 - a_0'^2 + f a_3'^2} = 3ba_3 + c$
  - ▶  $\frac{\sqrt{u^5 + b^2 u^2} f a'_3}{1 + u^3 f x_4'^2 - a_0'^2 + f a_3'^2} = 3ba_0 + d$
  - ▶  $\frac{\sqrt{u^5 + b^2 u^2} f u^3 x'_4}{1 + u^3 f x_4'^2 - a_0'^2 + f a_3'^2} = k$

# Deconfined phase of Sakai-Sugimoto model with external fields

- ▶ Different boundary conditions (and simplifications) for  $\chi$ SB and  $\chi$ SR configurations, e.g.,  
 $\chi$ SR:  $x_4' = 0 \rightarrow k = 0$ , only 2 remaining equations;  
 $\chi$ SB: Approximation  $f(u) \sim 1 \leftrightarrow u_0 \gg u_T$ .  
 This is the case for which PRS applied semi-analytic methods:

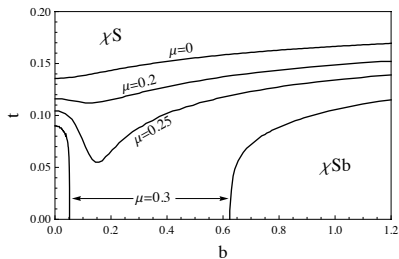
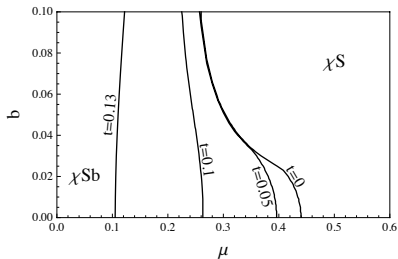
Preis, Rebhan, Schmitt (2011, 2012)

- ▶ Zero temperature limit:  $u_T \rightarrow 0$ . Since  $T > T_c \sim M_{KK} \sim R^{-1}$ , this is simultaneously a decompactification limit  $R \rightarrow \infty$  (with  $L$  fixed).
- ▶ Small  $L$  limit (large  $m_q$  mass):  $L \rightarrow 0$ ,  $R$  fixed, equivalent to decompactification limit.
- ▶ We perform general numerical analysis **without** going to the above-mentioned limit.

## Results: Low temperature regime

Preis, Rebhan, Schmitt (2011, 2012)

- ▶ Compute and compare free energies of  $\chi$ Sb and  $\chi$ SR configurations for a given set of parameters:  $\Delta\Omega = \Omega_{||} - \Omega_{\perp}$ .
- ▶ **Main observation:** There is a region of parameter space for which **IMC** occurs!
- ▶ 2D slices of the 3D phase space:



## Side remark: Towards understanding of IMC

- ▶ If IMC can happen is model dependent.
- ▶ Naively:  $B$  increases free energy gain from  $\bar{\psi}\psi$ -pairing (MC),  $\mu$  induces free energy cost  $\sim \mu^2 B$ .
- ▶ In weak coupling limit (e.g. NJL), or S-S at large  $B$ :

$$\Delta\Omega \propto B \left( c_{mod}.M_q(B)^2 - \mu^2 \right), \mu^2 < c_{mod}.M_q(B)^2$$

$\implies$  IMC not possible

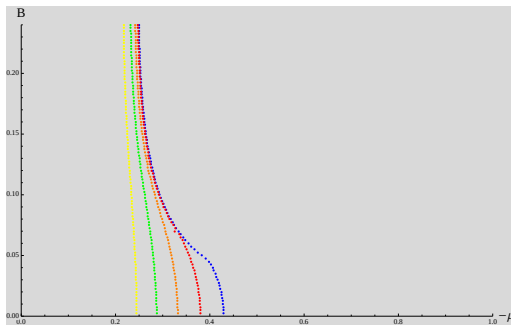
- ▶ S-S model at small  $B$ :

$$\Delta\Omega \propto c_{mod}.M_q(B)^{7/2} - \mu^2 B, M_q(B) \sim \text{const.} + \mathcal{O}(B^2)$$

$\implies$  IMC possible

## Preliminary results: Numerical analysis

- ▶ We generally find 10-20 % corrections to the zero (low) temperature results of PRS.
- ▶ We studied in detail a phase transition within the  $\chi$ SB phase (with a richer phase structure than at  $T = 0$ , PRS), but “unfortunately” only a **metastable** phase!



## Conclusions and Outlook

- ▶ Difficult numerics, no BIG news (yet).
- ▶ We can now easily repeat the full numerical analysis in other holographic models (e.g. non-critical  $AdS_{6,\dots}$ )
- ▶ There is a scaling symmetry  $t \rightarrow t\ell = TL$  ( $b \rightarrow b\ell^3, \mu \rightarrow \mu\ell^2$ ), under which the Hamiltonians are invariant. Changing  $L$  does not seem to change the phase structure in any interesting way. **However:** Relationship between bare quark mass, the chiral condensate and  $L$  somewhat non-standard in S-S model.
- ▶ Computation of magnetic susceptibility: Some indications that magnetic susceptibility changes drastically ( $T$ -dependence, change of sign) across phase transition !!?

Thank you!