Extremal Vanishing Horizon Black Holes

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Based on

- S. S. , M.M. Sheikh-Jabbari, M.H. Vahidinia, H. Yavartanoo, *Physics Letters* **B753** (2016)
- S. S. , H. Yavartanoo, *Class.Quant.Grav.* **33** (2016)
- S. S. , M.M. Sheikh-Jabbari, M.H. Vahidinia, H. Yavartanoo, *Nuclear Physics* **B 900** (2015)
- S. S. , M.M. Sheikh-Jabbari, H. Yavartanoo, *JHEP* **1410** (2014)

Black Hole Thermodynamics

Stationary Black Hole is a thermodynamic system

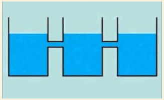
temperature (T) ↔ surface gravity (κ)

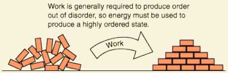
entropy (S) ↔ horizon area (A)

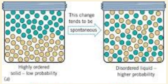
laws of black hole dynamics

- surface gravity is constant on the horizon.
- $dM = \kappa dA + \Omega_i dJ^i$
- $\frac{dA}{dt} \geq 0$

How about third law ?

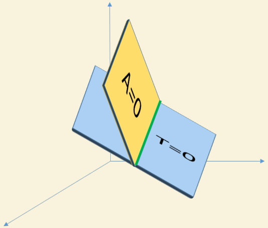






Extremal and EVH Black Holes

- Extremal black holes
  - have vanishing temperature (T)
  - are *unusual* under third law of thermodynamics ($S_0 \neq 0$)
- Extremal Vanishing Horizon (EVH) black holes
  - obey the third law of thermodynamics ($S_0 = 0$)
  - have vanishing temperature and area (A)



Near Horizon Extremal Geometry

- Changing coordinates $(r - r_h) \to \lambda r$
  (and all other needed changes)
- Taking the limit $\lambda \to 0$

on an Extremal Black hole geometry gives

- a geometry which is also a solution.

AdS_2 in near horizon of extremal black hole

Theorem

Consider an extremal black hole solution of **Einstein Maxwell Dilaton theory** in D=4,5 with $R \times U(1)^{(D-3)}$ symmetry. The near-horizon limit of this solution has $G \times U(1)^{(D-3)}$ symmetry, where G is either SO(2, 1) or the 2D Poincare group.

Theorem

Consider a D-dimensional spacetime containing a degenerate horizon, invariant under an $R \times U(1)^{(D-3)}$ isometry group, and satisfying the **Einstein equations** $R_{\mu\nu} = \Lambda g_{\mu\nu}$. Then the near-horizon geometry has $G \times U(1)^{(D-3)}$ symmetry, where G is either $O(2, 1)$ or the 2d Poincare group.

H. K. Kunduri, J. Lucietti,H. S. Reall, *Class.Quant.Grav.* **24** (2007)

AdS_3 in near horizon of EVH black hole

Theorem *

Near horizon of 4D EVH solution of Einstein-Maxwel-Dilaton theory contains an AdS_3 factor.

How about

- higher dimenstions?
- other theories?

* M. M. Sheikh-Jabbari and H. Yavartanoo, *JHEP* **1110** (2011)

EVH black holes are interesting since...

Geometry

Presence of AdS_3 space allows us to classify solutions.

(thermo)dynamics

In near-EVH case, dynamical mode can exist.
(in contrast to no-dynamics for generic extremal case)

AdS_3/CFT_2

There is a complete CFT_2 dual to these solutions
rather than a chiral CFT in generic Extremal black hole.

Outline

- **E**xtremal **V**anishing **H**orizon **(EVH)** Black Hole
- **Theorems** on Near Horizon of EVH
- **Classification** of Solutions with $SO(2,2)$ Symmetry

***E**xtremal **V**anishing **H**orizon Black Hole*

- EVH Definition
- Examples of EVH
- EVH in Gaussian Null coordinates

EVH Definition

- Extremal Vanishing Horizon (EVH) Black Holes
  - have vanishing **temperature** (T) and **area** (A).
  - A/T remains finite as $T \to 0$ and $A \to 0$.
  - have a vanishing one cycle in a Killing direction.

Examples of EVH

- (3d) BTZ

Generic BTZ

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2\, (d\varphi - \frac{r_+ r_-}{\ell\, r^2} dt)^2, \qquad f(r) = \frac{(r^2 - r_+^2)\,(r^2 - r_-^2)}{r^2\, \ell^2}$$

$$T = \frac{r_+^2 - r_-^2}{2\pi r_+ \ell^2}, \qquad A_h = 2\pi\, r_+, \qquad \text{EVH} : r_+ - r_- \sim \epsilon, \qquad r_+ \sim \epsilon$$

- (4d) rotating KK black hole

Examples of EVH

- (5d) Myers Perry

$$S = \frac{\pi M}{G_5} r_+ \,, \qquad T = \frac{r_+^4 - a^2 b^2}{4\pi M r_+^3}$$

Extremal case

$$M = \frac{1}{2}(a^2 + b^2)\,, \qquad r_+ = \sqrt{ab}$$

EVH case : $a = 0$

$$\begin{aligned} ds^2 &= -\frac{r^2 - b^2\cos^2\theta}{r^2 + b^2\sin^2\theta} dt^2 + (r^2 + b^2\sin^2\theta)(\frac{dr^2}{r^2} + d\theta^2) + r^2 \sin^2\theta d\phi^2 \\ &+ \frac{(r^2+b^2)^2 - r^2 b^2\cos^2\theta}{r^2 + b^2\sin^2\theta}\cos^2\theta d\psi^2 - \frac{2b^3\cos^2\theta}{r^2 + b^2\sin^2\theta} dt d\psi \,. \end{aligned}$$

Examples of EVH

- (5d)
  - Black ring (Balanced/Unbalanced)
  - 1 Dipole double rotating Black Ring
  - Black Hole solutions to N=2 SUGRA (Wu)
  - 3 Dipole single rotating Black Ring solution to N=2 SUGRA
- (d=2n+1) Myers Perry

EVH case:

$$a_1 = 0 \,, \qquad \mu = \prod_{i=2}^{n} a_i$$

EVH in Gaussian Null coordinates

- $ds^2 = -r\, \mathsf{F}\, dv^2 + 2\, dv\, dr + r\, \mathsf{f}_i\, dy^i\, dv + h_{ij}\, dy^i dy^j$

EVH conditions

- $T \propto \mathsf{F}\,\Big|_{r=0} = \epsilon\, F^{(1)}$
- $A_h = \oint_h \sqrt{det\,(h_{ij})} \sim \epsilon$
- $h_{ij}\, dy^i dy^j = G\, d\phi^2 + 2\, g_a\, d\phi\, dx^a + \gamma_{ab}\, dx^a dx^b$

$$det\,(h_{ij}) = \begin{vmatrix} G & g_a \\ g_a & \gamma_{ab} \end{vmatrix} = G\,\gamma - g_a\, g_b\, \gamma^{ab}$$

- $G\,\Big|_{r=0} = \epsilon^2\, G^{(2)}\,,\ \ g_a\,\Big|_{r=0} = \epsilon\, g_a^{(1)}$

- ϵ is near EVH parameter.
- Horizon is located at $r = 0$.
- ∂_ϕ is the vanishing Killing direction.

Theorems on Near Horizon of EVH

- Theorems on Near Horizon EVH
- Theorem on Near Horizon Near-EVH

Near Horizon Geometry of EVH

- EVH metric ansatz
  - $ds^2 = -r\,\mathsf{F}\,dv^2 + 2\,dr\,dv + 2\,r\,\mathsf{f}_i\,dy^i dv + \mathsf{h}_{ij}\,dy^i dy^j$
  - $\mathsf{h}_{ij}\,dy^i dy^j = \mathsf{G}\,d\phi^2 + 2\,\mathsf{g}_a\,d\phi\,dx^a + \hat{\gamma}_{ab}\,dx^a dx^b$
  - $\mathsf{f}_i\,dy^i = \mathsf{H}\,d\phi + f_a\,dx^a$
- Near Horizon is defined through
  $r \to \lambda\,r, \qquad v \to \frac{v}{\lambda}, \qquad \phi \to \frac{\phi}{\lambda}\,, \qquad \lambda \to 0$
- Double expansion in powers of ϵ and λ
  - $\mathsf{F}(r,y) = \epsilon\,F^{(1)} + \lambda\,r\,F(x^a) + \cdots$
  - $\mathsf{G}(r,y) = \epsilon^2\,G^{(2)}(x^a) + \epsilon\,\lambda\,r\,G^{(1)}(x^a) + \lambda^2\,r^2 G(x^a) + \cdots$
  - ...

Near Horizon Geometry of EVH

$$
\begin{aligned}
ds^2 = &-r\,\left(\frac{\epsilon}{\lambda}\;F^{(1)} + r\;F\right)dv^2 + 2\;dr\;dv \\
&+2\;r\;\left(\frac{\epsilon}{\lambda}\;H^{(1)} + r\;H\right)d\phi\;dv + 2\;r\,f_a\;dx^a dv + \gamma_{ab}\;dx^a dx^b \\
&+\left(\frac{\epsilon^2}{\lambda^2}\;G^{(2)} + \frac{\epsilon}{\lambda}\;r\;G^{(1)} + r^2\;G\right)d\phi^2 + 2\;\left(\frac{\epsilon}{\lambda}\;g_a^{(1)} + r\;g_a\right)dx^a d\phi
\end{aligned}
$$

- $\frac{\epsilon}{\lambda} \ll 1$ Near horizon EVH geometry
- $\frac{\epsilon}{\lambda} \sim 1$ Near horizon near EVH geometry
- $\frac{\epsilon}{\lambda} \gg 1$ ...(trivially) Rindler space...

Near Horizon Geometry of EVH

- $\frac{\epsilon}{\lambda} \ll 1$:

NHEVH metric :

$$g_{\mu\nu} = \begin{pmatrix} -r^2 F & 1 & r^2 H & r f_a \\ 1 & 0 & 0 & 0 \\ r^2 H & 0 & r^2 G & r g_a \\ r f_a & 0 & r g_a & \gamma_{ab} \end{pmatrix}$$

- This is given by a process on both parameter space and spacetime.
- ∂_v and ∂_ϕ are Killing.
- r-dependence is completely fixed.
- Setting $\phi = const.$, we recover NHEG.

Implications of smoothness and Einstein equations

Lemma 1

Finiteness of $T_{\mu\nu}$ implies $T_{rr} = T_{ra} = 0$.

Lemma 2

Einstein equations imply $g_a = 0$ and $f_a = G^{-1}\partial_a G \equiv 2\,\partial_a K$.

Using $\rho = r\,e^{2K}$, NHEVH metric reduces to

$$ds^2 = e^{-2K}\left[\rho^2\,(-\tilde{F}\,dv^2 + 2\,\tilde{H}\,dv\,d\phi + d\phi^2) + 2\,dv\,d\rho\right] + \gamma_{ab}\,dx^a dx^b$$

where $\tilde{H} = e^{-2K}\,H$, $\tilde{F} = e^{-2K}\,F$.

Implications of Einstein equations

Lemma 3

For Einstein Maxwell scalar theories, $\tilde{H}$ and $\tilde{F}$ are constants.

Theorem 1

The near-horizon geometry of any EVH black hole in Einstein Maxwell scalar theories with a finite energy momentum tensor $T_{\mu\nu}$ at the horizon, is given by

$$ds^2 = e^{-2K}\left[A_0\,\rho^2 dv^2 + 2\,dv\,d\rho + \rho^2 d\phi^2\right] + \gamma_{ab}\,dx^a dx^b;$$
$$A_0 = -(\tilde{H}^2 + \tilde{F})$$

the 3d (ρ, v, ϕ) part of the metric is maximally symmetric.

Implications of Strong Energy condition

Strong Energy Condition (SEC)

For any time-like vector field t^μ, SEC implies

$$\left(T_{\mu\nu} - \frac{T}{(d-2)} g_{\mu\nu}\right) t^\mu t^\nu \geq 0$$

Using Einstein equations, SEC reads

$$\left(R_{\mu\nu} - \frac{2\Lambda}{(d-2)} g_{\mu\nu}\right) t^\mu t^\nu \geq 0$$

$$R_{\mu\nu} = \left[e^{-2K}\left(\nabla^2 K - 3\,(\nabla K)^2\right) + 2\,A_0\right] \tilde{g}_{\mu\nu}, \quad \mu, \nu = v, r, \phi$$

Theorem2

Strong energy condition implies the 3d part of NHEVH with $\Lambda \leq 0$ is either flat or AdS_3. Flat case can only occur for $\Lambda = 0$.

Near Horizon Geometry of near-EVH

- $\frac{\epsilon}{\lambda} \sim 1$ near EVH case
  - Finiteness of $T_{\mu\nu}$
  - Einstein Equations
  - Strong Energy Condition

$$\mathsf{F}(r, y) = \epsilon\, F^{(1)} + r\, \lambda\, F(x^a) + \cdots$$

Theorem 3

The 3d part of near horizon of a near-EVH black hole with non-positive cosmological constant is either a BTZ black hole or a rotating massive particle on the flat spacetime.

Summary

Theorem1

Finiteness of $T_{\mu\nu}$ at the horizon for Einstein Maxwell scalar theories, implies that NHEVH is given by

$$ds^2 = e^{-2K}\left[A_0\rho^2 dv^2 + 2dvd\rho + \rho^2 d\phi^2\right] + \gamma_{ab}dx^a dx^b,\ (A_0\text{=const.})$$

the 3d (ρ, v, ϕ) part of the metric is maximally symmetric.

Theorem2

Strong energy condition implies the 3d part of NHEVH with $\Lambda \leq 0$ is either flat or AdS_3. Flat case can only occur for $\Lambda = 0$.

Theorem3

The 3d part of near horizon of a near-EVH black hole with non-positive cosmological constant is either a BTZ black hole or a rotating massive particle on the flat spacetime.

Classification of Solutions with SO(2,2) Symmetry

- 4 dimensions
- 5 dimensions
- higher dimensions

Implications of SO(2,2) Symmetry

The only invariant tensors of SO(2,2) are $g_{\mu\nu}^{(AdS_3)}$ and $\epsilon_{\mu\nu\rho}$.

- Generic form of SO(2,2) invariant geometry in **d-dim**

$$ds^2 = e^R g_{\mu\nu}^{(AdS_3)} dx^\mu dx^\nu + g_{ij}\, dx^i dx^j$$

- R , g_{ij} are just functions of x^i.
- Geometry is necessarily **static**.
- Scalar field is just a function of x^i.
- $F_{\mu\nu}$ and $F_{\mu i}$ should be zero and $F_{ij} = F_{ij}(x^k)$

4d solutions with SO(2,2) symmetry

Action

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left(R - 2\partial_\mu \Phi \partial^\mu \Phi - \sum_I \mathcal{F}_I(\Phi)(F^I)^2 - V(\Phi) \right)$$

Metric

$$ds^2 = e^{f(\theta)}(ds_3^2 + \beta^2 d\theta^2), \quad ds_3^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + r^2 d\varphi^2$$

Matter

$$F^I_{\mu\nu} = 0, \qquad \Phi = \Phi(\theta)$$

5d solutions with SO(2,2) symmetry

Metric and Matter Ansatz

- $ds^2 = e^{2R} ds_3^2 + e^{2f} d\theta^2 + e^{2h} d\psi^2$
- $F^I_{\mu\nu} = F^I_{\theta\psi}(\theta, \psi), \quad \Phi = \Phi(\theta, \psi)$

where R, f and h are functions of both θ and ψ.

A Rigidity Theorem

All SO(2,2) invariant solutions to 5d Einstein-Maxwell-Λ theory have *necessarily* an extra $U(1)$ isometry.

Summary

dimensions	SO(2,2) inv. solution	N.H. EVH of
4D	EMD	rotating K.K.
4D	Scalar+Potential	———
5D	Pure Einstein	MP & Black Ring
5D	Einstein-Λ	AdS-MP
5D	Einstein-Maxwell-Λ	Black String
5D	Einstein-Dilaton	———
5D	EMD	rotating K.K.(*)
5D	$U(1)^3$ SUGRA	Wu(**) & Black Ring
(n+4)D	Pure Ein. with $U(1)^n$	———
(n+3)D	Pure Ein. with $SO(n)$	———
(2n+3)D	Pure Ein. with $U(1)^n$	MP

(*) Jutta Kunz, Dieter Maison, Francisco Navarro-Lerida, Jan Viebahn, Phys.Lett.B639:95-100,2006

(**) S. Q. Wu, Phys. Lett.B707, 286 (2012)

Conclusion

- Extremal Vanishing Horizon (EVH) Black Holes do exist in various theories and arbitrary dimensions.
- Like Extremal black hole, we can always find the near horizon geometry of EVH black holes.
- Symmetry enhancement in the near horizon EVH (NHEVH) geometries is larger than NHEG case.
- Under sufficient conditions, an AdS_3 space would appear in NHEVH geometries.
- Near horizon near EVH metric contains a BTZ subspace. So we expect (thermo)dynamics around EVH geometries.