Bjorken flow in Holographic First Order Phase Transition

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In collaboration with:
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Outline

- Motivation and Introduction
- Gravity setup dual to non-CFT
- Linearized Gravity (QNMs)
- Dynamics of 1st order phase transition
- Boost invariant expansion
- Summary and Future directions

Motivation and Introduction

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→ The AdS/CFT has built a bridge between problems from the Quantum world and methods from General Relativity.

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→ Linearized Einstein EoMs for gauge invariant perturbations identifies the dispersion relation $\omega(k)$ =?

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2-Full time evolution of a given initial configuration

→ Solving the time dependent Einstein EoMs

→ reading the data corresponds to the dual FT from near boundary expansion(like 1-pt functions)

Gravity Setup dual to non-CFT

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2-Bottom-up: (a bit of engineering)

→ Assuming AdS/CFT works

→ Model the gravity+matter with a potential $V(\varphi)$

→ We may choose $V(\varphi)$ such that it reproduces the physics of interest
(like l-QCD EoS, 1st or 2nd order phase transitions)

Gravity Setup dual to non-CFT

The simplest setup dual to $\mathcal{L}_{\rm CFT} + \Lambda^{d-\Delta} < O_\phi >$

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$$\rightarrow \quad S = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x \sqrt{|g|} \left[R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

Gubser, Nellore '08 & Gursoy, Kiritsis, Mazzanti, Nitti '09

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→ The potential has a general form:

$$V(\phi) = -d(d-1)\cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4$$

$$\sim -d(d-1) + \frac{m^2}{2}\phi^2 + \mathcal{O}(\phi^4), \qquad m^2 = \Delta(\Delta - d)$$

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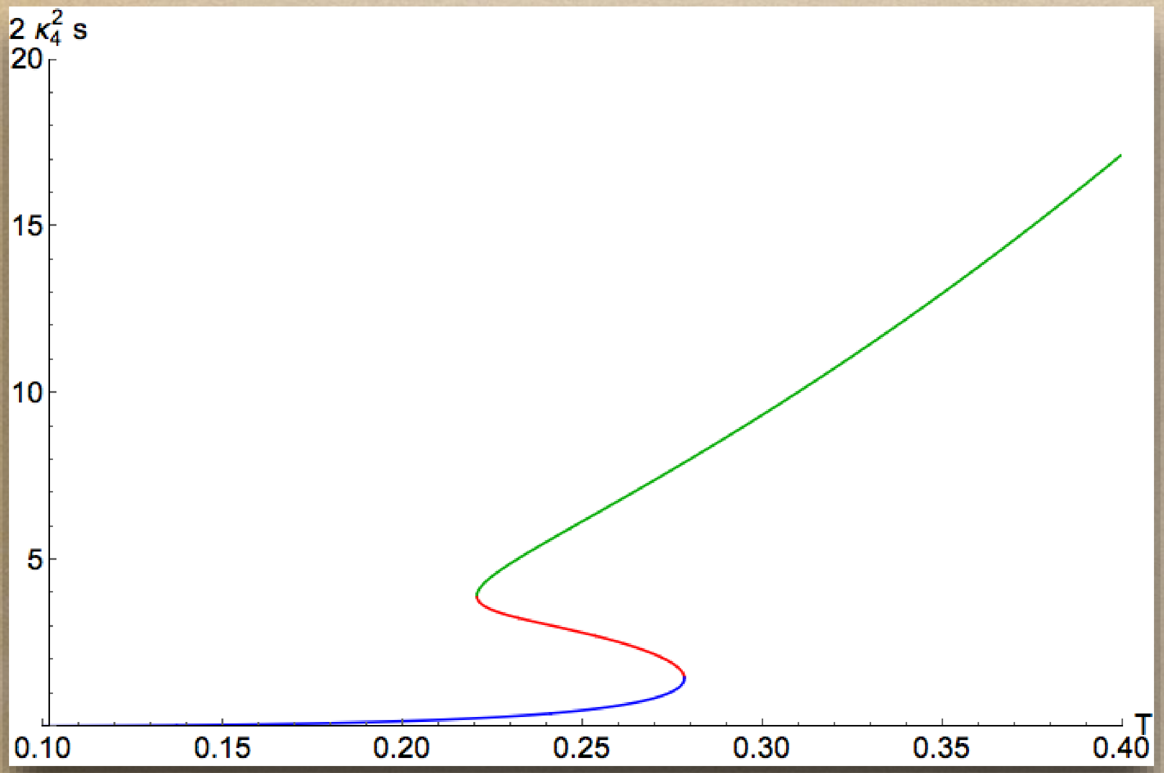
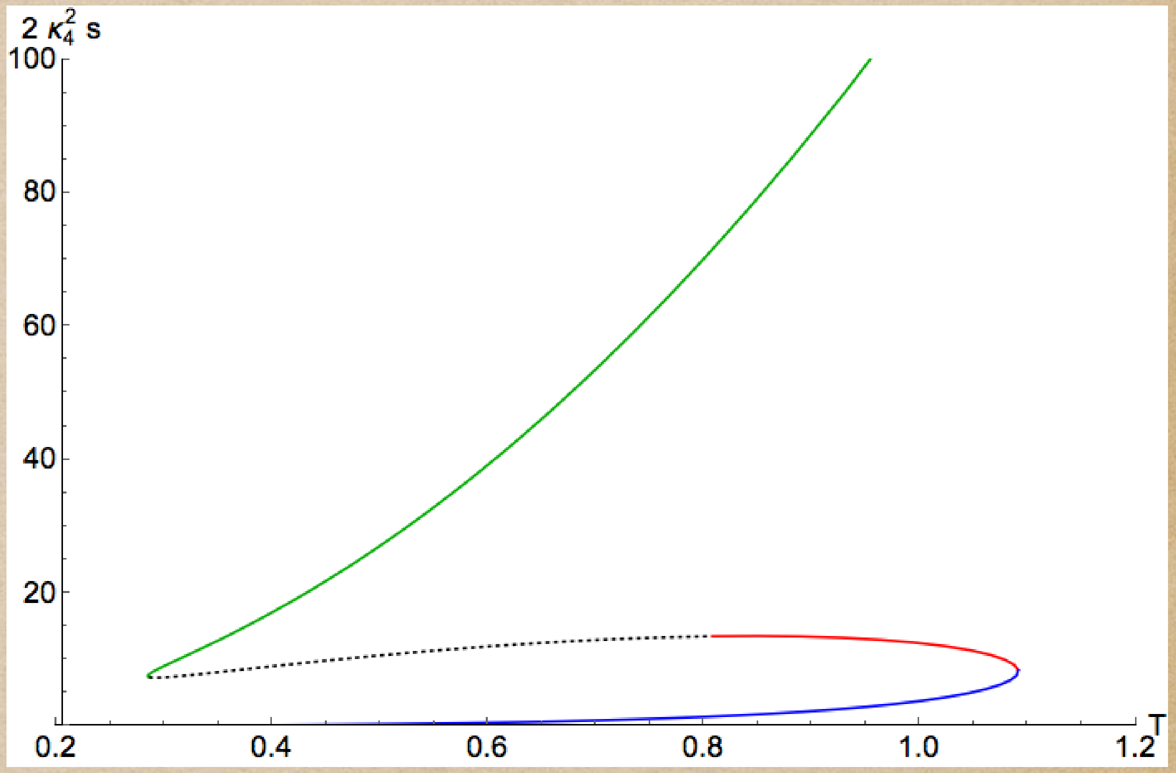
→ Equation of State: $s(T)\ , \qquad T_{ij}(T)\ , \qquad {C_s}^2 = \frac{s}{T}\,(ds/dT)^{-1}$

Gravity Setup dual to non-CFT

Examples: 3+1-gravity/CFT$_4$ ($\Delta = 2$)

$V_1 = -6\cosh(\phi/\sqrt{3}) - 0.2\,\phi^4,$ $\qquad$ $V_2 = -6\cosh(\phi/\sqrt{3}) - 0.3\,\phi^4$





Stable regimes:
Green: Large BH
Blue: Small BH

Unstable regimes:
Red: thermodynamic instability
Black-dotted: Dynamical instability

Linearized Gravity
Quasi Normal Mods (QNM)

Linearized Gravity (QNMs)

QNM equations: linearized Einstein equations $(\delta\Phi_i \sim e^{-i\omega t+ikx}\,\Psi_i(r))$

$$EoM(g_{\mu\nu} + \varepsilon\, h_{\mu\nu}, \varphi + \varepsilon\, \psi) = EoM(g_{\mu\nu}, \varphi) + \varepsilon\, QNM_{eqs}$$

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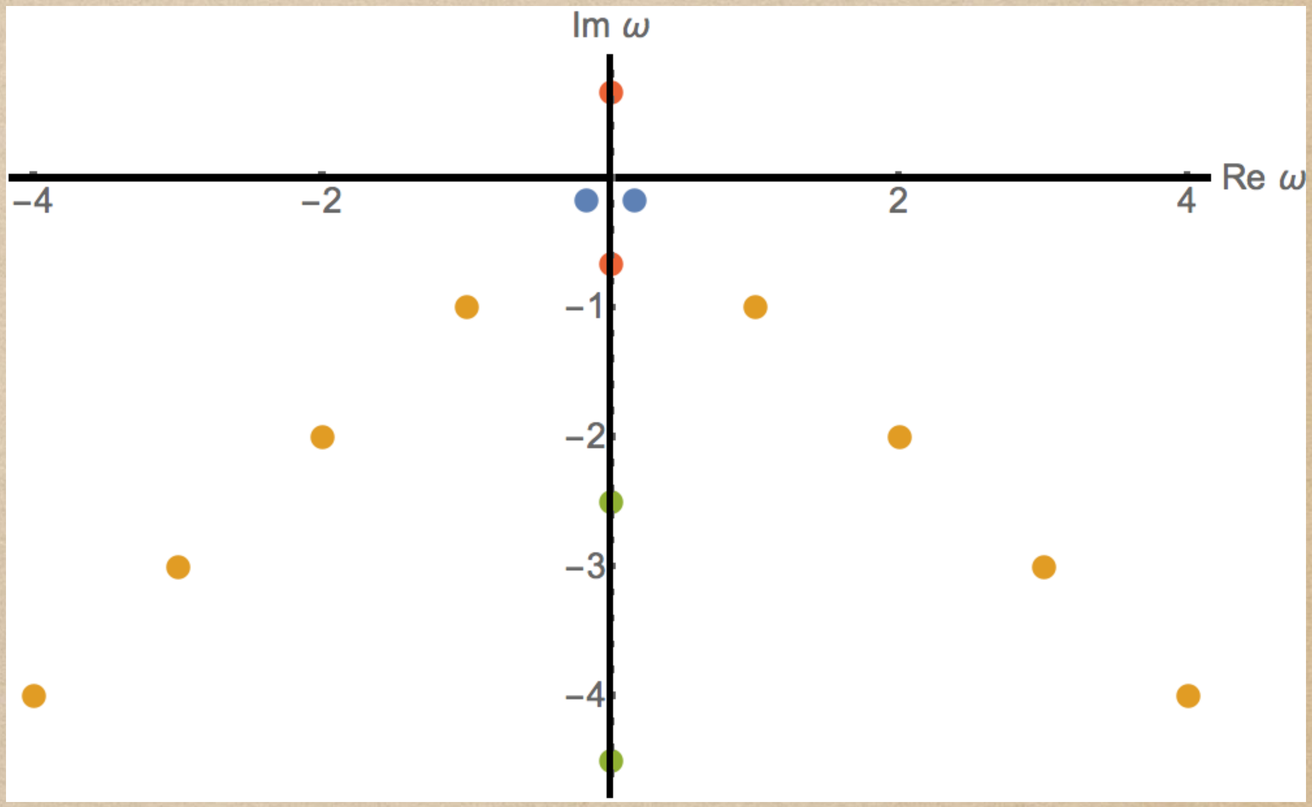
Typical structure of the QNMs in AAdS (k= finite)



Red: Unstable mode
Green: Diffusive modes
Yellow: Non-hydrodynamic modes
Blue: Hydrodynamic modes, $\omega_{(k\to 0)} \to 0$

Linearized Gravity (QNMs)

New features in non-CFT cases:

→ Sign for bubble formation in 1st order phase transitions!

Janik, Jankowski, HS, PRL '16

→ Dynamical instability: A thermodynamically stable black hole MAY suffers an unstable non-hydro mode!

Janik, Jankowski, HS, PRL '16
Gursoy, Jansen, Wilke, PRD '16

→ Crossing between hydro and nonhydro modes!
(limits applicability of hydrodynamics)

Janik, Jankowski, HS, JHEP '16, ...

→ Joining modes: Hydro modes and nonhydro modes might be indistinguishable!

Janik, Jankowski, HS, JHEP '16

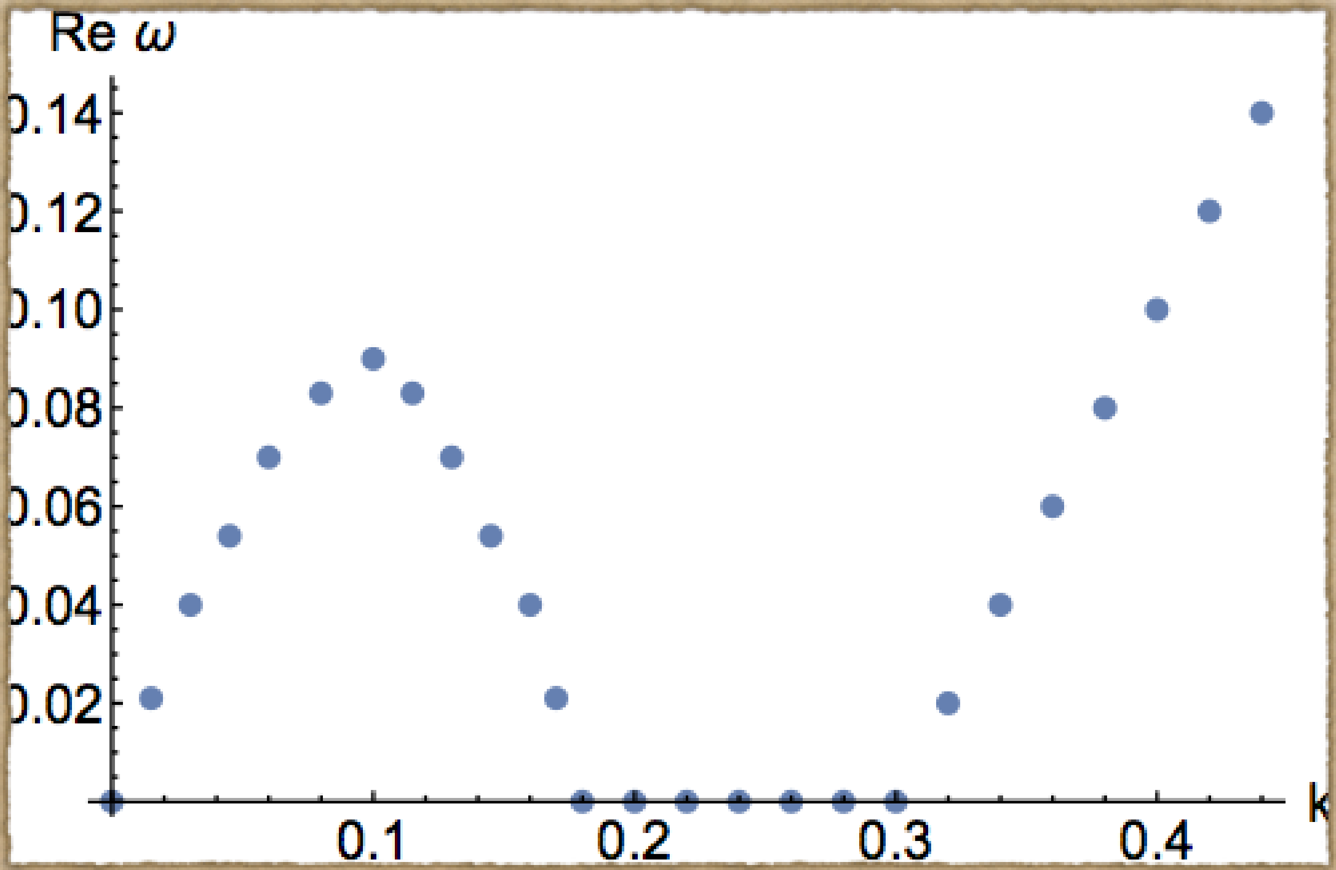
Linearized Gravity (QNMs)

New features in non-CFT cases:

→Sign for bubble formation in 1st order phase transitions!

(Re(ω)=0 for $k_{min}<k<k_{max}$)

Janik, Jankowski, HS, PRL '16



Dynamics of 1st order phase transition

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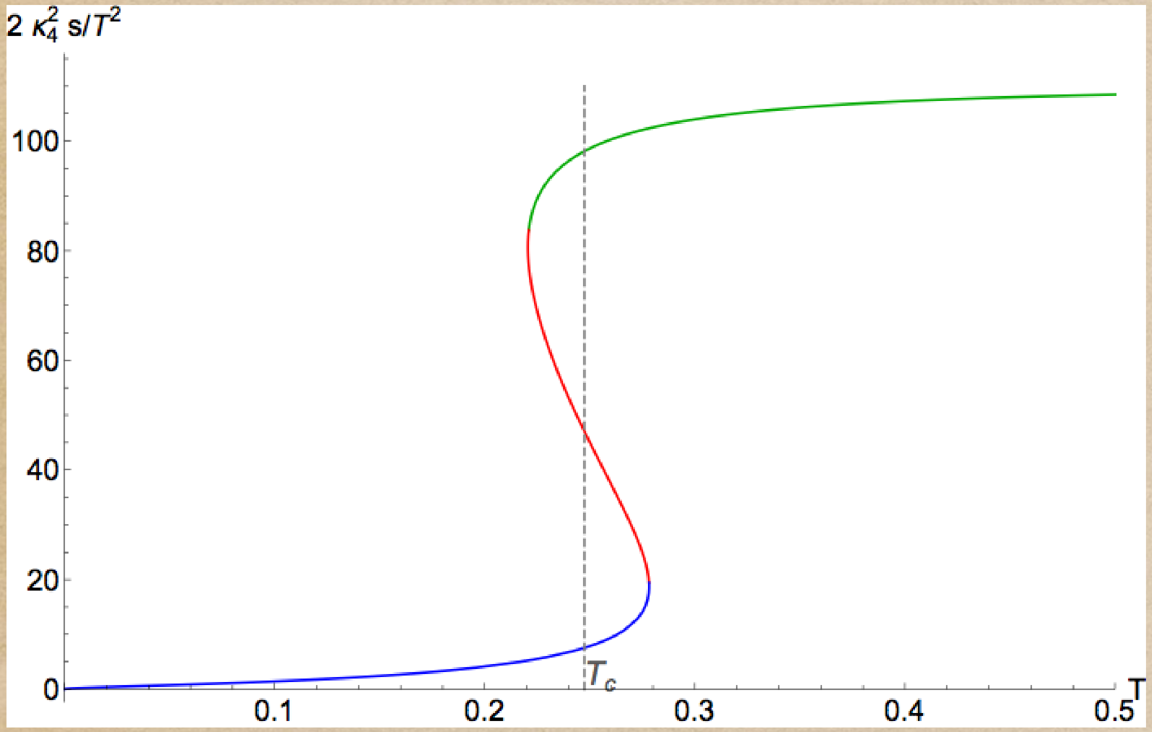
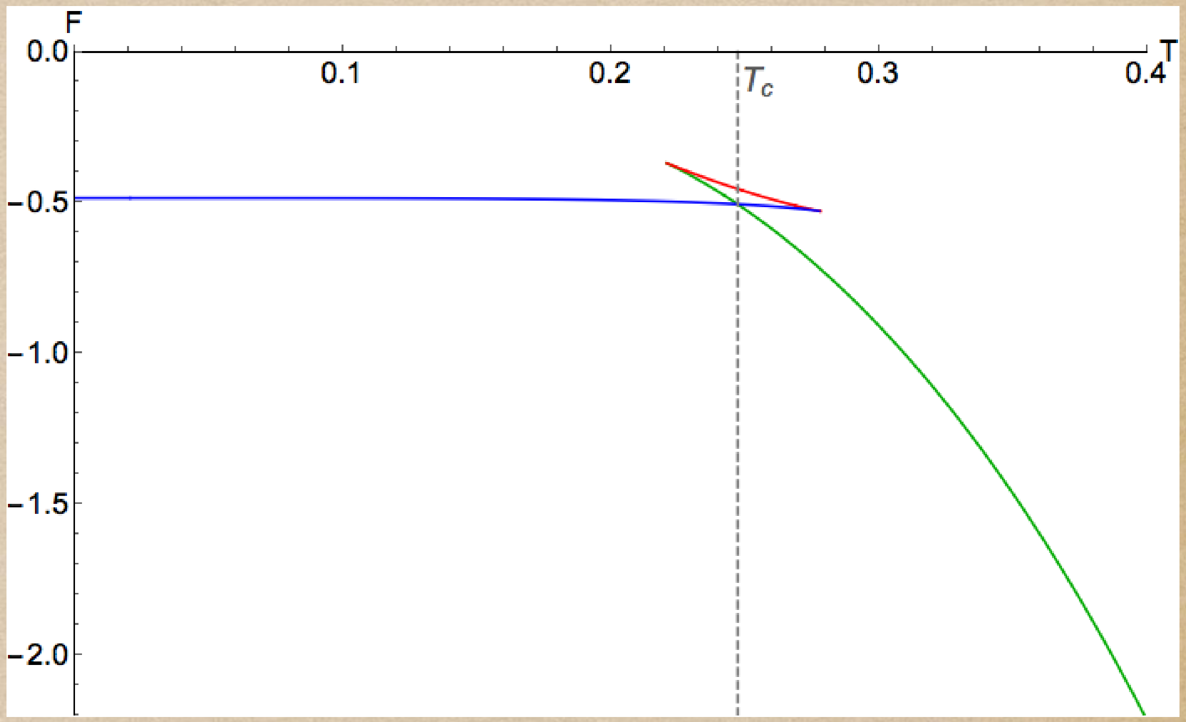
Potential in 4D: $V(\phi) = -6\cosh(\phi/\sqrt{3}) - 0.2\,\phi^4, \qquad \Delta = 2$

Janik, Jankowski, HS, PRL '17





Initial configurations: perturbed black holes in Spinodal region

Dynamics of 1st order phase transition

Some technical points:

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Metric Ansatz (in Eddington-Finkelstein): $x \sim x + 12\pi$

$$ds^2 = -A(u,x,t)\,dt^2 - \frac{2dtdu}{u^2} - 2B(u,x,t)\,dtdx + \Sigma(u,x,t)^2\left[H(u,x,t)\,dx^2 + H(u,x,t)^{-1}dy^2\right]$$

Commenets:

→ Time integration: Runge-Kutta and Adams-Bashforth methods

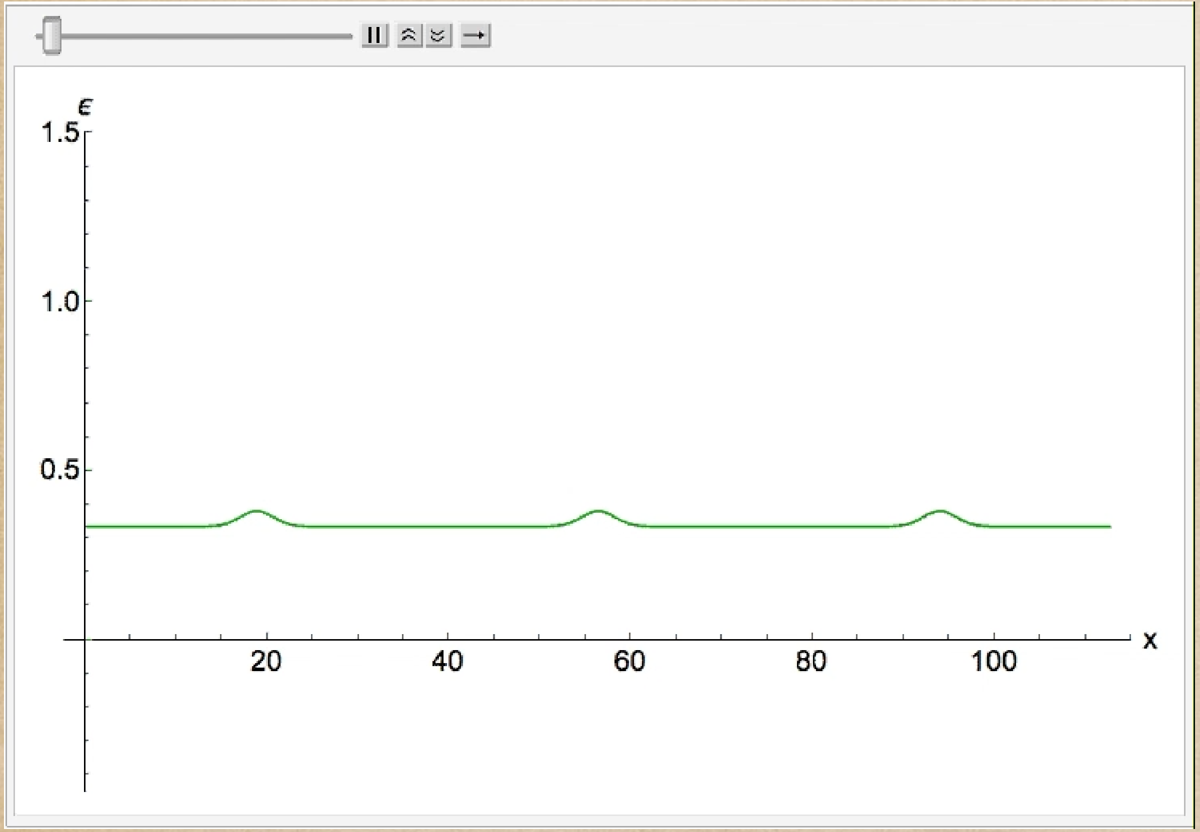
→ Proper boundary conditions at AAdS and at the apparent horizon

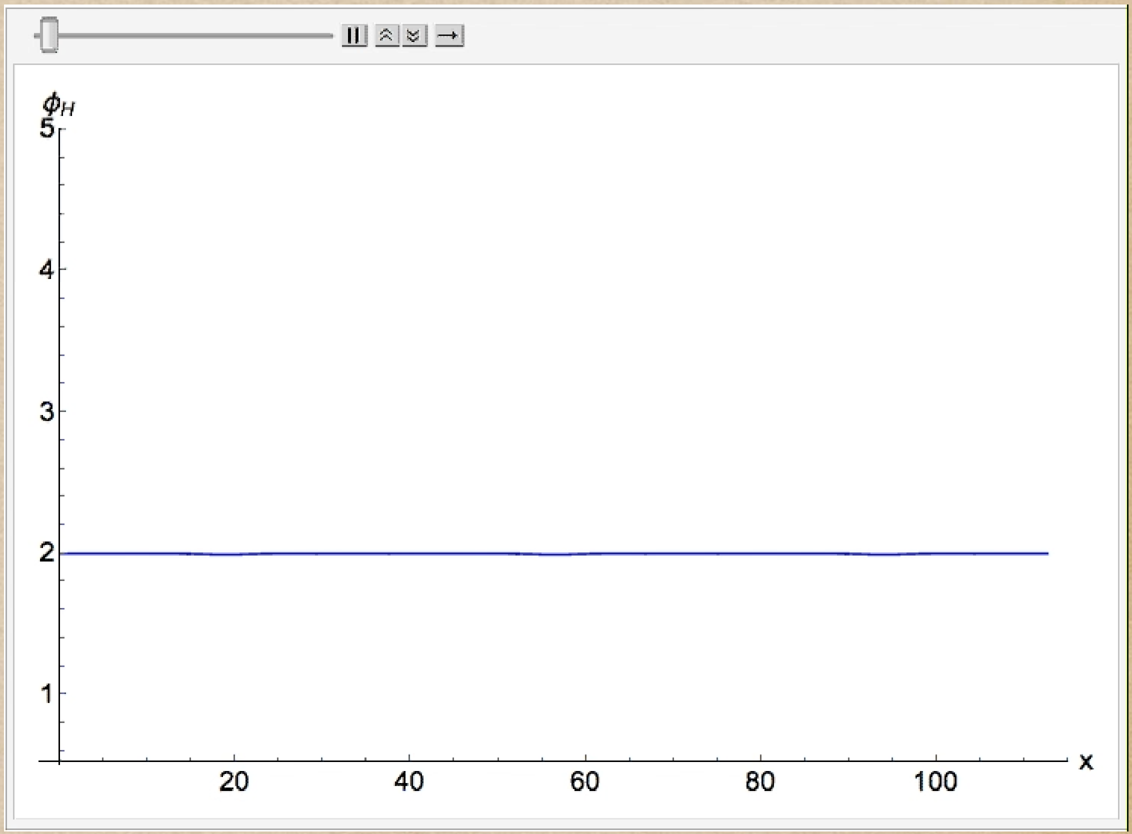
→ Holographic renormalization: $\langle T_{ij} \rangle$, $\langle O_\varphi \rangle$

→ Ward identities: $\langle T^i{}_i \rangle = \langle O_\varphi \rangle$, $\nabla^i \langle T_{ij} \rangle = 0$

Dynamics of 1st order phase transition

Results: initial config. : $\varphi_H = 2$ $\delta\Sigma \propto \exp\left(-\frac{\cos(k\,x)^2}{w}\right)$





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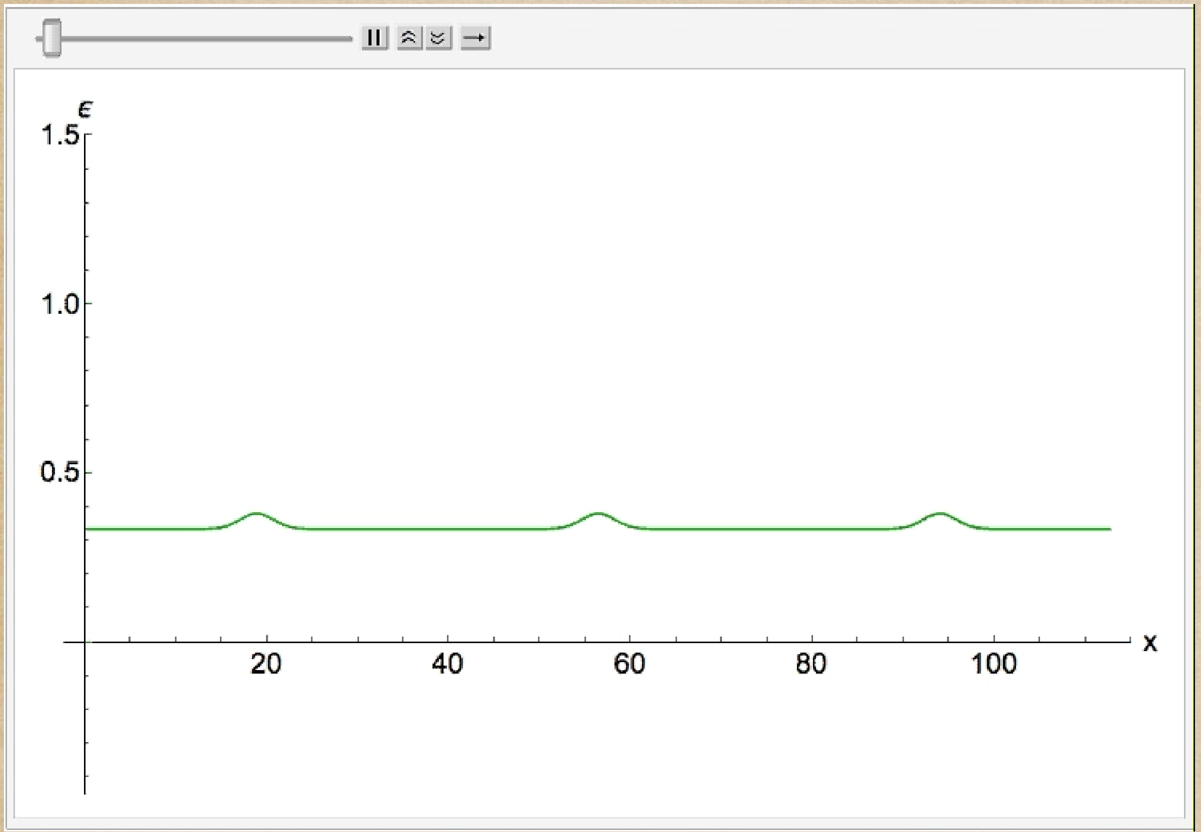

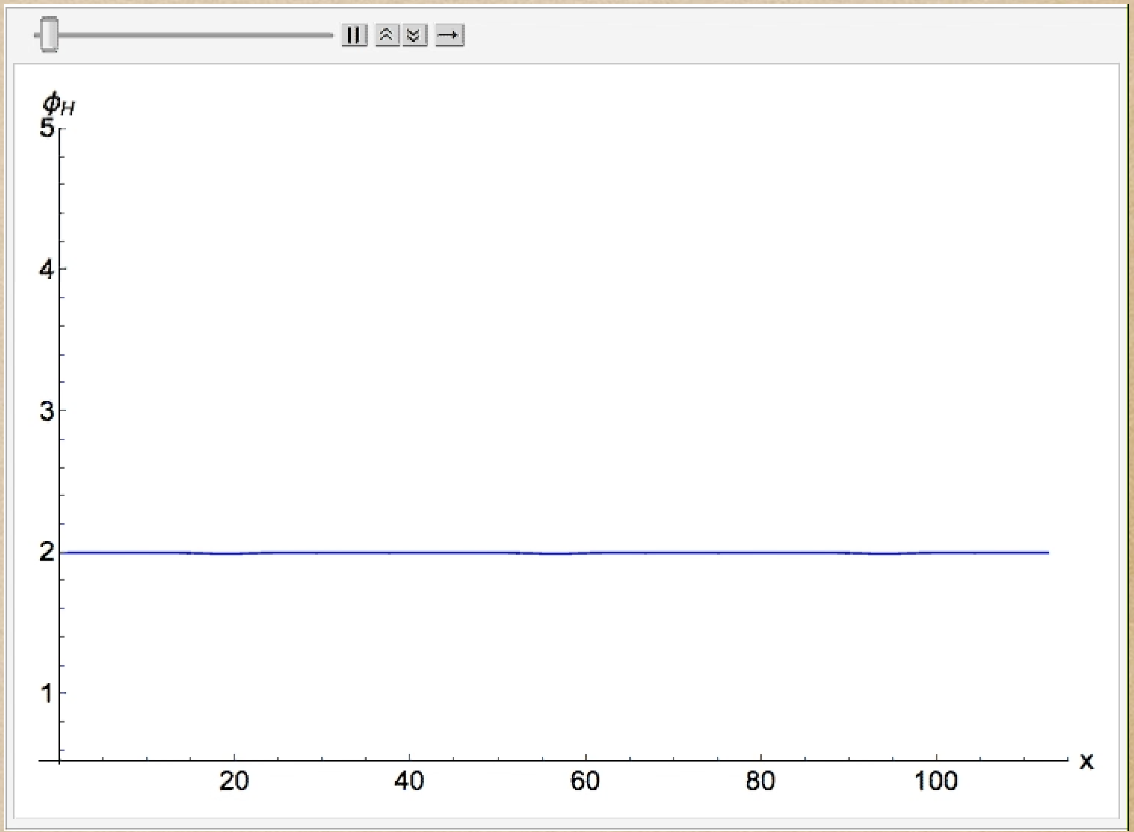

The final state: Static Black holes with inhomogenous horizons
With homogenous sources

Dynamics of 1st order phase transition

Results: New static black hole: inhomogenous horizon and temperature T_c

Dias, Santos, Way, JHEP '17

Dynamics of 1st order phase transition

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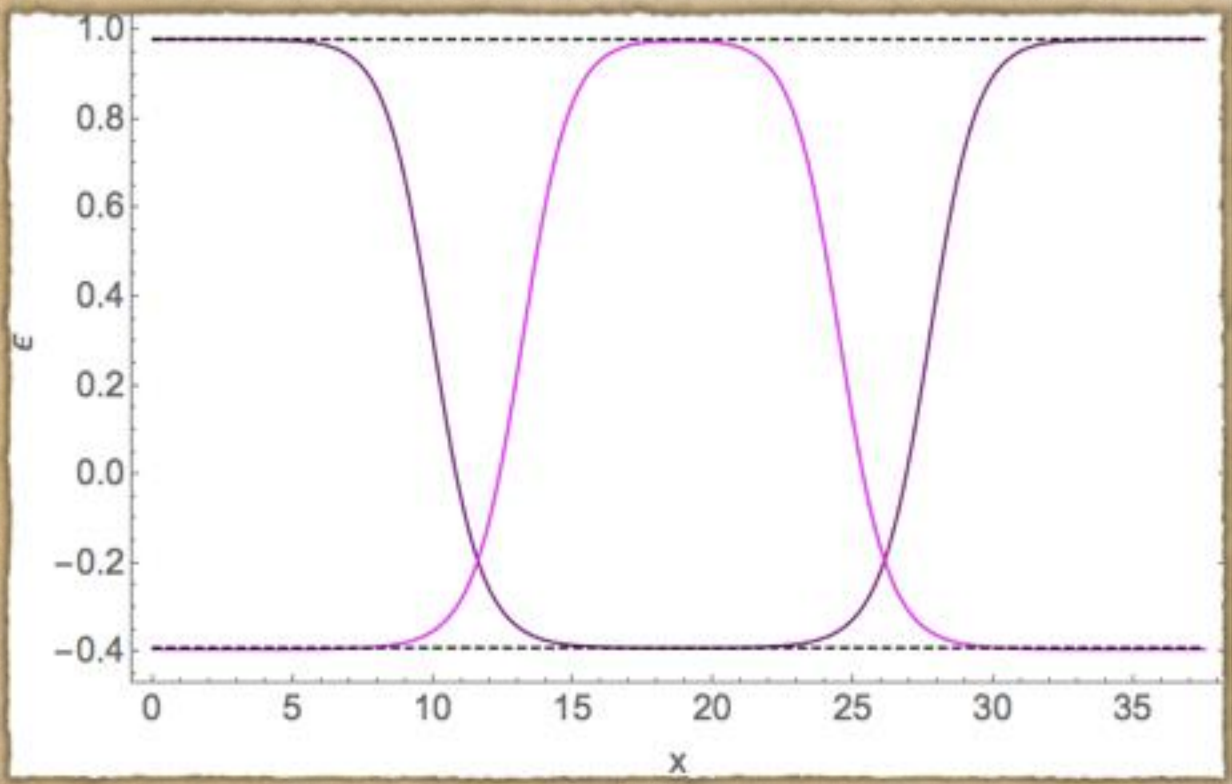

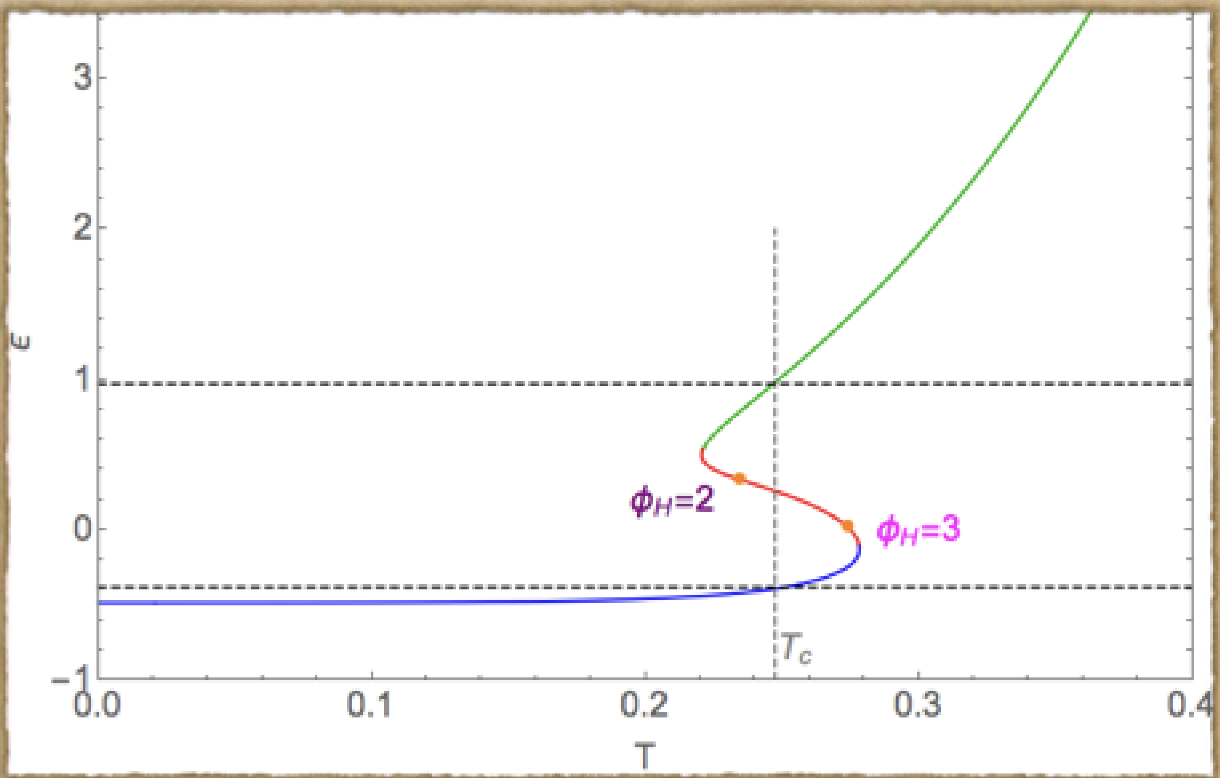

Green lines: Min and Max points on EoS at T_c

Red and Blue line: ε(x) for the final state of perturbations

Hydrodynamics is applicable with 2nd-order transport Coefficients

Boost invariant expansion

Expanding plasma: Boost-invariant expansion as a simple model

The physics of (d+1)-dim FT depends only on τ and not rapidity y.

$$ds^2 = -dt^2 + d\tilde{y}^2 + dx_i^2$$

$$t = \tau \cosh y, \qquad \tilde{y} = \tau \sinh y$$

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_i^2$$

i=1,…,d-1

Bjorken, PRD, '83

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Dual gravity Ansatz in 3+1 d:

$$ds^2 = -A(u,\tau)\, d\tau^2 - \frac{2d\tau du}{u^2} + \Sigma(u,\tau)^2 \left[H(u,\tau)\, dx^2 + \tau^2\, H(u,\tau)^{-1} dy^2\right]$$

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Janik, Jankowski, HS, "soon"

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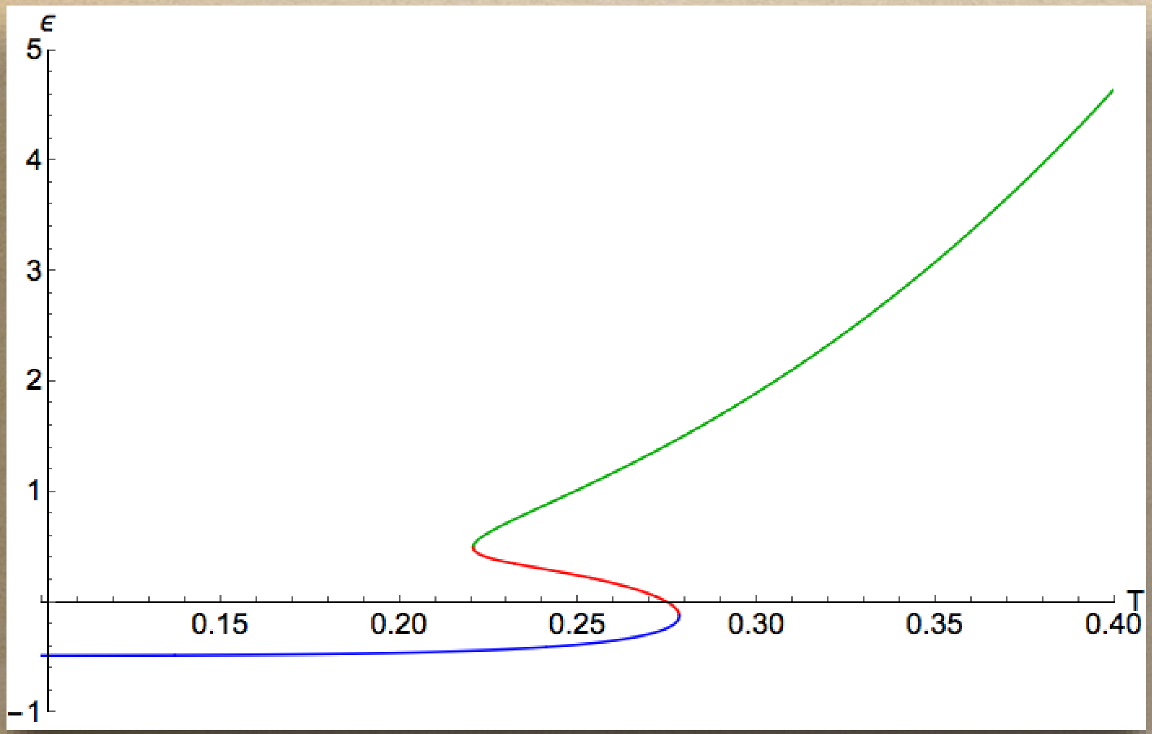

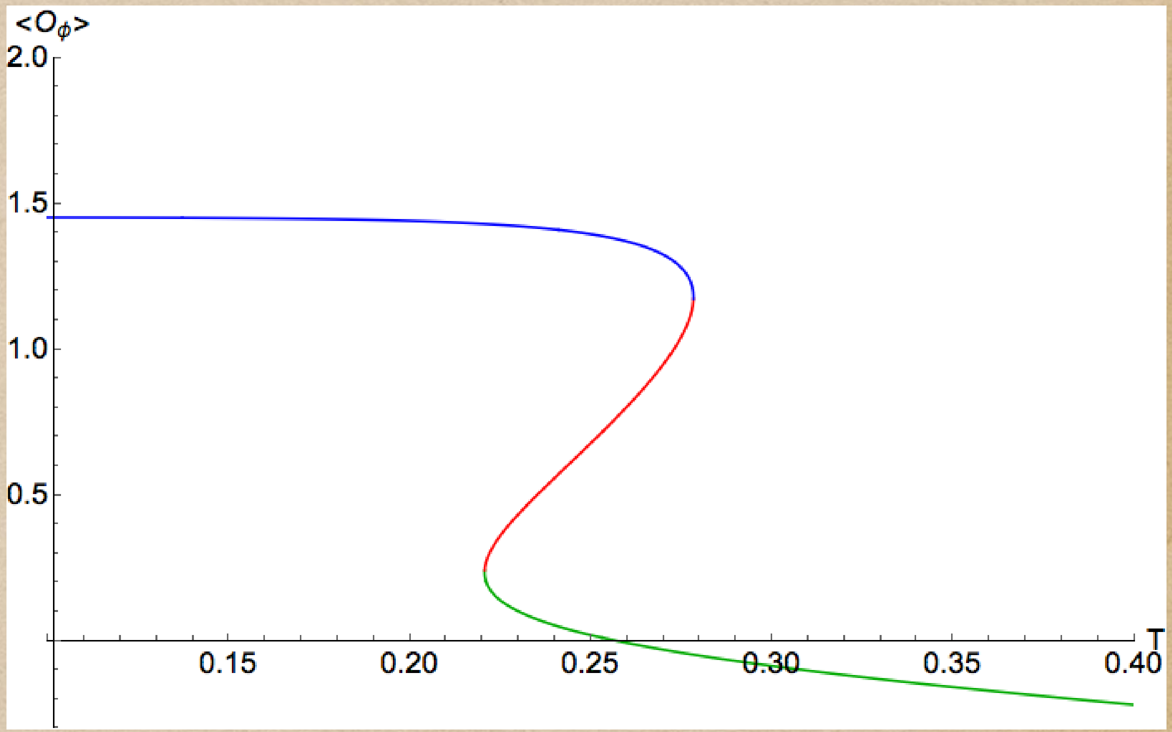

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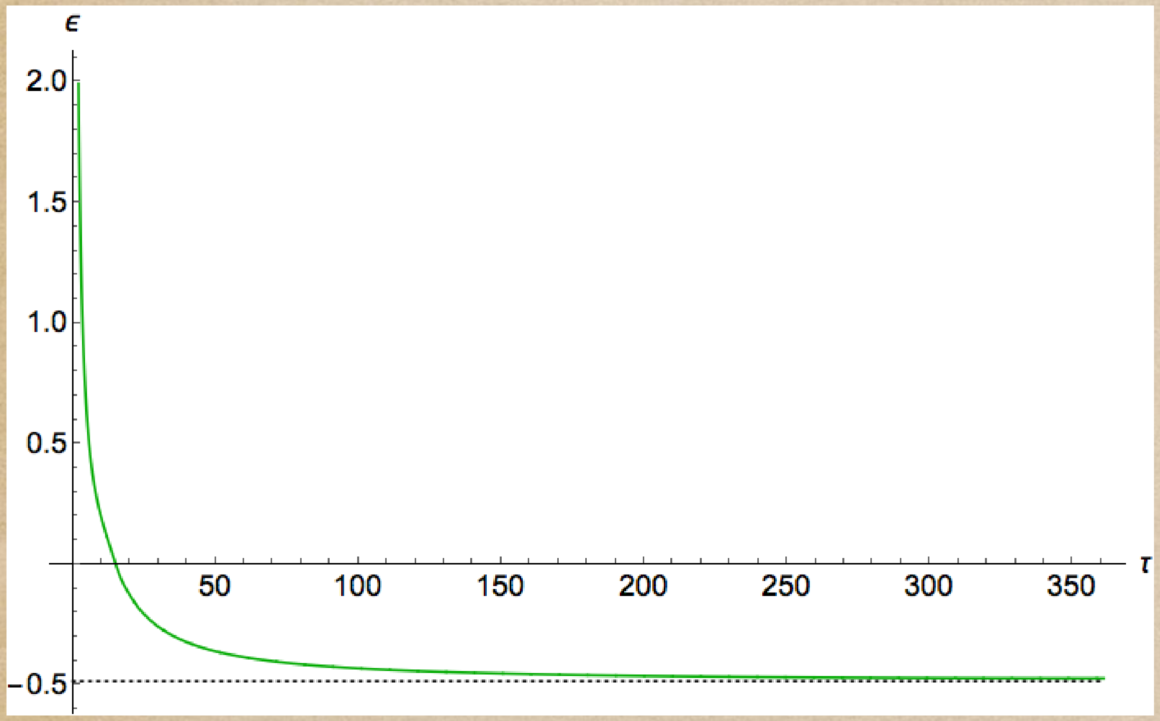

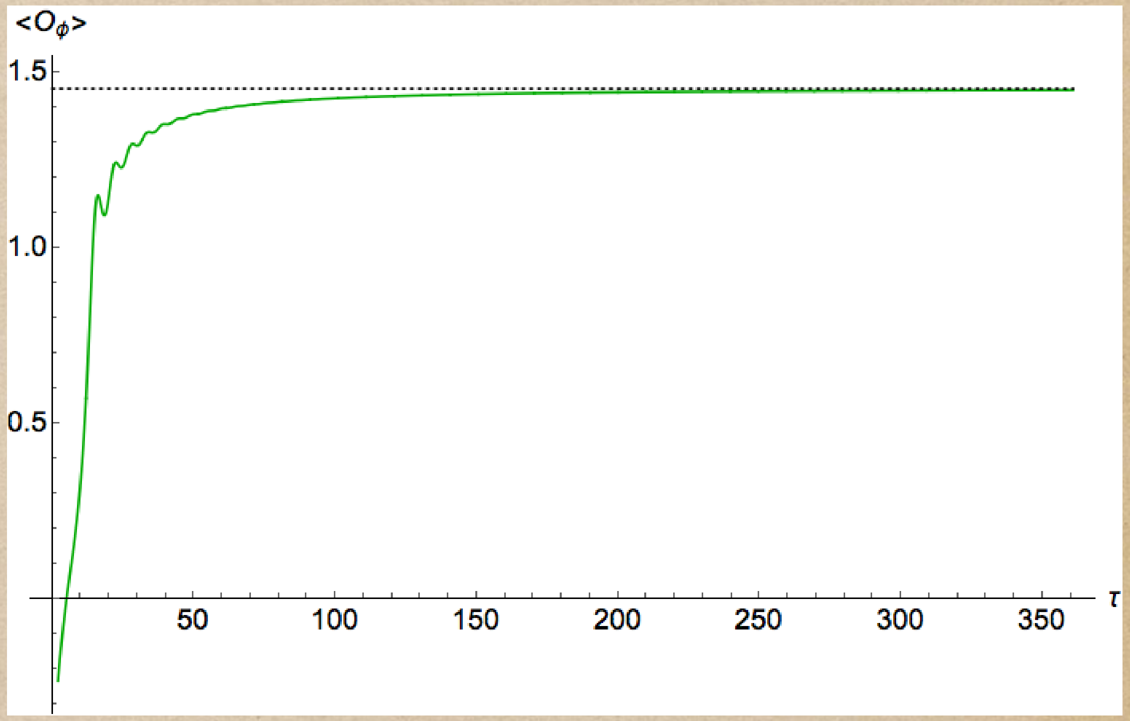

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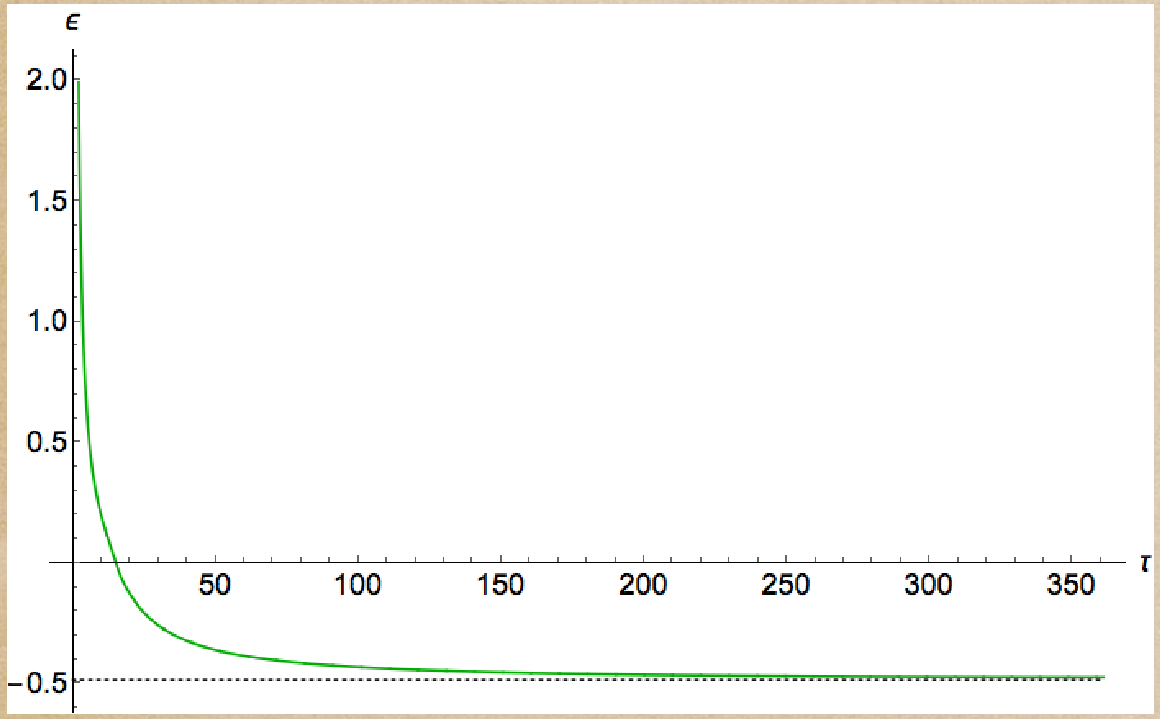

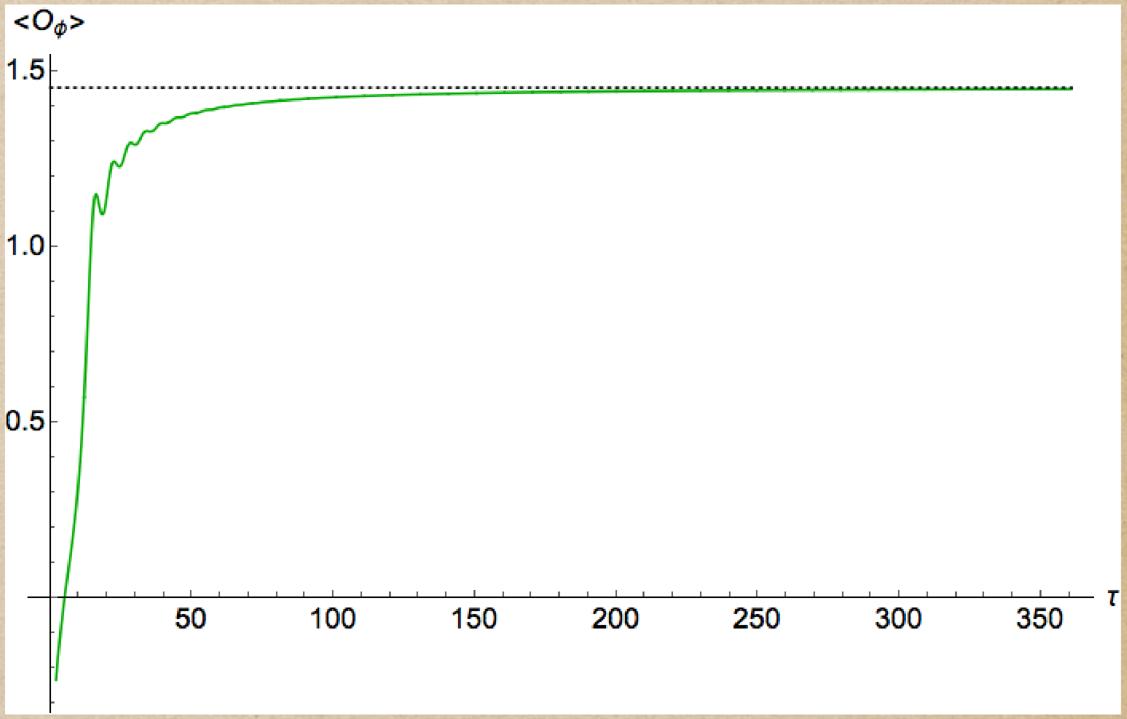

→The energy is decreasing

→The expansion passes the Spinodal region "trivially"

Expanding plasma: Boost-invariant expansion as a simple model

2nd Potential in 4D: $V_2(\phi) = -6\cosh(\phi/\sqrt{3}) - 0.3\,\phi^4, \qquad \Delta = 2$

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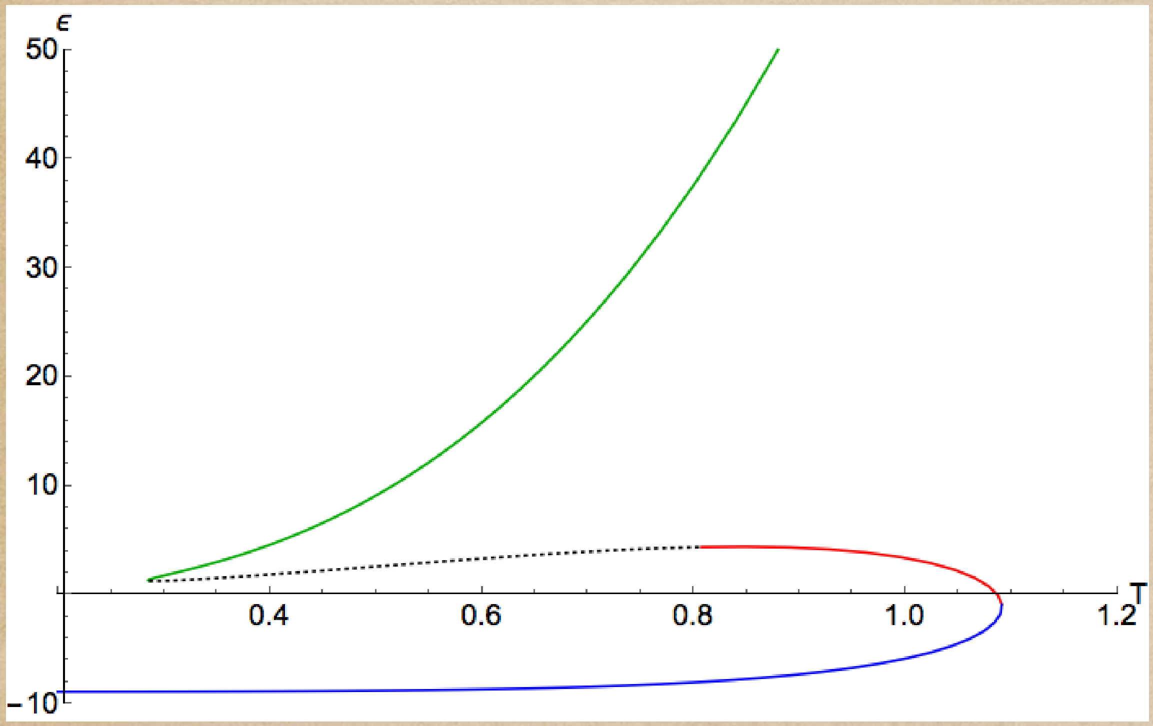

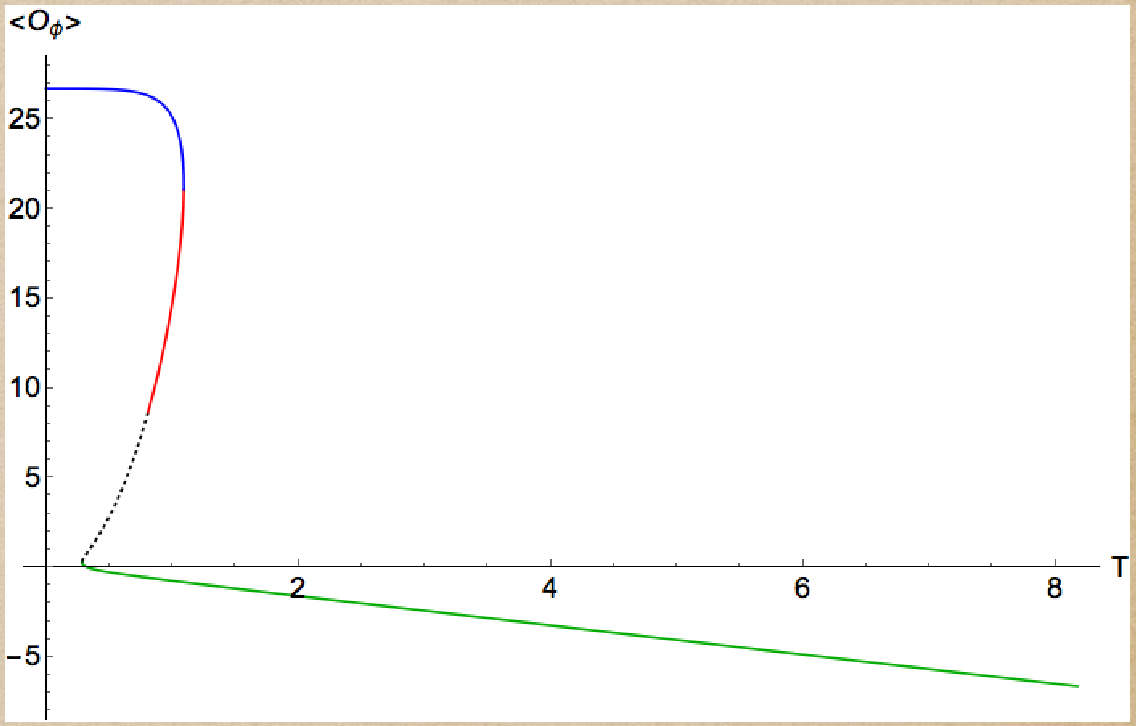

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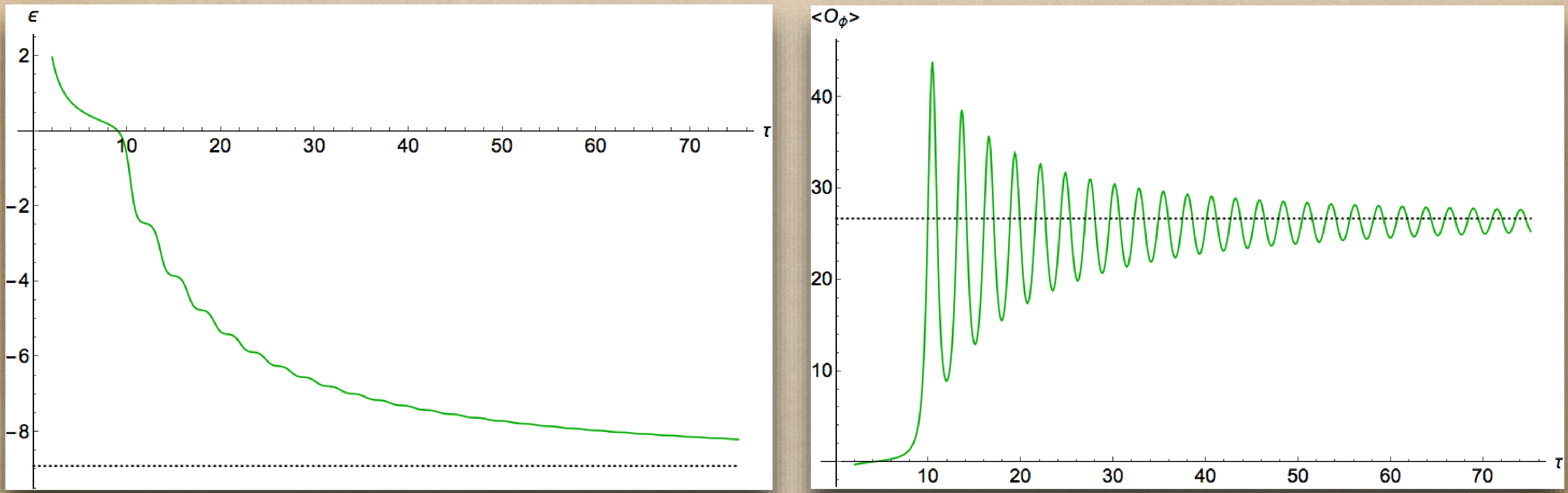

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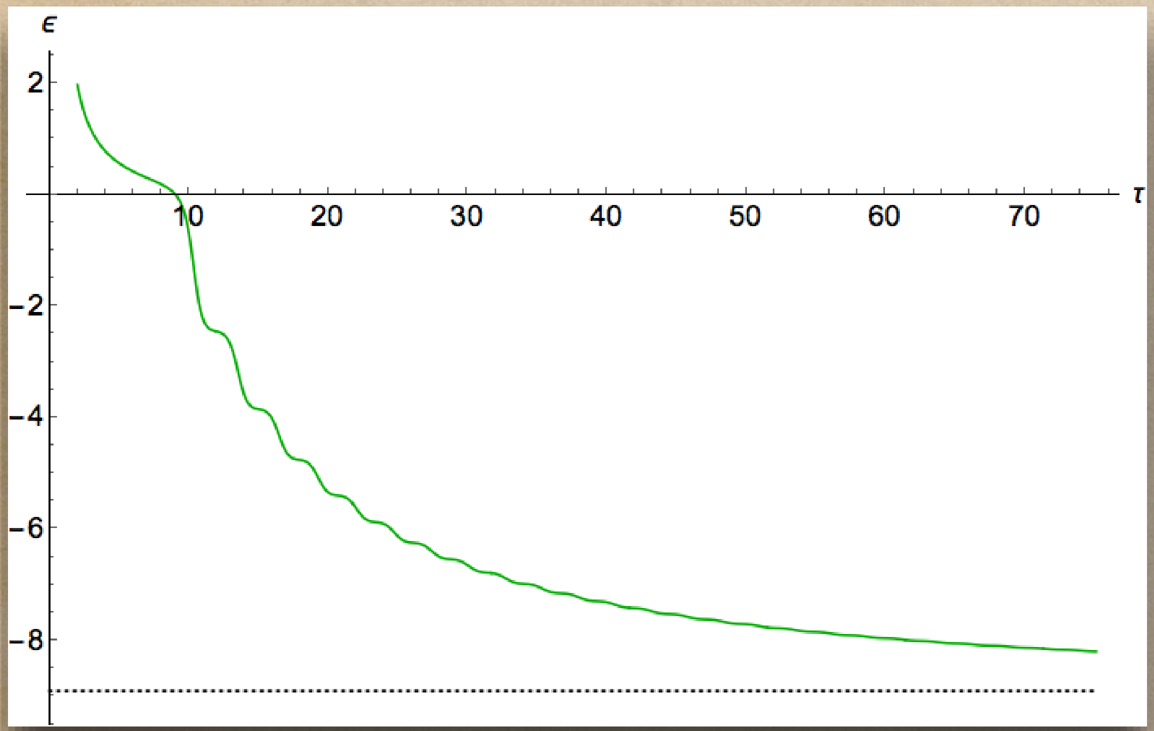

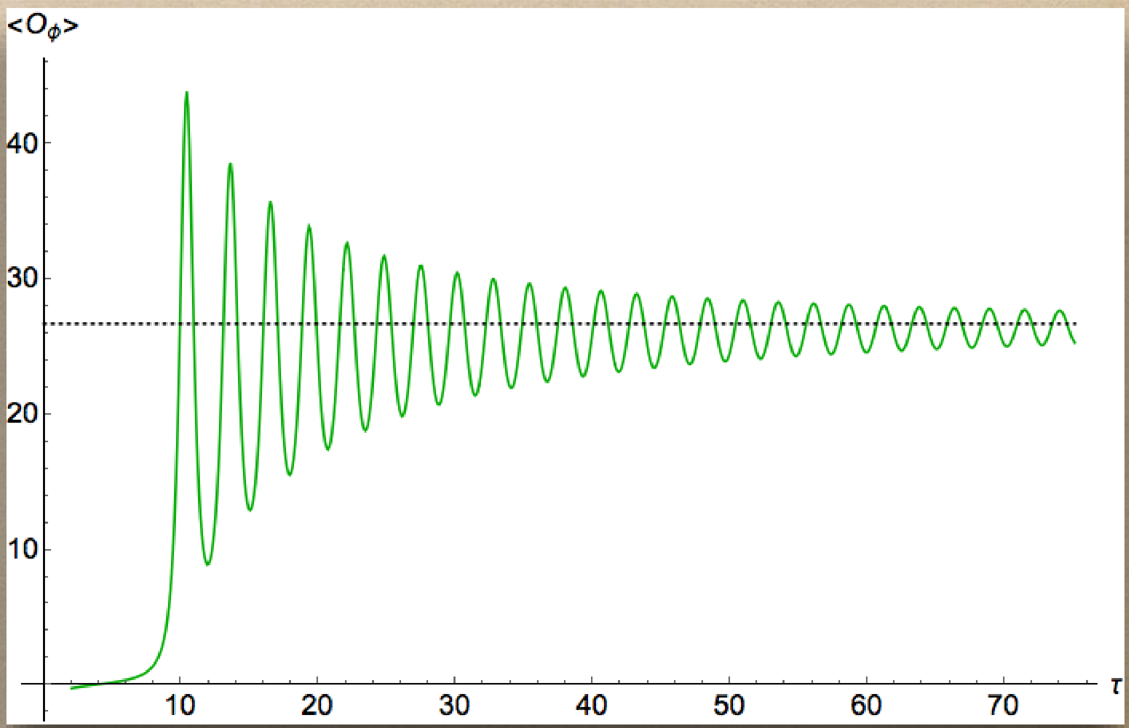

→ The energy is decreasing

→ There is a reminiscent of dynamical instability until the late time in expansion!!!

Summary

→ Higher QNMs introduce new instabilities in BHs

→ Bubble formation is studied (in strongly coupled regime, with universal properties)

→ New black holes with homogenous sources but inhomogenous horizon

→ Plasma expansion (boost invariant flow) is studied and needs more,...

Summary and future directions

→ Higher QNMs introduce new instabilities in BHs

→ Bubble formation is studied (in strongly coupled regime, with universal properties)

→ New black holes with homogenous sources but inhomogenous horizon

→ Plasma expansion (boost invariant flow) is studied and needs more,...

→ Different dimensions (in particular 3D gravity + matter)

→ Investigating anisotropic plasma

→ Collision of bubbles in 1st order phase transition

→ Using probes: strings, Wilson line/loop, EE, ...

→ New black holes: Domain wall solutions to GR

Thanks for your attention

Dynamics of 1st order phase transition

Some technical points:

Metric Ansatz (in Eddington-Finkelstein): $x \sim x + 12\pi$

$$ds^2 = -A(u,x,t)\,dt^2 - \frac{2dtdu}{u^2} - 2B(u,x,t)\,dtdx + \Sigma(u,x,t)^2\left[H(u,x,t)\,dx^2 + H(u,x,t)^{-1}dy^2\right]$$

EoMs: Using the new time derivative: $d_+ := \partial_t - \frac{u^2 A}{2}\partial_u$

$$\partial_u \phi = S_\phi(\Sigma, H), \qquad \partial_{u^2} B + p_1 \partial_u B + p_2 B = S_B(\Sigma, H, \phi)$$

$$\partial_u(d_+\Sigma) + p_3 d_+\Sigma = S_{d_+\Sigma}(\Sigma, H, \phi, B)$$

$$\partial_u(d_+H) + p_4 d_+H = S_{d_+H}(\Sigma, H, \phi, B, d_+\Sigma)$$

$$\partial_u(d_+\phi) + p_5 d_+\phi = S_{d_+\phi}(\Sigma, H, \phi, B, d_+\Sigma, d_+H)$$

$$\partial_{u^2} A + p_6 \partial_u A = S(\Sigma, H, \phi, B, d_+\Sigma, d_+H, d_+\phi)$$

Solve the equations at t=0 and then one step forward