Blackhole microstate counting in AdS_5/CFT_4

Arash Arabi Ardehali

Uppsala University

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Based on

Cardy-like asymptotics of the 4d $\mathcal{N} = 4$ index and AdS_5 blackholes, arXiv:1902.06619 [hep-th]

Recent papers on the subject

- **Cardy limit of the $\mathcal{N} = 4$ index**

  Choi-Kim-Kim-Nahmgoong 1811.08646
  Honda 1901.08091
  AAA 1902.06619

- **Large-N limit of the $\mathcal{N} = 4$ index**

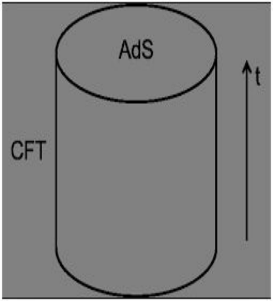
  Benini-Milan 1812.09613

- **Large-N/Cardy limit of a modified $\mathcal{N} = 1$ index**

  Cabo Bizet-Cassani-Martelli-Murthy 1810.11442
  Kim-Kim-Song 1904.03455
  Cabo Bizet-Cassani-Martelli-Murthy 1904.05865

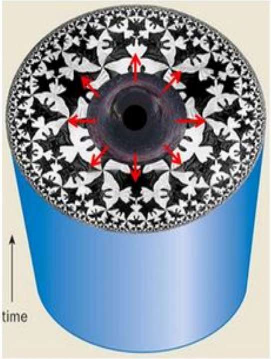
AdS_5/CFT_4

The SU(N) $\mathcal{N} = 4$ theory, as $\lambda, N \to \infty$, describes type IIB quantum gravity in the $\text{AdS}_5 \times S^5$ spacetime. $(N^2 = \frac{\pi \ell_{\mathrm{AdS}_5}}{2G_{\mathrm{AdS}_5}})$



Blackholes in AdS_5/CFT_4

$$S_{GR\ '04} = \frac{\text{Area}}{4} = 2\pi\sqrt{\frac{Q_1 Q_2 Q_3 + \frac{N^2}{2} J_1 J_2}{Q_1+Q_2+Q_3}} \stackrel{?}{=} \ln d(J_{1,2}, Q_{1,2,3})$$



Blackhole microstate counting in AdS/CFT I: historical AdS_3/CFT_2 developments

- Strominger-Vafa '96, Strominger '97 (Cardy-limit derivation) Explained in the famous '02 review of David *et. al.*
- Sen and collaborators, *e.g.* 1109.3706 (holographic scaling limits)
- Hartman-Keller-Stoica '14 (large-c derivation)

Blackhole microstate counting in AdS/CFT II: methodology

i) **Partition function**:

$$Z^{\mathrm{SUSY}}(p,q;y_k) = \sum d^{\mathrm{SUSY}}(J_{1,2};Q_k)p^{J_1}q^{J_2}y_k^{Q_k}$$

ii) **Asymptotic analysis**:

$$Z^{\mathrm{SUSY}}(p,q;y_k) \overset{N\to\infty \text{ or Cardy}}{\longrightarrow} e^{\mathcal{F}(\sigma,\tau;\Delta_k)}$$

iii) **Legendre transform and extremization**:

$$d(J_{1,2},Q_k) = \oint Z^{\mathrm{SUSY}}(p,q;y_k)p^{-J_1}q^{-J_2}y_k^{-Q_k}\,\frac{\mathrm{d}p}{2\pi ip}\frac{\mathrm{d}q}{2\pi iq}\frac{\mathrm{d}y_k}{2\pi iy_k}$$
$$\sim e^{\left(\mathcal{F}(\sigma,\tau;\Delta_k)-2\pi i\sigma J_1-2\pi i\tau J_2-2\pi i\Delta_k Q_k\right)_{\mathrm{ext}}}$$

Recent advances in blackhole microstate counting I: Benini-Hristov-Zaffaroni '15 AdS_4/CFT_3

Used the tt-index of ABJM (computed a few months earlier by Benini-Zaffaroni via SUSY localization), took its large-N limit, Legendre transformed and extremized and found the entropy of magnetically charged (and a few months later dyonic) AdS_4 blackholes.

Significant aspects of the BHZ work:

1) the first large-N analysis of a blackhole counting index

2) formulation of $\mathcal{I}$-extremization

3) resolving the issue with the # of charges

Recent advances in blackhole microstate counting II: subsequent developments

- *AdS_4 hyperbolic black hole entropy*, Cabo-Bizet, Giraldo-Rivera, and Pando Zayas '17
- *Holographic attractor mechanism*, Cabo-Bizet, Kol, Pando Zayas, Papadimitriou, and Rathee '17
- *Universal counting*, Azzurli, Bobev, Crichigno, Min, and Zaffaroni '17
- *Log(Area) corrections*, Liu, Pando Zayas, Rathee, and Zhao '17
- and many more ...

Blackhole microstate counting in AdS_5/CFT_4 I: overview

The blackholes: BPS solutions of $D = 5$ $U(1)^3$ gauged supergravity [Gutowski-Reall '04]; their type IIB lift preserves two real supercharges, so they are dual to 1/16 BPS states in the $\mathcal{N} = 4$ theory; Hosseini-Hristov-Zaffaroni '17 found

$$\mathcal{F} = -i\pi N^2 \frac{\Delta_1 \Delta_2 \Delta_3}{\tau\sigma}$$

The index: Kinney-Maldacena-Minwalla-Raju '05 found

$$Z^{\mathrm{SUSY}}(p, q; y_{1,2}) = \mathrm{Tr}\left[(-1)^F p^{J_1} q^{J_2} y_1^{Q_1} y_2^{Q_2} y_3^{Q_3}\right] =$$

$$\frac{\left((p;p)(q;q)\right)^{N-1}}{N!} \prod_k \Gamma^{N-1}\left(y_k\right) \int_{-\frac{1}{2}}^{\frac{1}{2}} \prod_{n=1}^{N-1} \mathrm{d}x_n \prod_{1\leq i<j\leq N} \frac{\prod_k \Gamma\left(y_k e^{\pm 2\pi i(x_i - x_j)}\right)}{\Gamma\left(e^{\pm 2\pi i(x_i - x_j)}\right)}$$

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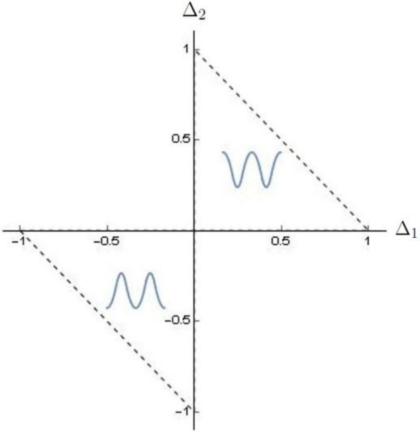
Benini-Milan took the large-N limit: equal angular momenta so far

CKKN, Honda, and Ardehali took a Cardy-like limit: equal charges so far (see upcoming work by AAA, J.T. Liu, and J. Hong)

Blackhole microstate counting in AdS_5/CFT_4 II: the Cardy-limit derivation

Take $y_{1,2} = e^{2\pi i \Delta_{1,2}}$ on the unit circle (so $\Delta_{1,2} \in \mathbb{R}$), and take $p = e^{-\beta b}$, $q = e^{-\beta b^{-1}}$ with $b > 0$ and $\arg\beta \in (-\frac{\pi}{2}, 0)$ fixed, then send $|\beta| \to 0$.



Blackhole microstate counting in AdS_5/CFT_4 III: open problems

- Problem 1: Deconfined objects on W-wing(s)?
- Problem 2: Non-equal charges?

Both addressed in upcoming work by AAA, J.T. Liu, and J. Hong

The chemical potentials $\Delta_{1,2}$ are control-parameters triggering infinite-temperature phase-transitions in the $\mathcal{N} = 4$ index.
The blackhole phase is only one of the possibilities!

Thanks for your attention!

Previous work on limits of the 4d $\mathcal{N} = 4$ index

- **Large-N limit**: Kinney-Maldacena-Minwalla-Raju '05 had evaluated the large-N limit of the index for *real-valued fugacities*, and had found that it simplifies to the $\mathcal{O}(N^0)$ index of the bulk multi-particle states.

- **Cardy-like limit**: in 1512.03376 I had computed the asymptotics of the index *without flavor fugacities* and for *real-valued p, q*, finding a $(1/\beta)^N$ behavior, which was of course much slower than the required e^{N^2/β^2} behavior that now we know arises with complexified p, q, y_k.

Thanks for your attention!