Magneto-transport of a chiral fluid in a system with weakly broken symmetries

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Recent Trends in String Theory and Related Topics

1812.11310[hep-th]

Outline of talk

- Motivation
- Chiral transport and Linear Response Theory (LRT)
  - Magneto Transport (MT) in weak regime
  - MT in strong regime
- Conclusion

Micro v.s. Macro scale

- **Is there any chance to explore the influences of microscopic phenomena onto the macroscopic scales?**

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Non-dissipative currents in weakly interacting system

- Weakly interacting fermions in presence of arbitrary global rotation:

$$J_\omega(0) = \vec{\omega}\left(\frac{T^2}{12} + \frac{\mu^2}{4\pi^2}\right).$$

This current density is developed in a system with large vorticity field.

A. Vilenkin, Phys. Rev. D **20**, 1807 (1979).

- Weakly interacting fermions in presence of arbitrary magnetic field:

$$J_B = \vec{B}\frac{e\mu}{4\pi^2}.$$

This current density is developed in a system with large magnetic field and only contributes from lowest Landau level.

A. Vilenkin, Phys. Rev. D **22**, 3080 (1980).

CKT

- **CKT**: A framework to involve spin DoF into the Boltzmann kinetic equation.
- Canonical forms of equations

$$\sqrt{G}\,\dot{\mathbf{x}} = \frac{\partial \epsilon_p}{\partial p_j} + e\,\vec{E} \times \vec{\Omega}_p + e\vec{B}(\hat{p}\cdot\vec{\Omega}_p),$$
$$\sqrt{G}\,\dot{\mathbf{p}} = e\vec{E} + e\,\hat{p} \times \vec{B} + e^2\vec{\Omega}_p(\vec{E}\cdot\vec{B}), \quad \sqrt{G} = 1 + e\vec{B}\cdot\vec{\Omega}_p.$$

  - Continuity equation $\Rightarrow \frac{\partial n}{\partial t} + \frac{\partial J_i}{\partial x_i} = \frac{e^2}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$,
  - Chiral magnetic and Chiral vortical effect

$$\mathbf{J}_{CME} = \int \frac{d^3p}{(2\pi)^3}\sqrt{G}\,\dot{\mathbf{x}} f(p) = \frac{e\mu}{4\pi^2}\mathbf{B},$$
$$\mathbf{J}_{CVE} = \int \frac{d^3p}{(2\pi)^3}\sqrt{G}\,\dot{\mathbf{x}} f(p) = \frac{\mu^2}{4\pi^2}\vec{\omega}.$$

M. A. Stephanov and Y. Yin, Phys. Rev. Lett. **109**, 162001 (2012).

Transport coefficients in the view of symmetry

Time reversal symmetry

- Dissipative transports

$$\underbrace{J}_{-} = \underbrace{\sigma}_{-} \underbrace{E}_{+}$$

This process is an irreversible one and contributes to the heat production and takes place in an out-off equilibrium state.

- Non-dissipative transports

$$\underbrace{J}_{-} = \underbrace{\kappa}_{+} \underbrace{B}_{-}$$

This process is a reversible one and do not contribute to the heat production and takes place in an equilibrium state.

D. E. Kharzeev and H. U. Yee, Phys. Rev. D **84**, 045025 (2011).

Anomaly in dissipative transports

- Non-dissipative transports appear only when anomalous effects turn on.
- **Can we follow the effect of anomaly in the dissipative transports?**

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- **Negative-Magneto Resistivity (NMR)** : The more the magnetic field, the more the electrical conductivity.
- CKT equations

$$\sqrt{G}\frac{\partial f}{\partial t} + \sqrt{G}\dot{\mathbf{x}}\frac{\partial f}{\partial \mathbf{x}} + \sqrt{G}\dot{\mathbf{p}}\frac{\partial f}{\partial \mathbf{p}} = -\sqrt{G}\frac{f - f_{eq}}{\tau}.$$

- Response to external $\vec{E}$

$$\mathbf{J} = \int \frac{d^3p}{(2\pi)^3}\sqrt{G}\,\dot{\mathbf{x}} f(p) = \underbrace{\frac{B^2 e^4 \tau}{15\pi^2\mu^2}}_{\sigma}\mathbf{E}.$$

D. T. Son and B. Z. Spivak, Phys. Rev. B **88**, 104412 (2013).

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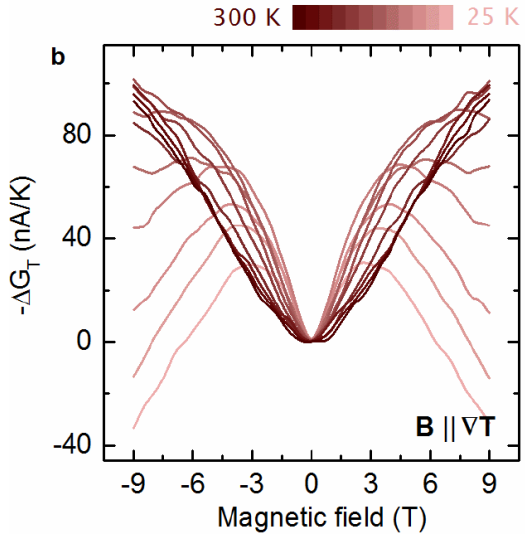
Evidence for mixed gauge-gravitational anomaly

- It has been observed that mixed gauge-gravitational anomaly can be seen in the thermo-electric transport coefficient in the Weyl semimetal NbP.

$$J_i = G^{ij} E_j + G_T^{ij} \nabla_j T,$$
$$G_T^{ij} = \tau T \frac{2 a_\chi a_g}{det(\Xi)} \frac{\partial \rho}{\partial T} B_i B_j.$$



J. Gooth *et al.*, Nature **547**, 324 (2017).

Importance of relaxation times

- **Observation**: In a translationally invariant system and in the absence of any mechanism of momentum dissipation, we get an infinite DC conductivity.
- To get a finite DC conductivity, we have to construct a set up for momentum dissipation.

$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha - \frac{T^{\nu\alpha}\tilde{u}_\alpha}{\tau_{em}},$$

$$\partial_\mu J^\mu = CE.B - \frac{J^\mu \tilde{u}_\mu}{\tau_c}.$$

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Our question

Using the linearized sets of hydro equations in order to derive transport coefficients

K. Landsteiner, Y. Liu and Y. W. Sun, JHEP **1503**, 127 (2015).

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Second order anomalous hydro

- Constitutive relations

$$
\begin{aligned}
T^{\mu\nu} &= (\epsilon + p)u^\mu u^\nu + p\,\eta^{\mu\nu} + \tau^{\mu\nu}, \\
J^\mu &= n\,u^\mu + \nu^\mu, \\
\tau^{\mu\nu} &= \sigma^\epsilon_{\mathcal{B}}(u^\mu B^\nu + u^\nu B^\mu) + C_1(B^\mu B^\nu - \frac{1}{3}P^{\mu\nu}B^2) \\
&\quad + \Pi^{\mu\nu}_{\alpha\beta}B^\alpha\big(C_2 E^\beta - C_3\mu\frac{\nabla^\beta T}{T}\big), \\
\nu^\mu &= \sigma_E\,E^\mu + \sigma_{\mathcal{B}}\,B^\mu, \\
\Pi^{\mu\nu}_{\alpha\beta} &= \frac{1}{2}(P^\mu{}_\alpha P^\nu{}_\beta + P^\mu{}_\beta P^\nu{}_\alpha - \frac{2}{3}P^{\mu\nu}P_{\alpha\beta}).
\end{aligned}
$$

D. E. Kharzeev and H. U. Yee, Phys. Rev. D **84**, 045025 (2011),

J. Hernandez and P. Kovtun, JHEP **1705**, 001 (2017).

Perturbation around equilibrium state

- Equilibrium state

$$u^\mu = u_0^\mu = (1, \vec{0}), \quad T = T_0, \quad \mu = \mu_0,$$
$$\tilde{T}^{tt} = \epsilon, \quad \tilde{T}^{zz} = p, \quad \tilde{T}^{ii} = p - \frac{C_1}{3} B^2 \ (i = x, y), \quad \tilde{J}^t = n$$
$$\tilde{T}^{tz} = \frac{1}{2}\left(c_g T^2 + c\mu^2\right) B, \quad \tilde{J}^z = c\mu B.$$

- Perturbations

$$\begin{aligned} u^\mu &\to \left(1, \delta u_x(\boldsymbol{x}, t), \delta u_y(\boldsymbol{x}, t), \delta u_z(\boldsymbol{x}, t)\right), \\ T &\to T + \delta T(\boldsymbol{x}, t), \\ \mu &\to \mu + \delta\mu(\boldsymbol{x}, t). \end{aligned}$$

We want to read the response of system to the external E and ∇T.

Conductivities

- Linearize constitutive relations w.r.t fluctuations
- Solve linearized sets of hydro equations for fluctuations $(\delta\mu, \delta T, \delta u^i)$.
- Read on-shell constitutive relations
- Derive transport coefficient

$$\begin{pmatrix} J_i \\ Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & T\alpha_{ij} \\ T\alpha_{ij} & T\kappa_{ij} \end{pmatrix} \begin{pmatrix} E_j \\ -\frac{\nabla_j T}{T} \end{pmatrix}.$$

- General form of conductivities in small B and $k \to 0$

$$\sigma \sim B^2 \left(c^2 \mathcal{A} + c\, c_g\, \mathcal{B} + c_g^2\, \mathcal{C} \right).$$

N. Abbasi, A. Ghazi, F. Taghinavaz and O. Tavakol, arXiv:1812.11310 [hep-th].

Ward identities

- **Non-anomalous case**: Relations among transport coefficients implied by invariance under the gauge and diff transformation (C. P. Herzog, J. Phys. A **42**, 343001 (2009).)
- **Anomalous case**:

$$W_{cov} = W[\partial\mathcal{M}_5] + \int d^5x\, \mathcal{I}_{CS}.$$

  - W_{cov} is invariant under the gauge and diff transformations
  - Ward identities are relations among **covariant** conserved currents.

$$\begin{aligned} T\,\alpha_{zz} &= \tau\left\{G_{\mathcal{R}}^{T_{0z}J_z} - e\mu\, G_{\mathcal{R}}^{J_zJ_z}\right\} = en\tau - \frac{\mu}{e}\,\sigma_{zz} \\ T\,\kappa_{zz} &= \tau\left\{G_{\mathcal{R}}^{T_{0z}T_{0z}} - 2\mu G_{\mathcal{R}}^{T_{0z}J_z} + \mu^2\, G_{\mathcal{R}}^{J_zJ_z}\right\} \\ &= \tau(\epsilon + p - 2\mu n) + \frac{\mu^2}{e^2}\,\sigma_{zz}. \end{aligned}$$

N. Abbasi, F. Taghinavaz and O. Tavakol, JHEP **1903**, 051 (2019).

Thermodynamics of spin $\frac{1}{2}$ particles

- Conserved currents of a chiral fluid in $B \ll T^2 \ll \mu^2$

$$\epsilon = T^4\left(\frac{\mu^4}{8\pi^2 T^4} + \frac{\mu^2}{4T^2} + \frac{7\pi^2}{120}\right) + \frac{e^2 B^2}{24\pi^2} - \left(log\frac{\mu}{\Delta_B} - \frac{\pi^2}{6}\frac{T^2}{\mu^2}\right)\frac{e^2 B^2}{16\pi^2},$$

$$p = T^4\left(\frac{\mu^4}{24\pi^2 T^4} + \frac{\mu^2}{12T^2} + \frac{7\pi^2}{360}\right) + \frac{e^2 B^2}{48\pi^2} + \left(log\frac{\mu}{\Delta_B} - \frac{\pi^2}{6}\frac{T^2}{\mu^2}\right)\frac{e^2 B^2}{16\pi^2},$$

$$n = T^3\left(\frac{\mu^3}{6\pi^2 T^3} + \frac{\mu}{6T}\right) + \frac{e^2 B^2}{16\pi^2\mu}\left(1 + \frac{\pi^2 T^2}{3\mu^2}\right).$$

N. Abbasi, F. Taghinavaz and O. Tavakol, JHEP **1903**, 051 (2019).

Symmetry constrains relaxation time

- Transport coefficients coincide with the classical one in the limit of $B \to 0$ and are proportional to τ_m.
- Onsager reciprocal relation (microscopic time reversal symmetry) dictates that $\alpha_1 = \alpha_2$

$$O(\frac{1}{\mu}): \qquad \tau_e - \tau_c = 0,$$

$$O(\frac{1}{\mu^3}): \qquad \frac{7}{48}\tau_m + \frac{7}{60}\tau_e - \frac{13}{40}\tau_c = -\frac{1}{48}(2\tau_e + \tau_m),$$

$$\boxed{\tau_e = \tau_m = \tau_c}.$$

N. Abbasi, A. Ghazi, F. Taghinavaz and O. Tavakol, arXiv:1812.11310 [hep-th].

Motivation for fluid-gravity

- **Membrane paradigm**: For an external observer, the black holes seems to be as a fluid membrane. It has shear viscosity, diffusion coefficient, $\cdots$. Dynamics of this membrane is described by laws of fluid dynamics.

  T. Damour, Phys. Rev. D **18**, 3598 (1978).
  R. H. Price and K. S. Thorne, Phys. Rev. D **33**, 915 (1986).

- **Defect in membrane paradigm**: It gives negative bulk viscosity, $\xi = -\frac{1}{16\pi}$. (M. Parikh and F. Wilczek, Phys. Rev. D **58**, 064011 (1998)).
- Gauge-gravity duality can resolve this problem.

Global hydro vs. gravity: boosted frame

- Boundary thermodynamics : (T, u^μ)
  $\Rightarrow$ Gravity:

$$
\begin{aligned}
ds^2 &= -2u_\mu dx^\mu dr - r^2 f(br) u_\mu u_\nu dx^\mu dx^\nu - r^2 \mathcal{P}_{\mu\nu} dx^\mu dx^\nu, \\
f(r) &= 1 - \frac{1}{r^4}, \\
\mathcal{P}_{\mu\nu} &= \eta_{\mu\nu} + u_\mu u_\nu, \\
E_{MN} &= R_{MN} - \frac{1}{2} g_{MN} - 6 g_{MN} = 0, \quad u_\mu u^\mu = -1.
\end{aligned}
$$

(μ, ν) are boundary coordinates,
It is a four-parameter solution of Einstein equation

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Conserved currents from AdS/CFT

- $T_{\mu\nu}$ of a conformal fluid on boundary:

$$T_{\mu\nu} = (\epsilon + \mathcal{P})u_\mu u_\nu + \mathcal{P}\eta_{\mu\nu} = \mathcal{P}\left(\eta_{\mu\nu} + 4u_\mu u_\nu\right).$$

- $T_{\mu\nu}$ on the boundary:

$$\begin{aligned} T_{\mu\nu} &= \lim_{r\to\infty} \frac{r^2}{8\pi G_5}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}\gamma_{\mu\nu}\right). \\ 8\pi G_5 T_{\mu\nu} &= \frac{1}{b^4}\left(\eta_{\mu\nu} + 4u_\mu u_\nu\right), \\ \epsilon &= 3\mathcal{P} = \frac{3\pi^4 T^4}{8\pi G_5}, \\ s &= \frac{\epsilon + \mathcal{P}}{T} = \frac{4\pi^4 T^3}{8\pi G_5}. \end{aligned}$$

V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208**, 413 (1999).

Towards FG

- Localized version of boosted brane

$$
\begin{aligned}
ds^2 &= -2u_\mu(x^\alpha)dx^\mu dr - r^2 f(b(x^\alpha)r)u_\mu(x^\alpha)u_\nu(x^\alpha)dx^\mu dx^\nu \\
&\quad -r^2\mathcal{P}_{\mu\nu}(x^\alpha)dx^\mu dx^\nu, \\
E_{MN} &\neq 0.
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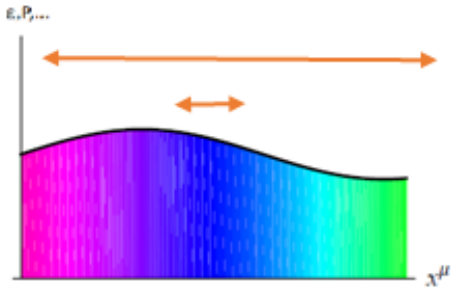
- However, we can solve the Einstein equation perturbatively by using the notion of slow variation of hydro fields



$$
u_\mu(x) = u_\mu(x_0) + x^\nu \partial_\nu u_\mu(x) + \cdots,
$$

$$
\theta = x^\mu \partial_\mu \sim \frac{l_{Mic}}{L_{Mac}} \sim \frac{1}{TL} \ll 1.
$$

V. E. Hubeny,Class. Quant. Grav. **28**, 114007 (2011)

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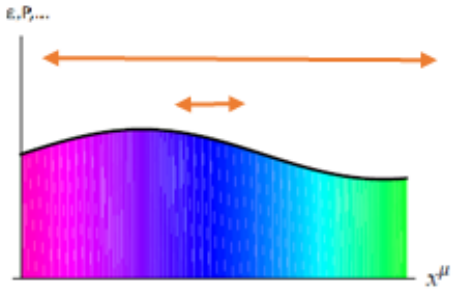

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Perturbative structure of metric

- In a non-perturbative manner

$$E_{MN}(g^0(r, x^\mu)) \neq 0.$$

- In a perturbative manner:

$$\begin{aligned} &E_{MN}(g^{(0)}(r, x^\mu)) = 0 + \mathcal{O}(\theta), \\ &E_{MN}(g^{(0)}(r, x^\mu) + \underbrace{g^{(1)}(r, x^\mu)}_{\mathcal{O}(\theta)}) = 0 + \mathcal{O}(\theta^2), \\ &, \cdots . \end{aligned}$$

- Structure of unknown metric components

$$g^{(1)}_{MN}(r, x^\mu) = F(r)\, \underbrace{G(x^\alpha)}_{\mathcal{O}(\theta)}.$$

How to solve

- Gauge fixing:

$$g_{rr} = 0, \quad g_{r\mu} = u_\mu, \quad Tr((g^0)^{-1} g) = 0.$$

$\Rightarrow$ 10 unknown components $g_{\mu\nu}$. We decompose them in terms of irreducible representations of local $SO(3)$

$$\mathbf{10} = \underbrace{\mathbf{2}}_{\text{scalar}} + \underbrace{\mathbf{3}}_{\text{vector}} + \underbrace{\mathbf{5}}_{\text{symmetric traceless tensor}}$$

- Number of Einstein equations

$$\mathbf{15} = \underbrace{\mathbf{1}}_{\text{trivial}} + \underbrace{\mathbf{4}}_{\text{constrained}} + \underbrace{\mathbf{10}}_{\text{have to be solved}}$$

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Solution channels

- Scalar channel

$$g_{ii}^{(1)}(r) \;=\; 3\,r^2\,h_1(r),\;\; g_{vv}^{(1)}(r) = \frac{k_1(r)}{r^2},\;\; g_{vr}^{(1)}(r) = -\frac{3}{2}\,h_1(r).$$

- Vector channel

$$g_{vi}^{(1)}(r) = r^2\,(1 - f(r))\,j_i^{(1)}(r).$$

- Tensor channel

$$g_{ij}^{(1)}(r) = r^2 \alpha_{ij}^{(1)}(r).$$

- Constrained equations

$$\boxed{\partial_v b^0 = \frac{\partial_i \beta_i^{(0)}}{3},\; \Leftrightarrow \partial_\mu T_{(0)}^{\mu 0} = 0}$$

$$\boxed{\partial_i b^0 = \partial_v \beta_i^{(0)} \;\; \Leftrightarrow \partial_\mu T_{(0)}^{\mu i} = 0.}$$

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Charged fluid in strong regime

- Bulk action corresponds to boundary charged fluid

$$S = \frac{1}{16\pi G_5}\int\sqrt{-g_5}\left[R + 12 - F_{AB}F^{AB} - \frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB}F_{CD}\right].$$

- Current after solving equations

$$J^\mu = J^\mu_{(0)} + J^\mu_{(1)} = \lim_{r\to\infty}\frac{r^2 A^\mu}{2\pi G_5} = nu^\mu + \xi_B B^\mu + \xi_\omega \omega^\mu,$$

$$\xi_\omega = -\frac{3q^2\kappa}{2\pi G_5 m} = C\mu^2\left(1 - \frac{2}{3}\frac{n\mu}{\epsilon + \mathcal{P}}\right),$$

$$\xi_B = -\sqrt{3}\frac{(m + 3r_+^4)q\kappa}{4\pi G_5 m r_+^2} = C\mu\left(1 - \frac{1}{2}\frac{n\mu}{\epsilon + \mathcal{P}}\right),$$

N. Banerjee, et al, JHEP **1101**, 094 (2011),
J. Erdmenger, M. Haack, M. Kaminski and A. Yarom, JHEP **0901**, 055 (2009),
D. T. Son and P. Surowka, Phys. Rev. Lett. **103**, 191601 (2009).

Lessons from FG

What have we learned from FG correspondence?

- FG correspondence is indeed long-wavelength regime of ADS/CFT.
- Einstein equations in the limit of long-wavelength correspond to the boundary Navier-Stokes equation. FG correspondence provides a map from solution space of fluid dynamics to solution space of Einstein space.
- **Thermodynamics of strongly interacting particles in magnetic field $\sim$ Magnetized brane**

Gravity set up

- Bulk action

$$S = -\frac{1}{16\pi G_5}\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R - \frac{12}{L^2} + F^{MN}F_{MN}\right) + S_{CS} + S_{bdy},$$
$$S_{CS} = \frac{k}{12\pi G_5}\int A\wedge F\wedge F = \frac{k}{192\pi G_5}\int d^5x\sqrt{-g}\epsilon^{MNPQE}A_M F_{NP}F_{QR},$$
$$S_{bdy} = -\frac{1}{8\pi G_5}\int_{\partial\mathcal{M}} d^4x\,\sqrt{-\gamma}\left(\mathcal{K} - \frac{3}{L} + \frac{L}{4}R(\gamma) + \frac{L}{2}\ln\left(\frac{r}{L}\right)F^{\mu\nu}F_{\mu\nu}\right).$$

- Equations of motion

$$\begin{aligned} 0 &= d*F + k\,F\wedge F, \\ R_{MN} &= 4g_{MN} + \frac{1}{3}g_{MN}F^{AB}F_{AB} - 2F_{MP}F_N^{\ P}. \end{aligned}$$

E. D'Hoker and P. Kraus, JHEP **1003**, 095 (2010).

Solution

- Solution ansatz

$$ds^2 = \frac{dr^2}{U(r)} - U(r)dt^2 + e^{2V(r)}(dx_1^2 + dx_2^2) + e^{2W(r)}(dx_3 + C(r)dt)^2.$$

- Structure of functions in weak B

$$\begin{aligned} U &= U_0 + B^2 U_2, & E &= E_0 + B^2 E_2, \\ W &= W_0 + B^2 W_2, & C &= C_0 + BC_1, \\ V &= V_0 + B^2 V_2, & P &= P_0 + BP_1. \end{aligned}$$

E. D'Hoker and P. Kraus, JHEP **1003**, 095 (2010).

Important points

- We have solved these unknown functions in the limit of small "B" and small μ in such a way that $B \ll \mu^2 \ll T^2$.
- Horizon's radius is modified due to the presence of "B".
- Due to presence of "B", T_0 and μ_0 are not the physical quantities of the boundary theory. They are modified and give rise the true ones.
- We introduce an energy scale Δ on the boundary in which our results make sense.

N. Abbasi, A. Ghazi, F. Taghinavaz and O. Tavakol, arXiv:1812.11310 [hep-th].

Thermodynamic Properties

- Conserved currents

$$T_{00} = \frac{N_c^2}{8\pi^2}\left(3(\pi T)^4 + 12(\pi T)^2\mu^2 + 8\mu^4\right) + \frac{N^2B^2}{4\pi^2}\left((1 - \ln\frac{\pi T}{\Delta}) - \frac{2}{3}\frac{\mu^2}{\pi T^2}(8\ln 2 - 3)\right),$$

$$T^{0z} = \frac{c\mu^2 B}{2},$$

$$T_{ii} = \frac{N_c^2}{24\pi^2}\left(3(\pi T)^4 + 12(\pi T)^2\mu^2 + 8\mu^4\right) + \frac{N^2B^2}{4\pi^2}\left(\ln\frac{\pi T}{\Delta} + \frac{2}{3}\frac{\mu^2}{\pi T^2}(8\ln 2 - 3)\right),$$

$$J^0 = \frac{N_c^2}{3\pi^2}\left(3(\pi T)^2\,\mu + 4\mu^3\right) + \frac{N^2B^2}{3\pi^2}\frac{\mu}{(\pi T)^2}\,(8\ln 2 - 3),$$

$$J^z = c\mu B.$$

- These currents satisfy Gibbs Duhem relation. Also they pass the entropy and number density check.

N. Abbasi, A. Ghazi, F. Taghinavaz and O. Tavakol, arXiv:1812.11310 [hep-th].

Conductivities in strong regime

- We read the response of system to the weak external electric field and temperature gradient by using general hydro formula

$$\sigma = \frac{2N_c^2\,\mu^2\,\tau}{\pi^2}\left(1+\frac{2B^2}{3\pi^2T^2\mu^2}\right),$$
$$T\alpha = N_c^2\,T^2\,\mu\,\tau\left(1-\frac{2\mu^2}{3\pi^2T^2}+\frac{B^2}{3\pi^4T^4}\,(8\ln 2-7)\right),$$
$$T\kappa = \frac{\pi^2N_c^2\,T^4\,\tau}{2}\left(1+\frac{4\mu^4}{3\pi^4T^4}+\frac{B^2}{2\pi^4T^4}\left(1-\frac{8\mu^2}{3\pi^2T^2}\,(8\ln 2-5)\right)\right).$$

- These transport coefficients respect to Anomalous Ward identities.

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Conclusion

- Chiral anomaly leads to the increase of electrical conductivity . This is the so called NMR. We study the magneto-transport for a system with weakly broken time, translation and charge symmetries.
- We utilize of anomalous second order hydrodynamics to obtain the most general relations for those transports. They are general and have relations to gauge and gravitational anomaly. They only depend on thermodynamic derivatives.
- We implement these results to the case of hydros which their particles are weakly and strongly interacting ones. In both regimes, the obtained transports respect to the Ward identities.