BMS4 Algebra, Its Stability and Deformations

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Motivation

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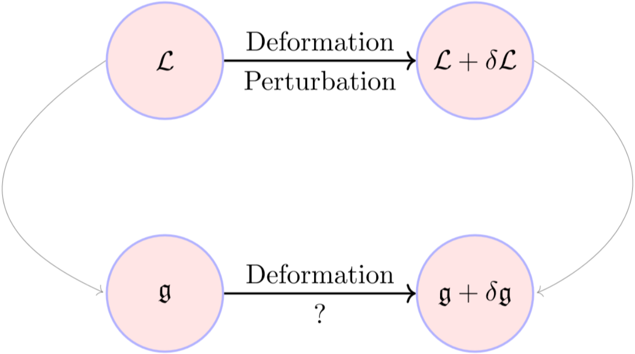
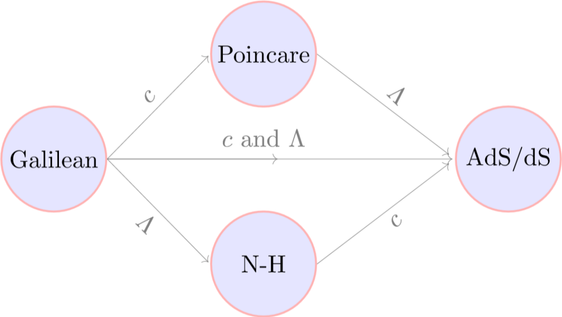


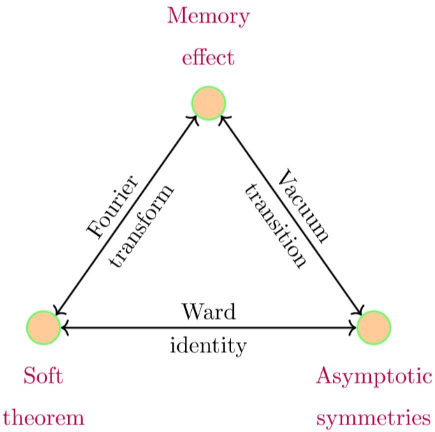
Is there a similar notion as deformations at the level of (symmetry) algebra???

Motivation



As an example, Galilean algebra can be deformed to Poincare, Newton-Hooke and AdS/dS algebras!

- The concept of asymptotic symmetries is one corner of triangular description of IR dynamics of gauge theories.



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- ASA of AdS_4 is just isometries algebra of AdS spacetime, $\mathfrak{so}(3,2)$
- ASA of $4d$ flat spacetime, $\mathfrak{bms}_4$ algebra, is infinite dimensional
- AS analysis depends very much on the choice of boundary falloff behavior

Why $\mathfrak{bms}_4$ algebra?

1. **The main question**

May $\mathfrak{bms}_4$ algebra come from contraction of an infinite dimensional asymptotic symmetry algebra of AdS_4 with another boundary falloff conditions?

Application of the deformation procedure to study asymptotic symmetry algebra of $4d$ flat spacetime.

2

Deformation theory of Lie algebras

Definition

Deformation of a certain Lie algebra is defined as

$$[g_i, g_j]_\varepsilon := \Psi(g_i, g_j; \varepsilon) = \Psi(g_i, g_j; \varepsilon = 0) + \psi_1(g_i, g_j)\varepsilon^1 + \psi_2(g_i, g_j)\varepsilon^2 + ...,$$

in which $\psi_r(g_i, g_j)$ is a bilinear and anti symmetric function and ε is called deformation parameter.
$\Psi(g_i, g_j; \varepsilon = 0)$, or $[g_i, g_j]_0$, denotes the Lie bracket of the original algebra.

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The deformed commutator must satisfy the Jacobi identity

$$[g_i, [g_j, g_k]_\varepsilon]_\varepsilon + \text{cyclic permutation of } (g_i, g_j, g_k) = 0,$$

order by order in ε.

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Definition

The *non trivial* deformations are whose can not be removed by change of the basis.

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- **Contraction vs deformation**

Definition

The *contraction* procedure is inverse of deformation. In fact by taking the limit $\varepsilon \to 0$ one can return to the original algebra $\mathfrak{g}$ from the deformed algebra.

- **Stable Lie algebra**

Definition

A Lie algebra $\mathfrak{g}$ is called formally *Stable or Rigid* if it does not admit any formal deformation.

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Whitehead's Lemma

All semisimple finite Lie algebras are stable.

- **Finite vs infinite dimensional Lie algebras**

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Definition

- In physics we are usually dealing with the Lie algebras which have semi direct sum structure, $\mathfrak{g} = \mathfrak{g}_1 \inplus \mathfrak{g}_2$, and its commutation relations are as follows

$$[\mathfrak{g}_1, \mathfrak{g}_1] = \mathfrak{g}_1,$$
$$[\mathfrak{g}_1, \mathfrak{g}_2] = \mathfrak{g}_2,$$
$$[\mathfrak{g}_2, \mathfrak{g}_2] = \mathfrak{g}_2.$$

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Hochschild-Serre factorization theorem

- All deformations of finite dimensional Lie algebra $\mathfrak{g}$ which has semi direct sum structure, are located in its ideal part.

- **Deformation of $3d$ Poincare algebra to $3d$ AdS/dS algebra**.

Commutation relations

$$\begin{aligned} i[\mathcal{J}_m, \mathcal{J}_n] &= (m-n)\mathcal{J}_{m+n}, \\ i[\mathcal{J}_m, \mathcal{P}_n] &= (m-n)\mathcal{P}_{m+n}, \\ i[\mathcal{P}_m, \mathcal{P}_n] &= 0. \end{aligned} \quad \longrightarrow \quad \begin{aligned} i[\mathcal{J}_m, \mathcal{J}_n] &= (m-n)\mathcal{J}_{m+n}, \\ i[\mathcal{J}_m, \mathcal{P}_n] &= (m-n)\mathcal{P}_{m+n}, \\ i[\mathcal{P}_m, \mathcal{P}_n] &= \pm(\Lambda^2)(m-n)\mathcal{J}_{m+n}. \end{aligned}$$

where $m, n = \pm 1, 0$.

- **Deformation of $3d$ Poincare algebra to $3d$ AdS/dS algebra**.

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where $m, n = \pm 1, 0$.

- The H-S factorization theorem does not work in the case of infinite dimensional algebras!

3

Review on deformations of $\mathfrak{bms}_3$ algebra

- $\mathfrak{bms}_3$ is an infinite dimensional algebra which has semi direct sum structure as

$$(Superrotations) \in\!\!\!\!+ (Supertranslations)$$

Commutation relations of $\mathfrak{bms}_3$

$$
\begin{aligned}
i[\mathcal{J}_m, \mathcal{J}_n] &= (m-n)\mathcal{J}_{m+n}, \\
i[\mathcal{J}_m, \mathcal{P}_n] &= (m-n)\mathcal{P}_{m+n}, \\
i[\mathcal{P}_m, \mathcal{P}_n] &= 0.
\end{aligned}
$$

- Deformations of centerless $\mathfrak{bms}_3$
- We have shown that $\mathfrak{bms}_3$ just have two independent deformations

- Deformations of centerless $\mathfrak{bms}_3$
- The $\mathfrak{bms}_3$ algebra can be deformed in its ideal part as

$$
\begin{aligned}
i[\mathcal{J}_m, \mathcal{J}_n] &= (m-n)\mathcal{J}_{m+n}, \\
i[\mathcal{J}_m, \mathcal{P}_n] &= (m-n)\mathcal{P}_{m+n}, \\
i[\mathcal{P}_m, \mathcal{P}_n] &= \varepsilon(m-n)f(m,n)\mathcal{J}_{m+n}.
\end{aligned}
$$

- Deformations of centerless $\mathfrak{bms}_3$

The Jacobi analysis gives rise to

$$i[\mathcal{J}_m,\mathcal{J}_n]=(m-n)\mathcal{J}_{m+n},$$
$$i[\mathcal{J}_m,\mathcal{P}_n]=(m-n)\mathcal{P}_{m+n},$$
$$i[\mathcal{P}_m,\mathcal{P}_n]=\tilde{\varepsilon}(m-n)\mathcal{J}_{m+n}.$$

- By an appropriate redefinition of generators one gets the asymptotic symmetry algebra of $3d$ AdS spacetimes ($\mathfrak{witt}\oplus\mathfrak{witt}$) in which deformation parameter is the cosmological constant Λ.

$$i[\mathcal{L}_m,\mathcal{L}_n]=(m-n)\mathcal{L}_{m+n},$$
$$i[\mathcal{L}_m,\bar{\mathcal{L}}_n]=0,$$
$$i[\bar{\mathcal{L}}_m,\bar{\mathcal{L}}_n]=(m-n)\bar{\mathcal{L}}_{m+n}.$$

- Deformations of centerless $\mathfrak{bms}_3$
- The $\mathfrak{bms}_3$ algebra has another deformation in commutators $[\mathcal{J},\mathcal{P}]$ which is in contrast with HSF theorem

$$
\begin{aligned}
&i[\mathcal{J}_m,\mathcal{J}_n]=(m-n)\mathcal{J}_{m+n},\\
&i[\mathcal{J}_m,\mathcal{P}_n]=(m-n)\mathcal{P}_{m+n}+\zeta K(m,n)\mathcal{P}_{m+n},\\
&i[\mathcal{P}_m,\mathcal{P}_n]=0.
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The Jacobi identities lead to

$$K(m,n) = \alpha + \beta m.$$

- Deformations of centerless $\mathfrak{bms}_3$

$W(a,b)$ algebra

$$[\mathcal{J}_m, \mathcal{J}_n] = (m-n)\mathcal{J}_{m+n},$$
$$[\mathcal{J}_m, \mathcal{P}_n] = -(a+bm+n)\mathcal{P}_{m+n},$$
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$$[\mathcal{P}_m, \mathcal{P}_n] = 0.$$

- The deformation parameter a is related to the periodicity of vector field $\mathcal{P}(\varphi)$ on circle.
- The deformation parameter b is related to the conformal weight of operator $\mathcal{P}$.

Theorem

The most general deformations of $\mathfrak{bms}_3$ are either $\mathfrak{witt} \oplus \mathfrak{witt}$ or $W(a,b)$ algebras.

Theorem

The most general deformations of $\widehat{\mathfrak{bms}_3}$ are either $\mathfrak{vir} \oplus \mathfrak{vir}$ or $\widehat{W}(a,b)$ algebras, (the latter has just one central terms in its Witt subalgebra).

4

Deformation of $\mathfrak{bms}_4$ algebra

- The $\mathfrak{bms}_4$ is ASA of $4d$ flat spacetime

- The original $\mathfrak{bms}_4$ algebra has semi direct sum structure as

$$(\mathfrak{bms}_4)_{old} = (Lorentz) \inplus (Supertranslations)$$

Deformation of $\mathfrak{bms}_4$ algebra

- The extended $\mathfrak{bms}_4$ algebra, which is infinite enhancement of the old version, has semi direct sum structure as

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$$\mathfrak{bms}_4 = (Superrotations) \inplus (Supertranslations)$$

- Commutation relations of $\mathfrak{bms}_4$ algebra

$$
\begin{aligned}
&[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n},\\
&[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n},\\
&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0,\\
&[\mathcal{L}_m, T_{p,q}] = (\frac{m+1}{2} - p)T_{p+m,q},\\
&[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q)T_{p,q+n},\\
&[T_{p,q}, T_{r,s}] = 0.
\end{aligned}
$$

- Commutation relations of $\mathfrak{bms}_4$ algebra
- It has direct sum of two Witt subalgebras which is infinite enhancement of Lorentz algebra

$$
\begin{aligned}
&[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n}, \\
&[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n}, \\
&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0, \\
&[\mathcal{L}_m, T_{p,q}] = (\frac{m+1}{2} - p)T_{p+m,q}, \\
&[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q)T_{p,q+n}, \\
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\end{aligned}
$$

- Commutation relations of $\mathfrak{bms}_4$ algebra
- $\mathcal{L}$ and $\bar{\mathcal{L}}$ act on first and second indeces of T generators respectively

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&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0, \\
&[\mathcal{L}_m, T_{p,q}] = (\frac{m+1}{2} - p)T_{p+m,q}, \\
&[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q)T_{p,q+n}, \\
&[T_{p,q}, T_{r,s}] = 0.
\end{aligned}
$$

- Commutation relations of $\mathfrak{bms}_4$ algebra
- The ideal part is Abelian

$$
\begin{aligned}
&[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n},\\
&[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n},\\
&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0,\\
&[\mathcal{L}_m, T_{p,q}] = (\frac{m+1}{2} - p)T_{p+m,q},\\
&[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q)T_{p,q+n},\\
&[T_{p,q}, T_{r,s}] = 0.
\end{aligned}
$$

- Deformation of centerless $\mathfrak{bms}_4$
- Deformation in the two Witt subalgebras

- Deformation of centerless $\mathfrak{bms}_4$

▶ Deformation in the two Witt subalgebras

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + (m-n)h(m,n)T_{m+n,0},$$
$$[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + (m-n)\bar{h}(m,n)T_{0,m+n},$$
$$[\mathcal{L}_m, \bar{\mathcal{L}}_n] = H(m,n)T_{m,n}.$$

- Deformation of centerless $\mathfrak{bms}_4$
  - Deformation in the two Witt subalgebras

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + (m-n)h(m,n)T_{m+n,0},$$
$$[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + (m-n)\bar{h}(m,n)T_{0,m+n},$$
$$[\mathcal{L}_m, \bar{\mathcal{L}}_n] = H(m,n)T_{m,n}.$$

The Jacobi analysis leads to

$$h(m,n) = constant = h, \quad \bar{h}(m,n) = constant = \bar{h}$$
$$H(m,n) = H_0(m+1)(n+1) + \bar{h}(m+1) - h(n+1).$$

- Deformation of centerless $\mathfrak{bms}_4$

▸ Deformation in the two Witt subalgebras

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + (m-n)h(m,n)T_{m+n,0},$$
$$[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + (m-n)\bar{h}(m,n)T_{0,m+n},$$
$$[\mathcal{L}_m, \bar{\mathcal{L}}_n] = H(m,n)T_{m,n}.$$

Redefinition of generators

$$\tilde{\mathcal{L}}_m \equiv \mathcal{L}_m + \sum X(m)T_{m,0},$$
$$\tilde{\bar{\mathcal{L}}}_m \equiv \bar{\mathcal{L}}_m + \sum Y(m)T_{0,m},$$
$$\tilde{T}_{m,n} \equiv T_{m,n},$$

where $X(m) = H_0(m+1) - 2h$ and $Y(m) = -H_0(m+1) - 2\bar{h}$.

- Deformation of centerless $\mathfrak{bms}_4$
- Deformation of $[\mathcal{L}, T]$ commutators

- Deformation of centerless $\mathfrak{bms}_4$

▶ Deformation of $[\mathcal{L}, T]$ commutators

$$[\mathcal{L}_m, T_{p,q}] = (\frac{m+1}{2} - p)T_{p+m,q} + K(m,p)T_{p+m,q},$$
$$[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q)T_{p,q+n} + \bar{K}(n,q)T_{p,n+q}.$$

Deformation of $\mathfrak{bms}_4$ algebra

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- Deformation of $[\mathcal{L}, T]$ commutators

$$[\mathcal{L}_m, T_{p,q}] = (\frac{m+1}{2} - p)T_{p+m,q} + K(m,p)T_{p+m,q},$$
$$[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q)T_{p,q+n} + \bar{K}(n,q)T_{p,n+q}.$$

The Jacobi identities $[\mathcal{L}_m, [\mathcal{L}_n, T_{p,q}]]$ + cyclic permutations $= 0$ and $[\bar{\mathcal{L}}_m, [\bar{\mathcal{L}}_n, T_{p,q}]]$ + cyclic permutations $= 0$ result in

$$K(m,n) = \alpha + \beta m,$$

$$\bar{K}(m,n) = \bar{\alpha} + \bar{\beta} m.$$

- Deformation of centerless $\mathfrak{bms}_4$

- Deformation of $[\mathcal{L}, T]$ commutators

Theorem

The $\mathfrak{bms}_4$ algebra is not stable and can be formally deformed into a four parameter family of algebras $W(a, b; \bar{a}, \bar{b})$.

- Deformation of centerless $\mathfrak{bms}_4$

▶ Deformation of $[\mathcal{L}, T]$ commutators

Commutation relations of $W(a, b; \bar{a}, \bar{b})$

$$
\begin{aligned}
&[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n}, \\
&[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n}, \\
&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0, \\
&[\mathcal{L}_m, T_{p,q}] = -(p + bm + a)T_{p+m,q}, \\
&[\bar{\mathcal{L}}_n, T_{p,q}] = -(q + \bar{b}n + \bar{a})T_{p,q+n}, \\
&[T_{p,q}, T_{r,s}] = 0.
\end{aligned}
$$

▶ We should note that $\mathfrak{bms}_4 = W(-\frac{1}{2}, -\frac{1}{2}; -\frac{1}{2}, -\frac{1}{2})$.

- Deformation of centerless $\mathfrak{bms}_4$
  - Deformation of $[\mathcal{L}, T]$ commutators

$$[\mathcal{L}_m, T_{p,q}] = (\frac{m+1}{2} - p)T_{p+m,q} + \eta f(m,p)\mathcal{L}_{p+m-1}\delta_{q,0} + \sigma g(m,p)\bar{\mathcal{L}}_{q-1}\delta_{m+p,0},$$

$$[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q)T_{p,n+q} + \bar{\eta}\bar{f}(n,q)\mathcal{L}_{p-1}\delta_{n+q,0} + \bar{\sigma}\bar{g}(n,q)\bar{\mathcal{L}}_{n+q-1}\delta_{p,0},$$

- Deformation of centerless $\mathfrak{bms}_4$

▸ Deformation of $[\mathcal{L}, T]$ commutators

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$$[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q)T_{p,n+q} + \bar{\eta}\bar{f}(n,q)\mathcal{L}_{p-1}\delta_{n+q,0} + \bar{\sigma}\bar{g}(n,q)\bar{\mathcal{L}}_{n+q-1}\delta_{p,0},$$

The Jacobi analysis leads to

$$f(m,p) = \bar{f}(n,q) = g(m,p) = \bar{g}(n,q) = 0.$$

- Deformation of centerless $\mathfrak{bms}_4$

  - Deformations of commutator of ideal part $[T, T]$

- Deformation of centerless $\mathfrak{bms}_4$

- Deformations of commutator of ideal part $[T, T]$

$$[T_{m,n}, T_{p,q}] = G(m, n; p, q) T_{m+p,n+q},$$

- Deformation of centerless $\mathfrak{bms}_4$

▶ Deformations of commutator of ideal part $[T, T]$

$$[T_{m,n}, T_{p,q}] = G(m, n; p, q)T_{m+p,n+q},$$

The Jacobi $[\mathcal{L}_r, [T_{m,n}, T_{p,q}]] + \textit{cyclic permutations} = 0$ leads to

$$G(m, n; p, q) = 0.$$

- Deformation of centerless $\mathfrak{bms}_4$
  - Deformations of commutator of ideal part $[T, T]$

$$[T_{m,n}, T_{p,q}] = A(m, n; p, q)\mathcal{L}_{m+p-1} + B(m, n; p, q)\bar{\mathcal{L}}_{n+q-1},$$

- Deformation of centerless $\mathfrak{bms}_4$

▶ Deformations of commutator of ideal part $[T, T]$

$$[T_{m,n}, T_{p,q}] = A(m,n;p,q)\mathcal{L}_{m+p-1} + B(m,n;p,q)\bar{\mathcal{L}}_{n+q-1},$$

The Jacobi identities $[\mathcal{L}_r, [T_{m,n}, T_{p,q}]] + \textit{cyclic permutations} = 0$ and $[\bar{\mathcal{L}}_r, [T_{m,n}, T_{p,q}]] + \textit{cyclic permutations} = 0$ lead to

$$A(m,n;p,q) = B(m,n;p,q) = 0.$$

- Deformation of centerless $\mathfrak{bms}_4$

Conclusion

Although the ideal part of $\mathfrak{bms}_4$ *algebra is STABLE, it is not stable generally and can be deformed into* $W(a, b; \bar{a}, \bar{b})$ *algebra.*

- Deformation of centrally extended $\mathfrak{bms}_4$, $\widehat{\mathfrak{bms}}_4$

  - The $\mathfrak{bms}_4$ algebra admits two independent central terms in its two Witt subalgebras

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$$
\begin{aligned}
&[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + \frac{C_{\mathcal{L}}}{12} m^3 \delta_{m+n,0},\\
&[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + \frac{C_{\bar{\mathcal{L}}}}{12} m^3 \delta_{m+n,0},\\
&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0,\\
&[\mathcal{L}_m, T_{p,q}] = (\frac{m+1}{2} - p) T_{p+m,q},\\
&[\bar{\mathcal{L}}_n, T_{p,q}] = (\frac{n+1}{2} - q) T_{p,q+n},\\
&[T_{p,q}, T_{r,s}] = 0.
\end{aligned}
$$

- Deformation of centrally extended $\mathfrak{bms}_4$, $\widehat{\mathfrak{bms}}_4$

▶ The most general deformation of $\widehat{\mathfrak{bms}}_4$ algebra is $\widehat{W}(a,b;\bar{a},\bar{b})$:

$$
\begin{aligned}
&[\mathcal{L}_m,\mathcal{L}_n]=(m-n)\mathcal{L}_{m+n}+\frac{C_{\mathcal{L}}}{12}m^3\delta_{m+n,0},\\
&[\bar{\mathcal{L}}_m,\bar{\mathcal{L}}_n]=(m-n)\bar{\mathcal{L}}_{m+n}+\frac{C_{\bar{\mathcal{L}}}}{12}m^3\delta_{m+n,0},\\
&[\mathcal{L}_m,\bar{\mathcal{L}}_n]=0,\\
&[\mathcal{L}_m,T_{p,q}]=-(p+bm+a)\;T_{p+m,q},\\
&[\bar{\mathcal{L}}_n,T_{p,q}]=-(q+\bar{b}n+\bar{a})\;T_{p,q+n},\\
&[T_{m,n},T_{p,q}]=0.
\end{aligned}
$$

5

Stability of deformed algebras

- **Deformations of W algebra**

- The most general deformation of $W(a, b; \bar{a}, \bar{b})$ algebra is $W(\tilde{a}, \tilde{b}; \tilde{\bar{a}}, \tilde{\bar{b}})$ with shifted parameters.

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- **Deformation of special W algebras**

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$W(0,0;0,0)$ algebra

$$
\begin{aligned}
&[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n},\\
&[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n},\\
&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0,\\
&[\mathcal{L}_m, T_{p,q}] = (-p)T_{p+m,q},\\
&[\bar{\mathcal{L}}_n, T_{p,q}] = (-q)T_{p,q+n},\\
&[T_{m,n}, T_{p,q}] = 0.
\end{aligned}
$$

- The $W(0,0;0,0)$ algebra is obtained (by Donnay et al.) as near horizon algebra of $4d$ Kerr black holes

- **Deformation of special W algebras**

Deformations of $W(0,0;0,0)$ algebra

$$
\begin{aligned}
&[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + \nu\ (m-n)T_{m+n,0},\\
&[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + \bar{\nu}\ (m-n)T_{0,m+n},\\
&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = H_0(\alpha+\beta m)(\bar{\alpha}+\bar{\beta}n)T_{m,n},\\
&[\mathcal{L}_m, T_{p,q}] = (-p)T_{p+m,q},\\
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&[\mathcal{L}_m, T_{p,q}] = -(a+bm+p)T_{p+m,q},\\
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- **Deformation of special W algebras**

$W(0,-1;0,0)$ algebra

$$\begin{aligned}
[\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\mathcal{L}_{m+n}, \\
[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] &= (m-n)\bar{\mathcal{L}}_{m+n}, \\
[\mathcal{L}_m, \bar{\mathcal{L}}_n] &= 0, \\
[\mathcal{L}_m, T_{p,q}] &= (m-p)T_{p+m,q}, \\
[\bar{\mathcal{L}}_n, T_{p,q}] &= (-q)T_{p,q+n}, \\
[T_{m,n}, T_{p,q}] &= 0.
\end{aligned}$$

- By setting the second index of the $T_{m,n}$ zero, one obtains the subalgebra $\mathfrak{bms}_3 \oplus \mathfrak{witt}$.

- **Deformation of special W algebras**

Deformations of $W(0,-1;0,0)$ algebra

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&[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0,\\
&[\mathcal{L}_m, T_{p,q}] = -(a + bm + p)T_{p+m,q},\\
&[\bar{\mathcal{L}}_n, T_{p,q}] = -(\bar{a} + \bar{b}n + q)T_{p,q+n},\\
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\end{aligned}
$$

- By appropriate redefinition of generators one obtains the direct sum of three Witt algebras as subalgebra.

- **Deformation of special W algebras**

  - *The most general formal deformations of* $W(0,-1;0,0)$ *algebra are* $\mathfrak{witt}\oplus\mathfrak{witt}\oplus\mathfrak{witt}$ *and* $W(a,b;\bar{a},\bar{b})$ *algebras.*

  - The $\mathfrak{witt}\oplus\mathfrak{witt}\oplus\mathfrak{witt}$ algebra and its contraction $\mathfrak{bms}_3\oplus\mathfrak{witt}$ algebra, can be obtained through deformation of infinite dimensional version of $3d$ Maxwell algebra.
  - These algebras can be obtained as asymptotic symmetry algebras of some Chern-Simons theories in $3d$.

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6. According to our computations, we have proposed a new version of HSF theorem for infinite dimensional algebras

1. Physical realization of deformation parameters $a, b, \bar{a}$ and $\bar{b}$

Future directions

1. Physical realization of deformation parameters $a, b, \bar{a}$ and $\bar{b}$
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4. From Mathematical point of view, consideration of our proposal for new version of HSF theorem

Thank You For Your Attention.