Probing the black hole interior in AdS/CFT

Kyriakos Papadodimas

ICTP

Recent Trends in String Theory and Related Topics
IPM, Tehran
April 2019
Motivation
- Black hole information paradox: fundamental conflict between GR and QM

Outline
- Review basic formulation of information paradox
- Explain the relevance of smoothness of horizon
- Formulate the problem in AdS/CFT
- A proposal for describing the BH interior from CFT
Motivation
- Black hole information paradox: fundamental conflict between GR and QM
- What happens to an infalling observer crossing the horizon?
Motivation

- Black hole information paradox: fundamental conflict between GR and QM

- What happens to an infalling observer crossing the horizon?

- IR paradox, quantum gravity effects/(non)-locality at large scales?
Motivation
- Black hole information paradox: fundamental conflict between GR and QM

- What happens to an infalling observer crossing the horizon?

- IR paradox, quantum gravity effects/(non)-locality at large scales?

- Concrete technical problem in AdS/CFT: how does the CFT describe the region behind the horizon?
Motivation
- Black hole information paradox: fundamental conflict between GR and QM
- What happens to an infalling observer crossing the horizon?
- IR paradox, quantum gravity effects/(non)-locality at large scales?
- Concrete technical problem in AdS/CFT: how does the CFT describe the region behind the horizon?
- Can we understand the BH singularity from the CFT?
Motivation
- Black hole information paradox: fundamental conflict between GR and QM
- What happens to an infalling observer crossing the horizon?
- IR paradox, quantum gravity effects/(non)-locality at large scales?
- Concrete technical problem in AdS/CFT: how does the CFT describe the region behind the horizon?
- Can we understand the BH singularity from the CFT?

Outline
- Review basic formulation of information paradox
- Explain the relevance of smoothness of horizon
- Formulate the problem in AdS/CFT
- A proposal for describing the BH interior from CFT
Hawking radiation

\[ \phi(x) = \sum f_i(x) a_i + h.c. \]

\[ \phi(x) = \sum g_i(x) b_i + \sum h_i(x) c_i + h.c. \]

\[ a_i \langle 0 \rangle_{in} = 0 \quad \text{but} \quad \langle 0 | b^\dagger b | 0 \rangle_{in} = \sum |\beta_{ij}|^2 \neq 0 \]
Properties of Hawking radiation

\[ \langle b^\dagger_\omega b_{\omega'} \rangle = \delta(\omega - \omega') P(\omega, l) \frac{1}{e^{\beta \omega} - 1} \]

where \( \beta^{-1} = T \) is

\[ T = \frac{1}{8\pi GM} \]

and where \( P(\omega, l) \) is the absorption probability (gray-body factor)

Moreover

\[ \langle bbbb... \rangle = \text{product of 2-point functions} \]

The Hawking particles seem to be \textbf{thermal} and \textbf{uncorrelated}.

The density matrix of the radiation is thermal (diagonal in occupation level basis)
Black Hole Evaporation

\[ t_{\text{evap}} \propto G^2 M^3 \]
Information Paradox

Hawking's computation seems to contradict Unitarity in Quantum Mechanics

It predicts that a pure state $|\Psi\rangle$ can evolve into a mixed state $\rho_{\text{thermal}}$ while in QM we have

$$\Psi(t) = e^{-iHt}\Psi(0)$$

Hawking's computation predicts that many initial states of same mass $M$, give the same final state $\Rightarrow$ fundamental irreversibility, information loss
Consider the first $N$ Hawking particles, compute reduced density matrix $\rho_N$ and its von Neumann entropy

$$S_N \equiv -\text{Tr}[\rho_N \log \rho_N]$$
Possible resolution

Information encoded in Hawking radiation in small correlations between particles

Can small corrections to Hawking’s computation resolve the information paradox?
Two clarifications:

1) We cannot yet calculate these corrections. This would be equivalent to computing an exact S-matrix in Quantum Gravity. We will only estimate what is the minimal size of these corrections necessary to restore unitarity and consider:
   i) whether it is reasonable to expect corrections of this size in the theory of Quantum Gravity and
   ii) whether the existence of these corrections is compatible with Effective Field Theory
Two clarifications:

1) We can not yet calculate these corrections. This would be equivalent to computing an exact S-matrix in Quantum Gravity. We will only estimate what is the minimal size of these corrections necessary to restore unitarity and consider:
   i) whether it is reasonable to expect corrections of this size in the theory of Quantum Gravity and
   ii) whether the existence of these corrections is compatible with Effective Field Theory

2) When talking about “small corrections to Hawking’s computation” it is important to be precise about the quantities to which these corrections apply.
General expectation: Unitarity can be restored in Hawking evaporation, at the price of introducing **exponentially small** (of order $e^{-S_{BH}}$) corrections to **simple** observables in effective field theory.

Corrections of this size are generally expected and do not invalidate effective field theory.

Corrections to complicated observables, for example $S_{BH}$-point functions, or “the quantum state of the entire Hawking radiation”, or the EE of the Hawking radiation may be very large.
General expectation: Unitarity can be restored in Hawking evaporation, at the price of introducing exponentially small (of order $e^{-S_{BH}}$) corrections to simple observables in effective field theory.

Corrections of this size are generally expected and do not invalidate effective field theory.

Corrections to complicated observables, for example $S_{BH}$-point functions, or “the quantum state of the entire Hawking radiation”, or the EE of the Hawking radiation may be very large

This claim relies on a basic property of Quantum Statistical Mechanics:
In a large quantum system most pure states look almost identical when probed by most observables — and almost identical to the maximally mixed state.
Pure vs Mixed states

[Lloyd]
In a large quantum system most pure states look almost identical when probed by most observables — and almost identical to the maximally mixed state.

For any observable $A$ we have the following identities

$$\overline{\langle \Psi | A | \Psi \rangle} = \text{Tr}[\rho_m A]$$
Pure vs Mixed states

In a large quantum system most pure states look almost identical when probed by most observables — and almost identical to the maximally mixed state.

For any observable $A$ we have the following identities

$$\langle \Psi | A | \Psi \rangle = \text{Tr}[\rho_m A]$$

$$\langle \overline{\Psi | A | \Psi} \rangle = \frac{1}{e^S + 1} \left( \text{Tr}[\rho_m A^2] - \text{Tr}[\rho_m A] \right)^2$$

where \text{"overline" denotes Haar-average over pure states, and } \rho_m = \frac{I}{e^S}.$$
Entanglement entropy of subsystem

Large system $A \otimes B$ in typical pure state $|\Psi\rangle$

Subsystem $A$: if $A$ is small, reduced density matrix $\rho_A$ is exponentially close to maximally mixed and its EE is $S_A \approx \log |A|$. This breaks down once $|A| > \frac{|B|}{2}$
Assuming evaporation is unitary we have that:

1. Before Page time: new Hawking particles mostly entangled with remaining black hole

2. After Page time: new Hawking particles mostly entangled with older radiation
Implications for Hawking radiation

If Quantum Gravity effects unitarize Hawking radiation, we generally expect that they will modify the predictions of Hawking to simple observables by $e^{-S_{BH}}$ corrections.

Hawking’s computation is reliable for the approximate computation of low-point functions of photons, up to exp-small corrections.

Hawking’s computation is not reliable for $S_{BH}$-point functions between photons, with sufficient accuracy to identify microstate.

The quantum information of the BH microstate is encoded in $S_{BH}$-point functions.

So far we have not said anything about the effect of these small corrections to the black hole interior.
Quantum cloning on nice slices

Importance of BH interior for information paradox
Black Hole complementarity

[‘tHooft, Susskind-Thorlacius]

In gravity:

$$\mathcal{H} \neq \mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}$$
Observer $A$ needs at least $\beta \log S$ to extract information of qubit (Hayden-Preskill) and then dive into BH

Qubit $B$ would have to be sent with transplanckian energy to reach $A$
Theorem: strong subadditivity of Entanglement entropy

\[ S_{AB} + S_{BC} \geq S_A + S_C \]

+ Mathur’s theorem for small corrections
Observing the paradox
Quantum chaos vs specific entanglement

Smooth horizon requires **specific** pattern of entanglement between field operators at $B$ and $C$

Fragile under perturbations due to chaotic nature of system

Hard to imagine how **typical states** will end up with the correct, **specific** entanglement needed for smoothness
Eternal black hole

\[ H_{\text{total}} = H_L + H_R \]

\[ |\text{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{2E_i}{\epsilon}} |E_i\rangle_L \otimes |E_i\rangle_R \]

Smoothness of horizon depends on correct entanglement

\[ \langle \text{TFD} | O_L \ O_R | \text{TFD} \rangle \sim O(1) \]

But, for a typical state \(|\Psi\rangle\) (with same amount, but different details of entanglement) we find using ETH

\[ \langle \Psi | O_L \ O_R | \Psi \rangle \sim O(e^{-S}) \]
We start with $|\text{TFD}\rangle$ and perturb it by a small operator (energy of $O(1)$) at time $t_L = -T$.

For small $T$, effect on infalling observer $A$ is small. But center of mass collision energy grows exponentially with $T$.

For $T > \beta \log S$ (scrambling time) we can no longer ignore backreaction.

The “correct entanglement” of the TFD disrupted even by small perturbations due to chaos.
What if there is no entanglement of Hawking particles with interior modes after Page time?
Entanglement and smoothness of spacetime
Entanglement and smoothness of spacetime
If, after Page time, the Hawking particles are no longer entangled with something inside the black hole, playing the role of the *interior partner* $C$, then an infalling observer would not see the vacuum but rather a highly excited state of the quantum field.

This corresponds to a very large $\langle T_{\mu\nu} \rangle$ which would backreact and dramatically modify the region behind the horizon (firewall/fuzzball?)

In particular the infalling observer would not experience a smooth horizon.

This effect would be there even for black holes of very large mass, thus violating the predictions of classical GR in a regime of low curvatures.
Summary

1. Information paradox from the point of view of asymptotic observer: natural (in-principle) resolution, consistent with generic expectations from quantum statistical mechanics

2. Preserving smoothness of the horizon: more challenging. Seems to violate monogamy of entanglement. Seems to contradict generic expectations from quantum statistical mechanics (requires specific detailed enganglement for typical states).

3. We will make some of these paradoxes more precise in AdS/CFT
Information paradox in AdS/CFT

CFT dual $\Rightarrow$ Evaporation manifestly unitary

What about smoothness of horizon?
Large BHs in AdS

In flat space we emphasized the relation between the information paradox and the smoothness of the horizon.

A natural question is whether large black holes in AdS have a smooth interior.

These black holes do not evaporate, so one might expect that there would be no paradox to address.

We will see that even for these black holes it is difficult to reconcile the smoothness of their horizon with unitarity in the CFT.

Mathematically precise version of the paradox.
Comments on bulk reconstruction

-CFT with gravity dual, large N, large $\lambda$

-HKLL construction

\[
(\Box_{\text{AdS}} - m^2)\phi = 0 \quad \lim_{z \to 0} z^{-\Delta} \phi(x, z) = O(x)
\]

\[
\phi_{\text{CFT}}(t, \Omega, z) = \sum_m \int_{\omega > 0} d\omega \ (O_{\omega,m} f_{\omega,m}(t, \Omega, z) + \text{h.c.})
\]

\([\phi_{\text{CFT}}(P_1), \phi_{\text{CFT}}(P_2)] = 0\), if points $P_1, P_2$ spacelike with respect to AdS metric

-On-shell, uses bulk EOMs

-perturbative in $1/N$
$$\phi_{\text{CFT}}(t, \Omega, z) = \int dt' d\Omega' \ K(t, \Omega, z ; t', \Omega') \mathcal{O}(t', \Omega')$$

where $K$ is some known kernel
Consider big black hole in AdS. Expectation from bulk effective field theory (EFT) for a free scalar

\[ \phi(t, r, \Omega) = \int_0^\infty d\omega \sum_{lm} b_{\omega lm} e^{-i\omega t} f_{\omega, l}(r) Y_{lm}(\Omega) + \text{h.c.} \]

where (dropping \(l,m\) indices) we have

\[ [b_\omega, b_{\omega'}^\dagger] = \delta(\omega - \omega') \]
\[ [H, b_\omega] = -\omega b_\omega \]

and

\[ \langle b_{\omega}^\dagger b_{\omega} \rangle \sim \frac{1}{e^{\beta \omega} - 1} \]

How do we reconstruct this from the CFT?
In typical QGP pure state $|\Psi\rangle$ (energy $O(N^2)$), single trace correlators factorize at large $N$

$$\langle \Psi | \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) | \Psi \rangle = \langle \Psi | \mathcal{O}(x_1) \mathcal{O}(x_2) | \Psi \rangle \cdots \langle \Psi | \mathcal{O}(x_{n-1}) \mathcal{O}(x_n) | \Psi \rangle + \ldots$$

The 2-point function in which they factorize is the thermal 2-point function

$$G(t, x) \equiv Z^{-1} \text{Tr} \left[ e^{-\beta H} \mathcal{O}(t, x) \mathcal{O}(0, 0) \right]$$

which is hard to compute, but obeys KMS condition

$$G(t - i\beta, x) = G(-t, -x)$$
Consider single-trace operator $\mathcal{O}$ in CFT, dual to bulk field $\phi$. Define Fourier modes

$$\mathcal{O}_{\omega lm} = \int dt d\Omega \mathcal{O}(t, \Omega) e^{i\omega t} Y^*_{lm}(\Omega)$$
Consider single-trace operator $\mathcal{O}$ in CFT, dual to bulk field $\phi$. Define Fourier modes

$$\mathcal{O}_{\omega lm} = \int dt d\Omega \mathcal{O}(t, \Omega) e^{i\omega t} Y^*_{lm}(\Omega)$$

The KMS condition implies

$$\langle \mathcal{O}^\dagger_{\omega lm} \mathcal{O}_{\omega lm} \rangle = e^{-\beta \omega} \langle \mathcal{O}_{\omega lm} \mathcal{O}^\dagger_{\omega lm} \rangle$$
Consider single-trace operator $\mathcal{O}$ in CFT, dual to bulk field $\phi$. Define Fourier modes

$$O_{\omega lm} = \int dt d\Omega \, \mathcal{O}(t, \Omega) e^{i\omega t} Y^*_l m(\Omega)$$

The KMS condition implies

$$\langle O^\dagger_{\omega lm} O_{\omega lm} \rangle = e^{-\beta \omega} \langle O_{\omega lm} O^\dagger_{\omega lm} \rangle$$

then we identify

$$b_{\omega lm} = \frac{1}{\sqrt{\langle [O_{\omega lm}, O^\dagger_{\omega lm}] \rangle}} O_{\omega lm}$$
Consider single-trace operator $O$ in CFT, dual to bulk field $\phi$. Define Fourier modes

$$O_{\omega lm} = \int dt d\Omega \, O(t, \Omega) \, e^{i\omega t} Y_{lm}^*(\Omega)$$

The KMS condition implies

$$\langle O_{\omega lm}^\dagger O_{\omega lm} \rangle = e^{-\beta \omega} \langle O_{\omega lm} O_{\omega lm}^\dagger \rangle$$

then we identify

$$b_{\omega lm} = \frac{1}{\sqrt{\langle [O_{\omega lm}, O_{\omega lm}^\dagger] \rangle}} O_{\omega lm}$$

so

$$[b_{\omega lm}, b_{\omega lm}^\dagger] = 1$$

and, using KMS and large N, we have

$$\langle b_{\omega lm}^\dagger b_{\omega lm} \rangle = \frac{1}{e^{\beta \omega} - 1}$$
Local bulk field outside horizon of AdS black hole

\[ \phi_{\text{CFT}}(t, \Omega, z) = \sum_m \int_0^\infty d\omega \, \mathcal{O}_{\omega,m} f_{\omega,m}(t, \Omega, z) + \text{h.c.} \]

At large \( N \) (and late times) the correlators

\[ \langle \Psi | \phi_{\text{CFT}}(t_1, \Omega_1, z_1) \ldots \phi_{\text{CFT}}(t_n, \Omega_n, z_n) | \Psi \rangle \]

reproduce those of semiclassical QFT on the BH background (in AdS-Hartle-Hawking state).
Some subtleties with spacelike modes

Fix $\omega$, take $\vec{k} \to \infty$. If we normalize bulk wavefunctions so that $f \to 1$ as $r \to \infty$, then at any finite $r$ they grow like

$$e^{\alpha \beta |\vec{k}|/2}$$

For BTZ $\alpha = 1$, for higher dimensional AdS-BHs $\alpha < 1$

HKLL reconstruction is possible because of exponential decay of spacelike thermal correlators.

[S.Banerjee, KP, S.Raju, P.Samantray, P. Shrivastava]

We recently derived a bound for spacelike thermal correlators in any QFT

$$\langle \mathcal{O}(\omega_1, \vec{k}_1)\ldots \mathcal{O}(\omega_n, \vec{k}_n) \rangle_\beta \leq e^{-R\beta}$$

where $R$ is smallest sphere enclosing $\vec{k}_i$. 
Local analysis near the horizon

Demanding that low-point correlators of local fields at late times look locally like flat space we find some conditions which must hold at large $N$

\[
[O_\omega, O_\omega^\dagger] = 1, \quad [\tilde{O}_\omega, \tilde{O}_\omega^\dagger] = 1
\]

\[
[H, O_\omega^\dagger] = \omega O_\omega^\dagger, \quad [H, \tilde{O}_\omega^\dagger] = -\omega \tilde{O}_\omega^\dagger,
\]

\[
\langle O_\omega^\dagger O_\omega \rangle = \langle \tilde{O}_\omega^\dagger \tilde{O}_\omega \rangle = \frac{1}{e^{\beta \omega} - 1}
\]

\[
\langle O_\omega \tilde{O}_\omega \rangle = \frac{e^{\beta \omega}}{e^{\beta \omega} - 1}
\]
On TFD state

\[ \tilde{\mathcal{O}} = \mathcal{O}_L \]
On TFD state

\[ \tilde{O} = O_L \]

Collapsing BH

\[ \tilde{O} = O_{\text{early}} (?) \]
Collapsing vs typical black holes

Black holes formed by (simple) gravitational collapse are \textit{a-typical}.
Collapsing vs typical black holes

Black holes formed by (simple) gravitational collapse are \textit{a-typical}

Typical black hole microstates are defined by “microcanonical measure”

\[ |\Psi\rangle = \sum_i c_i |E_i\rangle \]

where \( E_i \in E_0 \pm \delta E \) and \( c_i \) selected randomly by Haar measure

Notice that typical states are almost time-independent

\[ \langle \Psi | \frac{dA}{dt} | \Psi \rangle = \sum_{ij} c_i^* c_j A_{ij} dA_i dE_{ij} t = O\left( e^{-S/2} \right) \]

Typical states are equilibrium states.
Collapsing vs typical black holes

Typical black hole microstates are defined by “microcanonical measure”

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

where $E_i \in E_0 \pm \delta E$ and $c_i$ selected randomly by Haar measure

Notice that typical states are almost time-independent

$$\langle \Psi| \frac{dA}{dt} |\Psi\rangle = \sum_{ij} c_i^* c_j A_{ij} \frac{d}{dt} e^{iE_{ij}t} = O(e^{-S/2})$$
Collapsing vs typical black holes

Black holes formed by (simple) gravitational collapse are a-typical.

Typical black hole microstates are defined by “microcanonical measure”

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

where $E_i \in E_0 \pm \delta E$ and $c_i$ selected randomly by Haar measure

Notice that typical states are almost time-independent

$$\langle \Psi | \frac{dA}{dt} | \Psi \rangle = \sum_{ij} c_i^* c_j A_{ij} \frac{d}{dt} e^{iE_{ij}t} = O(e^{-S/2})$$

Typical states are equilibrium states.
A problem

It is challenging to identify the operators $\tilde{O}$ in the CFT, with satisfy desired properties on most states counted by

$$S = \frac{A}{4G}$$

because these properties seem to imply [AMPSS, Marolf-Polchinski]

$$\text{Tr}[e^{-\beta H} \tilde{O}^\dagger \tilde{O}] < 0$$
A problem

It is challenging to identify the operators $\tilde{O}$ in the CFT, with satisfy desired properties on most states counted by

$$S = \frac{A}{4G}$$

because these properties seem to imply [AMPSS, Marolf-Polchinski]

$$\text{Tr}[e^{-\beta H}\tilde{O}^\dagger_\omega \tilde{O}_\omega] < 0$$

There is no problem to find these operators in particular, special states.
\[ [\tilde{O}, \tilde{O}^\dagger] = 1 \quad \Rightarrow \quad \tilde{O}^\dagger = \text{“creation operator”} \]

\[ \Rightarrow \tilde{O}^\dagger \text{ should not annihilate (typical) states of the CFT} \quad (*) . \]

On the other hand

\[ [H, \tilde{O}^\dagger] = -\omega \tilde{O}^\dagger \]

implies that \( \tilde{O}^\dagger \) lowers the energy so it maps CFT states of energy \( E \) to \( E - \omega \).

But in CFT, we have \( S(E) > S(E - \omega) \).

\[ \Rightarrow \text{if } \tilde{O}^\dagger \text{ is an ordinary linear operator, it must have a nontrivial kernel.} \]

Inconsistent with statement \( (*) \).

\[ \Rightarrow \text{The CFT does not contain } \tilde{O} \text{ operators and cannot describe the BH interior (?)} \]
Consider the number operator $N_a$ for some modes relevant for the infalling observer. If typical states have a smooth horizon then we expect

$$\text{Tr}_E [N_a] = 0$$

where the trace is over states of energy $E \pm \delta E$.

A trace can be evaluated in any basis of orthonormal vectors.

The number operator of Schwarzschild modes $N_b = \mathcal{O}^\dagger \mathcal{O}$ obeys $[H, N_b] = 0$ and can be simultaneously diagonalized with $H$.

$$\text{Tr}_E [N_a] = \sum \langle n_b | N_a | n_b \rangle > 0$$

since eigenstates $|n_b\rangle$ of Schwarzschild number operator have excited horizon
Paradox for eternal black hole
Using entanglement to go behind the horizon

[KP, S. Raju]

The quantum fields outside the horizon appear to be in an entangled state. They are entangled with certain CFT d.o.f. which can play the modes of the interior. There is a natural mathematical construction allowing us to identify those.
Tomita-Takesaki modular theory

Consider Hilbert space $\mathcal{H}$, a state $|\Psi\rangle \in \mathcal{H}$ and an algebra $\mathcal{A}$ acting on $\mathcal{H}$ with the properties:
1) The state is cyclic wrt the algebra $\mathcal{A}$ i.e.

$$\mathcal{H} = \text{span} \mathcal{A} |\Psi\rangle$$

Moreover $\mathcal{A}'$ is isomorphic to $\mathcal{A}$. Finally, the algebras $\mathcal{A}$, $\mathcal{A}'$ are entangled in a particular way.
Tomita-Takesaki modular theory

Consider Hilbert space $\mathcal{H}$, a state $|\Psi\rangle \in \mathcal{H}$ and an algebra $\mathcal{A}$ acting on $\mathcal{H}$ with the properties:

1) The state is cyclic wrt the algebra $\mathcal{A}$ i.e.

$$\mathcal{H} = \text{span} \mathcal{A}|\Psi\rangle$$

2) The state is separating wrt the algebra $\mathcal{A}$ i.e. $\forall a \in \mathcal{A}, a \neq 0$

$$a|\Psi\rangle \neq 0$$
Tomita-Takesaki modular theory

Consider Hilbert space $\mathcal{H}$, a state $|\Psi\rangle \in \mathcal{H}$ and an algebra $A$ acting on $\mathcal{H}$ with the properties:

1) The state is cyclic wrt the algebra $A$ i.e.

$$\mathcal{H} = \text{span}A|\Psi\rangle$$

2) The state is separating wrt the algebra $A$ i.e. $\forall a \in A, a \neq 0$

$$a|\Psi\rangle \neq 0$$

Then the Tomita-Takesaki theorem says (among other things) that:

*The representation of the algebra $A$ on $\mathcal{H}$ is reducible, and the algebra has a non-trivial commutant $A'$ also acting on $\mathcal{H}$. Moreover $A'$ is isomorphic to $A$. Finally, the algebras $A, A'$ are entangled in a particular way.*
Tomita-Takesaki modular theory

We define an antilinear map

\[ Sa |\Psi\rangle = a^\dagger |\Psi\rangle \quad a \in \mathcal{A} \]
Tomita-Takesaki modular theory

We define an antilinear map

\[ Sa|\Psi\rangle = a^\dagger|\Psi\rangle \quad a \in \mathcal{A} \]

Consider the polar decomposition

\[ S = J \Delta^{1/2} \quad \Delta = S^\dagger S \]

where \( \Delta = e^{-K} \) and \( K \) = modular Hamiltonian.
Tomita-Takesaki modular theory

We define an antilinear map

\[ Sa |\Psi\rangle = a^\dagger |\Psi\rangle \quad a \in \mathcal{A} \]

Consider the polar decomposition

\[ S = J \Delta^{1/2} \quad \Delta = S^\dagger S \]

where \( \Delta = e^{-K} \) and \( K \) = modular Hamiltonian.

Then we have:

1. \( \mathcal{A}' = J\mathcal{A}J \): the commutant \( \mathcal{A}' \) is isomorphic to \( \mathcal{A} \) (notice \( J^2 = 1 \)).
We define an antilinear map
\[ Sa|\Psi\rangle = a^\dagger|\Psi\rangle \quad a \in \mathcal{A} \]

Consider the polar decomposition
\[ S = J\Delta^{1/2} \quad \Delta = S^\dagger S \]

where \( \Delta = e^{-K} \) and \( K \) = modular Hamiltonian.

Then we have:

1. \( \mathcal{A}' = JA\mathcal{J} \): the commutant \( \mathcal{A}' \) is isomorphic to \( \mathcal{A} \) (notice \( J^2 = 1 \)).

2. \( \Delta^{is}\mathcal{A}\Delta^{-is} = \mathcal{A}, \quad \Delta^{is}\mathcal{A}'\Delta^{-is} = \mathcal{A}' \) \( s \in \mathbb{R} \)
Tomita-Takesaki modular theory

We define an antilinear map

\[ Sa|Ψ\rangle = a^{\dagger}|Ψ\rangle \quad a \in \mathcal{A} \]

Consider the polar decomposition

\[ S = J\Delta^{1/2} \quad \Delta = S^{\dagger}S \]

where \( \Delta = e^{-K} \) and \( K \) = modular Hamiltonian.

Then we have:

1. \( \mathcal{A}' = J\mathcal{A}J \): the commutant \( \mathcal{A}' \) is isomorphic to \( \mathcal{A} \) (notice \( J^2 = 1 \)).
2. \( \Delta^{is} \mathcal{A} \Delta^{-is} = \mathcal{A}, \quad \Delta^{is} \mathcal{A}' \Delta^{-is} = \mathcal{A}' \quad s \in \mathbb{R} \)
3. KMS-like condition: \( F(z) \equiv \langle Ψ|a\Delta^{iz}b\Delta^{-iz}|Ψ\rangle \), then \( F(-i) = \langle Ψ|ba|Ψ\rangle \)
Consider a general, possibly strongly coupled, relativistic QFT in the Minkowski ground state $|0\rangle$. Suppose we have only access to right Rindler wedge. How can we use the entanglement to recover the rest of space-time?
Consider a general, possibly strongly coupled, relativistic QFT in the Minkowski ground state $|0\rangle$. Suppose we have only access to right Rindler wedge. How can we use the entanglement to recover the rest of space-time?

Reeh-Schlieder theorem: The Minkowski vacuum $|0\rangle$ is a cyclic and separating state for the algebra $\mathcal{A}$:

1. States of form $a_1...a_n|0\rangle$ $a_i \in \mathcal{A}$, span dense subspace of $\mathcal{H}$
2. There is no $a \in \mathcal{A}$ such that $a|0\rangle = 0$. 

Example: Rindler space
Example: Rindler space

Consider Lorentz boost $U = e^{iKs}$ on $t - x$ plane

$$t' = t \cosh s + x \sinh s$$

$$x' = t \sinh s + x \cosh s$$
Example: Rindler space

Consider Lorentz boost $U = e^{iKs}$ on $t - x$ plane

\[ t' = t \cosh s + x \sinh s \]
\[ x' = t \sinh s + x \cosh s \]

A complexified Lorentz boost by $s = i\pi$ maps $(t, x, \vec{y}) \rightarrow (-t, -x, \vec{y})$

\[ e^{-\pi K} \phi(t, x, \vec{y}) |0\rangle = \phi(-t, -x, \vec{y}) |0\rangle \]
Example: Rindler space

Consider Lorentz boost $U = e^{iKs}$ on $t - x$ plane

\[
\begin{align*}
  t' &= t \cosh s + x \sinh s \\
  x' &= t \sinh s + x \cosh s
\end{align*}
\]

A complexified Lorentz boost by $s = i\pi$ maps $(t, x, \vec{y}) \rightarrow (-t, -x, \vec{y})$

\[
e^{-\pi K} \phi(t, x, \vec{y}) |0\rangle = \phi(-t, -x, \vec{y}) |0\rangle
\]

Combine this with a rotation $R_1$ by $\pi$ around $x$ which takes $\vec{y} \rightarrow -\vec{y}$ and finally CPT transformation $\Theta$ which maps $(-t, -x, -\vec{y})$ back to $(t, x, \vec{y})$. All in all we find

\[
\Theta R_1 e^{-\pi K} \phi(t, x, \vec{y}) |0\rangle = \phi^\dagger(t, x, \vec{y}) |0\rangle
\]
Example: Rindler space

Consider Lorentz boost \( U = e^{iKs} \) on \( t - x \) plane

\[
\begin{align*}
t' &= t \cosh s + x \sinh s \\
x' &= t \sinh s + x \cosh s
\end{align*}
\]

A complexified Lorentz boost by \( s = i\pi \) maps \((t, x, \vec{y})\) to \((-t, -x, \vec{y})\)

\[
e^{-\pi K} \phi(t, x, \vec{y})|0\rangle = \phi(-t, -x, \vec{y})|0\rangle
\]

Combine this with a rotation \( R_1 \) by \( \pi \) around \( x \) which takes \( \vec{y} \rightarrow -\vec{y} \) and finally CPT transformation \( \Theta \) which maps \((-t, -x, -\vec{y})\) back to \((t, x, \vec{y})\). All in all we find

\[
\Theta R_1 e^{-\pi K} \phi(t, x, \vec{y})|0\rangle = \phi^\dagger(t, x, \vec{y})|0\rangle
\]

Generalizing to more operators (Bisognano-Wichmann thm.) it follows that the desired modular conjugation implementing \( S a|0\rangle = a^\dagger|0\rangle \) is

\[
S = \Theta R_1 e^{-\pi K}
\]
Example: Rindler space

\[ S = \Theta R_1 e^{-\pi K} \]

From this follows that

\[ \Delta = S^\dagger S = e^{-2\pi K} \]
Example: Rindler space

\[ S = \Theta R_1 e^{-\pi K} \]

From this follows that

\[ \Delta = S^\dagger S = e^{-2\pi K} \]

The modular Hamiltonian is the Lorentz boost generator with effective temperature \( \frac{1}{2\pi} \).
Example: Rindler space

\[ S = \Theta R_1 e^{-\pi K} \]

From this follows that

\[ \Delta = S^\dagger S = e^{-2\pi K} \]

The modular Hamiltonian is the Lorentz boost generator with effective temperature \( \frac{1}{2\pi} \). The antiunitary operator \( J \) mapping \( \mathcal{A} \) to \( \mathcal{A}' \) and allowing us to recover the left wedge is

\[ J = \Theta R_1 \]

The fact that each of the algebras \( \mathcal{A}, \mathcal{A}' \) remain invariant under conjugation by \( \Delta \) is obvious in this example. The KMS condition implies the Unruh temperature (even at strong coupling).
Tomita-Takesaki and the black hole

We do not have a decomposition of the algebra in **physical space**, but rather in the “space of operators” (simple vs complicated).

Introduce a “small algebra” $\mathcal{A}$ of simple operators (single trace + small products).
Tomita-Takesaki and the black hole

We do not have a decomposition of the algebra in **physical space**, but rather in the “space of operators” (simple vs complicated).

Introduce a “small algebra” $\mathcal{A}$ of simple operators (single trace + small products).

We define the small Hilbert space (also called “code-subspace” in later works)

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle$$
Tomita-Takesaki and the black hole

We do not have a decomposition of the algebra in **physical space**, but rather in the “space of operators” (simple vs complicated).

Introduce a “small algebra” \( \mathcal{A} \) of simple operators (single trace + small products).

We define the small Hilbert space (also called “code-subspace” in later works)

\[
\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle
\]

The algebra \( \mathcal{A} \) probes the typical pure state \(|\Psi\rangle\) as a thermal state

\[
\langle\Psi|\mathcal{O}(x_1)\ldots\mathcal{O}(x_n)|\Psi\rangle = Z^{-1}\operatorname{Tr}[e^{-\beta H}\mathcal{O}(x_1)\ldots\mathcal{O}(x_n)] + O(1/N)
\]
Tomita-Takesaki and the black hole

We do not have a decomposition of the algebra in **physical space**, but rather in the "space of operators" (simple vs complicated). Introduce a "small algebra" $\mathcal{A}$ of simple operators (single trace + small products).

We define the small Hilbert space (also called "code-subspace" in later works)

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle$$

The algebra $\mathcal{A}$ probes the typical pure state $|\Psi\rangle$ as a thermal state

$$\langle\Psi|\mathcal{O}(x_1)\ldots\mathcal{O}(x_n)|\Psi\rangle = Z^{-1}\text{Tr}[e^{-\beta H} \mathcal{O}(x_1)\ldots\mathcal{O}(x_n)] + O(1/N)$$

**No annihilation operators in $\mathcal{A} \Rightarrow |\Psi\rangle$ is a cyclic and separating vector.**
Tomita-Takesaki and the black hole

We do not have a decomposition of the algebra in physical space, but rather in the “space of operators” (simple vs complicated).

Introduce a “small algebra” $\mathcal{A}$ of simple operators (single trace + small products).

We define the small Hilbert space (also called “code-subspace” in later works)

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle$$

The algebra $\mathcal{A}$ probes the typical pure state $|\Psi\rangle$ as a thermal state

$$\langle\Psi|\mathcal{O}(x_1)\ldots\mathcal{O}(x_n)|\Psi\rangle = Z^{-1}\text{Tr}[e^{-\beta H}\mathcal{O}(x_1)\ldots\mathcal{O}(x_n)] + O(1/N)$$

No annihilation operators in $\mathcal{A} \Rightarrow |\Psi\rangle$ is a cyclic and separating vector.

An analogue of the Tomita-Takesaki construction applies.

Using large $N$ factorization and the KMS condition, we find the modular Hamiltonian for the small algebra

$$\Delta \equiv S^\dagger S = e^{-\beta(H-E_0)} + O(1/N)$$
The mirror operators

This leads to the “mirror operators”

$$
\tilde{O}_\omega |\Psi\rangle = e^{-\frac{\beta H}{2}} O^\dagger_\omega e^{\frac{\beta H}{2}} |\Psi\rangle
$$

$$
\tilde{O}_\omega O \ldots O |\Psi\rangle = O \ldots O \tilde{O}_\omega |\Psi\rangle
$$

$$
[H, \tilde{O}_\omega] O \ldots O |\Psi\rangle = \omega \tilde{O}_\omega O \ldots O |\Psi\rangle
$$

These equations define the operators $\tilde{O}$ on the code-subspace $H_\Psi \subset H_{CFT}$, which is relevant for EFT experiments around BH microstate $|\Psi\rangle$.
The mirror operators

This leads to the “mirror operators”

\[ \tilde{\mathcal{O}}_\omega |\Psi\rangle = e^{-\frac{\beta H}{2}} \mathcal{O}_\omega^\dagger e^{\frac{\beta H}{2}} |\Psi\rangle \]

\[ \tilde{\mathcal{O}}_\omega \mathcal{O} \ldots \mathcal{O} |\Psi\rangle = \mathcal{O} \ldots \mathcal{O} \tilde{\mathcal{O}}_\omega |\Psi\rangle \]

\[ [H, \tilde{\mathcal{O}}_\omega] \mathcal{O} \ldots \mathcal{O} |\Psi\rangle = \omega \tilde{\mathcal{O}}_\omega \mathcal{O} \ldots \mathcal{O} |\Psi\rangle \]

These equations define the operators \( \tilde{\mathcal{O}} \) on the code-subspace \( \mathcal{H}_\Psi \subset \mathcal{H}_{\text{CFT}} \), which is relevant for EFT experiments around BH microstate \( |\Psi\rangle \)

- Operators defined only on \( \mathcal{H}_\Psi \), not on full CFT Hilbert space - they are state-dependent operators.
The mirror operators

This leads to the “mirror operators”

\[ \tilde{O}_\omega |\Psi\rangle = e^{-\frac{\beta H}{2}} O_\omega^\dagger e^{\frac{\beta H}{2}} |\Psi\rangle \]

\[ \tilde{O}_\omega O \ldots O |\Psi\rangle = O \ldots O \tilde{O}_\omega |\Psi\rangle \]

\[ [H, \tilde{O}_\omega] O \ldots O |\Psi\rangle = \omega \tilde{O}_\omega O \ldots O |\Psi\rangle \]

These equations define the operators \( \tilde{O} \) on the code-subspace \( \mathcal{H}_\Psi \subset \mathcal{H}_{\text{CFT}} \), which is relevant for EFT experiments around BH microstate \( |\Psi\rangle \)

- Operators defined only on \( \mathcal{H}_\Psi \), not on full CFT Hilbert space - they are \textit{state-dependent} operators.
- \( [O, \tilde{O}] = 0 \) only inside \( \mathcal{H}_\Psi \), not as operator equation
The mirror operators

This leads to the “mirror operators”

\[ \tilde{O}_\omega |\Psi\rangle = e^{-\frac{\beta H}{2}} O_\omega^\dagger e^{\frac{\beta H}{2}} |\Psi\rangle \]

\[ \tilde{O}_\omega O ... O |\Psi\rangle = O ... O \tilde{O}_\omega |\Psi\rangle \]

\[ [H, \tilde{O}_\omega] O ... O |\Psi\rangle = \omega \tilde{O}_\omega O ... O |\Psi\rangle \]

These equations define the operators \( \tilde{O} \) on the code-subspace \( \mathcal{H}_\Psi \subset \mathcal{H}_{\text{CFT}} \), which is relevant for EFT experiments around BH microstate \( |\Psi\rangle \)

- Operators defined only on \( \mathcal{H}_\Psi \), not on full CFT Hilbert space - they are state-dependent operators.
- \([O, \tilde{O}] = 0\) only inside \( \mathcal{H}_\Psi \), not as operator equation
- Due to Boltzman factors \( \langle O_\omega^\dagger O_\omega \rangle \propto e^{-\beta \omega} \), we define these operators for \( \omega < \omega_* \), where \( \omega_* \) does not grow too fast with \( N \)
The mirror operators

The small algebra $\mathcal{A}$ is not an exact algebra, hence the Tomita-Takesaki theorem can not be applied exactly. Hence $\mathcal{A}'$ is not an exact commutant.

From a physical point of view this is a desirable feature of the construction. It realizes the idea of black hole complementarity in a precise setting.

It also naturally implies that there is some non-locality in the construction of the interior.

Finally, notice the operators $\widetilde{\mathcal{O}}$ defined by the Tomita-Takesaki construction are state-dependent, since they are “defined by the enganglement”.
Infalling observer

\[ \phi(t, r, \Omega) = \int_0^\infty d\omega \left[ \mathcal{O}_\omega f_\omega(t, \Omega, r) + \tilde{\mathcal{O}}_\omega g_\omega(t, \Omega, r) + \text{h.c.} \right] \]
Extended geometry

\[ \tilde{\mathcal{O}} \quad \mathcal{O} \]
The cutoff on the left is determined by $\omega_\ast$.

Since $\tilde{O}$ do not fundamentally commute with $O$, left region should not be thought as a fundamentally independent part of the Hilbert space (BH complementarity)
State-dependence

- Interior operators defined by

\[ \tilde{\mathcal{O}}_\omega |\Psi\rangle = e^{-\frac{\beta_\omega}{2}} \mathcal{O}_\omega^{\dagger} |\Psi\rangle \]

\[ \tilde{\mathcal{O}}_\omega \mathcal{O} \ldots \mathcal{O} |\Psi\rangle = \mathcal{O} \ldots \mathcal{O} \tilde{\mathcal{O}}_\omega |\Psi\rangle \]

- Solution defined only on \( \mathcal{H}_\Psi \), depends on reference state \( |\Psi\rangle \)

- Operators cannot be upgraded to “globally defined” operators

- Solves previous paradoxes (Negative trace, Chaos vs Entanglement problem, ...)

56
Complementarity and Non-locality

\( \tilde{\mathcal{O}} \) were constructed based on the fact that we restricted our attention to a “small algebra” of \( \mathcal{O} \)’s. The construction breaks down if the “small algebra” is enlarged to include all operators.
Complementarity and Non-locality

\( \tilde{O} \) were constructed based on the fact that we restricted our attention to a “small algebra” of \( O \)’s. The construction breaks down if the “small algebra” is enlarged to include all operators

\[
[O, \tilde{O}] = 0 \quad \text{only on } \mathcal{H}_\Psi, \text{ not as operator equation}
\]
Complementarity and Non-locality

\( \tilde{O} \) were constructed based on the fact that we restricted our attention to a “small algebra” of \( O \)’s. The construction breaks down if the “small algebra” is enlarged to include all operators

\[ [\mathcal{O}, \tilde{\mathcal{O}}] = 0 \text{ only on } \mathcal{H}_\Psi, \text{ not as operator equation} \]

Operators \( \tilde{\mathcal{O}} = \text{complicated combinations of } \mathcal{O} \). Realization of BH complementarity
Complementarity and Non-locality

\( \widetilde{O} \) were constructed based on the fact that we restricted our attention to a “small algebra” of \( O \)’s. The construction breaks down if the “small algebra” is enlarged to include all operators

\[ [O, \widetilde{O}] = 0 \text{ only on } H_\Psi, \text{ not as operator equation} \]

Operators \( \widetilde{O} = \text{complicated combinations of } O \). Realization of BH complementarity

\[
[\phi(P), \phi(Q)] \sim 0
\]

\[
[\phi(P), \Phi^{\text{complex}}(Q)] = O(1)
\]
Complementarity and Non-locality

\( \tilde{O} \) were constructed based on the fact that we restricted our attention to a “small algebra” of \( O \)’s. The construction breaks down if the “small algebra” is enlarged to include all operators

\[ [O, \tilde{O}] = 0 \text{ only on } \mathcal{H}_\Psi, \text{ not as operator equation} \]

Operators \( \tilde{O} = \text{complicated combinations of } O \). Realization of BH complementarity

\[ [\phi(P), \phi(Q)] \sim 0 \]

\[ [\phi(P), \Phi^{\text{complex}}(Q)] = O(1) \]

The Hilbert space of Quantum Gravity does not factorize as \( \mathcal{H}_{\text{inside}} \otimes \mathcal{H}_{\text{outside}} \)

1) Solves problem of Monogamy of Entanglement (and avoids Mathur’s theorem)

2) Is consistent with locality in EFT, concrete mathematical realization of complementarity
Two identical non-interacting CFTs

\[ H = H_L + H_R \]

in an entangled state

\[ |TFD\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\frac{\beta E}{2}} |E\rangle_L \otimes |E\rangle_R \]
Eternal AdS black hole

In the bulk they are connected by a wormhole (Einstein-Rosen bridge).

It is not traversable, consistent with the fact that CFTs are non-interacting
Eternal AdS black hole

\[ |\text{TFD} \rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle_L \otimes |E_i\rangle_R \]

\[ |\Psi \rangle = \sum_{ij} c_{ij} |E_i\rangle_L \otimes |E_j\rangle_R \]

\( c_{ij} = \text{generic} \)
Gao-Jafferis-Wall protocol

at $t = 0$ we briefly couple the CTFs by a double-trace interaction

$$H = H_L + H_R + g f(t) O_L O_R$$

For given sign of $g$ this creates negative energy shockwaves in the bulk. Probe undergoes time advance when crossing shockwaves

Wormhole becomes traversable
Gao-Jafferis-Wall protocol

Change of CFT energy

$$\delta \langle H_R \rangle \propto g \langle O_L O_R \rangle + O(g^2)$$

Black hole horizon shrinks somewhat, probe can cross the wormhole
CFTs briefly interacted via $O_L O_R$ at $t = 0$, so information can be exchanged
Notice $\phi$ vs $O$
Quantum Teleportation Interpretation

Measure $O_L$ on CFT$_L$, then apply

$$e^{i\gamma_0 L O_R}$$

on CFT$_R$. The probe $\phi$ is teleported.
We create the probe on the left by

\[ e^{i\epsilon\phi_L(-t)}|\text{TFD}\rangle \]

At \( t = 0 \) we apply double-trace perturbation coupling the two CFTs

\[ e^{igO_LO_R(0)} e^{i\epsilon\phi_L(-t)}|\text{TFD}\rangle \]

We measure the operator \( \phi_R(t) \) on this state. To leading order in \( \epsilon \) we need

\[ \langle \text{TFD} | [\phi_L(-t), e^{-igO_LO_R(0)}\phi_R(t)e^{igO_LO_R(0)}] |\text{TFD}\rangle \]

Expanding in \( g \)

\[ \langle \text{TFD} | [\phi_L(-t), O_L(0) ] [\phi_R(t), O_R(0)] |\text{TFD}\rangle \]
Traversable wormholes and quantum chaos

Growth of out-of-time-order-correlators (OTOC) due to quantum chaos

\[ \langle TFD | [\phi_L(-t), O_L(0)] [\phi_R(t), O_R(0)] | TFD \rangle \sim \frac{1}{N^2} e^{\frac{2\pi}{\beta} t} \]

Including higher orders in \( g \), we find that the commutator is zero up to scrambling time \( t \approx \beta \log S \), when it becomes nonzero and we get a nontrivial signal, corresponding to the probe appearing in the right CFT.
Gao-Jafferis-Wall identified an S-matrix-like experiment which probes the interior of eternal black hole.

CFT correlators contain information about geometry inside horizon.

Computations provide evidence for smoothness of horizon of eternal black hole, dual to the TFD state, and ER/EPR proposal.

However, the real difficulty in reconciling unitarity with the smoothness of the black hole horizon is not for the TFD (which is a very special, atypical state), but rather for typical black hole microstates.

Can we find a way of applying a similar protocol to (1-sided) typical black hole microstates, which will allow us to probe their interior?
Exciting the left region

Mirror quench: we perturb the CFT Hamiltonian by $\tilde{O}$ at $-t$

Excitation is invisible by simple CFT operators
Creating negative energy shockwaves for 1-sided black hole


At $t = 0$ we perturb CFT Hamiltonian by

$$g f(t) \tilde{O}\tilde{O}(0)$$

Compute effect on bulk correlators $\Rightarrow$ generates negative energy shockwaves for appropriate choice of $g$
Some subtleties (in progress)

Operators $\tilde{\mathcal{O}}$ are gravitationally dressed wrt the right $\Rightarrow$ Wilson lines extending across geometry

Backreaction and Einstein equations at subleading order?
We create a probe in the left region of the black hole by acting with $\phi(-t)$.

Then at $t = 0$ we perturb the CFT by $gf(t)O(0)\phi(0)$. Finally we detect the probe by measuring $\phi(t)$.

The postulated Penrose diagram makes a prediction about CFT correlators (singal around $t = \beta \log S$)

$$
\langle \Psi_0 | [\phi(-t), e^{-ig\tilde{O}(0)}\phi(t)e^{ig\tilde{O}(0)}] | \Psi_0 \rangle
$$
Comparison

Using properties of the TFD state and the mirror operators we find that both experiments are governed by the expectation value of exactly the same string of ordinary CFT operators $\chi(\phi, O)$. Moreover, in stat-mech we have

$$C' = \text{Tr}[\rho_m \chi(\phi, O)] + O(e^{-S})$$
Condition for CFT correlators

\[ C = \frac{1}{Z} \text{Tr}[e^{-\beta H} \mathcal{X}(\phi, O)] \quad C'' = \text{Tr}[\rho_m \mathcal{X}(\phi, O)] \]

A necessary condition for horizon of typical BH microstate to be smooth is

\[ \lim_{N \to \infty} C = \lim_{N \to \infty} C'' \]

keeping frequencies \( \omega < \omega_* \).

- Not obvious, trace-distance \( ||\rho_\beta - \rho_m|| \) between ensembles is almost maximal.
- \( \mathcal{X}(\phi, O) \) is a complicated observable, product of operators at time separation \( \Delta t \sim \beta \log S \)
- Condition is related to whether \( \mathcal{X}(\phi, O) \) obeys Eigenstate Thermalization Hypothesis (ETH)

\[ \langle E_i | \mathcal{X} | E_j \rangle = f(E_i) \delta_{ij} + R_{ij} e^{-S/2} \]  

(1)

with \( \frac{df}{dE} \sim O(1/S) \)
Condition for CFT correlators

- Interesting effect comes from subleading corrections of the form

\[ \frac{1}{N^2 e^{\frac{2\pi t}{\beta}}} \]

At scrambling time they become $O(1)$.

- Are these “chaos-enhanced” $1/N^2$ corrections the same in typical pure states and thermal ensemble?

- Our condition requires that correlators agree even after analytic continuation by $t \to t - \frac{i\beta}{2}$ (keeping frequencies up to $\omega_*$)
Evidence

1. ETH holds for products of operators at small time separation. We can show that it also holds for very large time separations (when chaos saturates). It is natural to expect that it holds for intermediate times of order $\beta \log S$

2. In 2d CFTs with large $c$ and sparse spectrum correlators are dominated by Virasoro identity block. In this case the conjecture is true.

3. Numerical evidence in SYK model
The SYK model

\(N\)-Majorana fermions in 0 + 1d

\[\{\psi^i, \psi^j\} = \delta^{ij}\]

\[H = \sum_{ijkl} J_{ijkl} \psi^i \psi^j \psi^k \psi^l\]

where \(J_{ijkl}\) random couplings

\[\dim \mathcal{H} = 2^\frac{N}{2}\]

Flows to strongly coupled CFT in IR

Model of black hole in AdS\(_2\)
The mirror operators in the SYK model

Typical state in SYK

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

Introduce the spin operators [Kourkoulou, Maldacena]

$$S_k = 2i \psi_{2k-1} \psi_{2k}$$
The mirror operators in the SYK model

$$|1\rangle = |\Psi_0\rangle,$$

$$|2\rangle = S_{1,\omega_1} |\Psi_0\rangle,$$

$$|3\rangle = S_{1,\omega_2} |\Psi_0\rangle,$$

$$\vdots$$

$$|n\rangle = S_{2,\omega_1} |\Psi_0\rangle,$$

$$|n + 1\rangle = S_{2,\omega_2} |\Psi_0\rangle,$$

$$\vdots$$

$$|l\rangle = S_{2,\omega_2} S_{1,\omega_1} |\Psi_0\rangle,$$

$$\vdots$$
The mirror operators in the SYK model

To simplify the notation, we denote these states as

$$|I\rangle \equiv \mathcal{O}_I |\Psi_0\rangle,$$

where $\mathcal{O}_I$ is a combination of the spin operators introduced above. We define

$$G_{IJ} \equiv \langle I|J \rangle$$

and

$$B_{IJ,k\omega} \equiv \langle I|\tilde{S}_{k,\omega}|J\rangle,$$

or using the equations for the mirror operators

$$B_{IJ,k\omega} = \langle \Psi_0 | O_I^\dagger O_J e^{\frac{\beta H}{2}} S_{k,\omega} e^{\frac{\beta H}{2}} |\Psi_0\rangle.$$

Finally we can represent the mirror operators explicitly as

$$\tilde{S}_{k,\omega} = G^{IJ} B_{JK,k\omega} G^{KL} |I\rangle \langle L|.$$
Extracting particle from behind the horizon
Relation to Kourkoulou-Maldacena

They consider a class of \textbf{a-typical}, non-equilibrium states in the SYK model

\[ e^{-\frac{\beta H}{2}} |B_s\rangle \quad \text{where} \quad S_k |B_s\rangle = s_k |B_s\rangle \]

On these states they consider the (state-dependent) perturbation of the form

\[ \delta H = g \sum_k s_k S_k \]

and they argue that this exposes part of the region behind the horizon. This thought experiment is closely related to the perturbations

\[ \delta H = g \mathcal{O} \tilde{\mathcal{O}} \]

that we discussed earlier.
Pure vs thermal state OTOC in SYK

\[ \langle \{ \psi^i(t), \psi^i(0) \}^2 \rangle \]

on thermal state (red) vs typical pure state (blue).
ETH for chaotic observables in SYK

Matrix elements in SYK of

$$\{\psi^i(t), \psi^i(0)\}^2$$

for $t \approx \beta \log S$
Recovering information from a black hole

We throw a qubit into black hole. How long do we need to wait to recover the information from Hawking radiation?

\[ t_{evap} \sim G^2 M^3 \]

**Hayden Preskill (2007):** if we have access to more than half of Hawking radiation we only need to wait scrambling time

\[ t_S \sim GM \log S \]

to recover information. For the protocol to work we need to know the initial state of the black hole.
Reformulated by Maldacena-Stanford-Yang in terms of traversable wormholes
A realization of Hayden-Preskill

We throw qubit $\phi(-t_s)$ into black hole.

At $t = 0$ we act with $\mathcal{O}\widetilde{\mathcal{O}}$

After scrambling time we can extract the quantum information of the qubit my measuring operator $\widetilde{\phi}(t_s)$.

This provides an explicit decoding Hayden-Preskill protocol.

Knowledge of the quantum state related to state-dependent $\widetilde{\mathcal{O}}$. 

85
Summary

- The nature of space-time behind the horizon remains mysterious.
- This question becomes particularly sharp for typical black hole microstates in AdS.
- Presented a proposal for their geometry, by making use of state-dependent operators.
- Developments related to traversable wormholes: new calculational tools to probe BH interior.
- Interesting connections with quantum teleportation and quantum chaos in pure states.