Probing the black hole interior in AdS/CFT

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-Black hole information paradox: fundamental conflict between GR and QM

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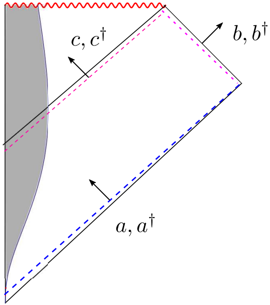
Outline

-review basic formulation of information paradox
-explain the relevance of smoothness of horizon
-formulate the problem in AdS/CFT
-a proposal for describing the BH interior from CFT

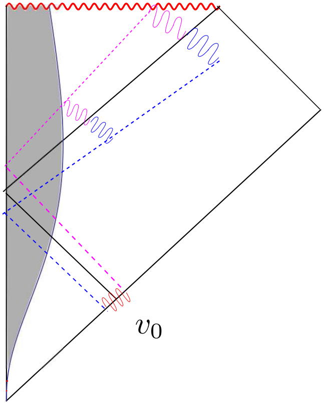
Hawking radiation



$$\phi(x) = \sum f_i(x)\,\mathbf{a_i} + h.c.$$

$$\phi(x) = \sum g_i(x)\,\mathbf{b_i} + \sum h_i(x)\,\mathbf{c_i} + h.c.$$

$$\mathbf{a}_i|0\rangle_{\rm in} = 0 \qquad \text{but} \qquad {}_{\rm in}\langle 0|\mathbf{b}^\dagger\mathbf{b}|0\rangle_{\rm in} = \sum |\beta_{ij}|^2 \neq 0$$


v_0

Properties of Hawking radiation

$$\langle b_\omega^\dagger b_{\omega'} \rangle = \delta(\omega - \omega') P(\omega, l) \frac{1}{e^{\beta\omega} - 1}$$

where $\beta^{-1} = T$ is

$$T = \frac{1}{8\pi GM}$$

and where $P(\omega, l)$ is the absorption probability (gray-body factor)

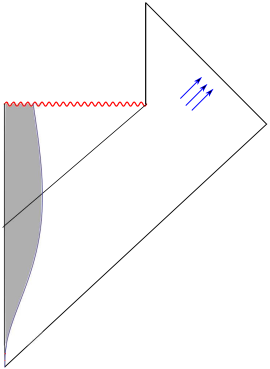
Moreover

$$\langle bbbb... \rangle = \text{product of 2-point functions}$$

The Hawking particles seem to be <u>thermal</u> and <u>uncorrelated</u>.

The density matrix of the radiation is thermal (diagonal in occupation level basis)

Black Hole Evaporation



$$t_{\text{evap}} \propto G^2 M^3$$

Information Paradox

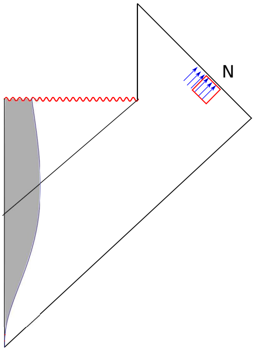
Hawking's computation seems to contradict Unitarity in Quantum Mechanics

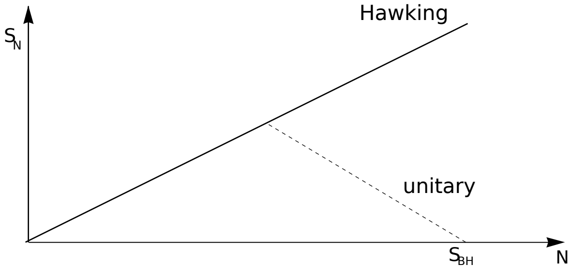
It predicts that a pure state $|\Psi\rangle$ can evolve into a mixed state ρ_{thermal} while in QM we have

$$\Psi(t) = e^{-iHt}\Psi(0)$$

Hawking's computation predicts that many initial states of same mass M, give the same final state $\Rightarrow$ fundamental irreversibility, information loss

Entropy of Hawking radiation





Consider the first N Hawking particles, compute reduced density matrix ρ_N and its von Neumann entropy

$$S_N \equiv -\mathrm{Tr}[\rho_N \log \rho_N]$$

Possible resolution

Information encoded in Hawking radiation in small correlations between particles

Can small corrections to Hawking's computation resolve the information paradox?

Two clarifications:

1) We can not yet calculate these corrections. This would be equivalent to computing an exact S-matrix in Quantum Gravity. We will only estimate what is the minimal size of these corrections necessary to restore unitarity and consider:
i) whether it is reasonable to expect corrections of this size in the theory of Quantum Gravity and
ii) whether the existence of these corrections is compatible with Effective Field Theory

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2) When talking about "small corrections to Hawking's computation" it is important to be precise about the quantities to which these corrections apply.

General expectation: Unitarity can be restored in Hawking evaporation, at the price of introducing **exponentially small** (of order $e^{-S_{BH}}$) corrections to **simple** observables in effective field theory.

Corrections of this size are generally expected and do not invalidate effective field theory.

Corrections to complicated observables, for example S_{BH}-point functions, or "the quantum state of the entire Hawking radiation", or the EE of the Hawking radiation may be very large

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This claim relies on a basic property of Quantum Statistical Mechanics:

Pure vs Mixed states

[Lloyd]

In a large quantum system most pure states look almost identical when probed my most observables — and almost identical to the maximally mixed state.

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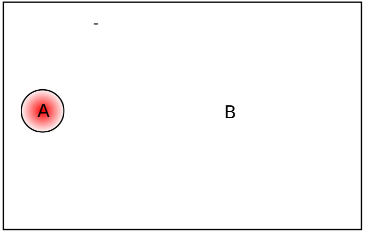
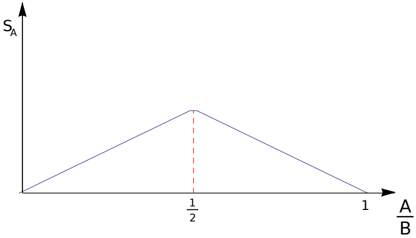
$$\overline{\left(\langle\Psi|A|\Psi\rangle - \mathrm{Tr}[\rho_m A]\right)^2} = \frac{1}{e^S+1}\left(\mathrm{Tr}[\rho_m A^2] - \mathrm{Tr}[\rho_m A]^2\right)$$

where "overline" denotes Haar-average over pure states, and $\rho_m = \frac{\mathbb{I}}{e^S}$.

Page curve

Entanglement entropy of subsystem

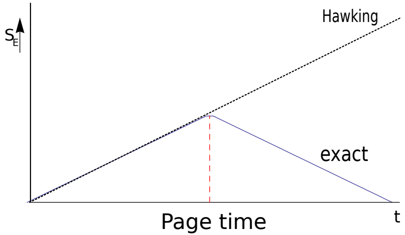




Large system $A \otimes B$ in typical pure state $|\Psi\rangle$

Subsystem A: if A is small, reduced density matrix ρ_A is exponentially close to maximally mixed and its EE is $S_A \approx \log |A|$. This breaks down once $|A| > \frac{|B|}{2}$

Page curve



Assuming evaporation is unitary we have that:

1. Before Page time: new Hawking particles mostly entangled with remaining black hole

2. After Page time: new Hawking particles mostly entangled with older radiation

Implications for Hawking radiation

If Quantum Gravity effects unitarize Hawking radiation, we generally expect that they will modify the predictions of Hawking to simple observables by $e^{-S_{BH}}$ corrections.

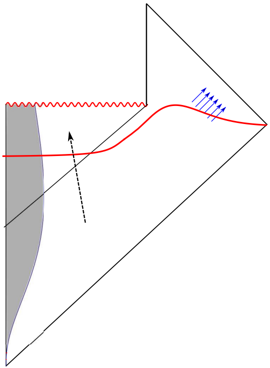
Hawking's computation is reliable for the approximate computation of low-point functions of photons, up to exp-small corrections.

Hawking's computation is not reliable for S_{BH}-point functions between photons, with sufficient accuracy to identify microstate.

The quantum information of the BH microstate is encoded in S_{BH}-point functions

So far we have not said anything about the effect of these small corrections to the black hole interior.

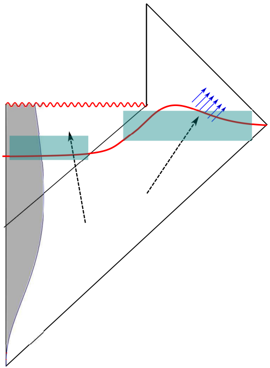
Quantum cloning on nice slices



Importance of BH interior for information paradox

Black Hole complementarity

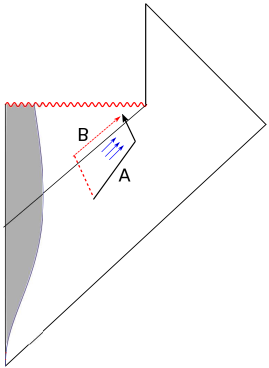
['tHooft, Susskind-Thorlacius]



In gravity:

$$\mathcal{H} \neq \mathcal{H}_{\rm in} \otimes \mathcal{H}_{\rm out}$$

A consistency check of complementarity

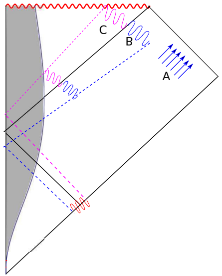


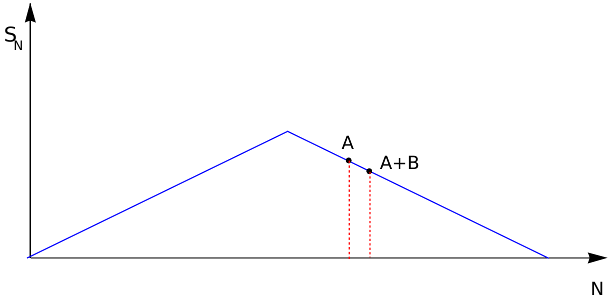
Observer A needs at least $\beta \log S$ to extract information of qubit (Hayden-Preskill) and then dive into BH

Qubit B would have to be sent with transplanckian energy to reach A

Strong subadditivity paradox

[Mathur, AMPS]





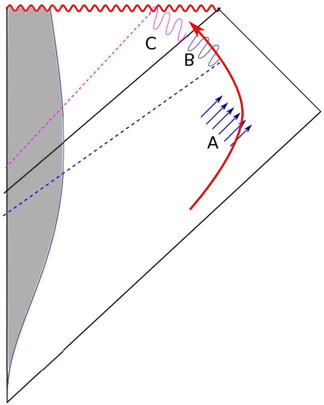
Theorem: strong subadditivity of Entanglement entropy

$$S_{AB} + S_{BC} \geq S_A + S_C$$

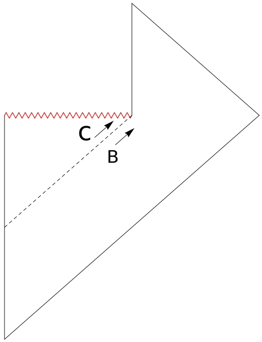
+Mathur's theorem for small corrections

Observing the paradox

[AMPS]



Quantum chaos vs specific entanglement

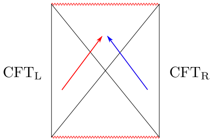


Smooth horizon requires **specific** pattern of entanglement between field operators at B and C

Fragile under perturbations due to chaotic nature of system

Hard to imagine how **typical states** will end up with the correct, **specific** entanglement needed for smoothness

Eternal black hole



$$H_{\text{total}} = H_L + H_R$$

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle_L \otimes |E_i\rangle_R$$

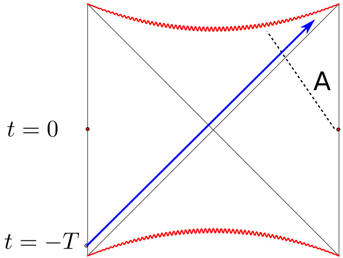
Smoothness of horizon depends on correct entanglement

$$\langle \mathrm{TFD}|\, \mathcal{O}_L \ \mathcal{O}_R |\mathrm{TFD}\rangle \sim O(1)$$

But, for a typical state $|\Psi\rangle$ (with same amount, but different details of entanglement) we find using ETH

$$\langle \Psi |\, \mathcal{O}_L \ \mathcal{O}_R |\Psi\rangle \sim O(e^{-S})$$

[Shenker,Stanford]



We start with $|\mathrm{TFD}\rangle$ and perturb it by a small operator (energy of $O(1)$) at time $t_L = -T$).

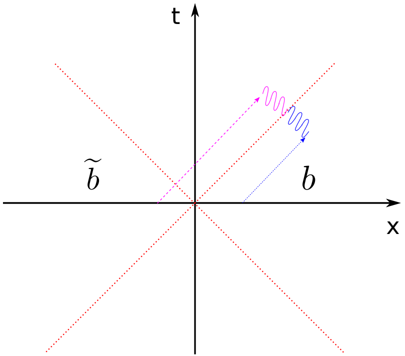
For small T, effect on infalling observer A is small. But center of mass collision energy grows exponentially with T

For $T > \beta \log S$ (scrambling time) we can no longer ignore backreaction.

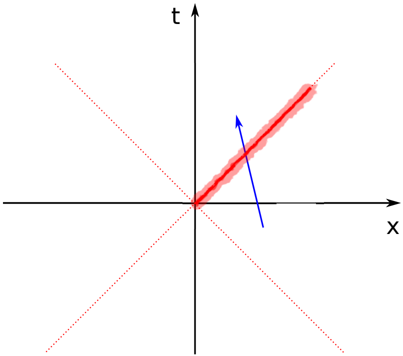
The "correct entanglement" of the TFD disrupted even by small perturbations due to chaos.

What if there is no entanglement of Hawking particles with interior modes after Page time?

Entanglement and smoothness of spacetime



Entanglement and smoothness of spacetime



If, after Page time, the Hawking particles are no longer entangled with something inside the black hole, playing the role of the *interior partner* C, then an infalling observer would not see the vacuum but rather a highly excited state of the quantum field.

This corresponds to a very large $\langle T_{\mu\nu} \rangle$ which would backreact and dramatically modify the region behind the horizon (firewall/fuzzball?)

In particular the infalling observer would not experience a smooth horizon.

This effect would be there even for black holes of very large mass, thus violating the predictions of classical GR in a regime of low curvatures.

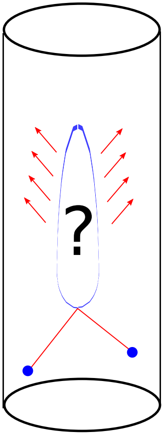
Summary

1. Information paradox from the point of view of asymptotic observer: natural (in-principle) resolution, consistent with generic expectations from quantum statistical mechanics

2. Preserving smoothness of the horizon: more challenging. Seems to violate monogamy of entanglement. Seems to contradict generic expectations from quantum statistical mechanics (requires specific detailed enganglement for typical states).

3. We will make some of these paradoxes more precise in AdS/CFT

Information paradox in AdS/CFT



CFT dual $\Rightarrow$ Evaporation manifestly unitary

What about smoothness of horizon?

Large BHs in AdS

In flat space we emphasized the relation betwen the information paradox and the smoothness of the horizon.

A natural question is whether large black holes in AdS have a smooth interior.

These black holes do not evaporate, so one might expect that there would be no paradox to address.

We will see that even for these black holes it is difficult to reconcile the smoothness of their horizon with unitarity in the CFT.

Mathematically precise version of the paradox.

Comments on bulk reconstruction

-CFT with gravity dual, large N, large λ

-HKLL construction

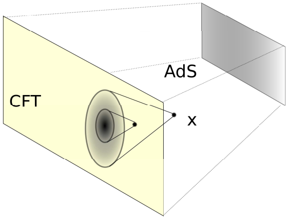
$$(\Box_{\mathrm{AdS}} - m^2)\phi = 0 \qquad \lim_{z\to 0} z^{-\Delta}\phi(x,z) = \mathcal{O}(x)$$

$$\phi_{\mathrm{CFT}}(t,\Omega,z) = \sum_m \int_{\omega>0} d\omega \;\; (\mathcal{O}_{\omega,m}\; f_{\omega,m}(t,\Omega,z) + \mathrm{h.c.})$$

$[\phi_{\mathrm{CFT}}(P_1), \phi_{\mathrm{CFT}}(P_2)] = 0$, if points P_1, P_2 spacelike with respect to AdS metric

-On-shell, uses bulk EOMs

-perturbative in 1/N



$$\phi_{\mathrm{CFT}}(t,\Omega,z) = \int dt' d\vec{\Omega}'\ K(t,\Omega,z\ ;\ t',\Omega')\mathcal{O}(t',\Omega')$$

where K is some known kernel

Black Hole Exterior

Consider big black hole in AdS. Expectation from bulk effective field theory (EFT) for a free scalar

$$\phi(t, r, \Omega) = \int_0^\infty d\omega \sum_{lm} b_{\omega lm}\, e^{-i\omega t} f_{\omega,l}(r) Y_{lm}(\Omega) + \text{h.c.}$$

where (dropping l, m indices) we have

$$[b_\omega, b^\dagger_{\omega'}] = \delta(\omega - \omega') \qquad [H, b_\omega] = -\omega b_\omega$$

and

$$\langle b^\dagger_\omega b_\omega \rangle \sim \frac{1}{e^{\beta\omega} - 1}$$

How do we reconstruct this from the CFT?

In typical QGP pure state $|\Psi\rangle$ (energy $O(N^2)$), single trace correlators factorize at large N

$$\langle\Psi|\mathcal{O}(x_1)...\mathcal{O}(x_n)|\Psi\rangle = \langle\Psi|\mathcal{O}(x_1)\mathcal{O}(x_2)|\Psi\rangle...\langle\Psi|\mathcal{O}(x_{n-1})\mathcal{O}(x_n)|\Psi\rangle + ...$$

The 2-point function in which they factorize is the thermal 2-point function

$$G(t,x) \equiv Z^{-1}\mathrm{Tr}[e^{-\beta H}\mathcal{O}(t,x)O(0,0)]$$

which is hard to compute, but obeys KMS condition

$$G(t - i\beta, x) = G(-t, -x)$$

Consider single-trace operator $\mathcal{O}$ in CFT, dual to bulk field ϕ. Define Fourier modes

$$\mathcal{O}_{\omega lm} = \int dt d\Omega \; \mathcal{O}(t, \Omega) \, e^{i\omega t} \, Y^*_{lm}(\Omega)$$

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then we identify

$$b_{\omega lm} = \frac{1}{\sqrt{\langle [\mathcal{O}_{\omega lm}, \mathcal{O}^\dagger_{\omega lm}] \rangle}} \mathcal{O}_{\omega lm}$$

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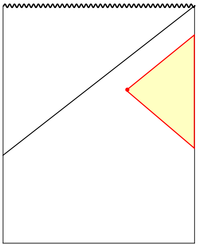
so

$$[b_{\omega lm}, b^\dagger_{\omega lm}] = 1$$

and, using KMS and large N, we have

$$\langle b^\dagger_{\omega lm} b_{\omega lm} \rangle = \frac{1}{e^{\beta\omega} - 1}$$



Local bulk field outside horizon of AdS black hole

$$\phi_{\rm CFT}(t,\Omega,z) = \sum_m \int_0^\infty d\omega\, \mathcal{O}_{\omega,m}\, f^\beta_{\omega,m}(t,\Omega,z) + {\rm h.c.}$$

At large N (and late times) the correlators

$$\langle\Psi|\phi_{\rm CFT}(t_1,\Omega_1,z_1)...\phi_{\rm CFT}(t_n,\Omega_n,z_n)|\Psi\rangle$$

reproduce those of semiclassical QFT on the BH background (in AdS-Hartle-Hawking state).

Some subtleties with spacelike modes

Fix ω, take $\vec{k} \to \infty$. If we normalize bulk wavefunctions so that $f \to 1$ as $r \to \infty$, then at any finite r they grow like

$$e^{\alpha \frac{\beta |\vec{k}|}{2}}$$

For BTZ $\alpha = 1$, for higher dimensional AdS-BHs $\alpha < 1$

HKLL reconstruction is possible because of exponential decay of spacelike thermal correlators.

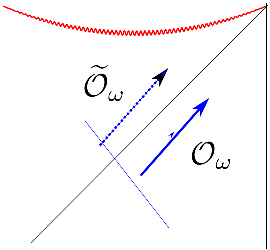
[S.Banerjee, KP, S.Raju, P.Samantray, P. Shrivastava]

We recently derived a bound for spacelike thermal correlators in any QFT

$$\langle \mathcal{O}(\omega_1, \vec{k}_1) ... \mathcal{O}(\omega_n, \vec{k}_n) \rangle_\beta \leq e^{-R\beta}$$

where R is smallest sphere enclosing $\vec{k}_i$.

Local analysis near the horizon



Demanding that low-point correlators of local fields at late times look locally like flat space we find some conditions which must hold at large N

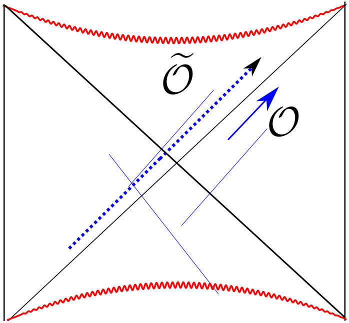
$$[\mathcal{O}_\omega, \mathcal{O}^\dagger_\omega] = 1, \qquad [\widetilde{\mathcal{O}}_\omega, \widetilde{\mathcal{O}}^\dagger_\omega] = 1$$

$$[\mathcal{O}_\omega, \widetilde{\mathcal{O}}_{\omega'}] = 0$$

$$[H, \mathcal{O}^\dagger_\omega] = \omega\mathcal{O}^\dagger_\omega, [H, \widetilde{\mathcal{O}}^\dagger_\omega] = -\omega\widetilde{\mathcal{O}}^\dagger_\omega,$$

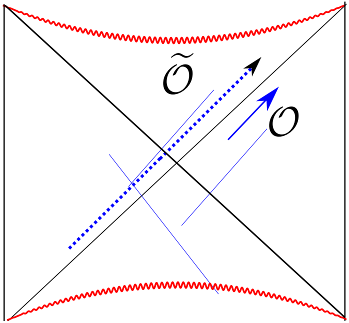
$$\langle \mathcal{O}^\dagger_\omega \mathcal{O}_\omega \rangle = \langle \widetilde{\mathcal{O}}^\dagger_\omega \widetilde{\mathcal{O}}_\omega \rangle = \frac{1}{e^{\beta\omega} - 1}$$

$$\langle \mathcal{O}_\omega \widetilde{\mathcal{O}}_\omega \rangle = \frac{e^{\frac{\beta\omega}{2}}}{e^{\beta\omega} - 1}$$



On TFD state

$$\widetilde{\mathcal{O}} = \mathcal{O}_L$$



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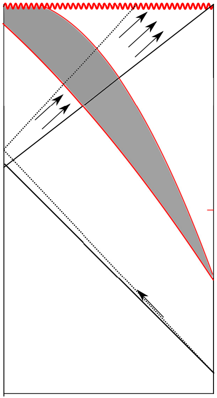
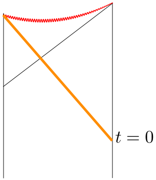


Collapsing BH

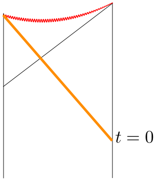
$$\widetilde{\mathcal{O}} = \mathcal{O}_{\text{early}} \ (?)$$

Collapsing vs typical black holes



Black holes formed by (simple) gravitational collapse are *a-typical*

Collapsing vs typical black holes



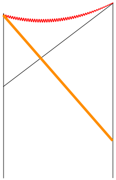
Black holes formed by (simple) gravitational collapse are *a-typical*

Typical black hole microstates are defined by “microcanonical measure”

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

where $E_i \in E_0 \pm \delta E$ and c_i selected randomly by Haar measure

Collapsing vs typical black holes



Black holes formed by (simple) gravitational collapse are *a-typical*

Typical black hole microstates are defined by "microcanonical measure"

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

where $E_i \in E_0 \pm \delta E$ and c_i selected randomly by Haar measure
Notice that typical states are almost time-independent

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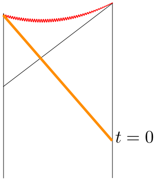

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Typical states are *equilibrium states*.

A problem

It is challenging to identify the operators $\widetilde{\mathcal{O}}$ in the CFT, with satisfy desired properties on $\mathbf{most}$ states counted by

$$S = \frac{A}{4G}$$

because these properties seem to imply [AMPSS, Marolf-Polchinski]

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There is no problem to find these operators in particular, special states.

$$[\widetilde{\mathcal{O}}, \widetilde{\mathcal{O}}^\dagger] = 1 \qquad \Rightarrow \qquad \widetilde{\mathcal{O}}^\dagger = \text{“creation operator”}$$

$\Rightarrow \widetilde{\mathcal{O}}^\dagger$ should not annihilate (typical) states of the CFT $\qquad (*)$.

On the other hand

$$[H, \widetilde{\mathcal{O}}^\dagger] = -\omega \widetilde{\mathcal{O}}^\dagger$$

implies that $\widetilde{\mathcal{O}}^\dagger$ lowers the energy so it maps CFT states of energy E to $E - \omega$.

But in CFT, we have $S(E) > S(E - \omega)$.

$\Rightarrow$ if $\widetilde{\mathcal{O}}^\dagger$ is an ordinary linear operator, it must have a nontrivial kernel.

Inconsistent with statement (*).

$\Rightarrow$ The CFT does not contain $\widetilde{\mathcal{O}}$ operators and cannot describe the BH interior (?)

A related paradox

[Bousso]

Consider the number operator N_a for some modes relevant for the infalling observer. If typical states have a smooth horizon then we expect

$$\mathrm{Tr}_E[N_a] = 0$$

where the trace is over states of energy $E \pm \delta E$.

A trace can be evaluated in any basis of orthonormal vectors.

The number operator of Schwarzchild modes $N_b = \mathcal{O}^\dagger \mathcal{O}$ obeys $[H, N_b] = 0$ and can be simultaneously diagonalized with H.

$$\mathrm{Tr}_E[N_a] = \sum \langle n_b | N_a | n_b \rangle > 0$$

since eigenstates $|n_b\rangle$ of Schwarzchild number operator have excited horizon

Paradox for eternal black hole

Using entanglement to go behind the horizon

[KP, S. Raju]

The quantum fields outside the horizon appear to be in an entangled state. They are entangled with certain CFT d.o.f. which can play the modes of the interior.
There is a natural mathematical construction allowing us to identify those.

Tomita-Takesaki modular theory

Consider Hilbert space $\mathcal{H}$, a state $|\Psi\rangle \in \mathcal{H}$ and an algebra $\mathcal{A}$ acting on $\mathcal{H}$ with the properties:
1) The state is *cyclic* wrt the algebra $\mathcal{A}$ i.e.

$$\mathcal{H} = \mathrm{span}\mathcal{A}|\Psi\rangle$$

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Then the Tomita-Takesaki theorem says (among other things) that:

The representation of the algebra $\mathcal{A}$ on $\mathcal{H}$ is reducible, and the algebra has a non-trivial commutant $\mathcal{A}'$ also acting on $\mathcal{H}$. Moreover $\mathcal{A}'$ is isomorphic to $\mathcal{A}$. Finally, the algebras $\mathcal{A}, \mathcal{A}'$ are entangled in a particular way.

Tomita-Takesaki modular theory

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$$Sa|\Psi\rangle = a^\dagger|\Psi\rangle \qquad a \in \mathcal{A}$$

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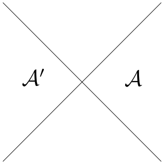
3. KMS-like condition: $F(z) \equiv \langle\Psi|a\Delta^{iz}b\Delta^{-iz}|\Psi\rangle$, then $F(-i) = \langle\Psi|ba|\Psi\rangle$

n

Example:Rindler space



Consider a general, possibly strongly coupled, relativistic QFT in the Minkowski ground state $|0\rangle$. Suppose we have only access to right Rindler wedge. How can we use the entanglement to recover the rest of space-time?

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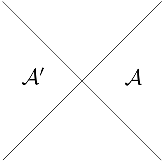

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Reeh-Schlieder theorem: The Minkowski vacuum $|0\rangle$ is a cyclic and separating state for the algebra $\mathcal{A}$:

1. States of form $a_1 ... a_n|0\rangle \qquad a_i \in \mathcal{A}$, span dense subspace of $\mathcal{H}$
2. There is no $a \in \mathcal{A}$ such that $a|0\rangle = 0$.

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Consider Lorentz boost $U = e^{iKs}$ on $t - x$ plane

$$t' = t \cosh s + x \sinh s$$
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Generalizing to more operators (Bisognano-Wichmann thm.) it follows that the desired modular conjugation implementing $Sa|0\rangle = a^\dagger|0\rangle$ is

$$S = \Theta R_1 e^{-\pi K}$$

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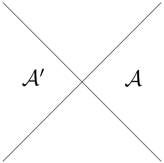

We have $S = \Theta R_1 e^{-\pi K}$. From this follows that

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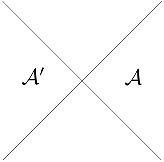

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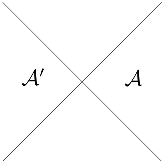

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The modular Hamiltonian is the Lorentz boost generator with effective temperature $\frac{1}{2\pi}$. The antiunitary operator J mapping $\mathcal{A}$ to $\mathcal{A}'$ and allowing us to recover the left wedge is

$$J = \Theta R_1$$

The fact that each of the algebras $\mathcal{A}, \mathcal{A}'$ remain invariant under conjugation by Δ^{is} is obvious in this example. The KMS condition implies the Unruh temperature (even at strong coupling).

Tomita-Takesaki and the black hole

We do not have a decomposition of the algebra in **physical space**, but rather in the "space of operators" (simple vs complicated).
Introduce a "small algebra" $\mathcal{A}$ of simple operators (single trace + small products).

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The algebra $\mathcal{A}$ probes the typical pure state $|\Psi\rangle$ as a thermal state

$$\langle\Psi|\mathcal{O}(x_1)...\mathcal{O}(x_n)|\Psi\rangle = Z^{-1}\mathrm{Tr}[e^{-\beta H}\mathcal{O}(x_1)...\mathcal{O}(x_n)] + O(1/N)$$

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An analogue of the Tomita-Takesaki construction applies.
Using large N factorization and the KMS condition, we find the modular Hamiltonian for the small algebra

$$\Delta \equiv S^\dagger S = e^{-\beta(H-E_0)} + O(1/N)$$

The mirror operators

This leads to the "mirror operators"

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta H}{2}}\mathcal{O}^\dagger_\omega e^{\frac{\beta H}{2}}|\Psi\rangle$$

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- Due to Boltzman factors $\langle\mathcal{O}_\omega^\dagger\mathcal{O}_\omega\rangle \propto e^{-\beta\omega}$, we define these operators for $\omega < \omega_*$, where ω_* does not grow too fast with N

The mirror operators

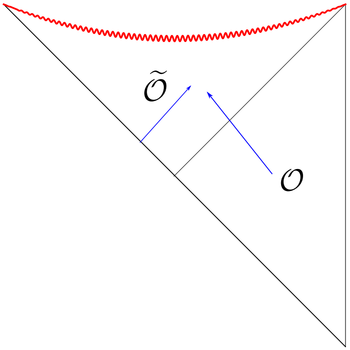
The small algebra $\mathcal{A}$ is not an exact algebra, hence the Tomita-Takesaki theorem can not be applied *exactly*. Hence $\mathcal{A}'$ is not an exact commutant.

From a physical point of view this is a **desirable feature** of the construction. It realizes the idea of black hole complementarity in a precise setting.

It also naturally implies that there is some non-locality in the construction of the interior.

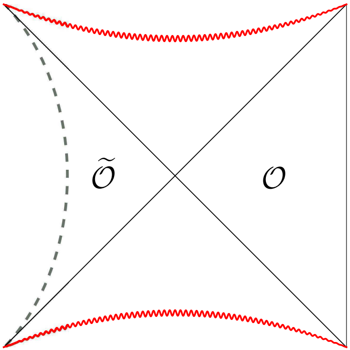
Finally, notice the operators $\widetilde{\mathcal{O}}$ defined by the Tomita-Takesaki constrcution are state-dependent, since they are "defined by the enganglement".

Infalling observer



$$\phi(t,r,\Omega) = \int_0^\infty d\omega \Big[\mathcal{O}_\omega\, f_\omega(t,\Omega,r) + \widetilde{\mathcal{O}}_\omega g_\omega(t,\Omega,r) + \text{h.c.}\Big]$$

Extended geometry



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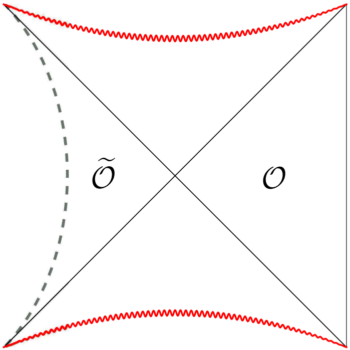

The cutoff on the left is determined by ω_*.

Since $\widetilde{\mathcal{O}}$ do not fundamentally commute with $\mathcal{O}$, left region should not be though as a fundamentally independent part of the Hilbert space (BH complementarity)

State-dependence

- Interior operators defined by

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta\omega}{2}}\mathcal{O}^\dagger_\omega|\Psi\rangle$$

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

- Solution defined only on $\mathcal{H}_\Psi$, depends on reference state $|\Psi\rangle$
- Operators cannot be upgraded to “globally defined” operators
- Solves previous paradoxes (Negative trace, Chaos vs Entanglement problem,...)

Complementarity and Non-locality

$\widetilde{\mathcal{O}}$ were constructed based on the fact that we restricted our attention to a "small algebra" of $\mathcal{O}$'s. The construction breaks down if the "small algebra" is enlarged to include all operators

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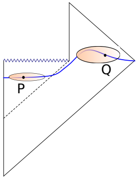

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$$[\phi(P), \Phi^{\text{complex}}(Q)] = O(1)$$

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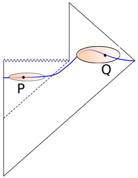

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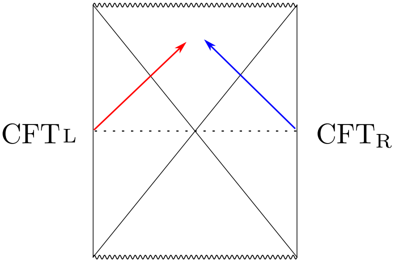
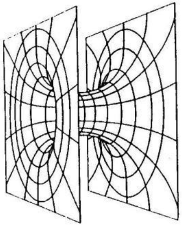
The Hilbert space of Quantum Gravity does not factorize as $\mathcal{H}_{\text{inside}} \otimes \mathcal{H}_{\text{outside}}$

1) Solves problem of Monogamy of Entanglement (and avoids Mathur's theorem)

2) Is consistent with locality in EFT, concrete mathematical realization of complementarity

Eternal AdS black hole





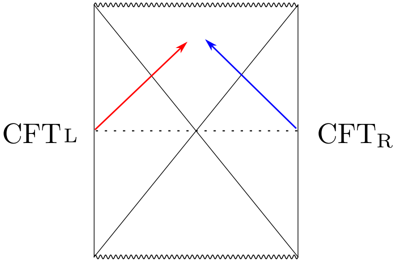
Two identical non-interacting CFTs

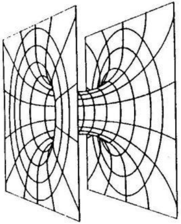
$$H = H_L + H_R$$

in an entangled state

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\frac{\beta E}{2}} |E\rangle_L \otimes |E\rangle_R$$

Eternal AdS black hole

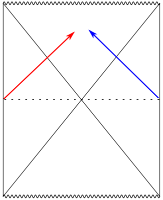




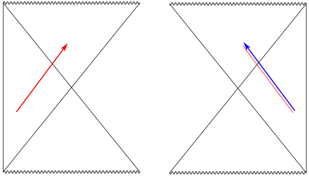
In the bulk they are connected by a wormhole (Einstein-Rosen bridge).

It is not traversable, consistent with the fact that CFTs are non-interacting

Eternal AdS black hole

CFT$_\mathrm{L}$  CFT$_\mathrm{R}$

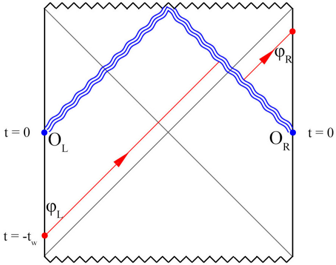
$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle_L \otimes |E_i\rangle_R$$

CFT$_\mathrm{L}$  CFT$_\mathrm{R}$

$$|\Psi\rangle = \sum_{ij} c_{ij} |E_i\rangle_L \otimes |E_j\rangle_R$$

$$c_{ij} = \text{generic}$$

Gao-Jafferis-Wall protocol



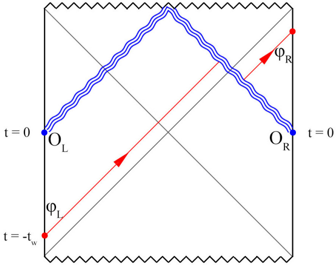
at $t = 0$ we briefly couple the CTFs by a double-trace interaction

$$H = H_L + H_R + g f(t) \mathcal{O}_L \mathcal{O}_R$$

For given sign of g this creates negative energy shockwaves in the bulk. Probe undergoes time advance when crossing shockwaves

Wormhole becomes traversable

Gao-Jafferis-Wall protocol

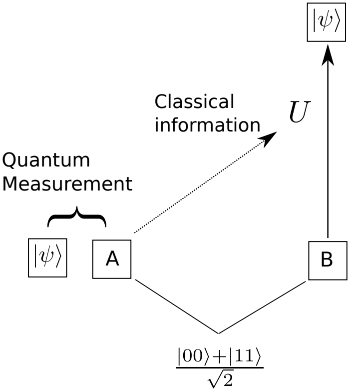


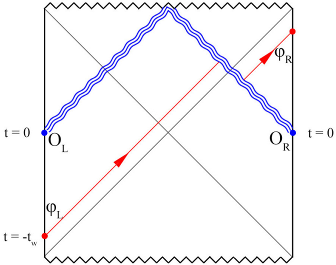
Change of CFT energy

$$\delta\langle H_R\rangle \propto g\langle \mathcal{O}_L\mathcal{O}_R\rangle + O(g^2)$$

Black hole horizon shrinks somewhat, probe can cross the wormhole
CFTs briefly interacted via $O_L O_R$ at $t = 0$, so information can be exchanged
Notice ϕ vs O

Quantum Teleportation Interpretation





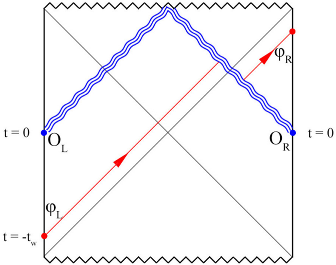
Measure O_L on CFT$_L$, then apply

$$e^{igo_L\mathcal{O}_R}$$

on CFT$_R$. The probe ϕ is teleported.

Gao-Jafferis-Wall protocol

analysis by [Maldacena-Stanford-Yang]



We create the probe on the left by

$$e^{i\epsilon\phi_L(-t)}|\mathrm{TFD}\rangle$$

At $t = 0$ we apply double-trace perturbation coupling the two CFTs

$$e^{igO_LO_R(0)}e^{i\epsilon\phi_L(-t)}|\mathrm{TFD}\rangle$$

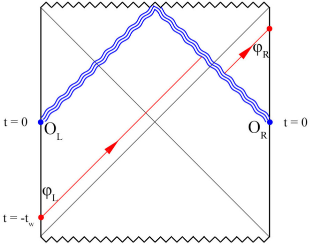
We measure the operator $\phi_R(t)$ on this state. To leading order in ϵ we need

$$\langle\mathrm{TFD}|[\phi_L(-t), e^{-igO_LO_R(0)}\phi_R(t)e^{igO_LO_R(0)}]|\mathrm{TFD}\rangle$$

Expanding in g

$$\langle\mathrm{TFD}|[\phi_L(-t), O_L(0)][\phi_R(t), O_R(0)]|\mathrm{TFD}\rangle$$

Traversable wormholes and quantum chaos



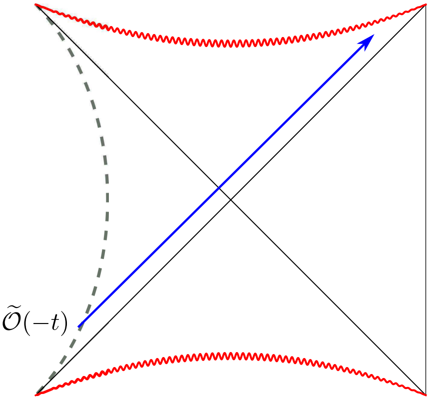
Growth of out-of-time-order-correlators (OTOC) due to **quantum chaos**

$$\langle \mathrm{TFD}|[\phi_L(-t), O_L(0)][\phi_R(t), O_R(0)]|\mathrm{TFD}\rangle \sim \frac{1}{N^2} e^{\frac{2\pi}{\beta}t}$$

Including higher orders in g, we find that the commutator is zero up to scrambling time $t \approx \beta \log S$, when it becomes nonzero and we get a nontrivial signal, corresponding to the probe appearing in the right CFT.

- Gao-Jafferis-Wall identified an S-matrix-like experiment which probes the interior of eternal black hole
- CFT correlators contain information about geometry inside horizon
- Computations provide evidence for smoothness of horizon of eternal black hole, dual to the TFD state, and ER/EPR proposal
- However, the real difficulty in reconciling unitarity with the smoothness of the black hole horizon is not for the TFD (which is a very special, atypical state), but rather for *typical black hole microstates*.
- Can we find a way of applying a similar protocol to (1-sided) typical black hole microstates, which will allow us to probe their interior?

Exciting the left region

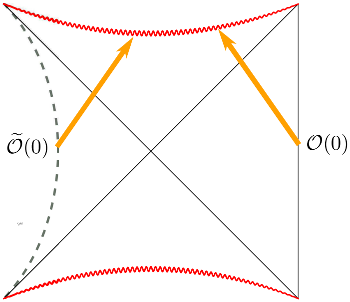


Mirror quench: we perturb the CFT Hamiltonian by $\widetilde{\mathcal{O}}$ at $-t$

Excitation is invisible by simple CFT operators

Creating negative energy shockwaves for 1-sided black hole

[J. de Boer, R. van Breukelen, S. Lokhande, KP, E. Verlinde, arXiv: 1804.10580, 1901.08527]

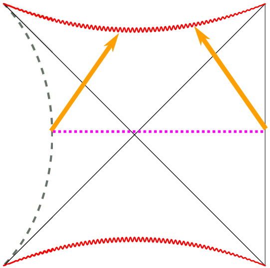


At $t = 0$ we perturb CFT Hamiltonian by

$$g f(t) \mathcal{O} \widetilde{\mathcal{O}}(0)$$

Compute effect on bulk correlators $\Rightarrow$ generates negative energy shockwaves for appropriate choice of g

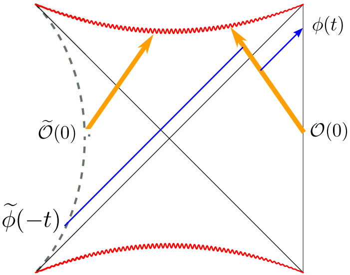
Some subtleties (in progress)



Operators $\widetilde{\mathcal{O}}$ are gravitationally dressed wrt the right $\Rightarrow$ Wilson lines extending across geometry

Backreaction and Einstein equations at subleading order?

The experiment



We create a probe in the left region of the black hole by acting with $\widetilde{\phi}(-t)$.

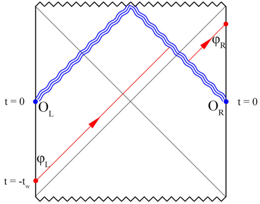
Then at $t = 0$ we perturb the CFT by $gf(t)\mathcal{O}(0)\widetilde{\mathcal{O}}(0)$. Finally we detect the probe by measuring $\phi(t)$.

The postulated Penrose diagram makes a prediction about CFT correlators (singal around $t = \beta \log S$)

$$\langle \Psi_0 | [\widetilde{\phi}(-t), e^{-ig\widetilde{\mathcal{O}}\mathcal{O}(0)} \phi(t) e^{ig\widetilde{\mathcal{O}}\mathcal{O}(0)}] | \Psi_0 \rangle$$

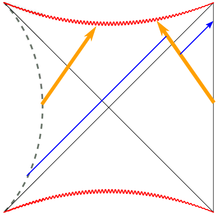
Comparison

Eternal BH



$$C = \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}\mathcal{X}(\phi, O)]$$

1-sided BH



$$C' = \langle\Psi_0|\mathcal{X}(\phi, O)|\Psi_0\rangle$$

Using properties of the TFD state and the mirror operators we find that both experiments are governed by the expectation value of **exactly the same** string of ordinary CFT operators $\chi(\phi, \mathcal{O})$. Moreover, in stat-mech we have

$$C' = \mathrm{Tr}[\rho_m \mathcal{X}(\phi, \mathcal{O})] + O(e^{-S})$$

Condition for CFT correlators

$$C = \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}\mathcal{X}(\phi, O)] \qquad C'' = \mathrm{Tr}[\rho_m \mathcal{X}(\phi, O)]$$

A necessary condition for horizon of typical BH mircostate to be smooth is

$$\boxed{\lim_{N\to\infty} C = \lim_{N\to\infty} C''}$$

keeping frequencies $\omega < \omega_*$.

- Not obvious, trace-distance $||\rho_\beta - \rho_m||$ between ensembles is almost maximal.
- $\mathcal{X}(\phi, O)$ is a complicated observable, product of operators at time separation $\Delta t \sim \beta \log S$
- Condition is related to whether $\mathcal{X}(\phi, O)$ obeys Eigenstate Thermalization Hypothesis (ETH)

$$\langle E_i|\mathcal{X}|E_j\rangle = f(E_i)\delta_{ij} + R_{ij}e^{-S/2} \tag{1}$$

with $\frac{df}{dE} \sim O(1/S)$

Condition for CFT correlators

- Interesting effect comes from subleading corrections of the form

$$\frac{1}{N^2} e^{\frac{2\pi t}{\beta}}$$

  At scrambling time they become O(1).

  Are these "chaos-enhanced" $1/N^2$ corrections the same in typical pure states and thermal ensemble?

- Our condition requires that correlators agree *even after analytic continuation* by $t \to t - i\frac{\beta}{2}$ (keeping frequencies up to ω_*)

Evidence

1. ETH holds for products of operators at small time separation. We can show that it also holds for very large time separations (when chaos saturates). It is natural to expect that it holds for intermediate times of order $\beta \log S$

2. In 2d CFTs with large c and sparse spectrum correlators are dominated by Virasoro identity block. In this case the conjecture is true.

3. Numerical evidence in SYK model

The SYK model

N-Majorana fermions in $0+1$d

$$\{\psi^i, \psi^j\} = \delta^{ij}$$

$$H = \sum_{ijkl} J_{ijkl} \psi^i \psi^j \psi^k \psi^l$$

where J_{ijkl} random couplings

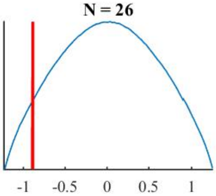
$$dim \mathcal{H} = 2^{\frac{N}{2}}$$

Flows to strongly coupled CFT in IR

Model of black hole in AdS_2

The mirror operators in the SYK model

Typical state in SYK



$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

Introduce the spin operators [Kourkoulou,Maldacena]

$$S_k = 2i\ \psi_{2k-1}\psi_{2k}$$

The mirror operators in the SYK model

$$
\begin{aligned}
|1\rangle &= |\Psi_0\rangle\,, \\
|2\rangle &= S_{1,\omega_1}\,|\Psi_0\rangle\,, \\
|3\rangle &= S_{1,\omega_2}\,|\Psi_0\rangle\,, \\
&\;\vdots \\
|n\rangle &= S_{2,\omega_1}\,|\Psi_0\rangle\,, \\
|n+1\rangle &= S_{2,\omega_2}\,|\Psi_0\rangle\,, \\
&\;\vdots \\
|l\rangle &= S_{2,\omega_2}S_{1,\omega_1}\,|\Psi_0\rangle\,, \\
&\;\vdots
\end{aligned}
\tag{2}
$$

The mirror operators in the SYK model

To simplify the notation, we denote these states as

$$|I\rangle \equiv \mathcal{O}_I|\Psi_0\rangle, \tag{3}$$

where $\mathcal{O}_I$ is a combination of the spin operators introduced above. We define

$$G_{IJ} \equiv \langle I|J\rangle$$

and

$$B_{IJ,k\omega} \equiv \langle I|\, \widetilde{S}_{k,\omega}\, |J\rangle\,, \tag{4}$$

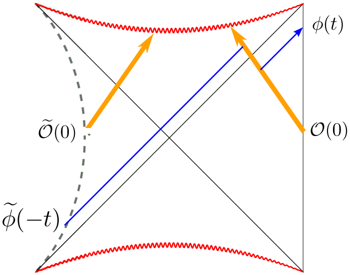
or using the equations for the mirror operators

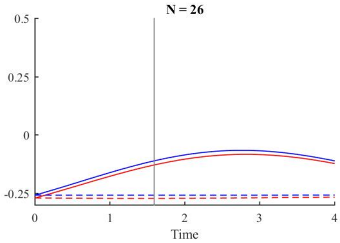
$$B_{IJ,k\omega} = \langle\Psi_0|O_I^\dagger O_J e^{-\frac{\beta H}{2}} S_{k,\omega} e^{\frac{\beta H}{2}}|\Psi_0\rangle. \tag{5}$$

Finally we can represent the mirror operators explicitly as

$$\widetilde{S}_{k,\omega} = G^{IJ} B_{JK,k\omega} G^{KL}\, |I\rangle\, \langle L|\,. \tag{6}$$

Extracting particle from behind the horizon





Relation to Kourkoulou-Maldacena

They consider a class of **a-typical**, non-equilibrium states in the SYK model

$$e^{-\frac{\beta H}{2}}|B_s\rangle \qquad \text{where} \qquad S_k|B_s\rangle = s_k|B_s\rangle$$

On these states they consider the (state-dependent) perturbation of the form

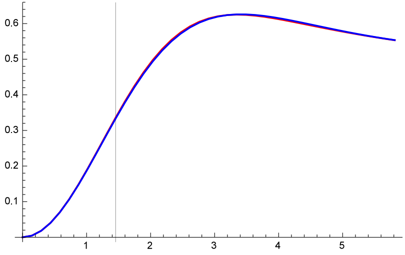
$$\delta H = g\sum_k s_k S_k$$

and they argue that this exposes part of the region behind the horizon.
This thought experiment is closely related to the perturbations

$$\delta H = g\mathcal{O}\widetilde{\mathcal{O}}$$

that we discussed earlier.

Pure vs thermal state OTOC in SYK

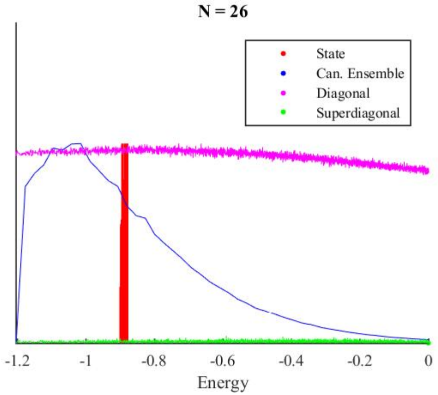


Time

$$\langle \{\psi^i(t), \psi^i(0)\}^2 \rangle$$

on thermal state (red) vs typical pure state (blue).

ETH for chaotic observables in SYK



Matrix elements in SYK of

$$\{\psi^i(t), \psi^i(0)\}^2$$

for $t \approx \beta \log S$

Recovering information from a black hole

We throw a qubit into black hole. How long do we need to wait to recover the information from Hawking radiation?

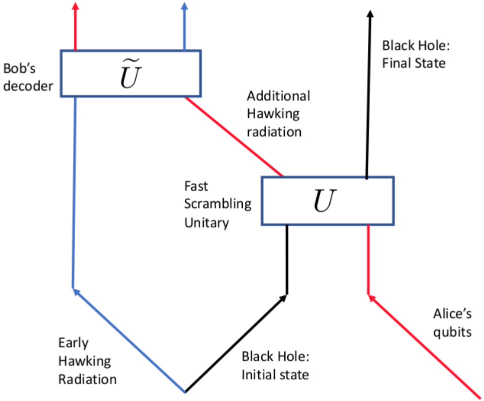
$$t_{evap} \sim G^2 M^3$$

Hayden Preskill (2007): if we have access to more than half of Hawking radiation we only need to wait scrambling time

$$t_S \sim GM \log S$$

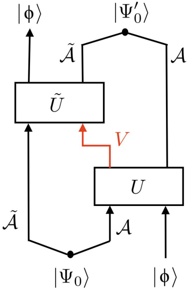
to recover information. For the protocol to work we need to know the initial state of the black hole.

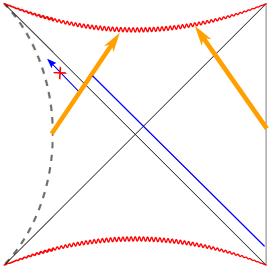
Hayden-Preskill protocol



Reformulated by Maldacena-Stanford-Yang in terms of traversable wormholes

A realization of Hayden-Preskill





We throw qubit $\phi(-t_s)$ into black hole

At $t = 0$ we act with $\mathcal{O}\widetilde{\mathcal{O}}$

After scrambling time we can extract the quantum information of the qubit my measuring operator $\widetilde{\phi}(t_s)$.

This provides an explicit decoding Hayden-Preskill protocol

Knowledge of the quantum state related to state-dependent $\widetilde{\mathcal{O}}$.

Summary

- The nature of space-time behind the horizon remains mysterious
- This question becomes particularly sharp for typical black hole microstates in AdS
- Presented a proposal for their geometry, by making use of state-dependent operators.
- Developments related to traversable wormholes: new calculational tools to probe BH interior
- Interesting connections with quantum teleportation and quantum chaos in pure states.