Symmetries in particle physics

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Noether

امی نوتر

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ترجمه‌ی ***حسن فتاحی***

انتشارات مازیار

Noether theorem

- Lagrangian $\mathcal{L}(\phi)$
- Continuous symmetry $\phi \to \phi + \epsilon\delta\phi$

J_μ $\partial_\mu J^\mu = 0$

- Conserved charge: $Q = \int d^3x J^0$
- All 'charges' of particles and antiparticles are opposite.
- Neutron is electrically neutral but has its antiparticle.
- Photon has no antiparticle.

Electrodynamics

- Coulomb force — Electric potential — $\nabla\phi$
- Magnetic force — A_i — $\vec{\nabla} \times \vec{A}$

Ancient knowledge

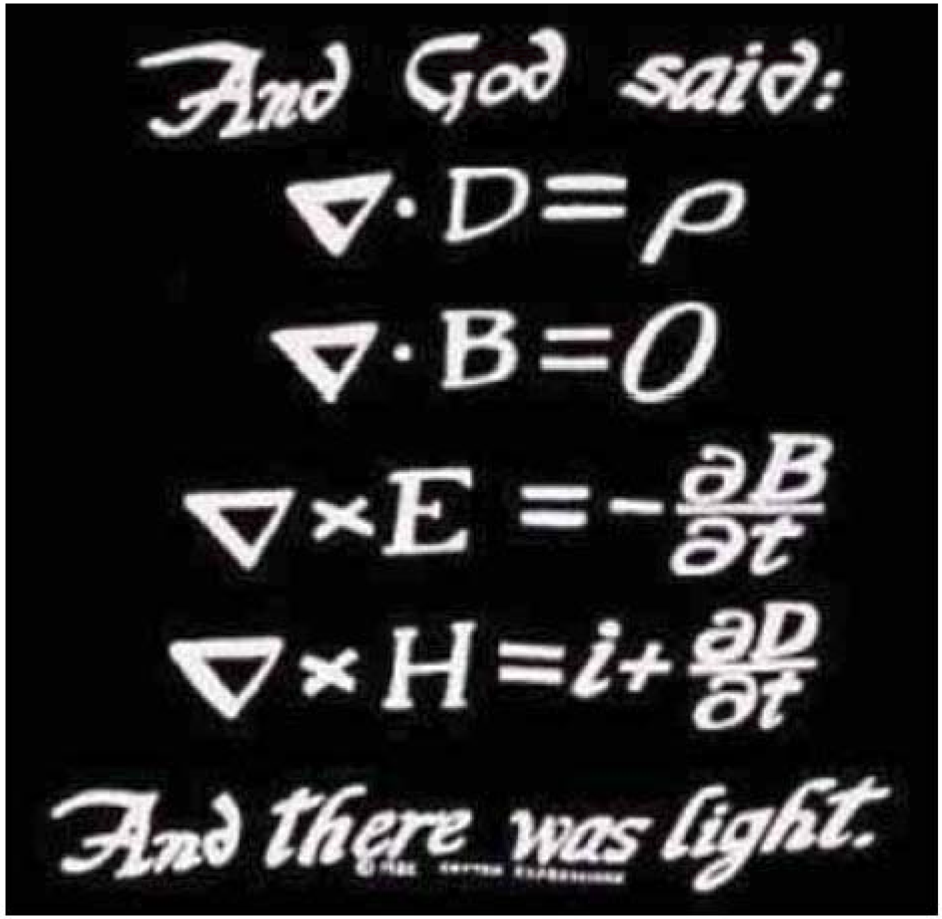
- Electricity

- Magnetism

Understanding Electricity

Formulation of electromagnetism

- Electricity+magnetism
- Maxwell’s equations
- Electromagnetic waves


And God said:
$\nabla \cdot D = \rho$
$\nabla \cdot B = 0$
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times H = i + \frac{\partial D}{\partial t}$
And there was light.

Relativistic

- Four-vector: $A^\mu = (\phi, A^i)$

Quantum mechanics→ field theory

- Invariant under $A^\mu(x) \to A^\mu(x) + \partial^\mu f(x)$

Dirac equation

- Fermions like electron (matter fields)

$$\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

$$\psi = \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix}$$

Dirac matrices

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

U(1) gauge theory

$$\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi + eA_\mu \bar{\psi}\gamma^\mu \psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- Field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- Invariant under $\begin{cases} \psi \to e^{ei\alpha}\psi \\ A_\mu \to A_\mu - \partial_\mu \alpha \end{cases}$
- Noether current: $\bar{\psi}\gamma^\mu \psi$

Massless photon

- The gauge boson is absolutely massless:

$$\cancel{m^2 A_\mu A^\mu} \qquad m_\gamma < 10^{-18}\ \mathrm{eV} \qquad m_e \sim 5 \times 10^5\ \mathrm{eV}$$

- The electric force is long range:

$$V \propto \frac{e^{-mr}}{r} \qquad \text{range} \propto \frac{1}{m}$$

Strong interaction

- $SU(3)_c$ symmetry
- Each quark comes in 3 colors.
- Quarks are in fundamental representation of $SU(3)_c$
- Gluons are in adjoint representations of $SU(3)_c$
- There are 8 gluons

The Gell-Mann matrices

$$\lambda_{1,2,3} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{4,5,6} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_{7,8} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

SU(n) Yang-Mills theory

- Invariant under

$$\begin{cases} \psi \to U\psi \\ G_\mu \cdot \lambda \to U(G_\mu \cdot \lambda)U^{-1} + \frac{i}{g}U\partial_\mu U^{-1} \end{cases}$$

$$U \in SU(n)$$

Gauging recipe

$$\partial_\mu \rightarrow D_\mu \qquad D_\mu = \partial_\mu - ig\sum_i G^i_\mu \lambda^i$$

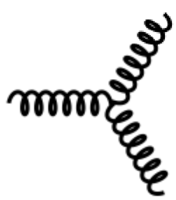
$$[D_\mu, D_\nu] = -iG^a_{\mu\nu}\lambda^a$$

$$[\lambda^a, \lambda^b] = if^{abc}\lambda^c$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + gf^{abc}G^b_\mu G^c_\nu$$

$$\bar{\psi}\partial_\mu\gamma^\mu\psi - m\bar{\psi}\psi + g\sum_i G_{i\mu}\bar{\psi}\gamma^\mu\lambda^i\psi - \frac{1}{4}\sum_i G^i_{\mu\nu}G^{i\mu\nu}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + gf^{abc}G^b_\mu G^c_\nu$$



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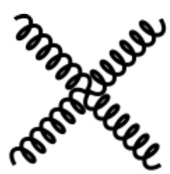
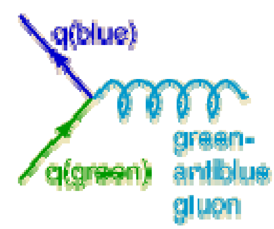


g^2



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Running of coupling

- Gluon is also massless but….
- Dependence of coupling on energy

$$\alpha = \frac{g^2}{4\pi}$$

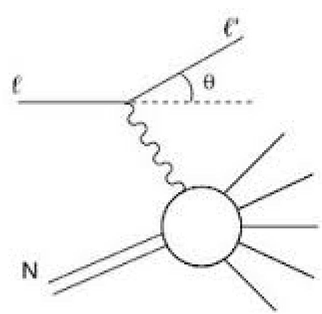
- QED coupling: $$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log(\frac{Q^2}{\mu^2})}$$
- QCD coupling: $$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{(33-2n_f)\alpha_s(\mu^2)}{12\pi}\log(\frac{Q^2}{\mu^2})}$$

First order approximation

Distinct Regimes

- Confinement

- DIS= Deep inelastic scattering (free parton)



$$Q \gg 200 \text{ MeV}$$

An energy scale in massless theory!

Low energy scheme

- Hadrons are all color singlets:

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \quad \Pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

- Yukawa interaction:

$$\bar{N}\Pi \cdot \tau\gamma_5 N$$

Short range: Range given by inverse of mass pion.

3 colors?

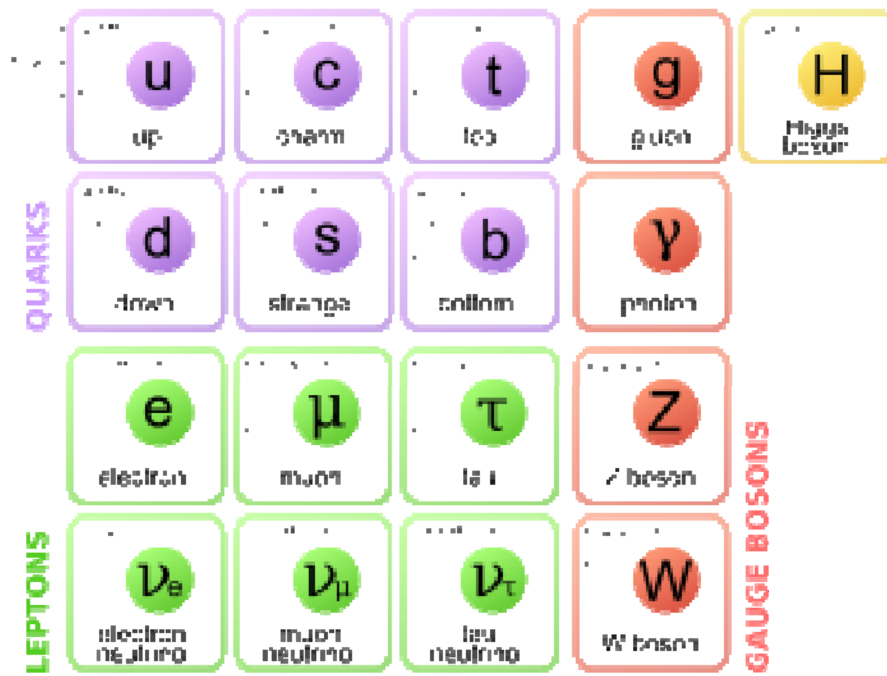
- spin 3/2 baryon $\Delta^{++}(1232)$ uuu

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to \text{lepons})}$$

Identification

Parton ↔ Quarks


u
c
t
g
H
d
s
b
γ
e
μ
τ
Z
νe
νμ
ντ
W
QUARKS
LEPTONS
GAUGE BOSONS

- 1) Gravity
- 2) Electromagnetism
- 3) Weak nuclear force
- 4) Strong nuclear force

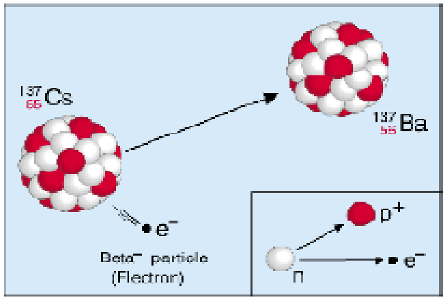
Is this all? What else?

- 1) Gravity
- 2) Electromagnetism
- 3) Weak nuclear force
- 4) Strong nuclear force

(2–4) → SM

Late 19th century history

New forces discovered



- Beta decay
- Alpha decay

- Have different time scales.

- Have short range.

- Strong interaction: $\alpha - ray$

$$time\ scale \sim 10^{-24}\ \mathrm{sec}$$

- Weak interaction: $\beta - ray$

$$time\ scale \gtrsim 10^{-8}\ \mathrm{sec}$$

Understanding weak interaction

- Fermi Effective formula

↓ Discovery of parity violation

- Correction

$$\left[\frac{g}{\sqrt{2}}\bar{u}_{\nu_\mu}\gamma_\rho\frac{1-\gamma_5}{2}u_\mu\right]\frac{-g^{\rho\sigma}+\frac{q^\rho q^\sigma}{M_W^2}}{q^2-M_W^2}\left[\frac{g}{\sqrt{2}}\bar{u}_e\gamma_\rho\frac{1-\gamma_5}{2}u_{\nu_e}\right]$$

$$[e\bar{u}_p\gamma_\mu u_p]\frac{-1}{q^2}[-e\bar{u}_e\gamma_\mu u_e]$$

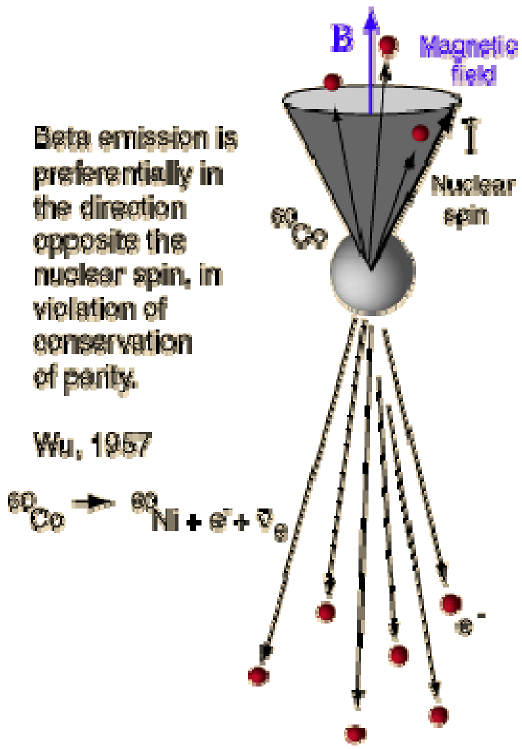
ED counterpart

$$M=-\frac{G_F}{\sqrt{2}}\left[\bar{u}_{\nu_\mu}\gamma_\rho\frac{1-\gamma_5}{2}u_\mu\right]\frac{g^{\rho\sigma}}{M_W^2}\left[\bar{u}_e\gamma_\rho\frac{1-\gamma_5}{2}u_{\nu_e}\right]$$

$$\frac{G_F}{\sqrt{2}}=\frac{g^2}{8M_W^2}$$

$$\gamma_5=i\gamma_0\gamma_1\gamma_2\gamma_3$$

Discovery of parity violation



In weak interaction

- P is violated.
- C is violated.
- T is violated.

- CP?
- T?

- CPT is anyway conserved!

Weak interaction and CP

$$W_\mu (J^\mu_{hadron} + J^\mu_{lepon})$$

$$J^\mu_{lepon} = \bar{e}\gamma^\mu(1-\gamma_5)\nu_e + \bar{\mu}\gamma^\mu(1-\gamma_5)\nu_\mu + \bar{\tau}\gamma^\mu(1-\gamma_5)\nu_\tau$$

$$J^\mu_{hadron} = (V_{CKM})_{ij}\bar{d}_i\gamma^\mu(1-\gamma_5)u_j$$

$$u_i = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \quad d_i = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Mass eigenstates $\neq$ Flavour (weak) eigenstates

$(V_{CKM})^\dagger \neq V_{CKM}$ → CP is violated

Establishing CP violation

$$K^0 = d\bar{s} \quad \bar{K}^0 = s\bar{d}$$

$$CP|K^0\rangle = -|\bar{K}^0\rangle \quad CP|\bar{K}^0\rangle = -|K^0\rangle$$

CP eigenstates

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|\bar{K}^0\rangle - |K^0\rangle)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|\bar{K}^0\rangle + |K^0\rangle)$$

$$CP|K_1\rangle = |K_1\rangle \quad CP|K_2\rangle = -|K_2\rangle$$

Mass eigenstates

$$|K_S\rangle \quad |K_L\rangle$$

$$K_S \to 2\pi^0, \pi^+\pi^- \qquad K_L \to 3\pi^0, \pi^+\pi^-\pi^0$$

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\bar{\epsilon}|^2)}}[|K_2\rangle + \bar{\epsilon}|K_1\rangle]$$

$$|K_S\rangle = \frac{1}{\sqrt{2(1+|\bar{\epsilon}|^2)}}[|K_1\rangle + \bar{\epsilon}|K_2\rangle]$$

$$Br(K_L \to \pi^+\pi^-) = 1.97 \times 10^{-3}$$

$$Br(K_L \to \pi^0\pi^0) = 8.64 \times 10^{-4}$$

From weak to electroweak

$$\frac{G_F}{\sqrt{2}} J_\mu^{hadron} J^\mu_{hadron}$$

$$\sigma \propto G_F^2 E^2 \qquad E \to \infty \quad \sigma \to \infty$$

Introduction of W

New problem

$$\sigma \propto \frac{E^2}{(E^2 + m_W^2)^2} \qquad \sigma(e^- e^+ \to W^- W^+) \to G_F^2 E^2$$

$$SU(2) \times U(1)$$

SU(2) Yang-Mills

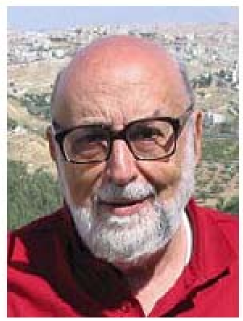
$$W^i \cdot \sigma^i/2 \qquad D_\mu = \partial_\mu - igW^i \cdot \sigma^i/2$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

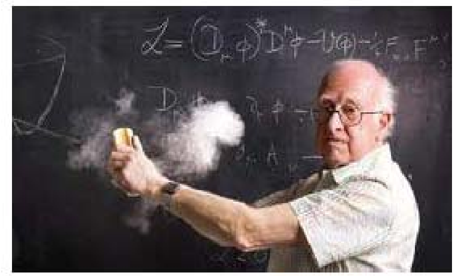
W is massless $\quad$ ~~$m_W^2 W^\mu W_\mu$~~

Short range?!

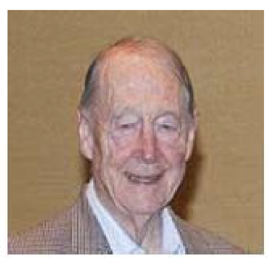
Brout



Englert



Guralnik



Kibble

Hagen

Higgs mechanism

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

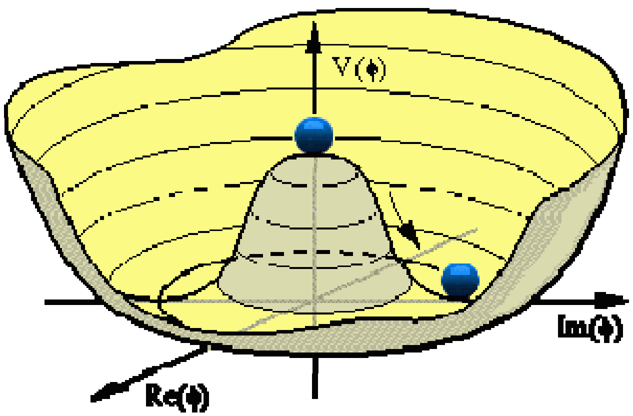
$$\partial^\mu H \cdot \partial_\mu H \rightarrow D^\mu H \cdot D_\mu H$$

$$SU(2) \times U(1)$$

$$D_\mu = \partial_\mu - ig' \frac{Y}{2} B_\mu - ig \sum_i \frac{\sigma^i W^i}{2}$$

Higgs potential

$$V(H) = -m^2|H|^2 + \lambda|H|^4$$



$$\langle H \rangle = \sqrt{\frac{m^2}{\lambda}}$$

Gauge bosons

$$W_\mu^\pm = \frac{W_\mu^1 \mp W_\mu^2}{\sqrt{2}}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$M_W = \frac{gv}{2} \quad M_Z = \frac{v}{2}\sqrt{g^2 + g'^2}$$

Photon remains massless

Matter fields

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

$$e_R \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad u_R, d_R \quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$D_\mu = \partial_\mu - ig'\frac{Y}{2}B_\mu - ig\sum_i \frac{\sigma^i W^i}{2}$$

$$Q = T^3 + \frac{Y}{2}$$

Fermions of the SM

Leptons

Quarks

1st generation

$$\begin{bmatrix} \nu_{eL} \\ e^-_L \end{bmatrix} \quad e^-_R \qquad\qquad \begin{bmatrix} u_L \\ d_L \end{bmatrix} \quad u_R \quad d_R$$

2nd generation

$$\begin{bmatrix} \nu_{\mu L} \\ \mu^-_L \end{bmatrix} \quad \mu^-_R \qquad\qquad \begin{bmatrix} c_L \\ s_L \end{bmatrix} \quad c_R \quad s_R$$

3rd generation

$$\begin{bmatrix} \nu_{\tau L} \\ \tau^-_L \end{bmatrix} \quad \tau^-_R \qquad\qquad \begin{bmatrix} t_L \\ b_L \end{bmatrix} \quad t_R \quad b_R$$

Notation:

$$L_\alpha \equiv \begin{bmatrix} \nu_{\alpha L} \\ \ell^-_{\alpha L} \end{bmatrix} \qquad \ell^-_{\alpha R}$$

Lagrangian of the leptons

EM: $$-eA_\mu \sum_\alpha (\bar{\ell}_{\alpha L}\gamma^\mu \ell_{\alpha L} + \bar{\ell}_{\alpha R}\gamma^\mu \ell_{\alpha R})$$

NC: $$\frac{eZ_\mu}{\sin\theta_w \cos\theta_w}\left[\sum_\alpha (\frac{\bar{\nu}_{\alpha L}\gamma^\mu \nu_{\alpha L}}{2} + \frac{\bar{\ell}_{\alpha L}(2\sin^2\theta_w - 1)\gamma^\mu \ell_{\alpha L}}{2} + \sin^2\theta_w \bar{\ell}_{\alpha R}\gamma^\mu \ell_{\alpha R})\right]$$

CC: $$\frac{e}{\sqrt{2}\sin\theta_w \cos\theta_w}\left[\sum_\alpha (\bar{\nu}_{\alpha L}\gamma^\mu \ell_{\alpha L} W_\mu^+ + \bar{\ell}_{\alpha L}\gamma^\mu \nu_{\alpha L} W_\mu^-)\right]$$

Weak interaction

- W boson is not massless!

- Gauge breaking mechanism

Spontaneous symmetry breaking

Scalar field

$\langle H \rangle \neq 0$ → Gauge masses

What gauge

- $SU(3) \times SU(2) \times U(1)$

$SU(3) \times U(1)$

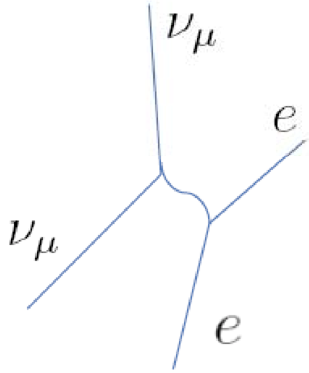
New gauge boson

Charged Current (CC) interactions: W^+ W^-

Neutral Current (NC) interactions: Z

Gargamelle experiment

1973 experiment at CERN



Direct discovery

- UA1 at SPS of CERN (1983)

- Carlo Rubbia and Simon van der Meer

Standard model Higgs

- SU(2) × U(1)

- Higgs is a doublet of SU(2)

TREE LEVEL
APPROXIMATION

$$\frac{m_W}{m_Z} = \frac{g}{\sqrt{g^2 + g'^2}}$$

Fermion masses

$$\bar{e}e = e_R^\dagger e_L + e_L^\dagger e_R$$

$$SU(2) \times U(1)$$

$$\lambda_e \bar{e}_R H^\dagger \cdot L + \lambda_d \bar{d}_R H^\dagger \cdot Q + \lambda_u \bar{u}_R H^T (i\sigma_2) Q$$

$$m_f = \lambda_f \langle H \rangle \quad \Gamma(H \to f\bar{f}) \propto m_f^2$$

Accidental symmetries of SM

- Lepton number U(1)
- Baryon number U(1)
- B-L
- Global → Gauge

GUTs

- 1) Gravity
- 2) Electromagnetism
- 3) Weak nuclear force
- 4) Strong nuclear force

SM

$$SU(3) \times SU(2) \times U(1)$$

SU(5) SO(10)

Proton decay!

Horizontal symmetry

- Predicting neutrino parameters
- Permutation symmetry
- A4 symmetry
- ….

Summary

- Symmetries helped to establish SM of particles

- Symmetries are guidelines for new physics beyond SM.