Cosmological Structure Formation

Maps and a brief history of time

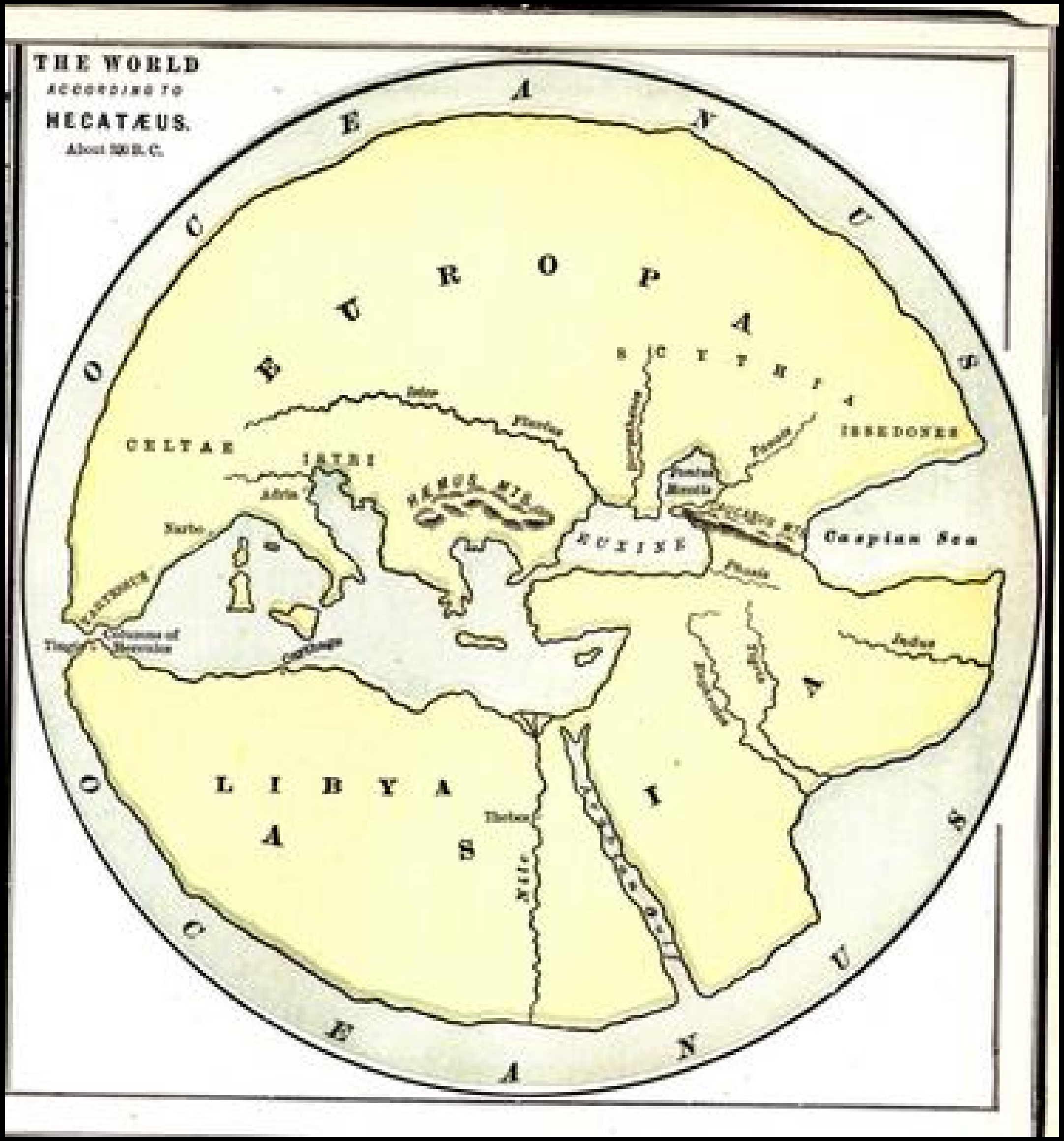
Spatial statistics

Baryon Acoustic Oscillations

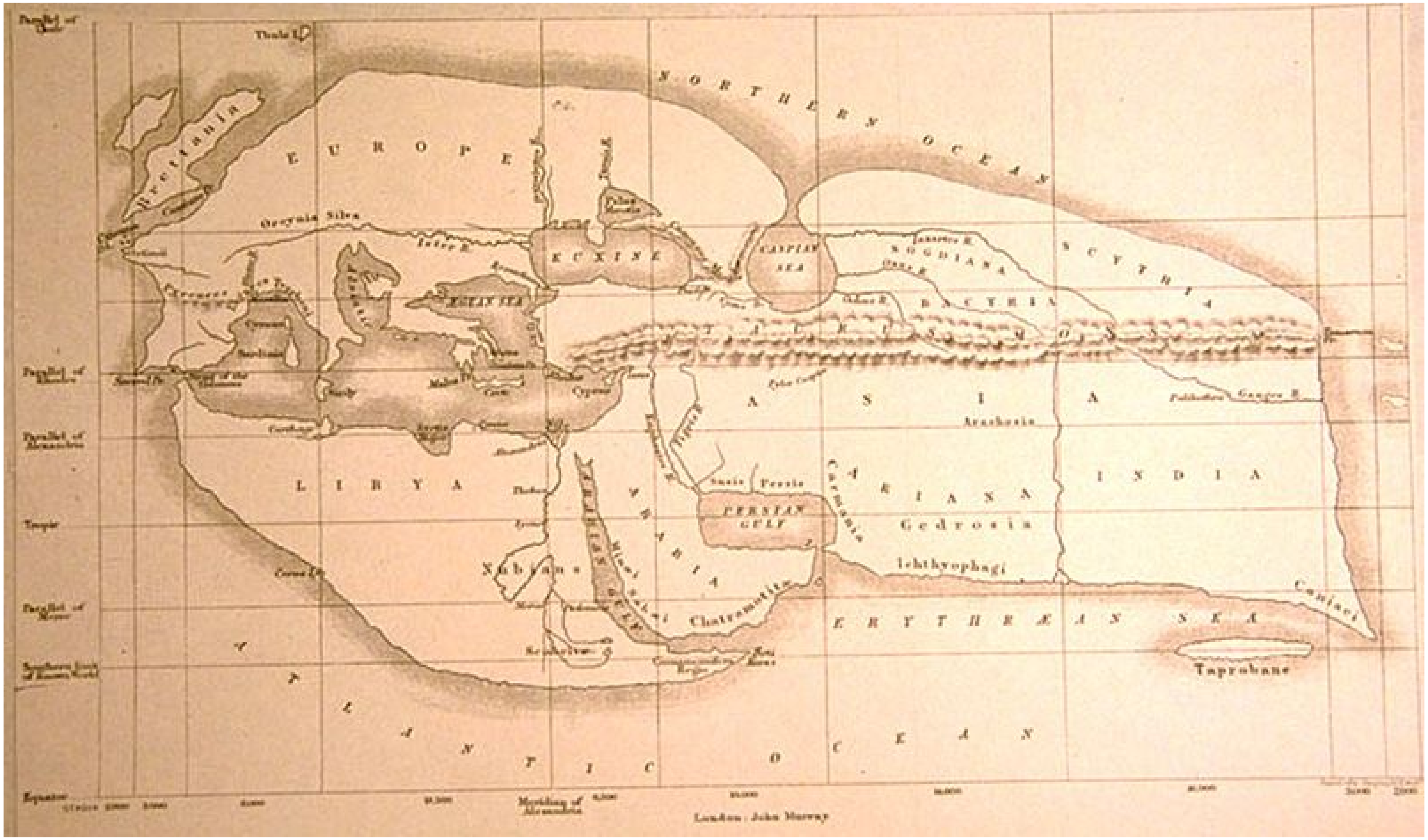
The transfer function

Linear theory

- Map what is known
- Assume simple model for unknown
- 600 BC: Earth is flat circle atop cylinder



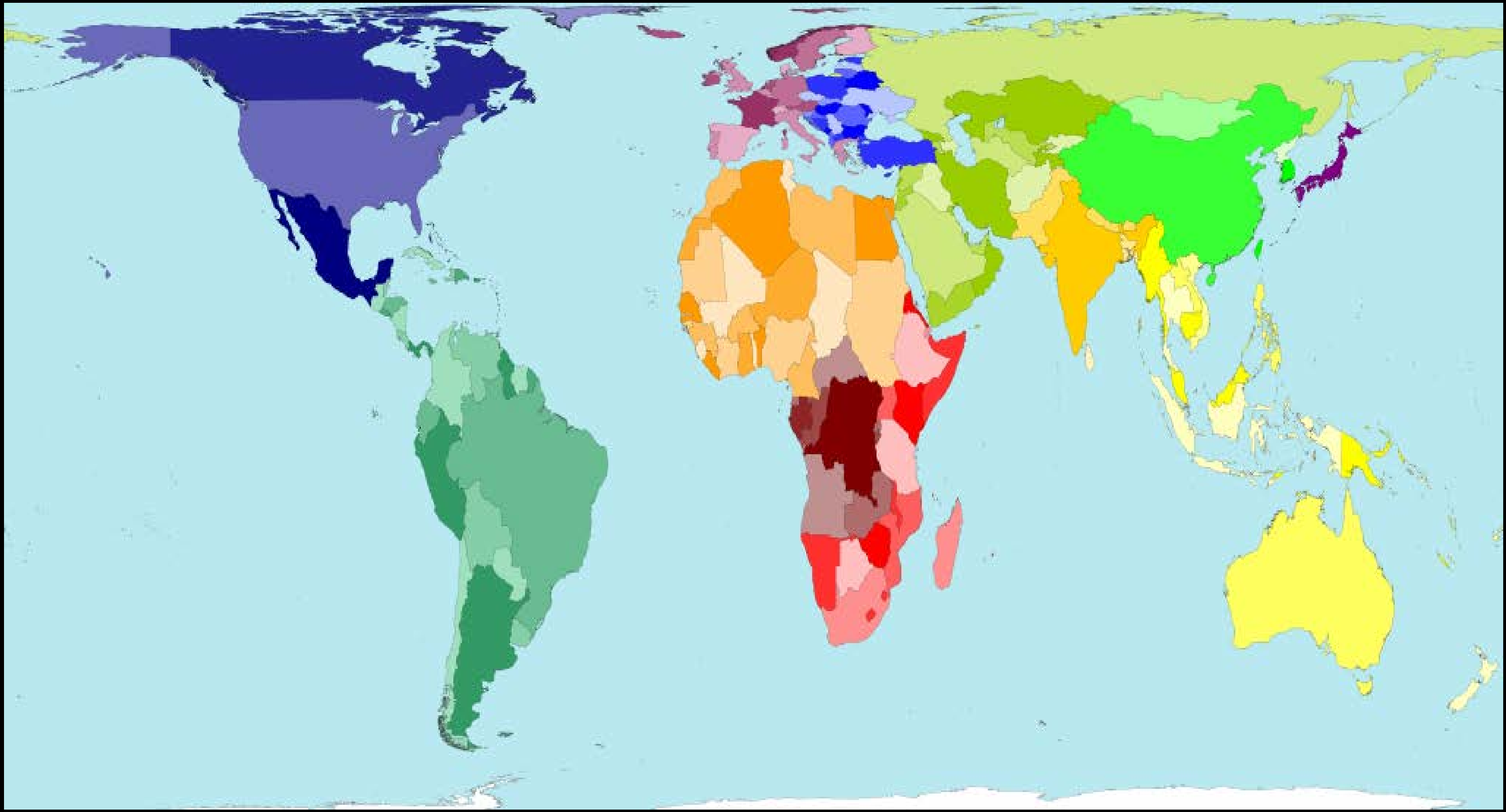
500 BC: Earth is flat, but not on cylinder surrounded by water! (Note similarity to human skull…)



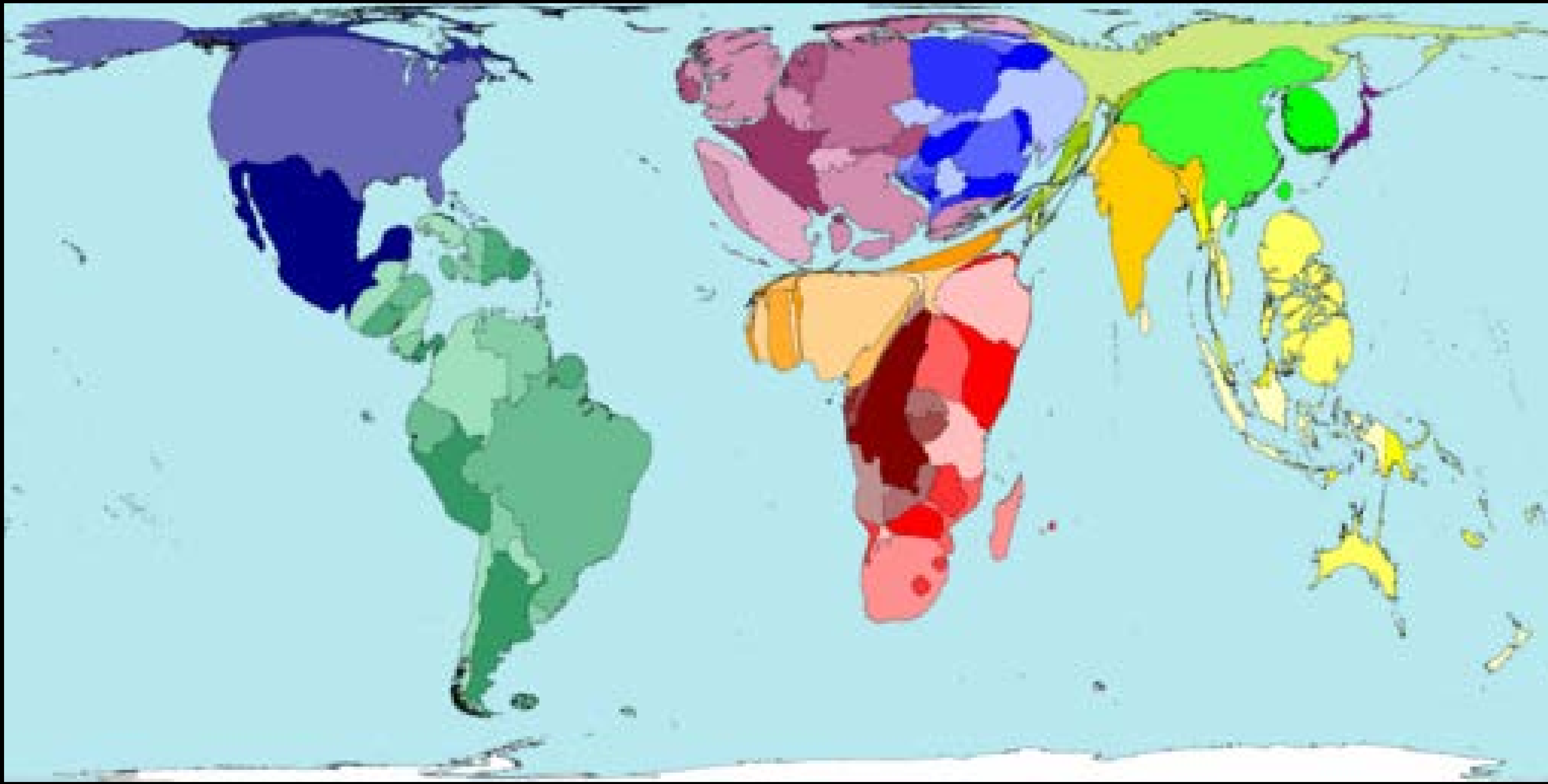
Alexander the Great's travels mean Asia larger than previously thought …

Known inhabited world much smaller than expected if radius computed by Eratosthenes correct

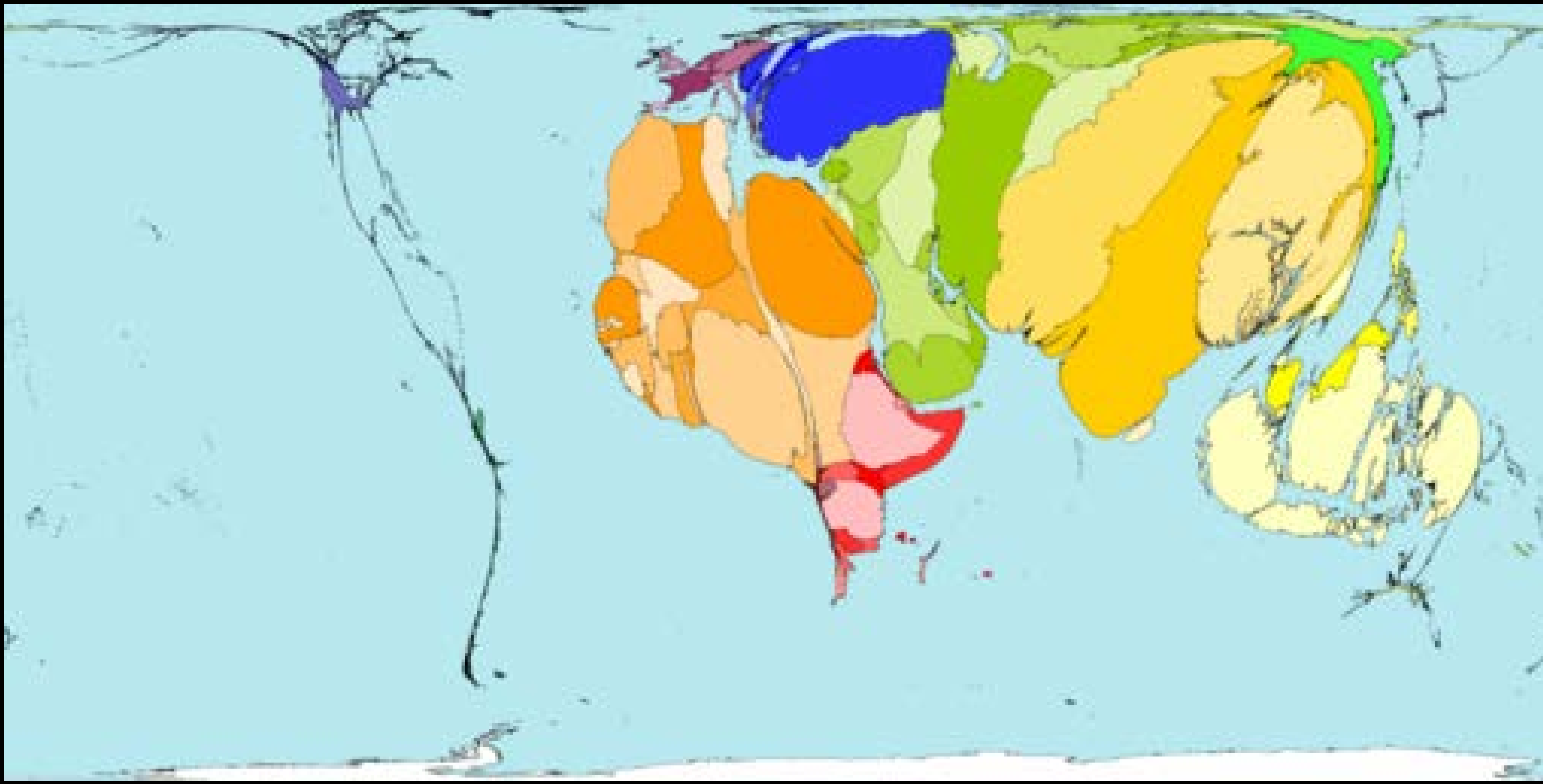
150 BC: Crates postulates three other identical landmasses, symmetrically located, separated by water



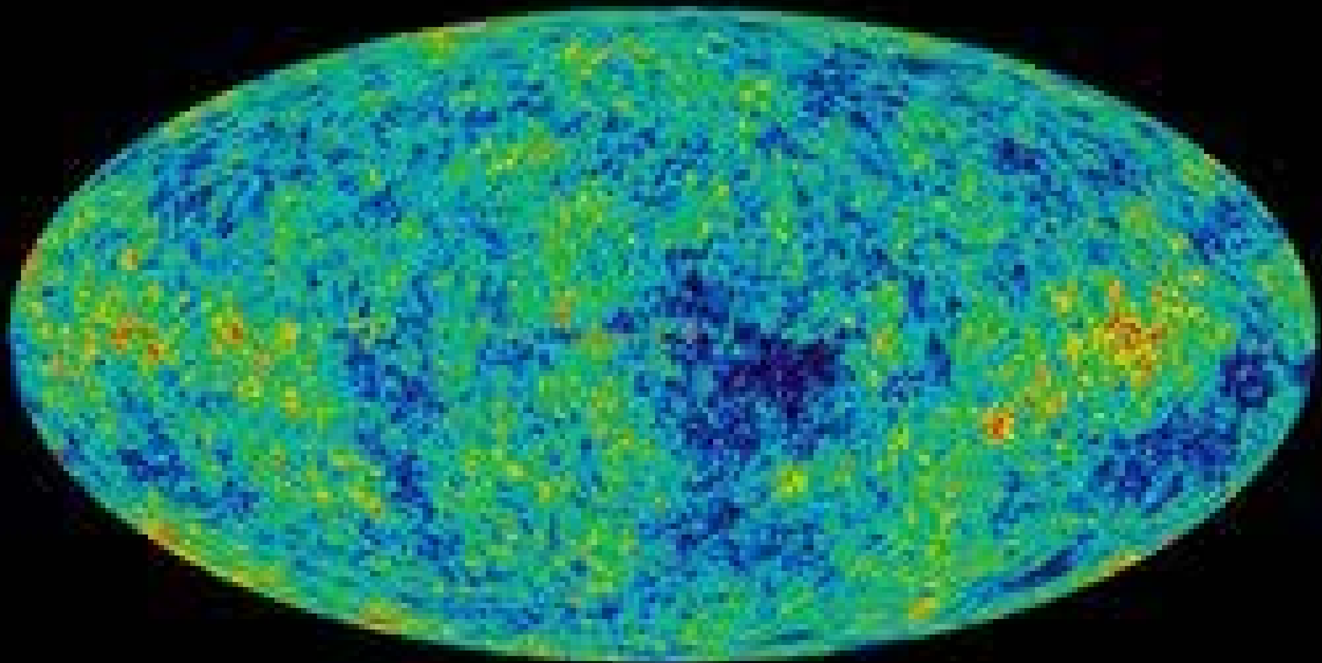
2007 AD: www.worldmapper.org

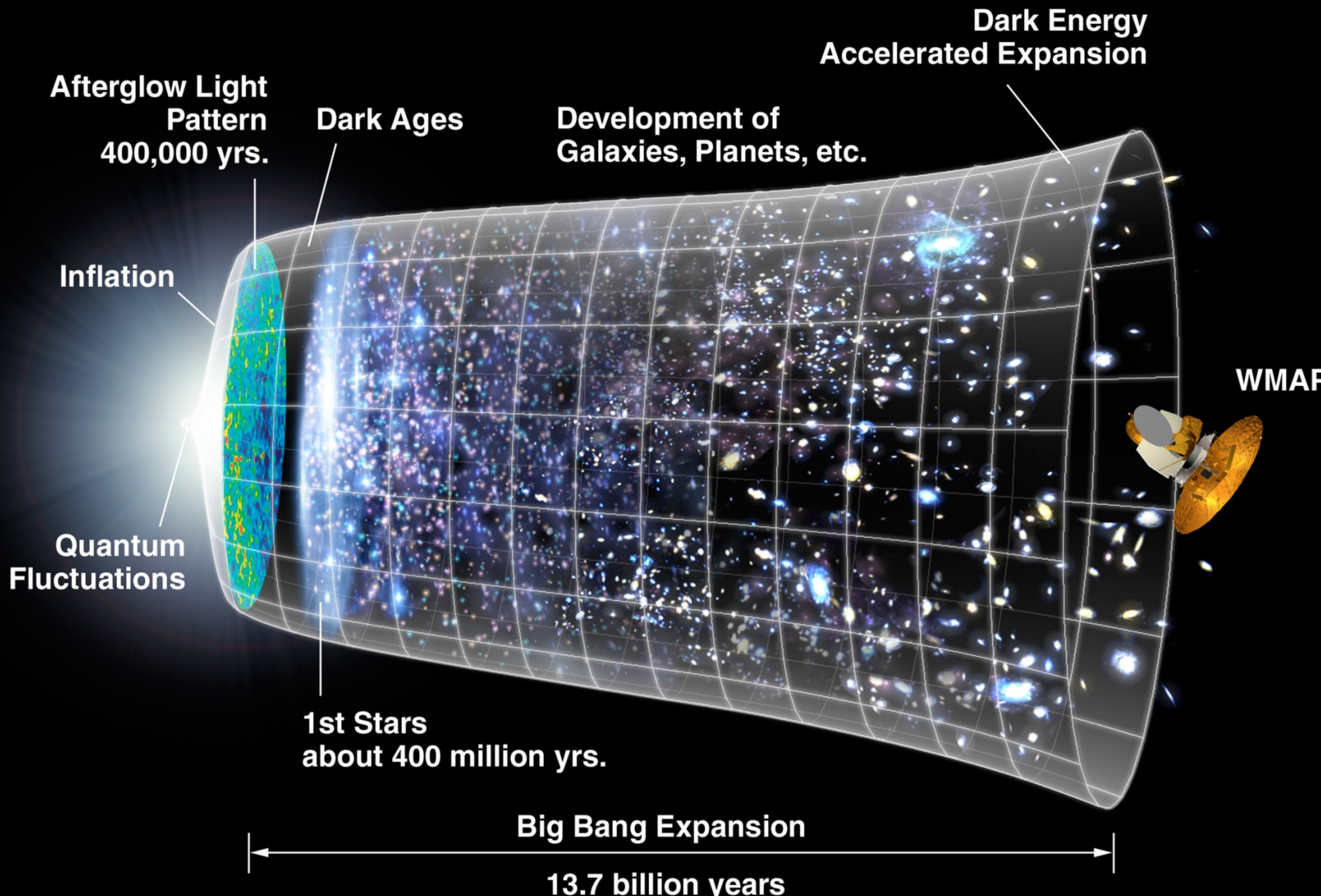


Christians



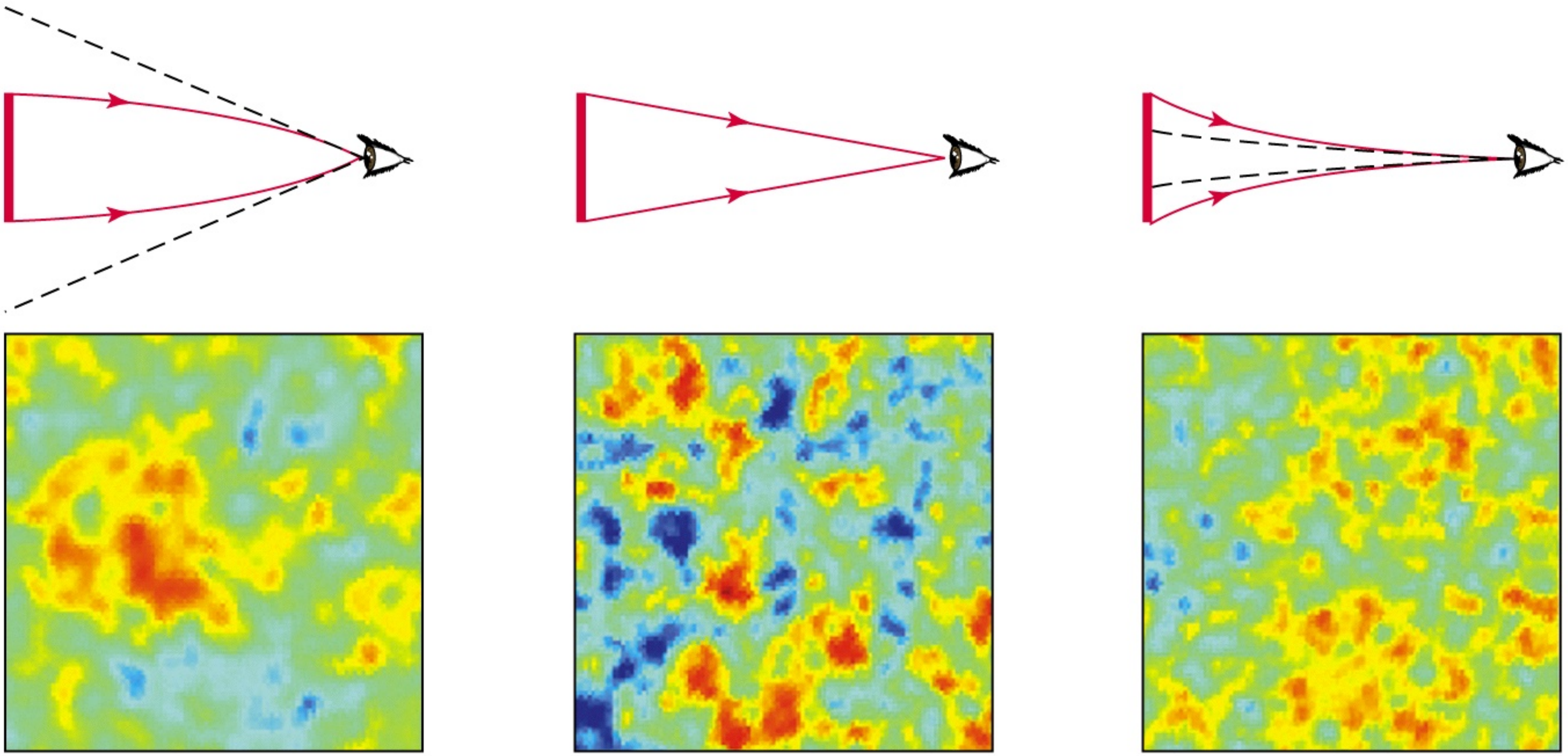
Muslims




Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Geometrical Test of curvature:

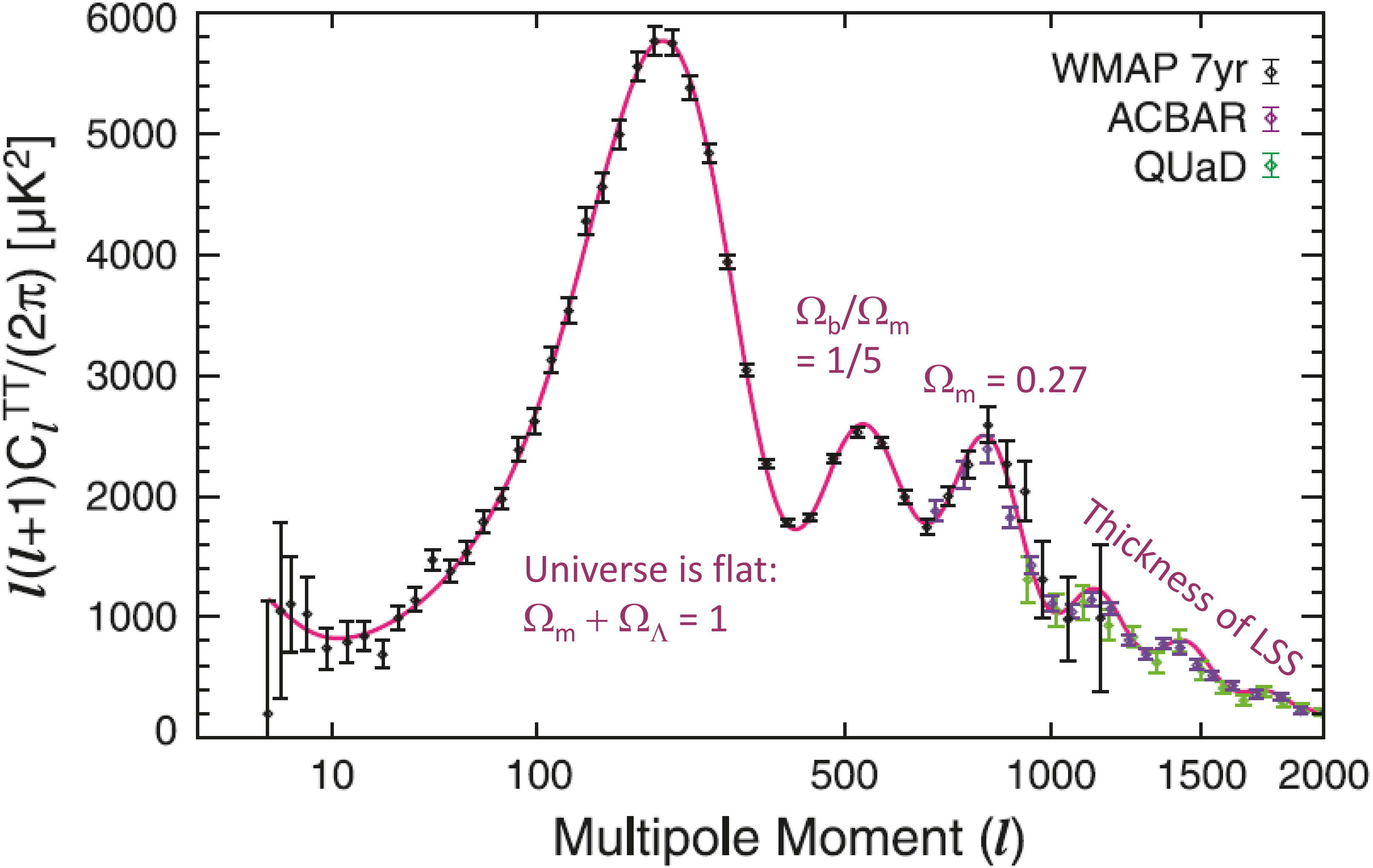
Standard Rod = Hubble volume at Last Scattering



a If universe is closed, "hot spots" appear larger than actual size

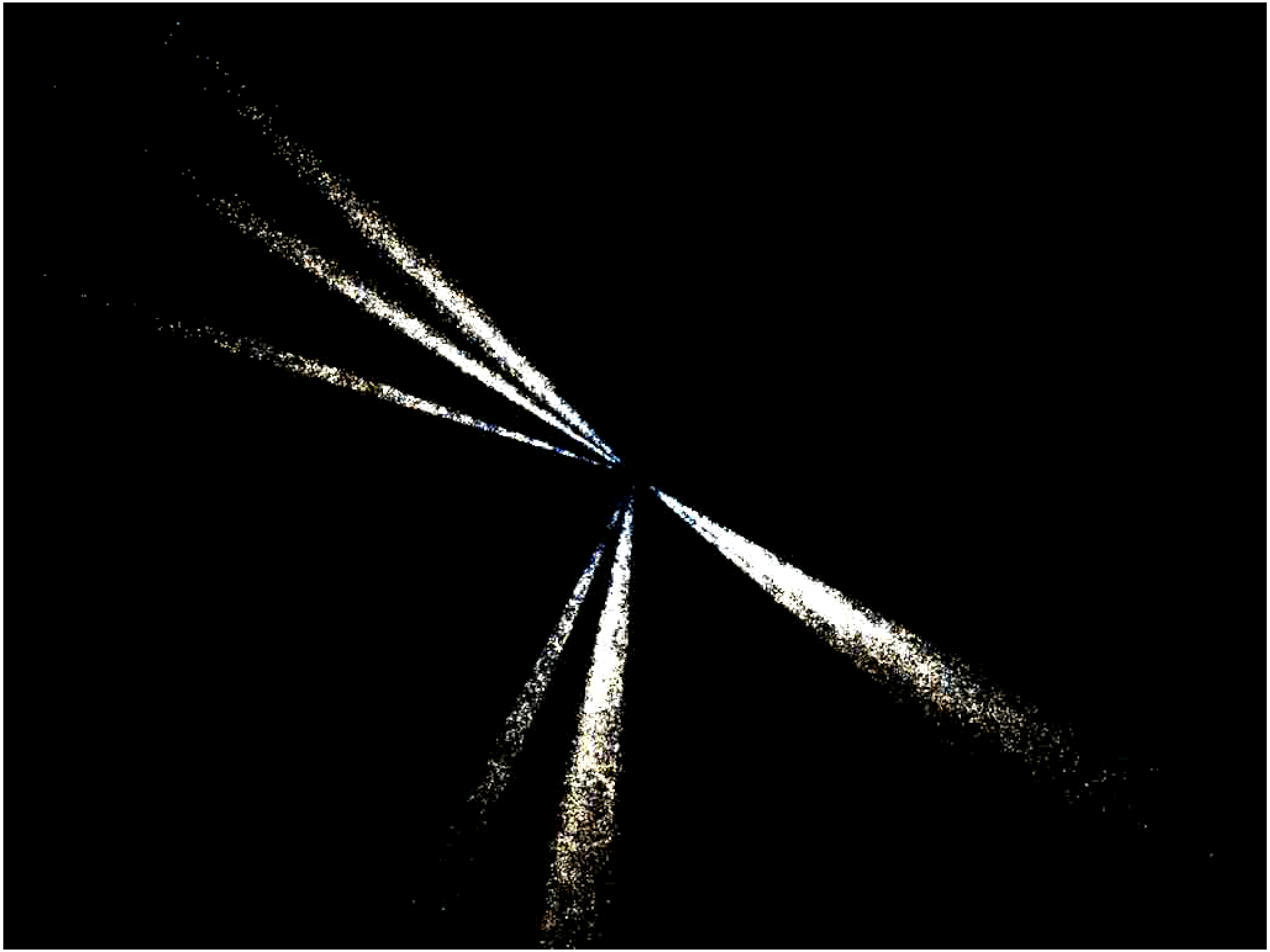
b If universe is flat, "hot spots" appear actual size

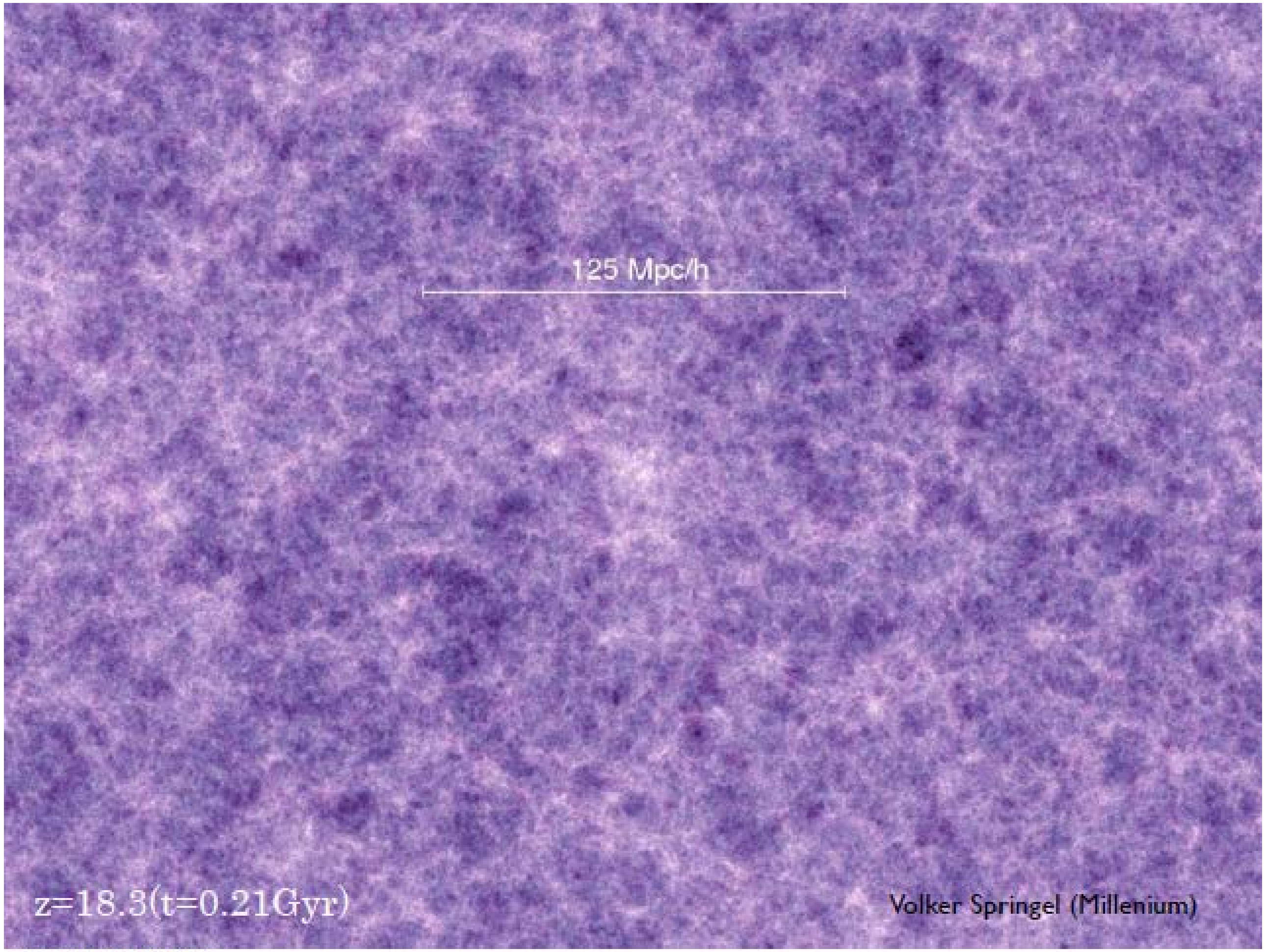
c If universe is open, "hot spots" appear smaller than actual size

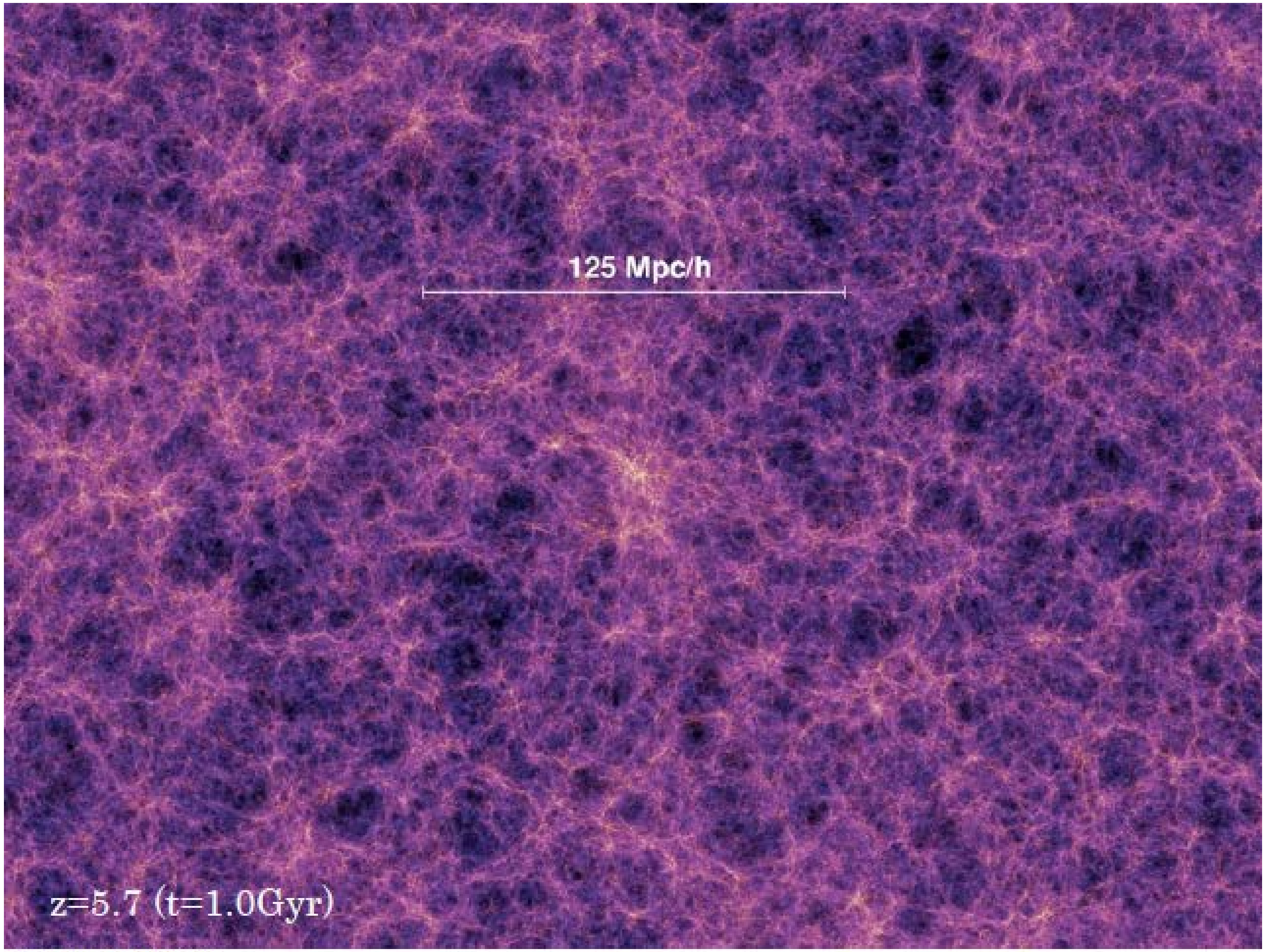


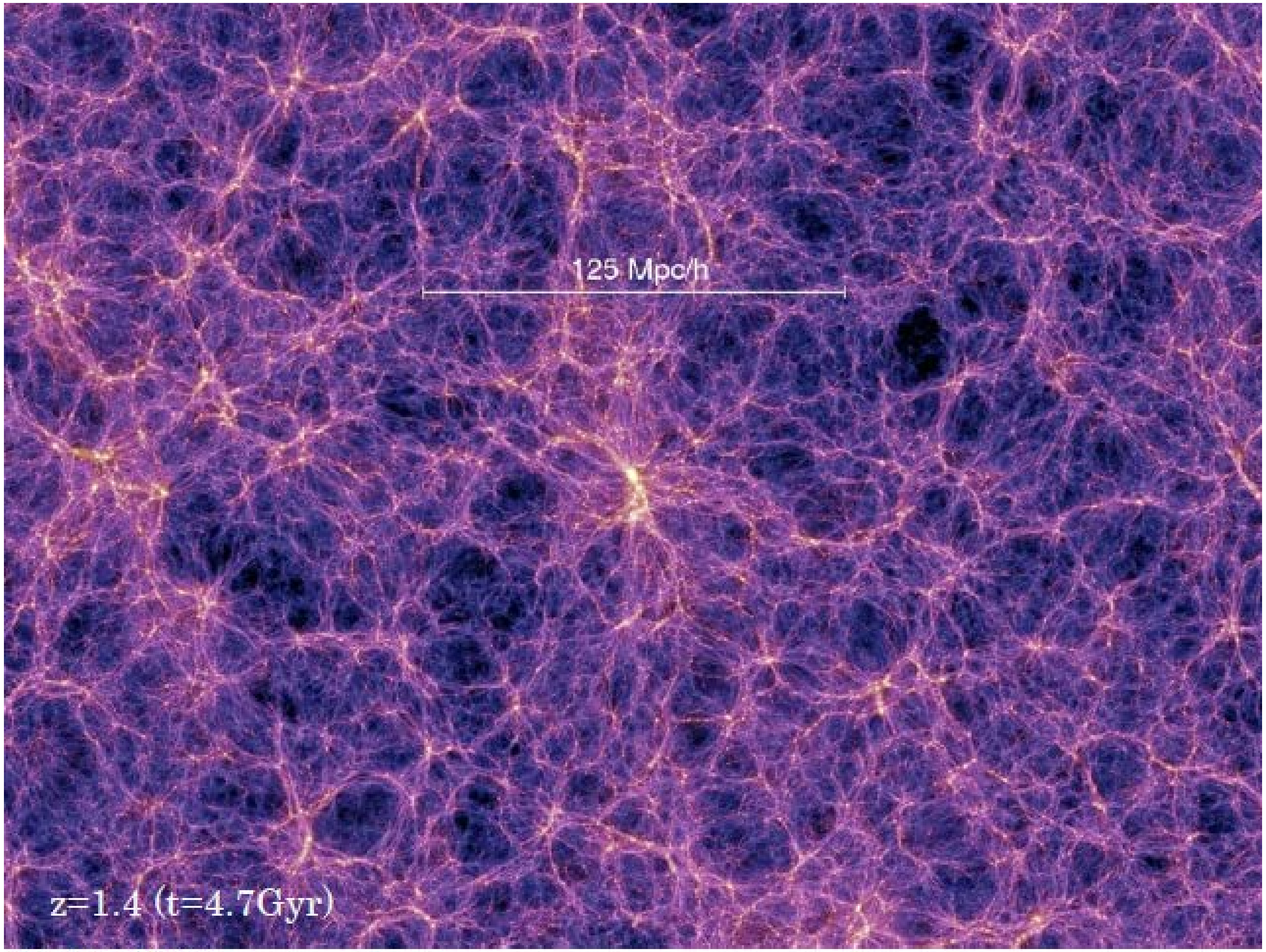
Adiabatic (same fluctuations in all components)

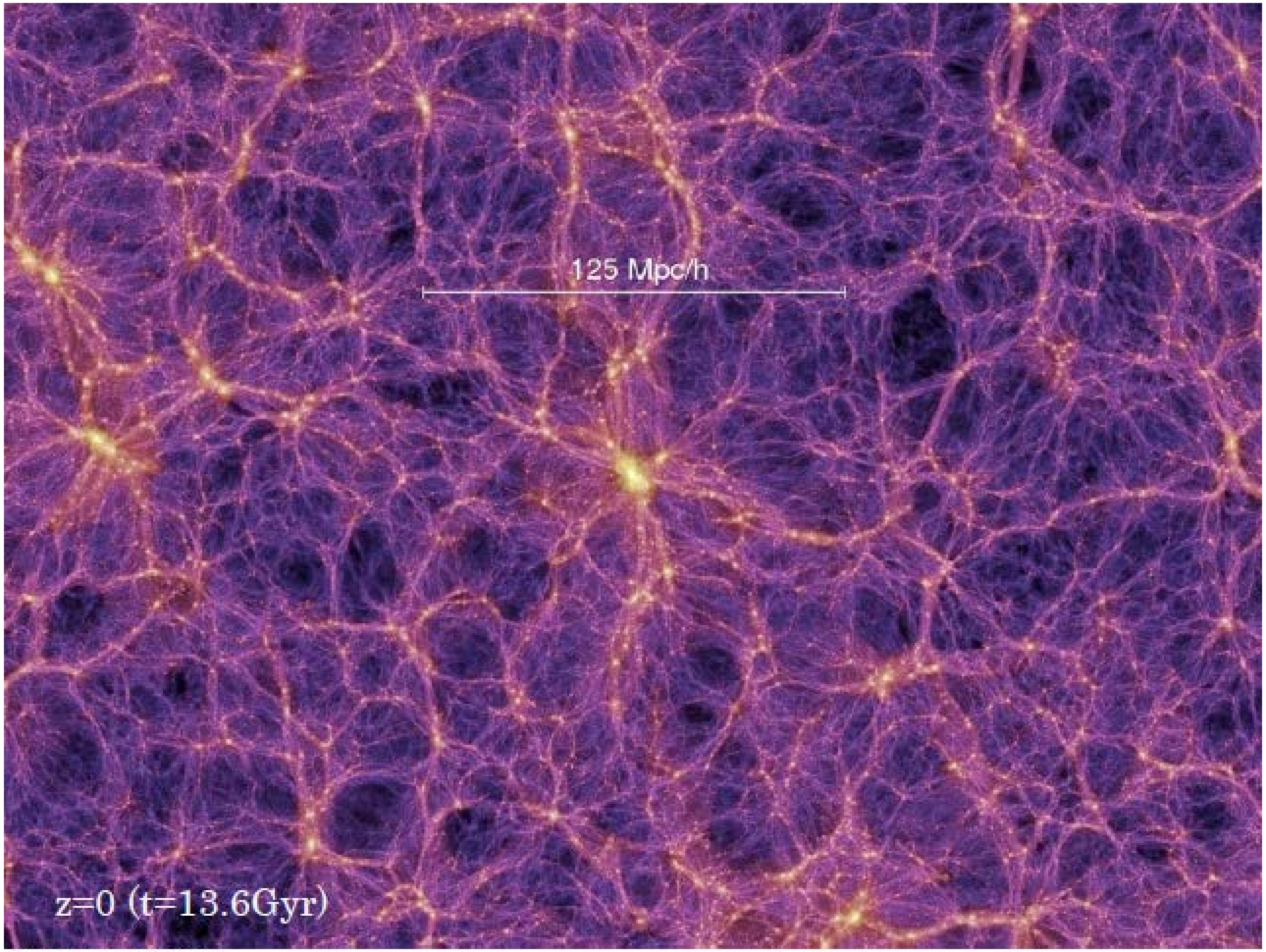
~~vs Iso-curvature (net fluctuation is zero)~~

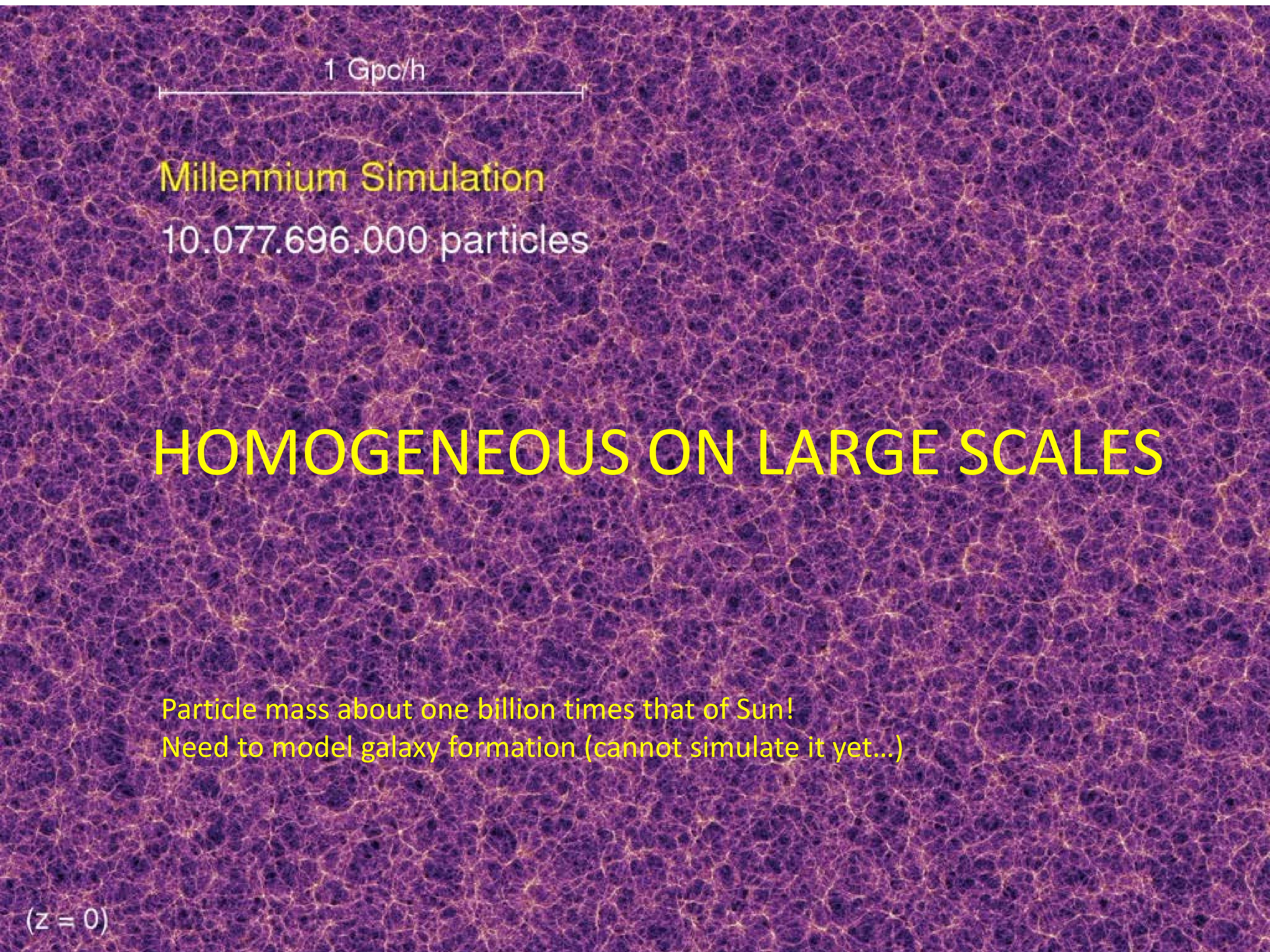



125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)


125 Mpc/h
z=5.7 (t=1.0Gyr)


125 Mpc/h
z=1.4 (t=4.7Gyr)


125 Mpc/h
z=0 (t=13.6Gyr)



HOMOGENEOUS ON LARGE SCALES

Particle mass about one billion times that of Sun!
Need to model galaxy formation (cannot simulate it yet...)

Cold Dark Matter

- **Cold:** speeds are non-relativistic
  - To illustrate, 1000 km/s ×10Gyr ≈ $10\mathrm{Mpc}$
  - From z~1000 to present, nothing (except photons!) travels more than ~ 10Mpc
- **Dark:** no idea (yet) when/where the stars light-up
- **Matter:** gravity the dominant interaction

Late-time field retains memory of initial conditions

STATISTICS OF RANDOM FIELDS

- Section 3.2-3.4 (p.32-38) in PT review (Bernardeau et al. 2002)
- Section 2.1 in Halo Model review (Cooray-Sheth 2002)

But first … some background

Continuous probability distributions

- $P(<x) = \int^x dx\, p(x)$
- m^{th} moment: $\langle x^m \rangle = \int dx\, p(x)\, x^m$
- Fourier transform: $F(t) = \int dx\, p(x) \exp(-itx)$
  - sometimes called Characteristic function
  - $d^mF/dt^m \sim i^m \langle x^m \rangle$, so F(t) is equivalent to knowledge of all moments
- If x>0, Laplace transform more useful:
- $L(t) = \int dx\, p(x) \exp(-tx)$

Distribution of sum of n independent random variates

- $p_2(s) = \int dx\, p(x) \int dy\, p(y)\ \delta_D(x+y = s)$
  $= \int dx\, p(x)\, p(s-x)$
- $F_2(t) = \int ds\, \exp(-its) \int dx\, p(x)\, p(s-x)$
  $= \int ds \int dx\, p(x) \exp(-itx)\, p(s-x) \exp[-it(s-x)]$
  $= F_1(t)\, F_1(t)$
- $F_n(t) = [F_1(t)]^n$

= Convolve PDFs = Multiply CFs

Gaussian PDF

- $p(x) = \exp[-(x-\mu)^2/2\sigma^2]/\sigma\sqrt{2\pi}$
- $F(t) = \exp(it\mu)\ \exp(-\ t^2\ \sigma^2)$
- $F_n(t) = \exp(it\ n\mu)\ \exp(-\ t^2\ n\sigma^2)$
- Distribution of sum of n Gaussians is Gaussian with mean $n\mu$ and variance $n\sigma^2$
- In general, PDFs are not ‘scale invariant’

Gaussian field

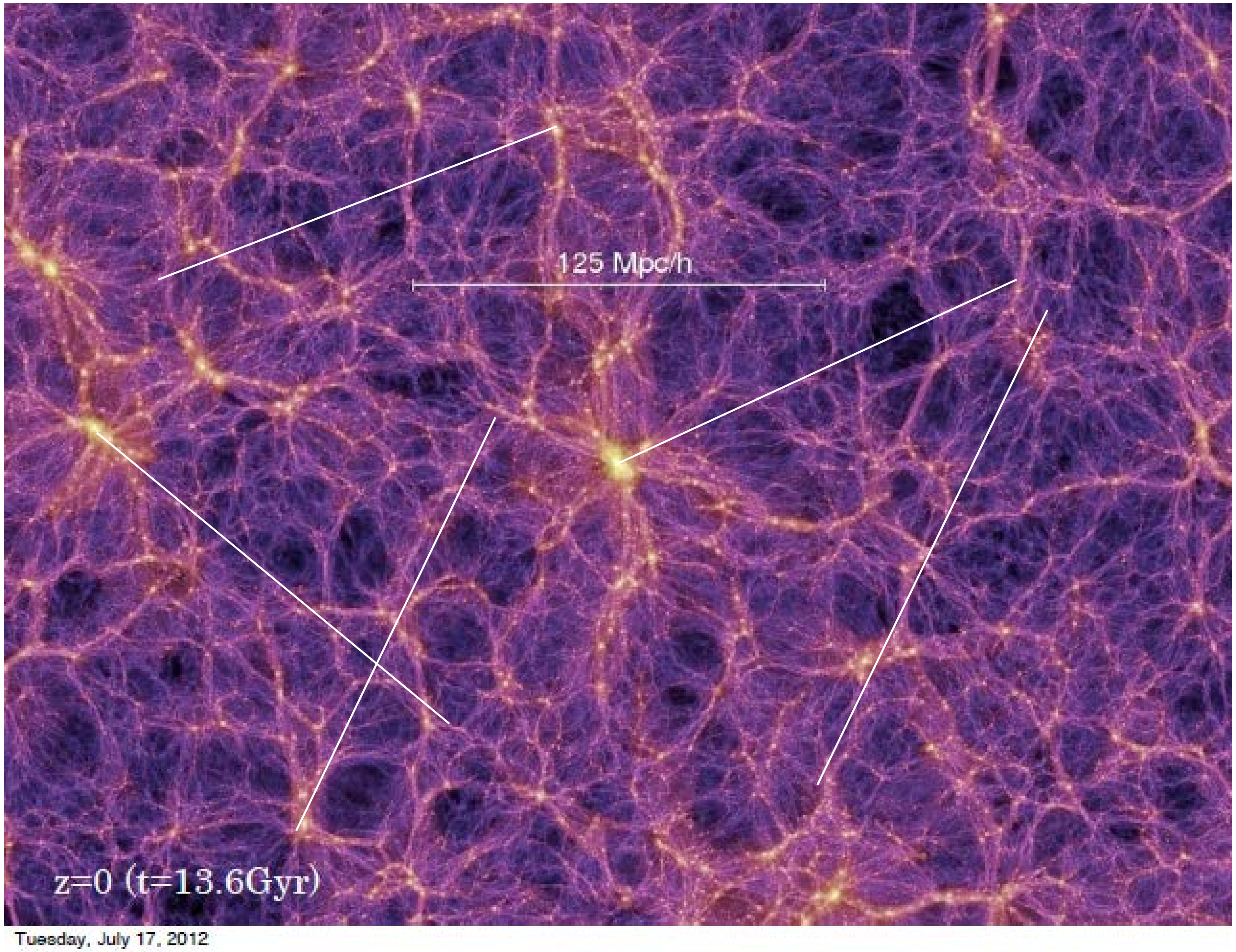
- $p(\mathbf{x}) = \exp(-\,\mathbf{x}^T\,\mathbf{C}^{-1}\,\mathbf{x}/2)/\,(2\pi)^{n/2}\sqrt{\mathrm{Det}[\mathbf{C}]}$
  where $\mathbf{x} = (x_1, \ldots\, x_n)$ with $x_1 = x(r_1) - \langle x(r_1)\rangle$
  and $\mathbf{C}_{ij} = \langle x_i\, x_j\rangle$

- HW: Show that $F(\mathbf{t}) = \exp(i\mathbf{m}^T\mathbf{t} - \mathbf{t}^T\mathbf{C}\mathbf{t}/2)$
  where $\mathbf{m} = (\langle x(r_1)\rangle, \ldots, \langle x(r_n)\rangle)$

- For Gaussian field **C** is diagonal, but $\mathbf{C}^{-1}$ can be complicated. Lesson: **C** may be much simpler (e.g. approximately band diagonal) than $\mathbf{C}^{-1}$.

Fourier transform exp(ikx) useful

- Convolutions become products
  - Smoothing on scale R: $\delta_R(\mathbf{x}) \rightarrow \delta(\mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}}\, W(kR)$
- Each derivative brings down a power of ik
  - Can transform differential equations into algebraic equations
- Integral brings 1/ik
  - divergence at k=0 ~ constant of integration



Quantify clustering by number of pairs compared to random (unclustered) distribution, triples compared to triangles (of same shape) in unclustered distribution, etc.

2pt spatial statistics

- $dP = \langle n_1 \rangle\, dV_1 \langle n_2 \rangle\, dV_2\, [1 + \xi(\mathbf{r}_1, \mathbf{r}_2)]$
  $= \langle n \rangle^2\, dV_1\, dV_2\, [1 + \xi(\mathbf{r}_1 - \mathbf{r}_2)]$ homogeneity
  $= \langle n \rangle^2\, dV_1\, dV_2\, [1 + \xi(|\mathbf{r}_1 - \mathbf{r}_2|)]$ isotropy

Define: $\delta(\mathbf{r}) = [n(\mathbf{r}) - \langle n \rangle]/\langle n \rangle$

Then: $\xi(r) = \langle \delta(\mathbf{x})\, \delta(\mathbf{x} + \mathbf{r}) \rangle$ ξ is the correlation function

Estimator: $\langle (D_1 - R_1)/R_1\, (D_2 - R_2)/R_2 \rangle \sim (DD - 2DR + RR)/RR$

And FT is: $\langle \delta(\mathbf{k}_1)\, \delta(\mathbf{k}_2) \rangle = (2\pi)^3\, \delta_D(\mathbf{k}_1 + \mathbf{k}_2)\, P(|\mathbf{k}_1|)$

P(k) is the power spectrum

Vol. 21, No. 3 Publications of the Astronomical Society of Japan 1969

The Correlation Function for the Distribution of Galaxies

Hiroo TOTSUJI and Taro KIHARA

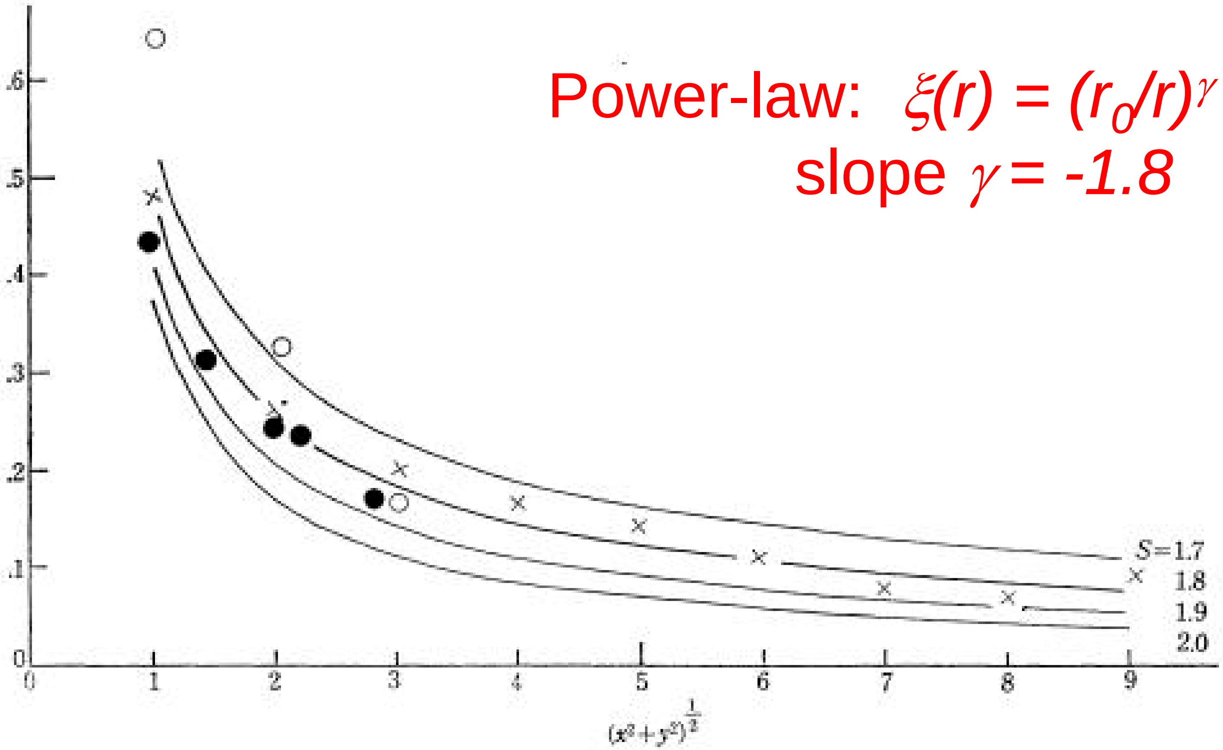
Department of Physics, Faculty of Science, University of Tokyo

(Received May 15, 1969; revised June 26, 1969)

Abstract

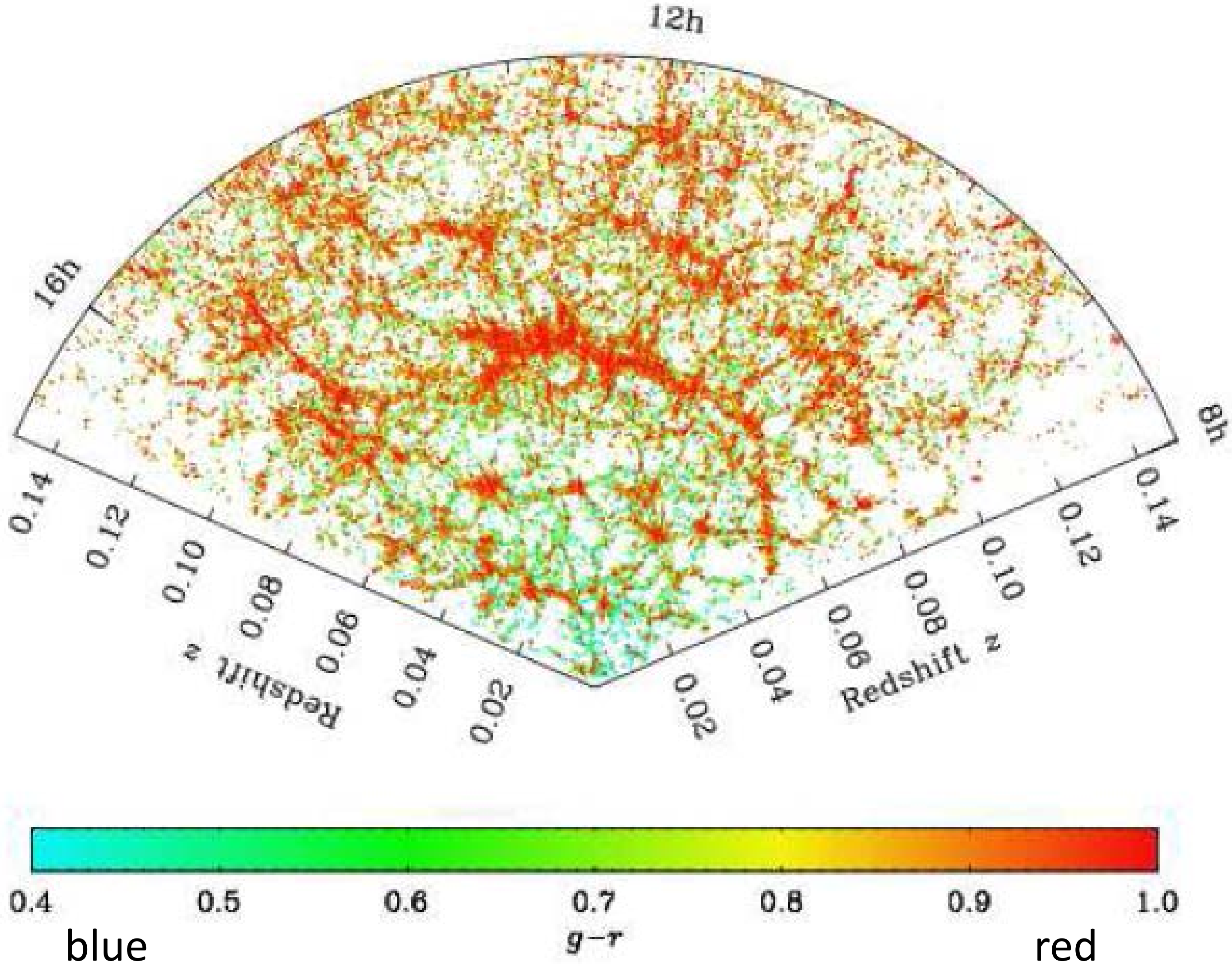
The correlation function for the spatial distribution of galaxies in the universe is determined to be $(r_0/r)^{1.8}$, r being the distance between galaxies. The characteristic length r_0 is 4.7 Mpc. This determination is based on the distribution of galaxies brighter than the apparent magnitude 19 counted by SHANE and WIRTANEN (1967). The reason why the correlation function has the form of inverse power of r is that the universe is in a state of "neutral" stability.

$$\frac{\text{Number of data pairs with separation } r}{\text{Number of random pairs with separation } r} \qquad \frac{DD(r)}{RR(r)} = 1 + \xi(r)$$



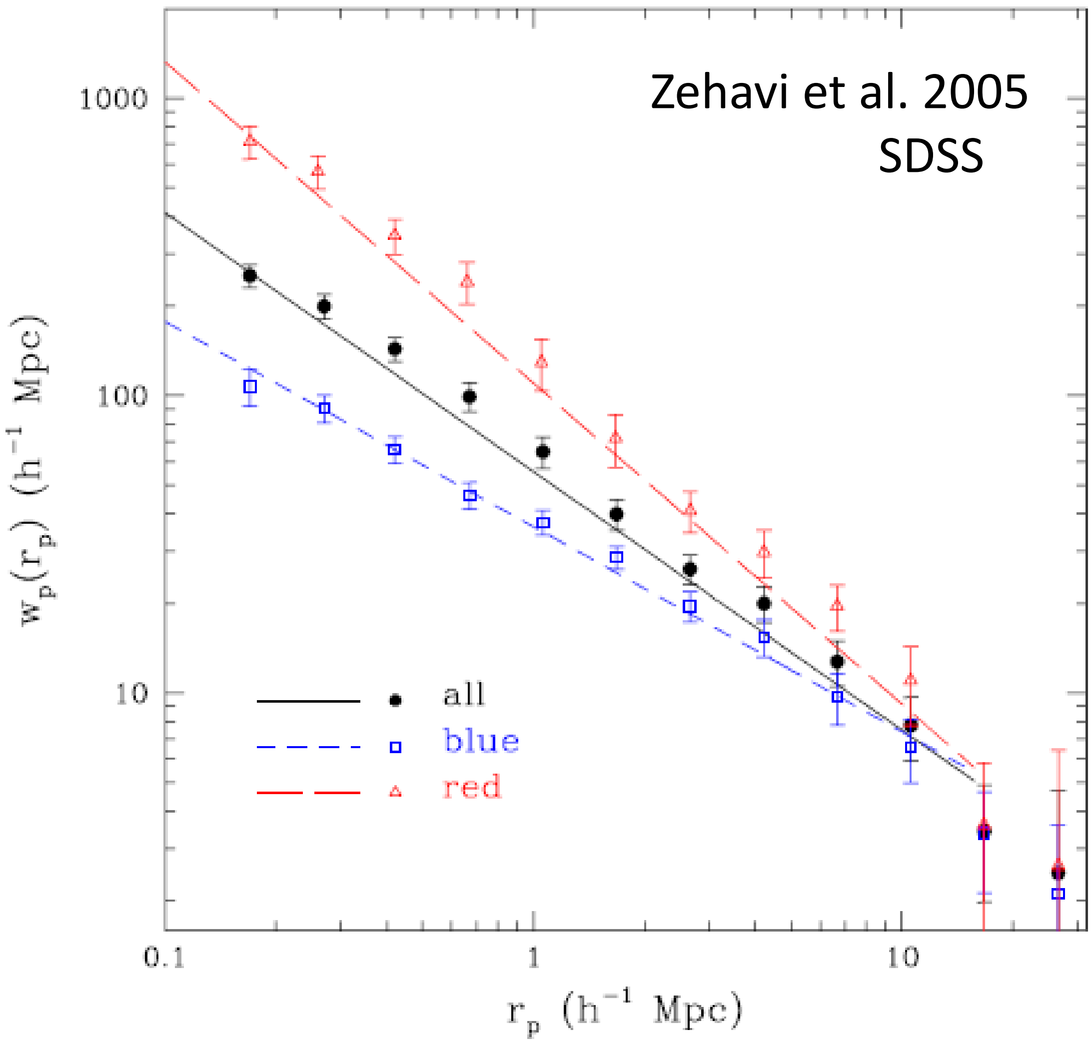
Power-law: $\xi(r) = (r_0/r)^\gamma$
slope $\gamma = -1.8$

FIG. 2. Comparison of the empirical and theoretical values of $\dfrac{\langle\{N_1-\langle N\rangle\}\{N_2-\langle N\rangle\}\rangle}{\langle\{N-\langle N\rangle\}^2\rangle-\langle N\rangle}$. The filled circles indicate the empirical values obtained by the authors, and the open circles and crosses by NEYMAN et al.; the unit solid angle is 1°×1° for the circles and 10′×10′ for the crosses. The curves are theoretical values for s=1.7, 1.8, 1.9, and 2.0.


12h
16h
8h
0.14
0.12
0.10
0.08
0.06
0.04
0.02
Redshift z
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Redshift z
0.4
0.5
0.6
0.7
0.8
0.9
1.0
g−r
blue
red

Galaxy clustering depends on galaxy type: luminosity, color, etc.

(Final lectures use Halo Model to describe this.)



2pt spatial statistics

- $dP = \langle n_1 \rangle\, dV_1 \langle n_2 \rangle\, dV_2\, [1 + \xi(\mathbf{r}_1, \mathbf{r}_2)]$
  $= \langle n \rangle^2\, dV_1\, dV_2\, [1 + \xi(\mathbf{r}_1 - \mathbf{r}_2)]$ homogeneity
  $= \langle n \rangle^2\, dV_1\, dV_2\, [1 + \xi(|\mathbf{r}_1 - \mathbf{r}_2|)]$ isotropy

Define: $\delta(\mathbf{r}) = [n(\mathbf{r}) - \langle n \rangle]/\langle n \rangle$

Then: $\xi(r) = \langle \delta(\mathbf{x})\, \delta(\mathbf{x} + \mathbf{r}) \rangle$ ξ is the correlation function

Estimator: $1 + \xi(r)$ = data-pairs/random-pairs = DD(r)/RR(r)

$= \sum_{I,j}^{Ndata} 1 \text{ (if } r_{ij} = r) / \sum_{I,j}^{Nrandom} 1 \text{ (if } r_{ij} = r \text{ in same volume)}$

or $\langle (D_1\text{-}R_1)/R_1\, (D_2\text{-}R_2)/R_2 \rangle \sim (DD\text{-}2DR\text{+}RR)/RR$

And FT is: $\langle \delta(\mathbf{k}_1)\, \delta(\mathbf{k}_2) \rangle = (2\pi)^3\, \delta_D(\mathbf{k}_1 + \mathbf{k}_2)\, P(|\mathbf{k}_1|)$

P(k) is the power spectrum

(Better) Estimator

$\xi(r) = < \delta(\mathbf{x})\ \delta(\mathbf{x} + \mathbf{r}) >$

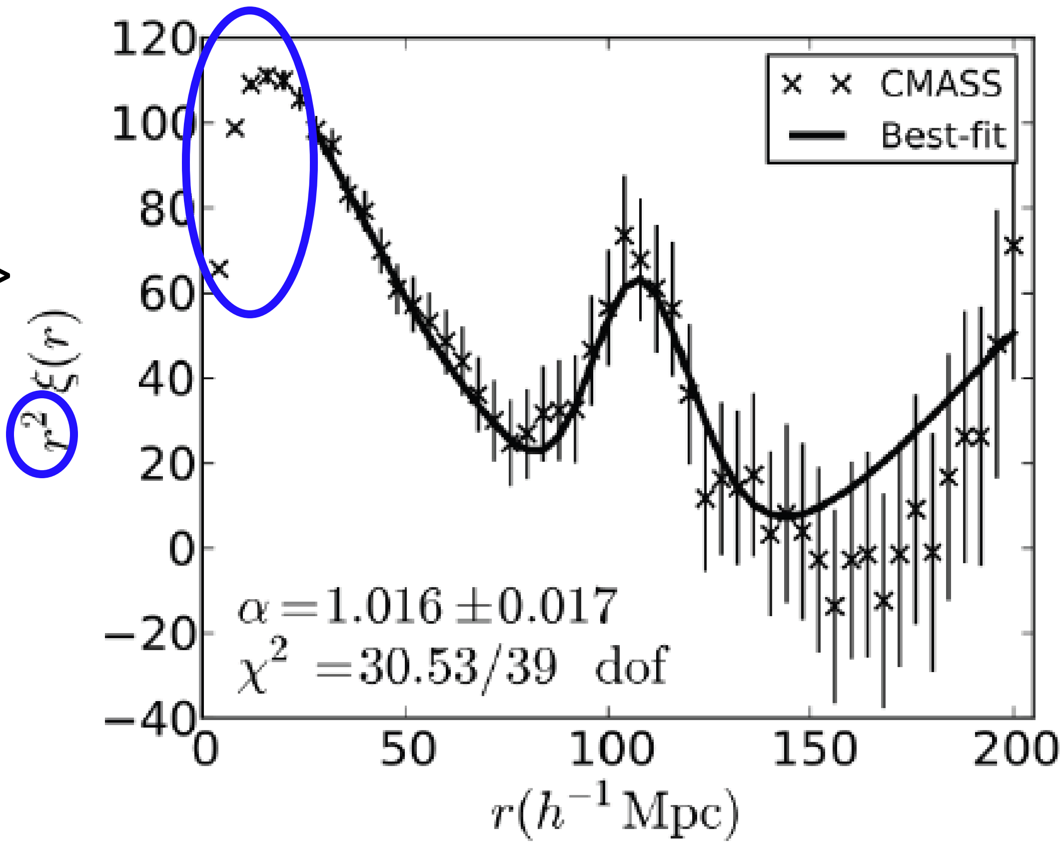
Since $\delta(\mathbf{r}) = [n(\mathbf{r}) - <n>]/<n>$

estimate using

$\xi = <(D_1-R_1)/R_1\ (D_2-R_2)/R_2>$

$\sim (DD-2DR+RR)/RR$

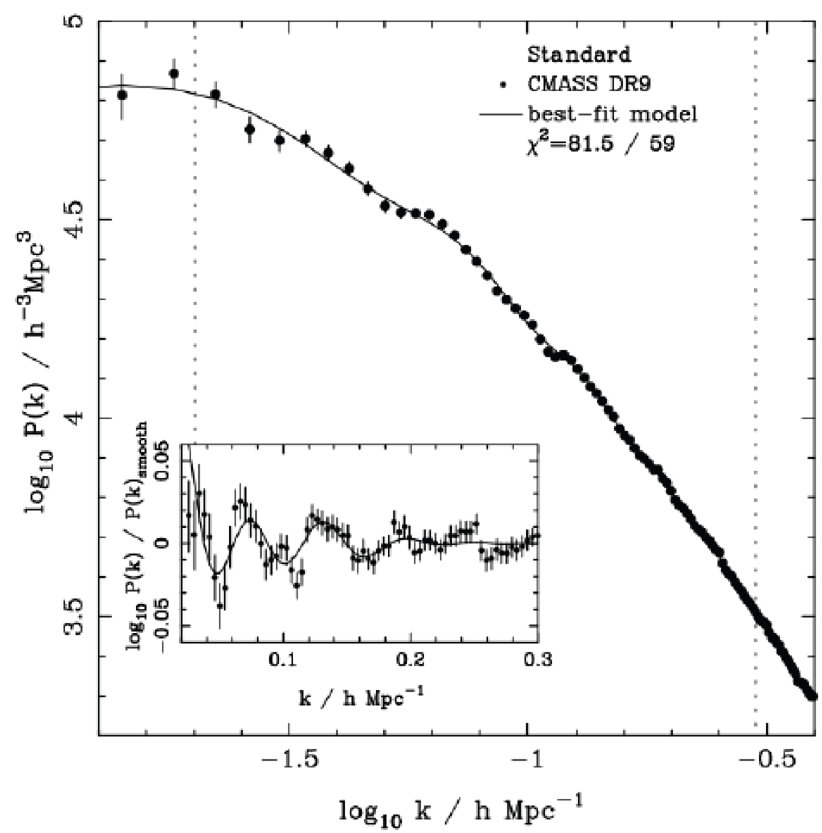
for pairs separated by r



$$\begin{aligned}
\xi(r) &= \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle \\
&= \lim_{V\to\infty}\frac{1}{V}\int_V \sum_{\mathbf{k}} \delta_{\mathbf{k}} \exp(i\mathbf{k}\cdot\mathbf{x}) \sum_{\mathbf{k}'} \delta^*_{\mathbf{k}'} \exp\left[-i\mathbf{k}'\cdot(\mathbf{x}+\mathbf{r})\right]\, d\mathbf{x} \\
&= \lim_{V\to\infty}\frac{1}{V}\int_V \sum_{\mathbf{k}} P(\mathbf{k}) \exp(-i\mathbf{k}\cdot\mathbf{r}) \\
&= \frac{1}{(2\pi)^3}\int P(k)\exp(-i\mathbf{k}\cdot\mathbf{r})d\mathbf{k} \\
P(k) &= \int \xi(\mathbf{r})\exp(i\mathbf{k}\cdot\mathbf{r})d\mathbf{r}
\end{aligned}$$

$$\int_\Omega \exp(-ikr\cos\theta)d\Omega = 4\pi\frac{\sin kr}{kr}$$

$$\boxed{\begin{aligned} P(k) &= \int_0^\infty \xi(r)\frac{\sin kr}{kr} r^2 dr \\ \xi(r) &= \frac{1}{2\pi^2}\int_0^\infty P(k)\frac{\sin kr}{kr} k^2 dk \end{aligned}}$$



P(k) and ξ(r) are FT pairs

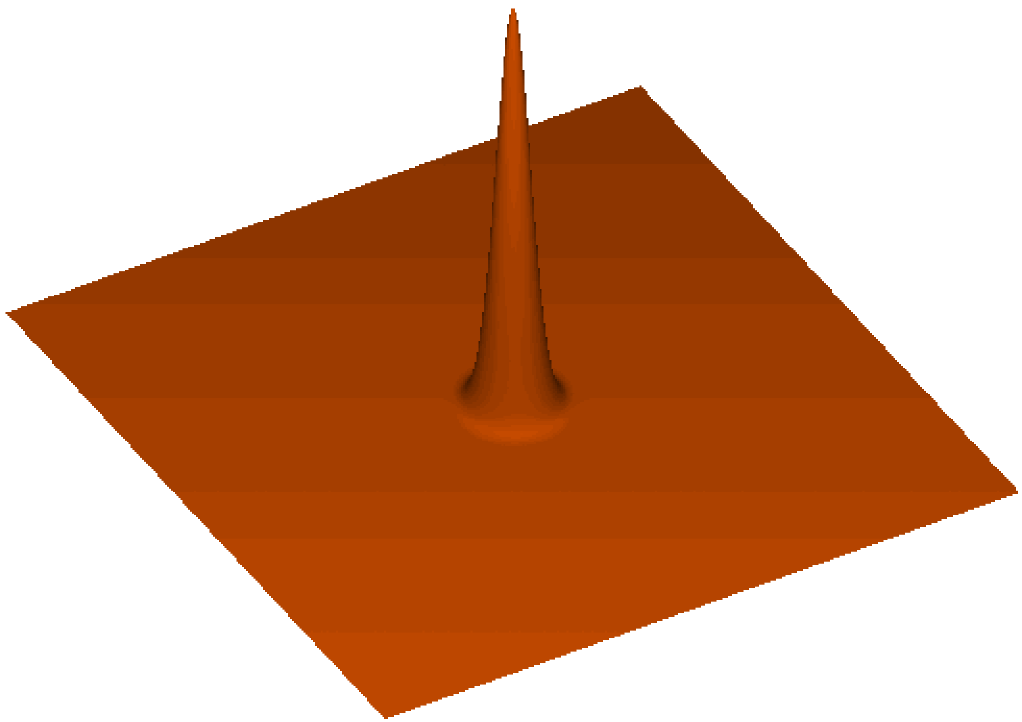
Cosmology from the same physics imprinted in the galaxy distribution at different redshifts:

Baryon Acoustic Oscillations

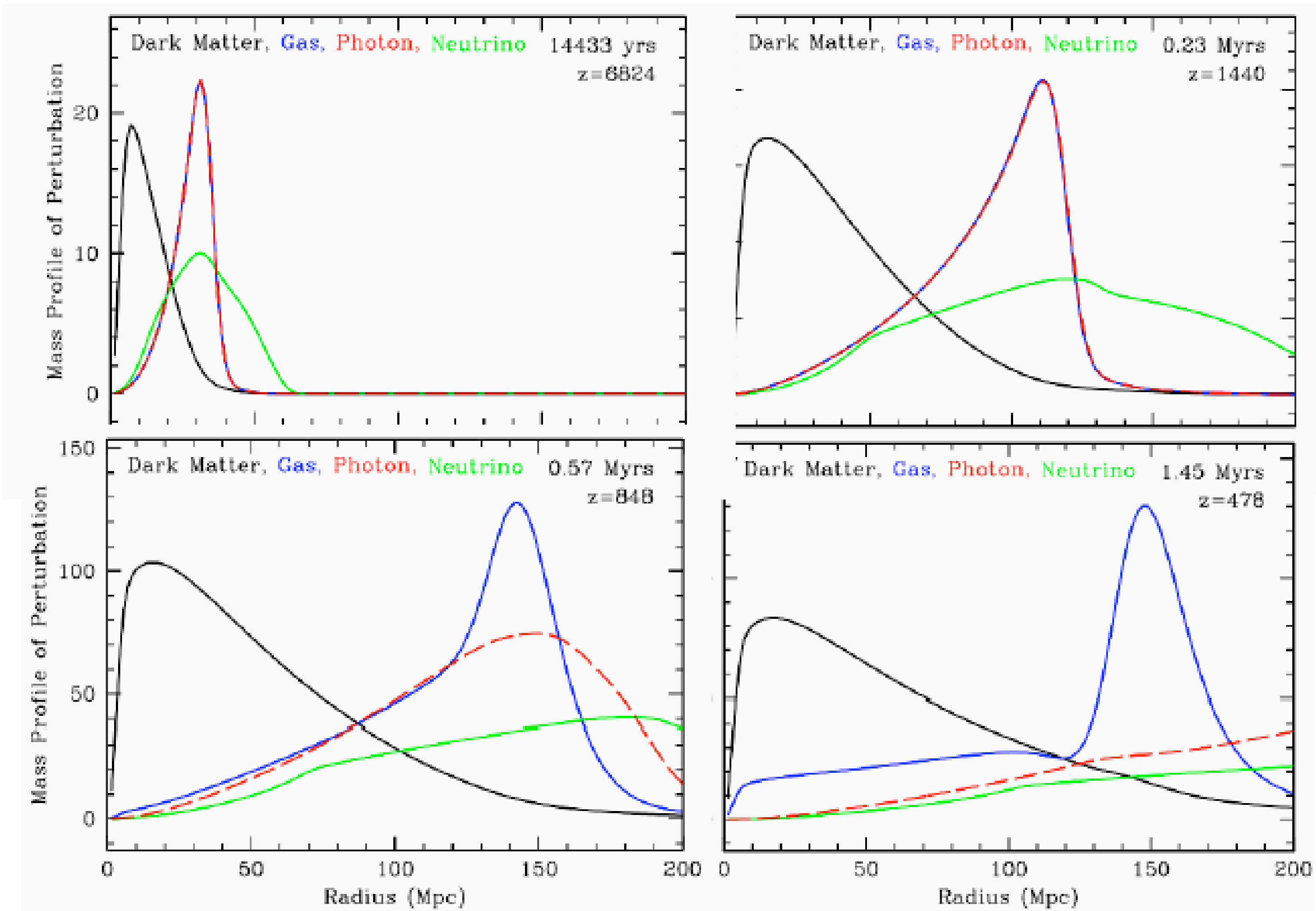
CMB from interaction between photons and baryons when Universe was 3,000 degrees (about 300,000 years old)

- Do galaxies which formed much later carry a memory of this epoch of last scattering?

Photons 'drag' baryons for ~400,000 years (time set by $\Omega_m h^2$) at speed ~ $c/[3(1 + 3\rho_b/4\rho_\gamma)]^{1/2}$ (set by $\Omega_b h^2$) …
300,000 light years ~ 100,000 pc ~ 100 kpc

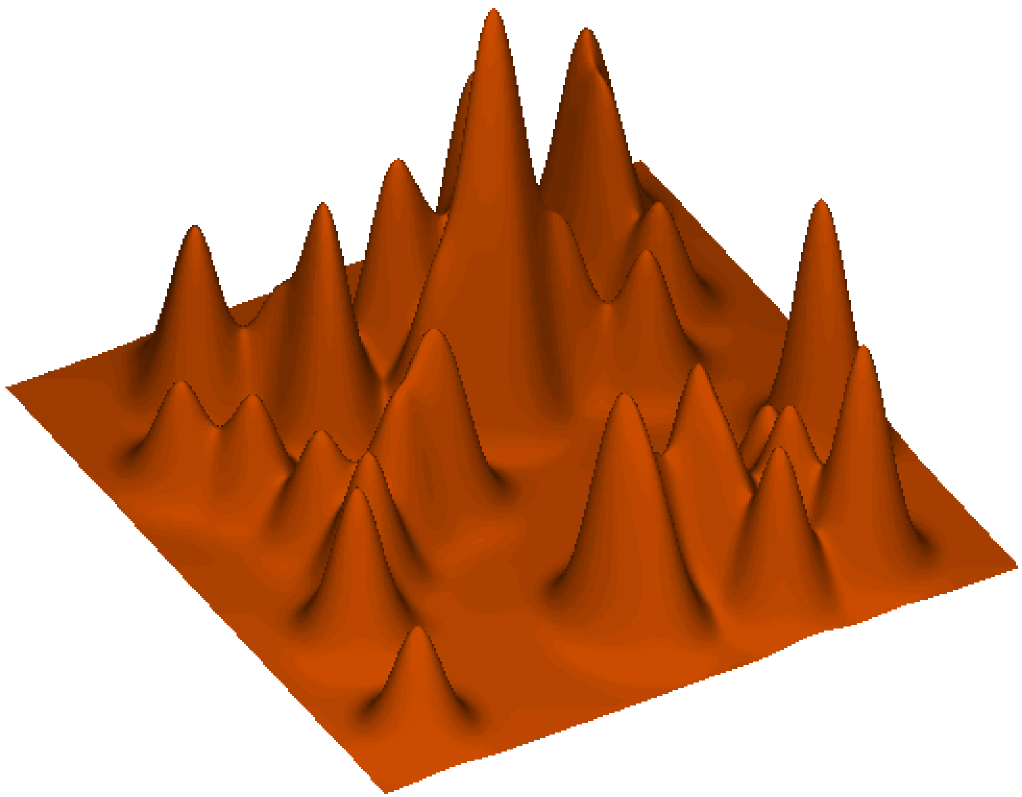


Expansion of Universe since then stretches this to (3000/2.725) ×100 kpc ~ 100 Mpc



Eisenstein, Seo, White 2007

Expect to see a feature in the Baryon distribution on scales of 100 Mpc today

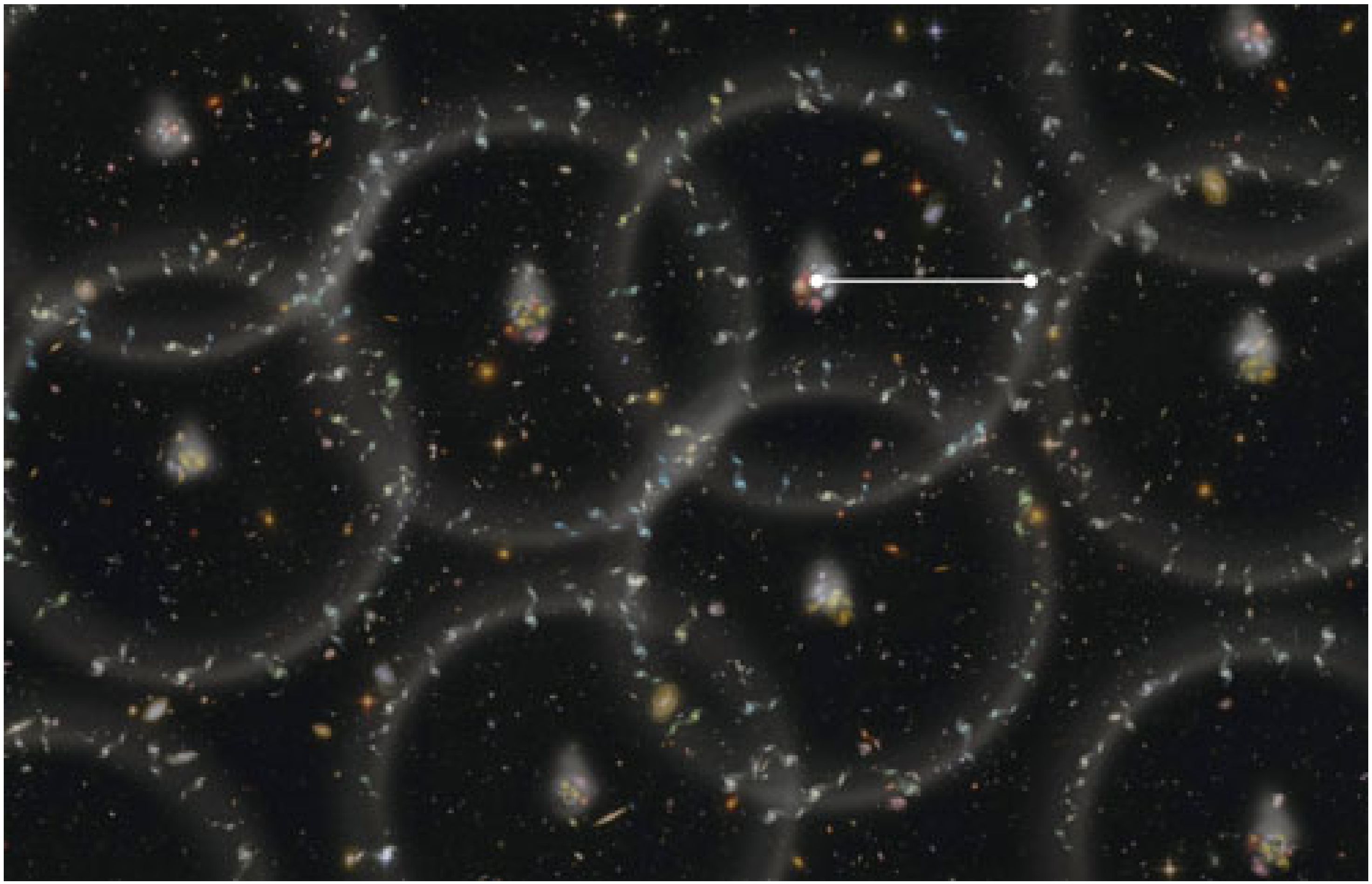


But this feature is like a standard rod:

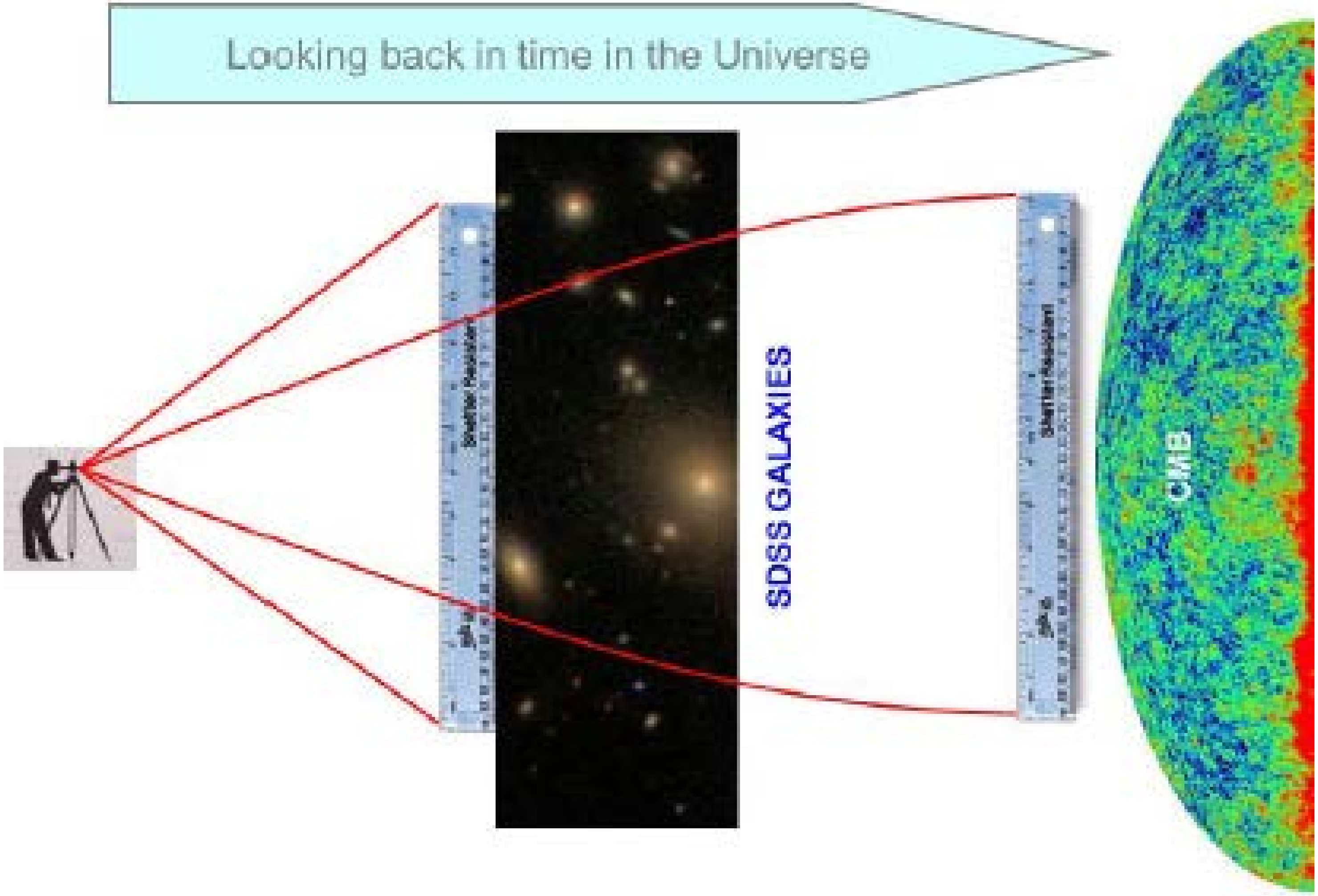
We see it in the CMB itself at z~1000

Should see it in the galaxy distribution at other z

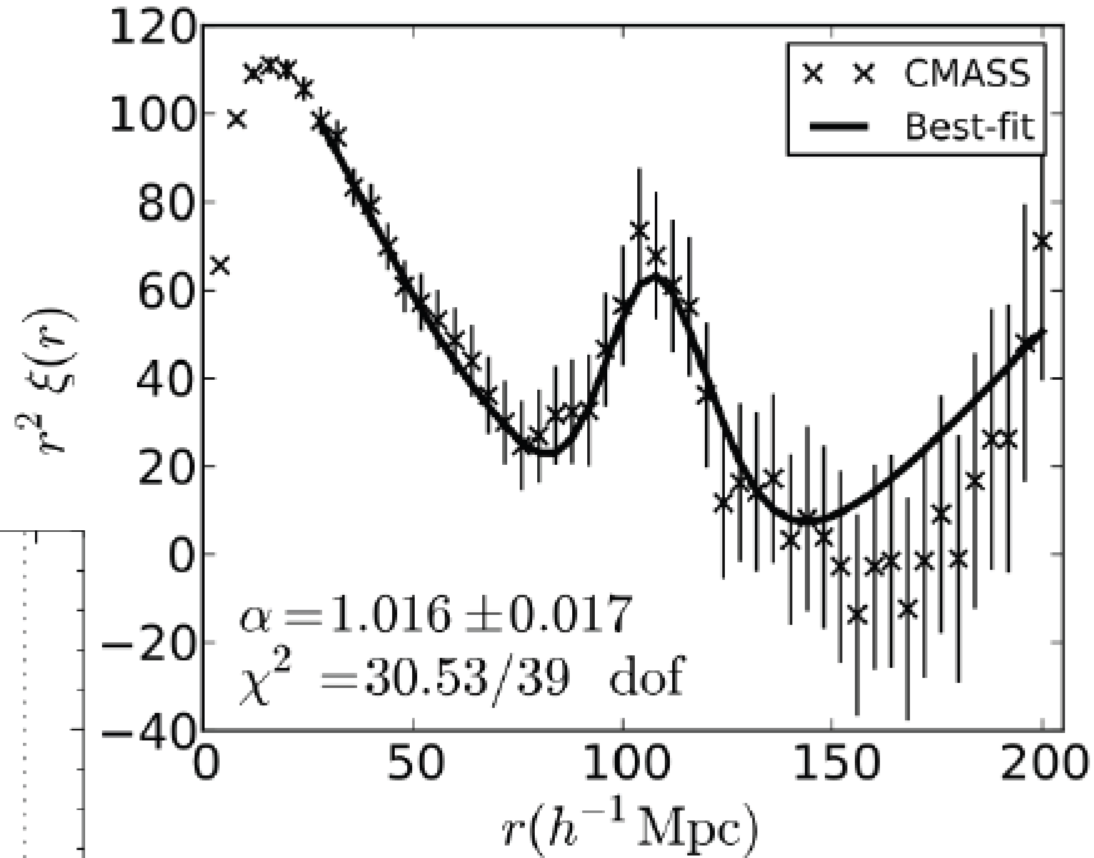
Cartoon of expected effect

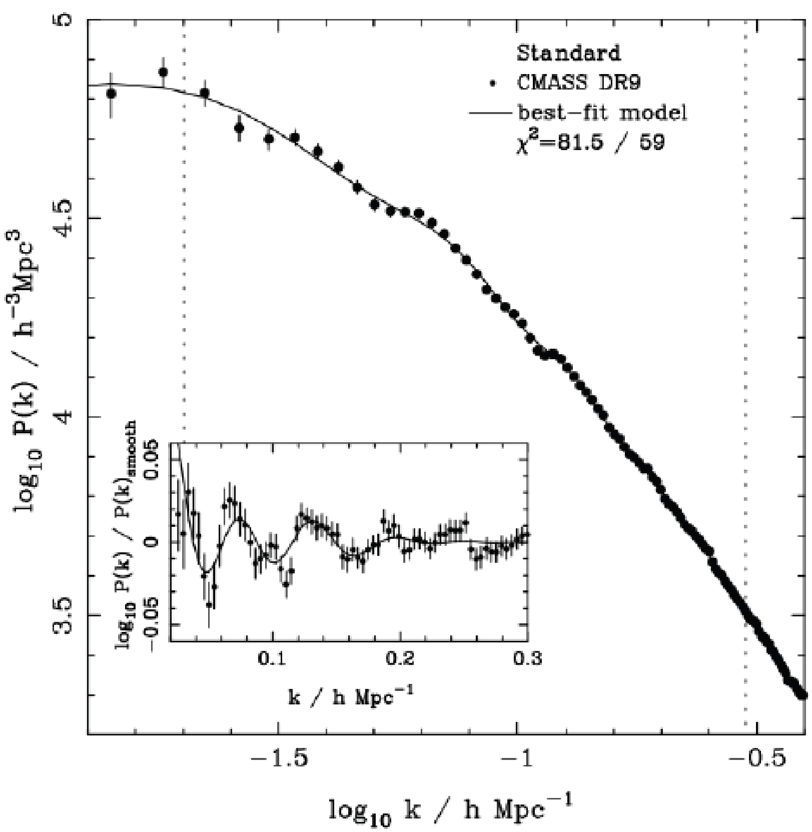


Baryon Oscillations in the Galaxy Distribution

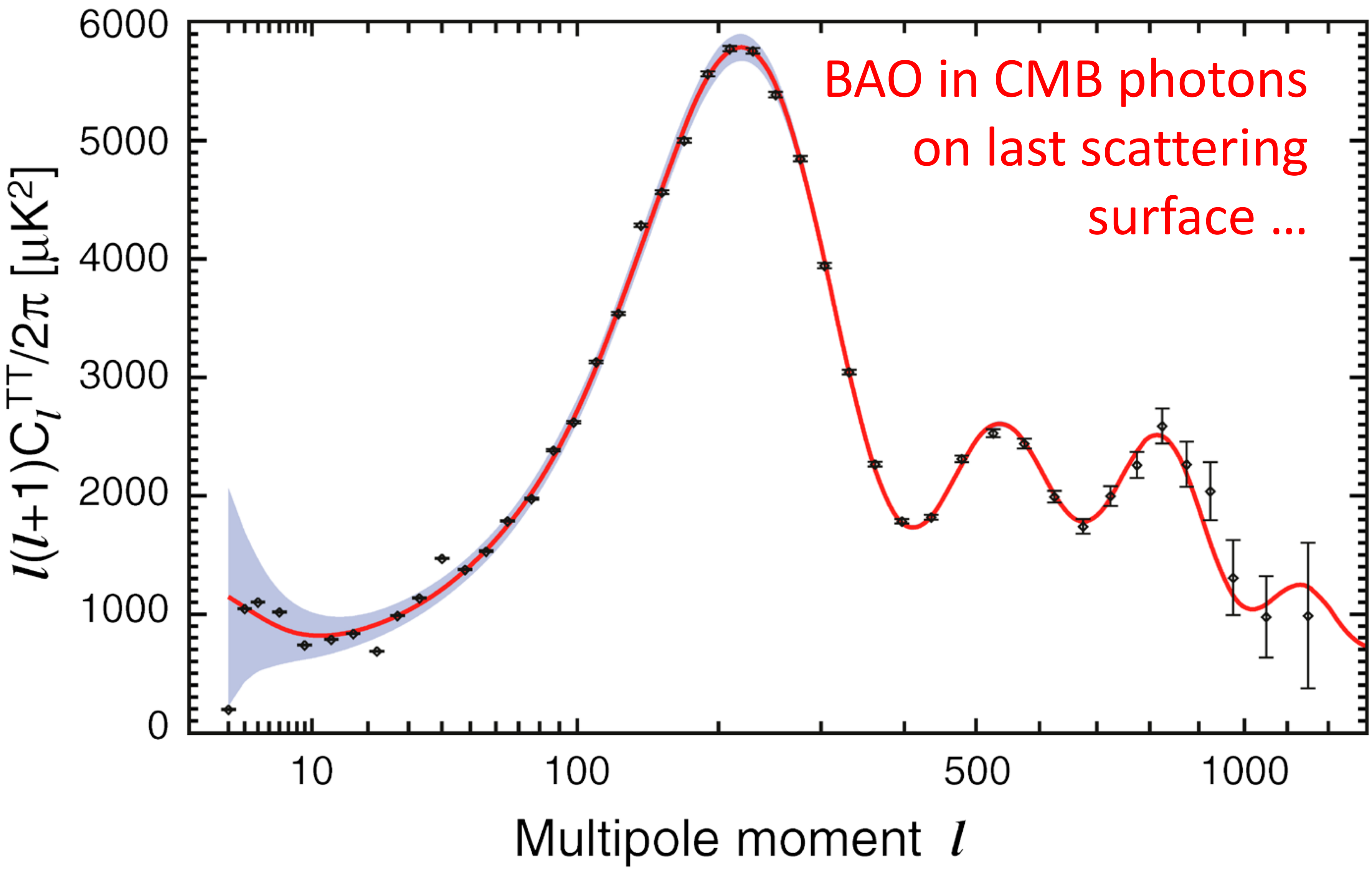


Spike in real space $\xi(r)$ means $\sin(kr_{BAO})/kr_{BAO}$ oscillations in Fourier space $P(k)$

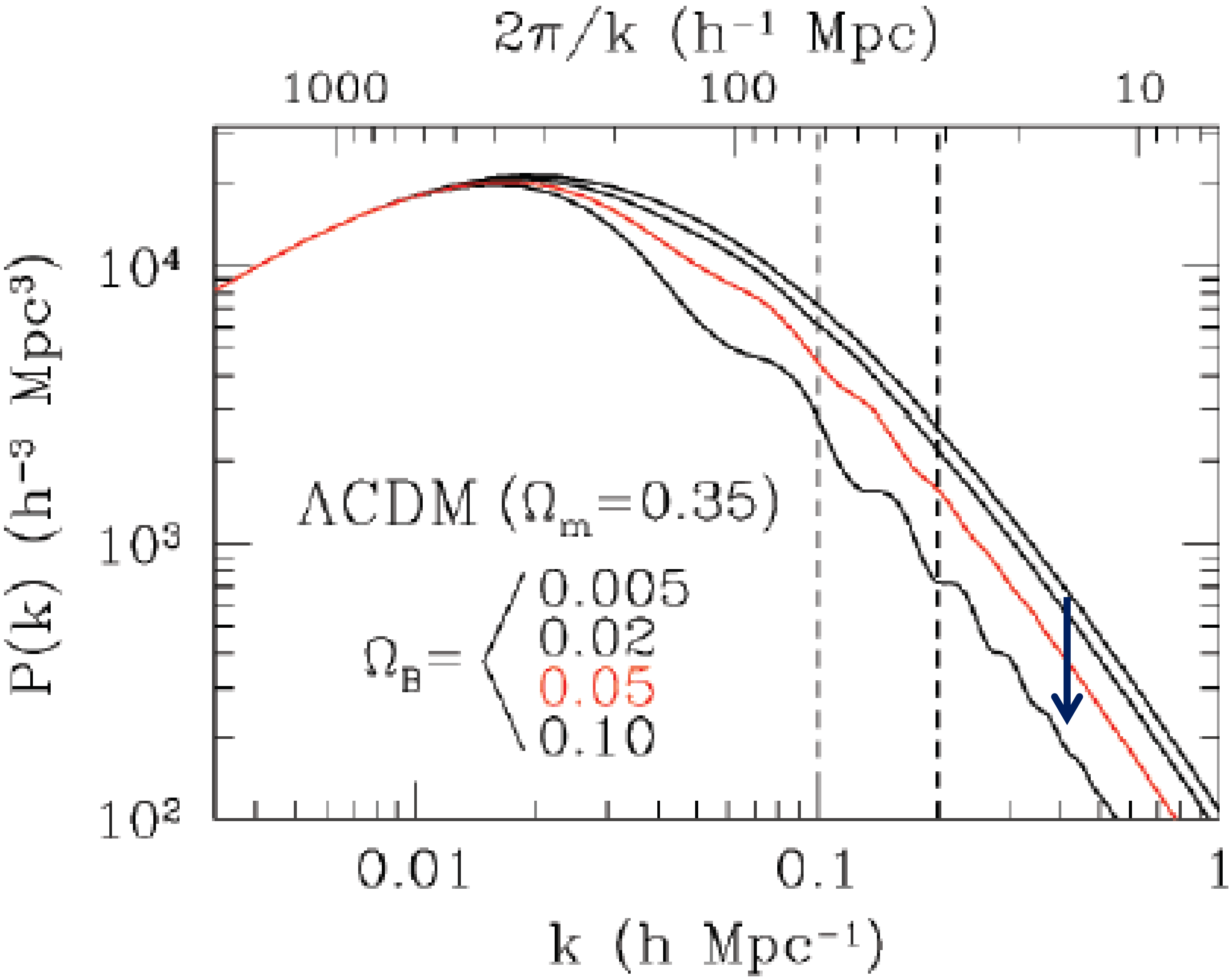




Spike not delta function as photon-baryon not perfectly coupled and surface of last scattering not instantaneous: $e^{-(k/k_{Silk})^{1.4}} \sin(kr_{BAO})/kr_{BAO}$



BAO in CMB photons
on last scattering
surface …
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
$l(l+1)C_l^{TT}/2\pi\ [\mu K^2]$
Multipole moment l



If all matter baryonic, power below 200 Mpc/h is suppressed

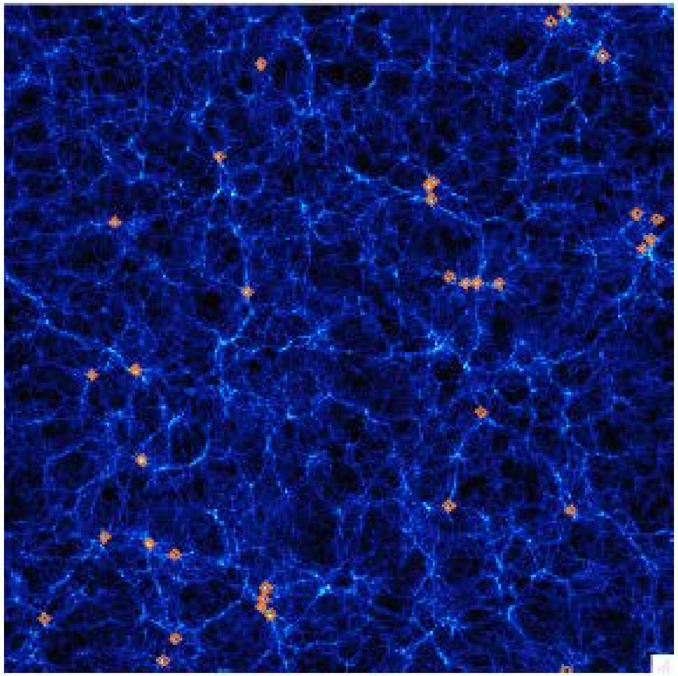
Need nonbaryonic gravitating dark matter to explain structure formation

… should/are seen in matter distribution at later times

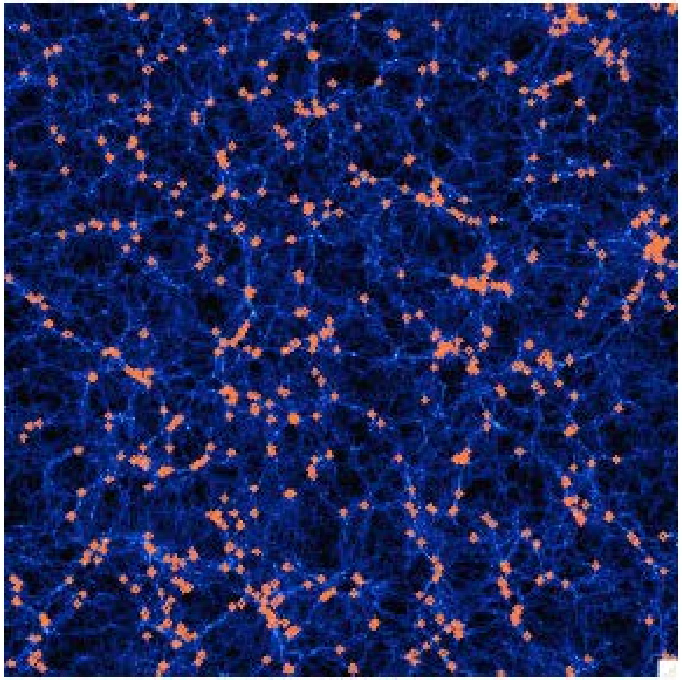
…we need a tracer of the baryons

- Luminous Red Galaxies
  - Luminous, so visible out to large distances
  - Red, presumably because they are old, so probably single burst population, so evolution relatively simple
  - Large luminosity suggests large mass, so probably strongly clustered, so signal easier to measure
  - Linear bias on large scales, so *length of rod* not affected by galaxy tracer!

The cosmic web at z~0.5, as traced by luminous red galaxies



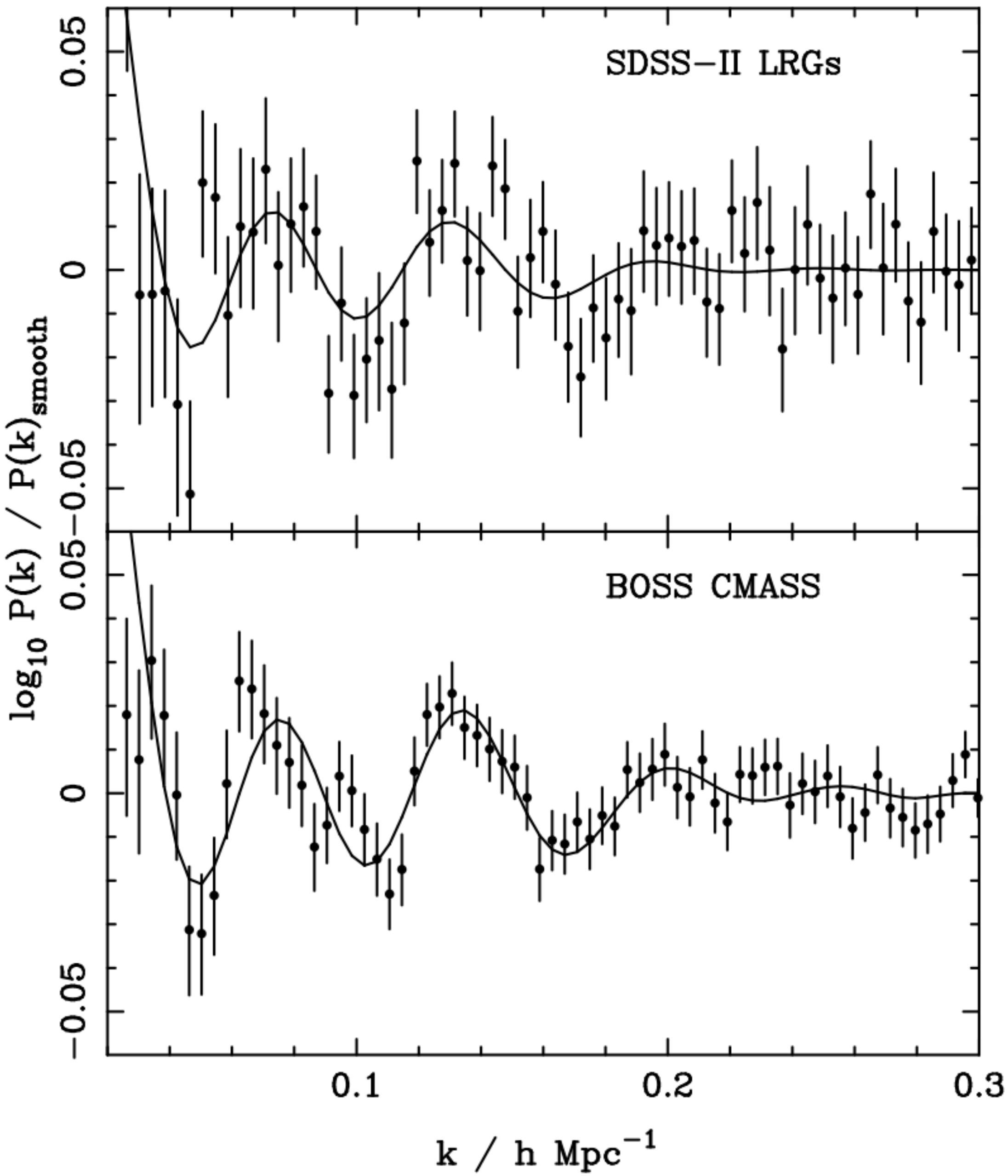
SDSS



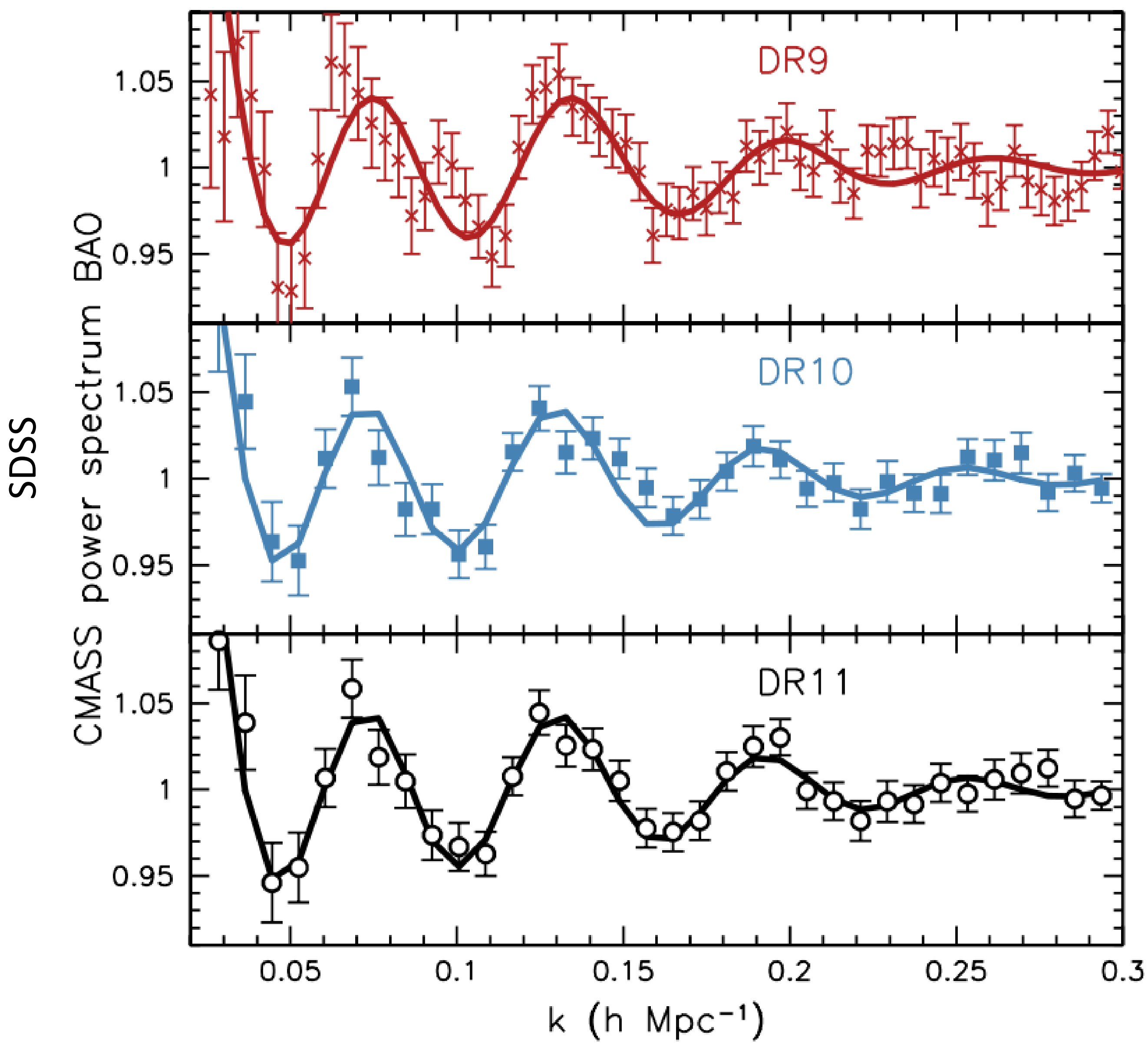
BOSS

(M. White 2010)

A slice $500h^{-1}$ Mpc across and $10\ h^{-1}$ Mpc thick

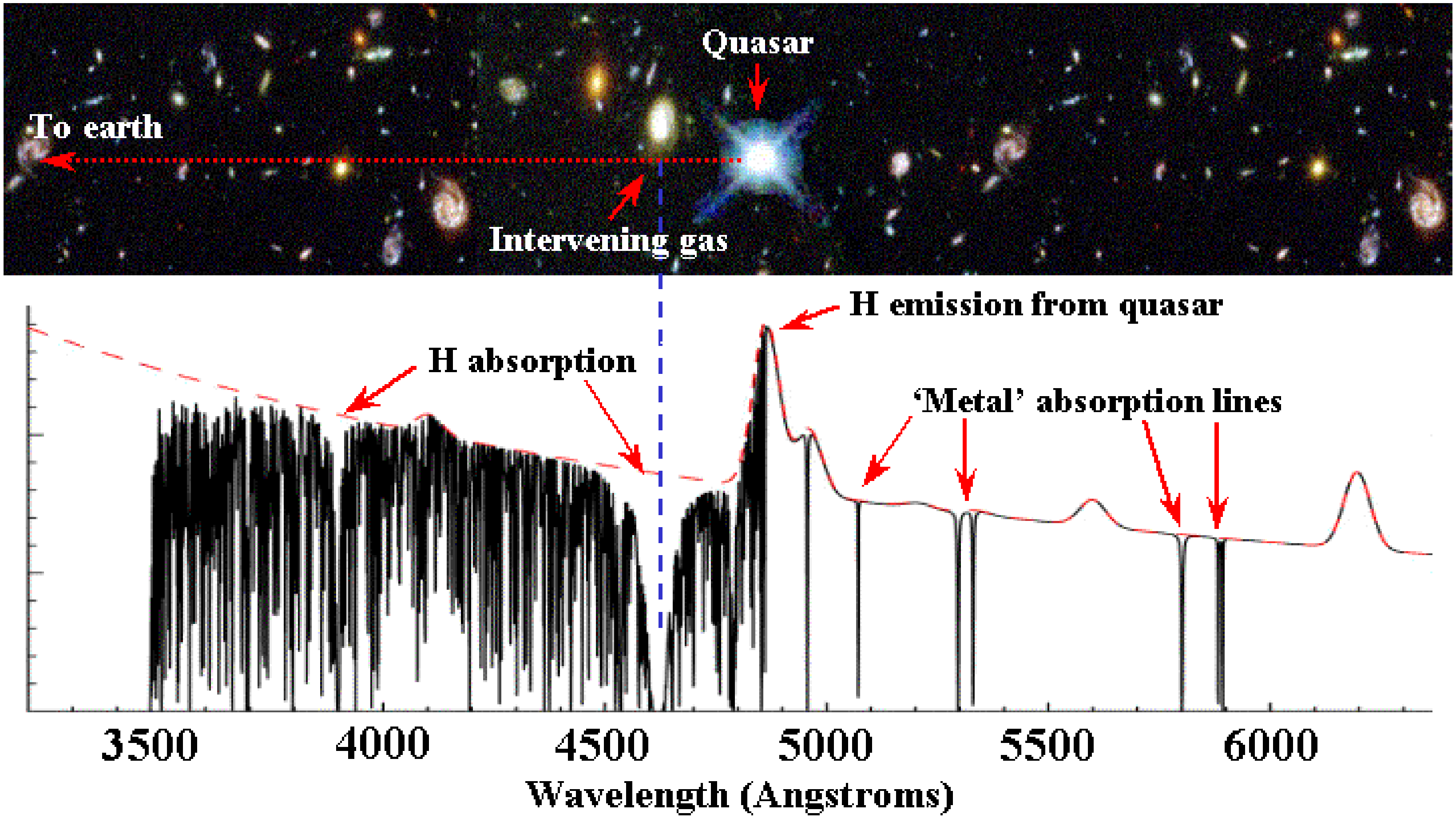


SDSS−II LRGs
BOSS CMASS
0.05
0
−0.05
0.1
0.2
0.3
$\log_{10}$ P(k) / P(k)$_{\rm smooth}$
k / h Mpc^{-1}



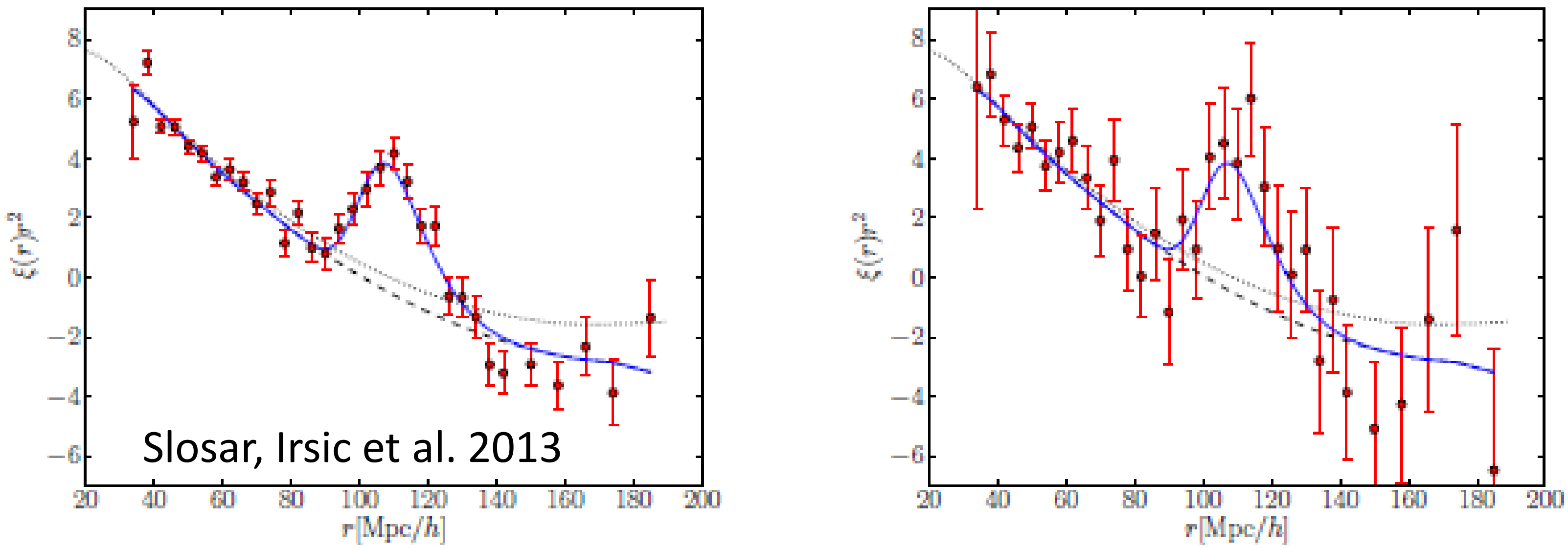
DR9
DR10
DR11
SDSS
CMASS power spectrum BAO
1.05
1
0.95
0.05
0.1
0.15
0.2
0.25
0.3
k (h Mpc⁻¹)

Can see baryons that are not in stars …



High redshift structures constrain neutrino mass

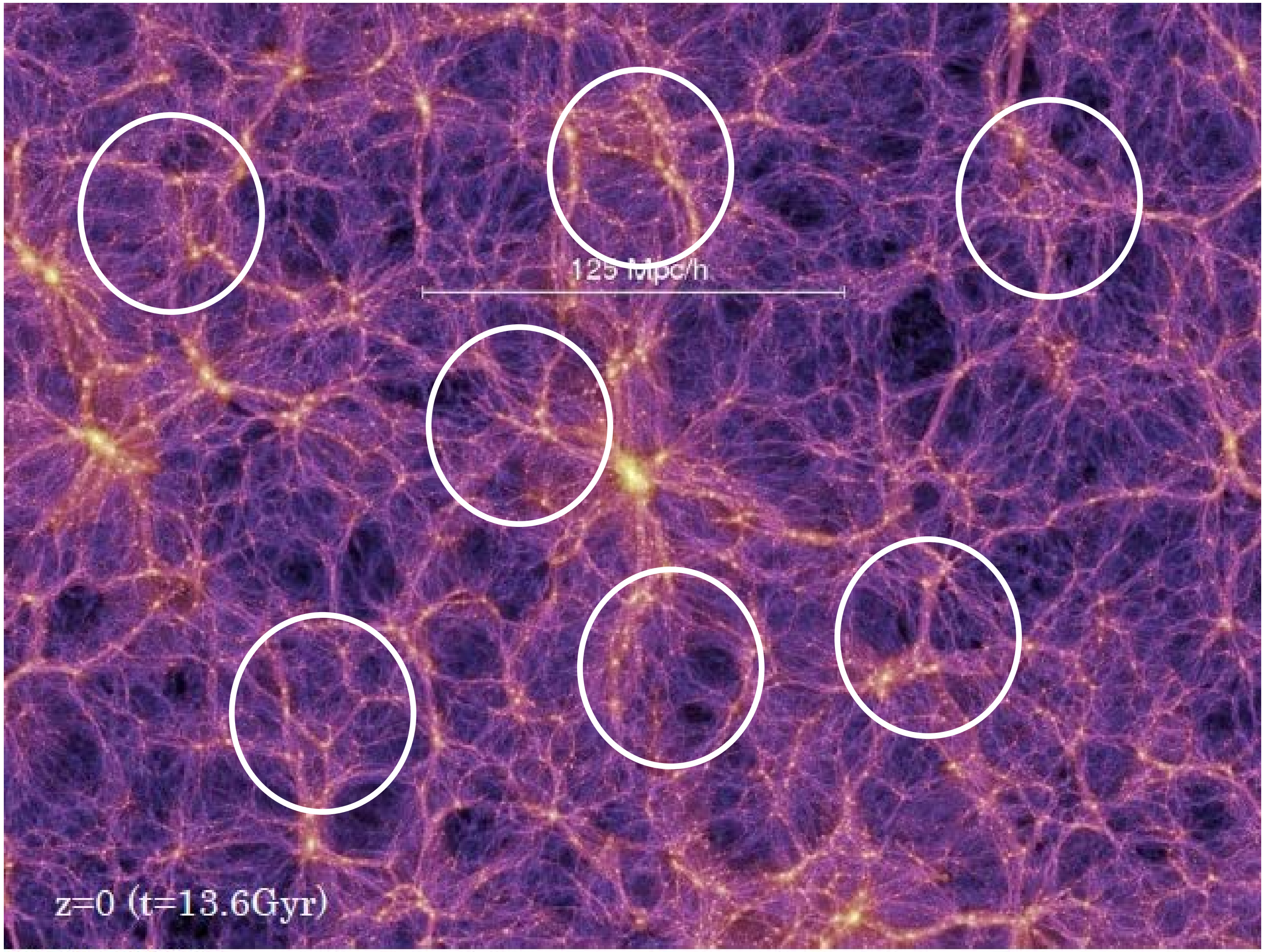
BAO in Ly-α forest at z~2.4

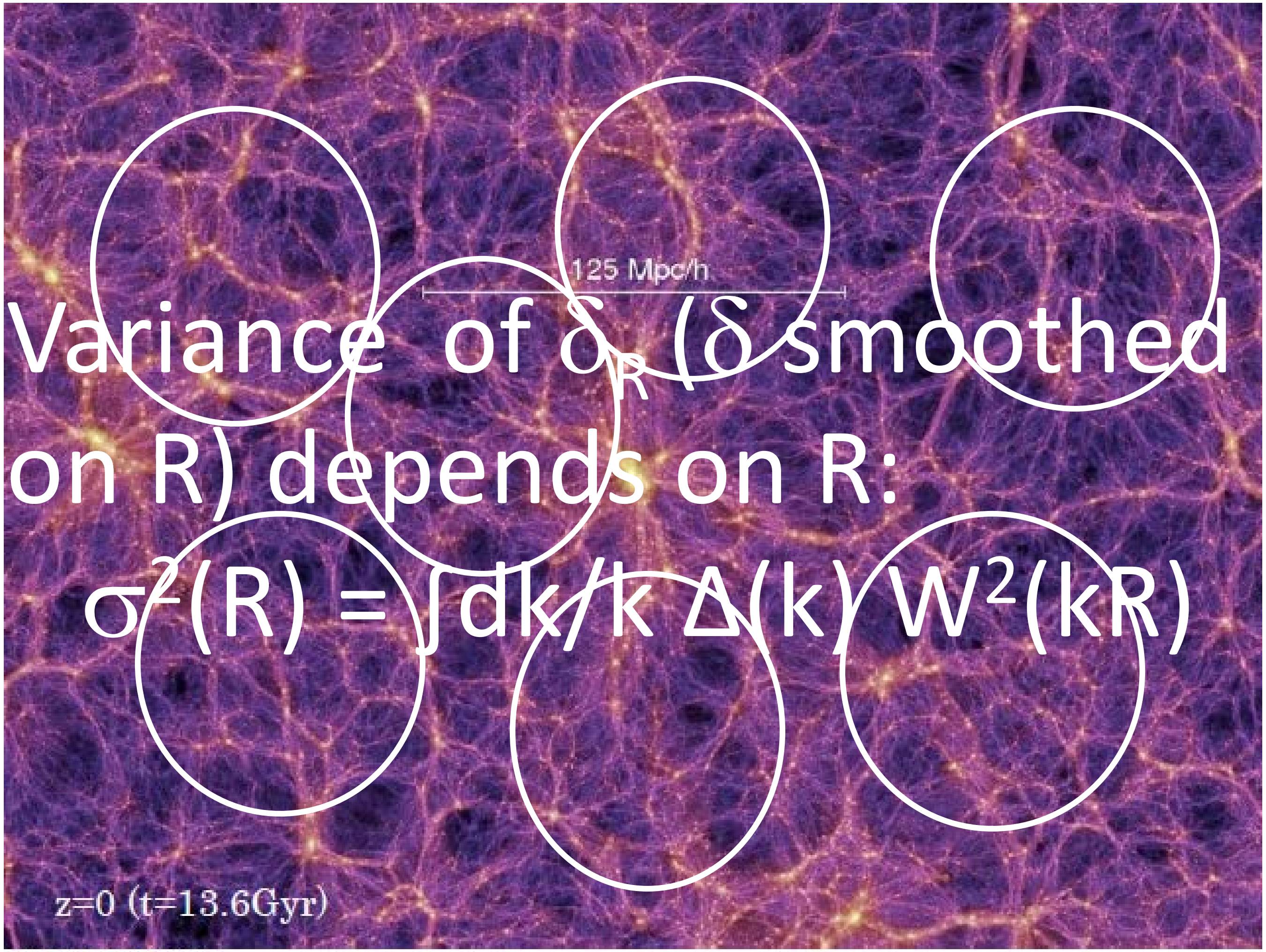


- Signal from cross-correlating different lines of sight

- The baryon distribution today ‘remembers’ the time of decoupling/last scattering; can use this to build a ‘standard rod’

- Next decade will bring observations of this standard rod out to redshifts $z \sim 2$ Constraints on model parameters from 10% to 1%


125 Mpc/h
z=0 (t=13.6Gyr)



Variance of δ_R (δ smoothed on R) depends on R:

$$\sigma^2(R) = \int dk/k\ \Delta(k)\ W^2(kR)$$

Variance of δ_R (δ smoothed on R):

$$\sigma^2(R) = \int dk/k\, \Delta(k)\, W^2(kR)$$

Correlations in smoothed field

$$\Delta(k) = k^3 P(k)/2\pi^2$$

$$\Delta_{R_1R_2}(k) = \Delta(k)\, W(kR_1)\, W(kR_2)$$

$$\xi_{R_1R_2}(r) = \int dk/k\, \Delta_{R_1R_2}(k)\, j_0(kr)$$

E.g. Power-law P(k)

- $\xi(r) = \int dk/k\ [k^3\ Ak^n/2\pi^2]\ j_0(kr) \ \propto\ r^{-3-n}$ if $n>-3$
- $\sigma^2(R) = (A/2\pi^2) \int dk/k\ k^{n+3}\ \exp(-k^2R^2)$

  $= (A/2\pi^2)\ \Gamma[(n+3)/2]/2\ R^{-3-n}$
- $\xi_R(r) = (A/2\pi^2) \int dk\ k^{2+n} \exp(-k^2R^2)\ j_0(kr)$

  $= (A/2\pi^2)\ (\pi/2r)\ \mathrm{erf}(r/2R)$ if $n=-2$

  $\rightarrow \xi_0(r)$ when $r \gg R$

(smoothing irrelevant on large scales? BAO …)

Structure formation:
The shape of P(k)

Three possible metrics for homogeneous and isotropic 3-space

$$ds^2 = dr^2 + S_\kappa(r)^2 d\Omega^2 \ ,$$

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$$

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1) \end{cases}$$

Changing from r to x = S_κ(r) makes this:

$$ds^2 = \frac{dx^2}{1 - \kappa x^2/R^2} + x^2 d\Omega^2$$

Robertson-Walker metric

(If homogeneity and isotropy did not exist, it would be necessary to invent them!)

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\Omega^2$$ Minkowski metric

$$\boxed{\begin{aligned} ds^2 &= -c^2dt^2 + a(t)^2\left[\frac{dx^2}{1-\kappa x^2/R_0^2} + x^2d\Omega^2\right] \\ ds^2 &= -c^2dt^2 + a(t)^2\left[dr^2 + S_\kappa(r)^2d\Omega^2\right] \end{aligned}}$$

Much of Observational Cosmology dedicated to determining κ, $a(t)$, R_0

Connection to GR

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} R/2 = 8\pi G\, T_{\mu\nu}$$

Homogeneity/isotropy:

$$T_{\mu\nu} = \text{diagonal} = (\rho,-p,-p,-p)$$

Conservation of stress-energy:

$$\nabla_\nu (T_{\mu\nu}) = 0$$

Using FRW metric:

$$d(\rho a^3) = -p\, d(a^3)$$

Since $a^3 \propto V$ this is like 1st Law of thermodynamics.

So, if $p(\rho)$ then can solve for $\rho(t)$:

Evolution depends on 'equation of state'

Equation of state

Consider: $p(t) = w\,\rho(t)$ w independent of t

Then $d(\rho V)/dt = V\,(d\rho/dt) + \rho\,(dV/dt) = -p\,(dV/dt)$

So $V\,(d\rho/dt) = -\,(\rho+p)\,(dV/dt)$

$(d\ln\rho/dt) = -\,(1+p/\rho)\,(d\ln V/dt)$

So $\rho(t) \propto a^{-3(1+w)}$

Special cases:

Non-relativistic matter: $p = 0$ so $w = 0$ so $\rho \propto a^{-3}$

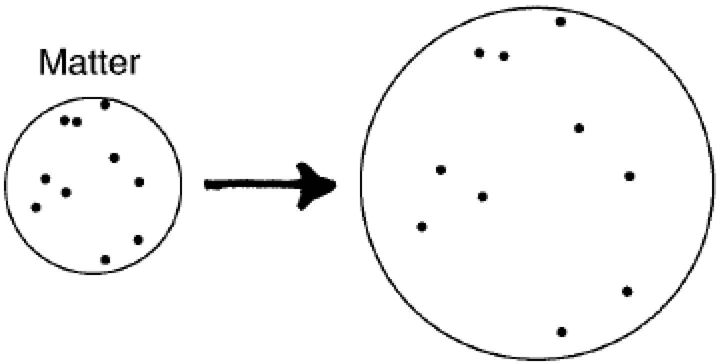
Radiation: $w = 1/3$ so $\rho \propto a^{-4}$

Vacuum energy: $w = -1$ so ρ constant

Special cases:

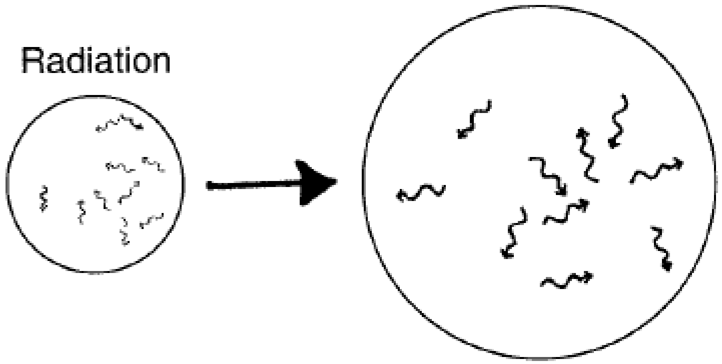
Non-relativistic matter:

w = 0 so $\rho \propto a^{-3}$



Radiation:

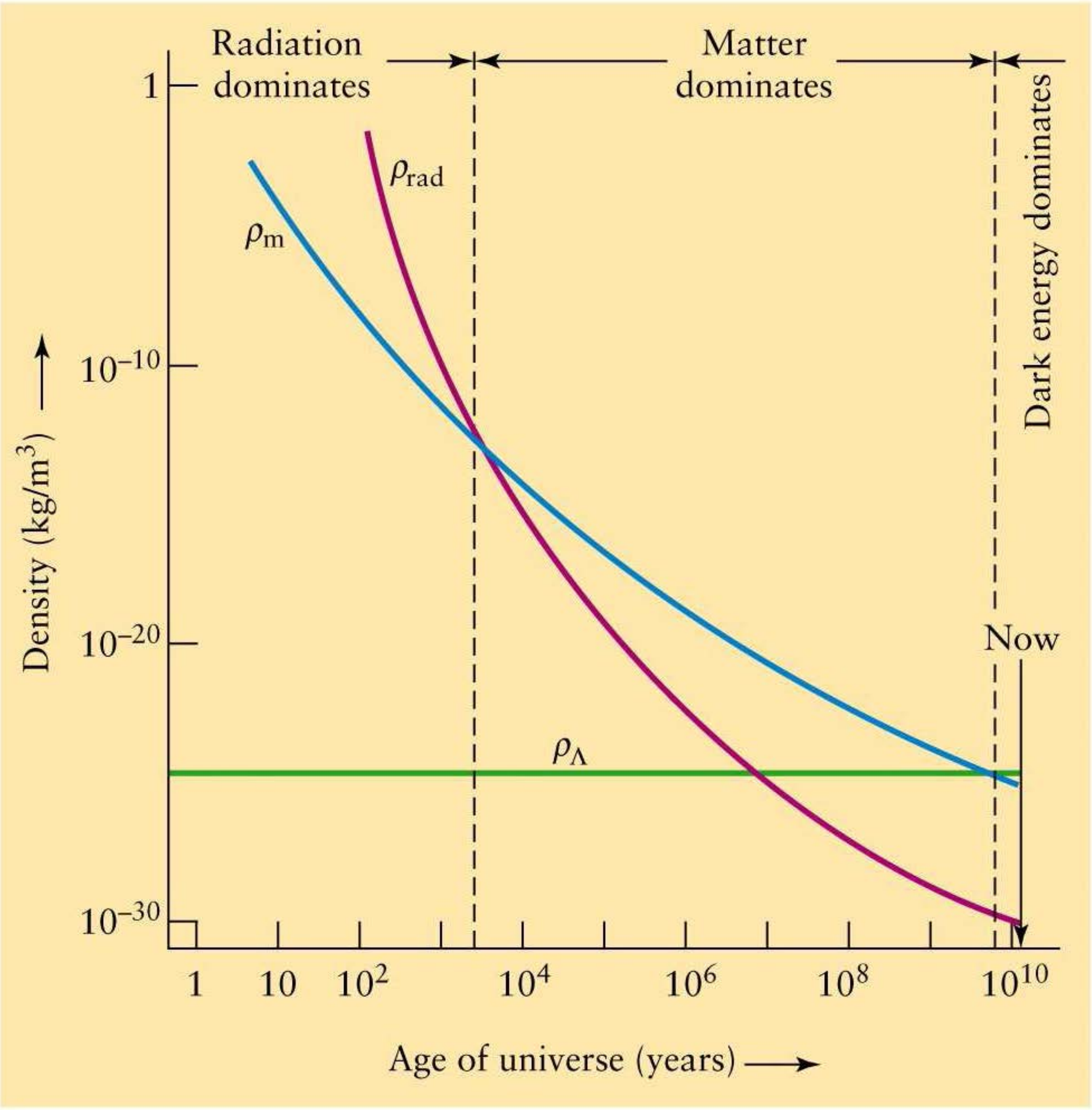
w = 1/3 so $\rho \propto a^{-4}$



Vacuum energy:

w = -1 so ρ constant

If Universe contains all three, then different ones dominate at different t



Conventional to define:

$\Omega_m = \rho_m/\rho_c$

$\Omega_r = \rho_r/\rho_c$

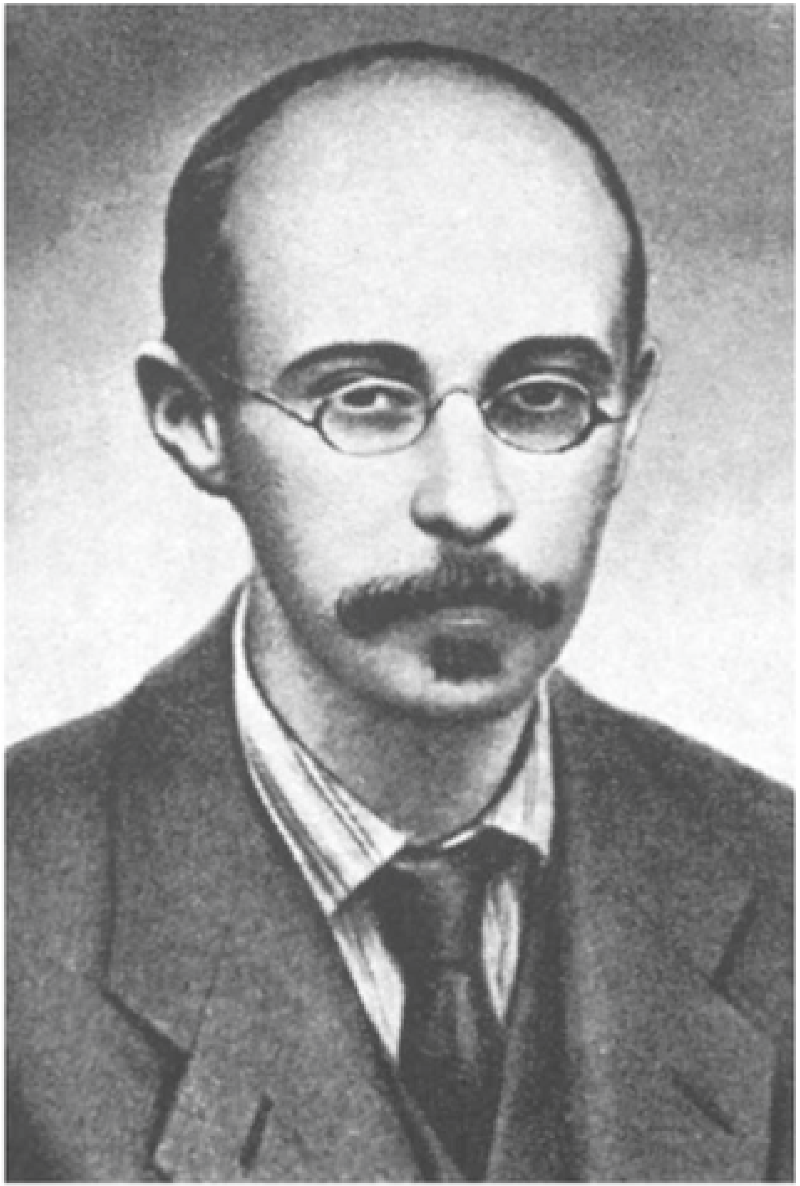
$\Omega_\Lambda = \rho_\Lambda/\rho_c$

$\rho_c = 3H^2/8\pi G$

Friedmann equations

From 00 element of Einstein equations with RW metric (relates expansion rate to density and curvature);

And from time derivative of it (relates acceleration to density and pressure).



Friedmann equation

$$(d\ln a/dt)^2 + (\kappa c^2/R_0^2 a(t)^2) = (8\pi G/3)\,\rho$$

$$H^2 = (8\pi G/3)\,\rho - (\kappa c^2/R_0^2 a(t)^2)$$

$$1 - \Omega(t) = -\kappa\,[c/H(t)]^2/R_0^2 a(t)^2$$

Knowing Ω = knowing sign of curvature

Flat Universe ($\kappa = 0$) has $\Omega(t) = 1$;

it has energy density $3H^2/(8\pi G)$.

Note that Ω is sum of all components (matter + radiation + dark energy) .

Empty Universe: $\Omega=0$

$$1 = -\kappa\,[c/H(t)]^2/R_0^{\,2}a(t)^2$$

$$(aH)^2 = -\kappa\,(c/R_0)^2$$

$\kappa=0$ requires a = constant

$\kappa=1$ not allowed

$\kappa=-1$ requires da/dt = constant; $a = ct/R_0$

Flat Universe: $\Omega = 1$

Suppose $a \propto t^q$

Then $H = q/t$ so $\rho \propto a^{-3(1+w)} \propto H^2 \propto t^{-2}$

means $q = 2/3(1+w)$

Matter ($w=0$): $a \propto t^{2/3}$

Radiation ($w=⅓$): $a \propto t^{1/2}$

Dark Energy ($w=-1$)?? $a \propto e^{Ht}$

(because $\rho \propto a^{-3(1+w)} \propto H^2 \propto$ constant)

Λ (w=-1):

$a \propto e^{Ht}$

Empty:

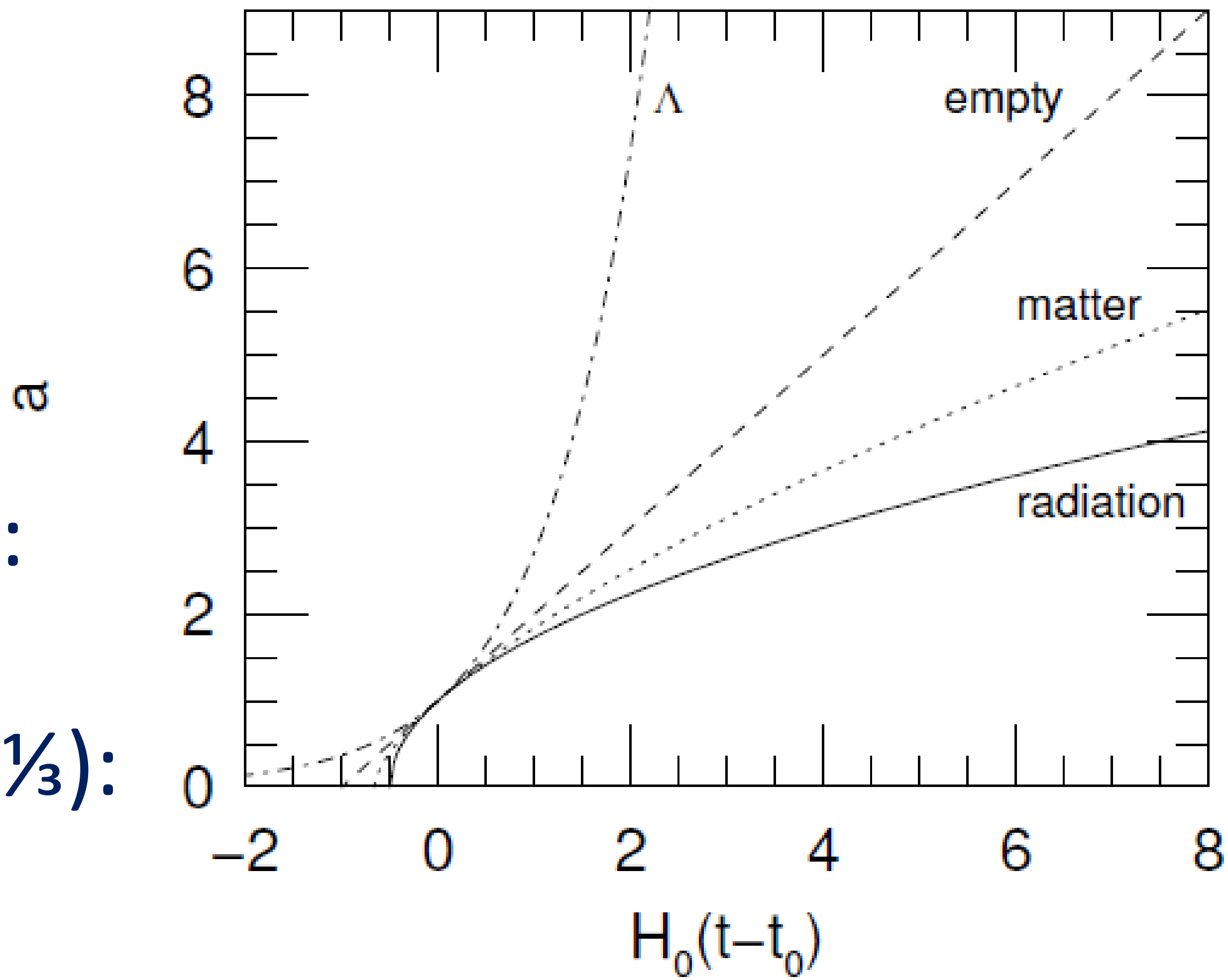
$a \propto t$

Matter (w=0):

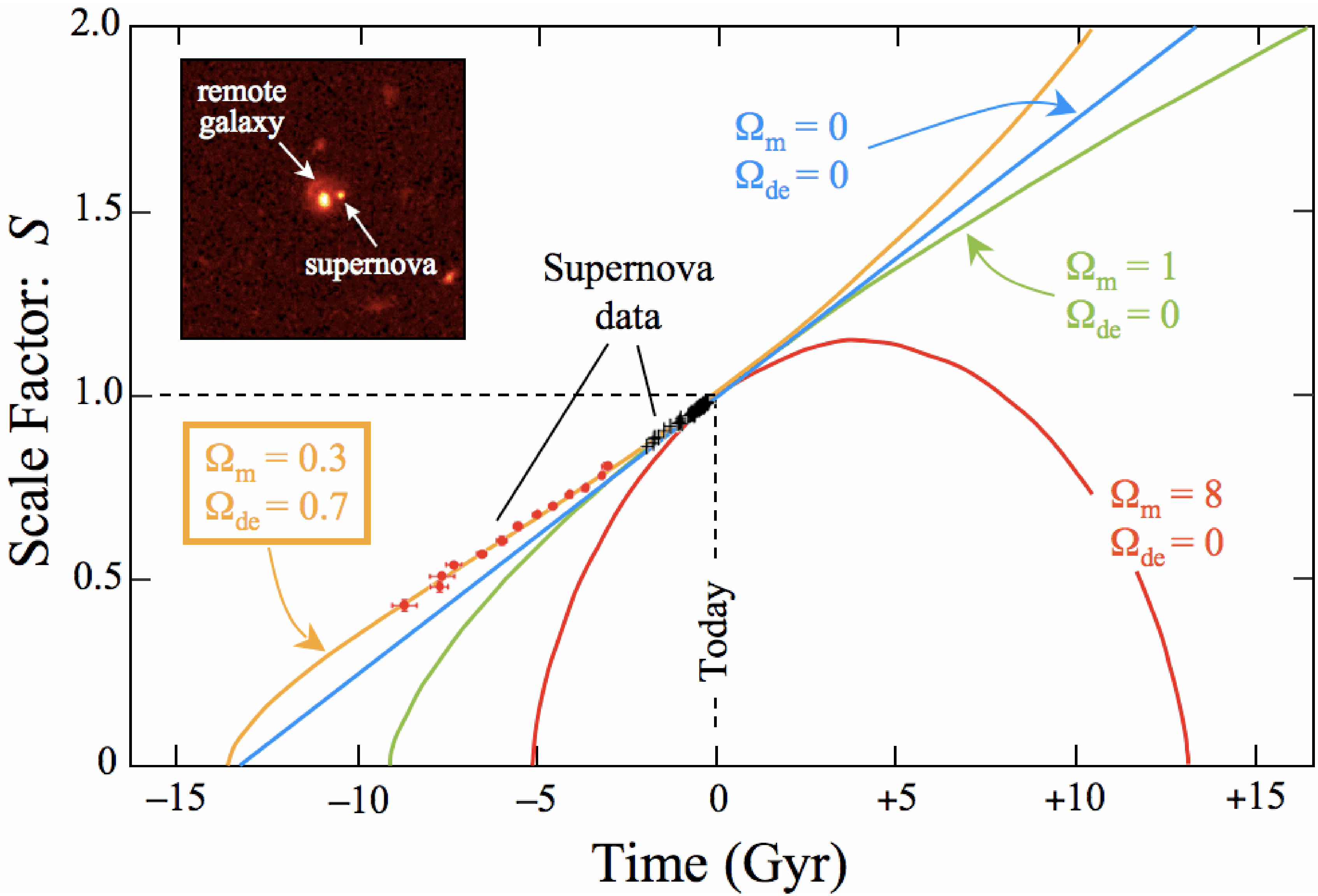
$a \propto t^{2/3}$

Radiation(w=⅓):

$a \propto t^{1/2}$



From these, can work out $d_L(z|\Omega,\Lambda)$



Scale Factor: S
2.0
1.5
1.0
0.5
0
−15
−10
−5
0
+5
+10
+15
Time (Gyr)
remote galaxy
supernova
Supernova data
Today
$\Omega_m = 0$
$\Omega_{de} = 0$
$\Omega_m = 1$
$\Omega_{de} = 0$
$\Omega_m = 0.3$
$\Omega_{de} = 0.7$
$\Omega_m = 8$
$\Omega_{de} = 0$

Matter + curvature + Λ

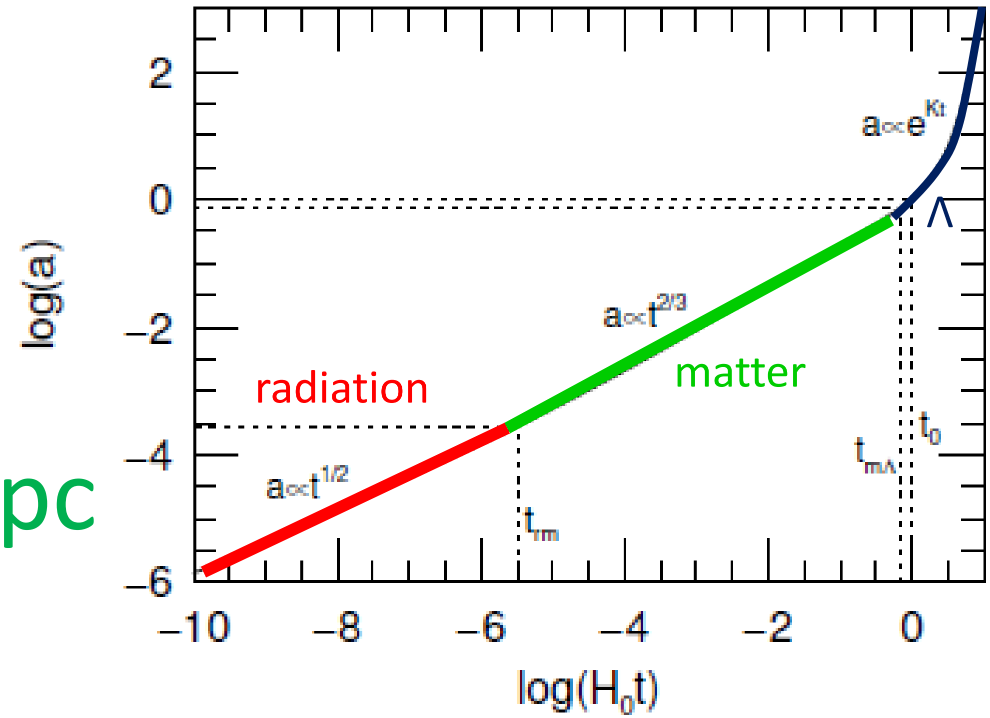
$$\boxed{\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \frac{1-\Omega_{m,0}-\Omega_{\Lambda,0}}{a^2} + \Omega_{\Lambda,0}}$$

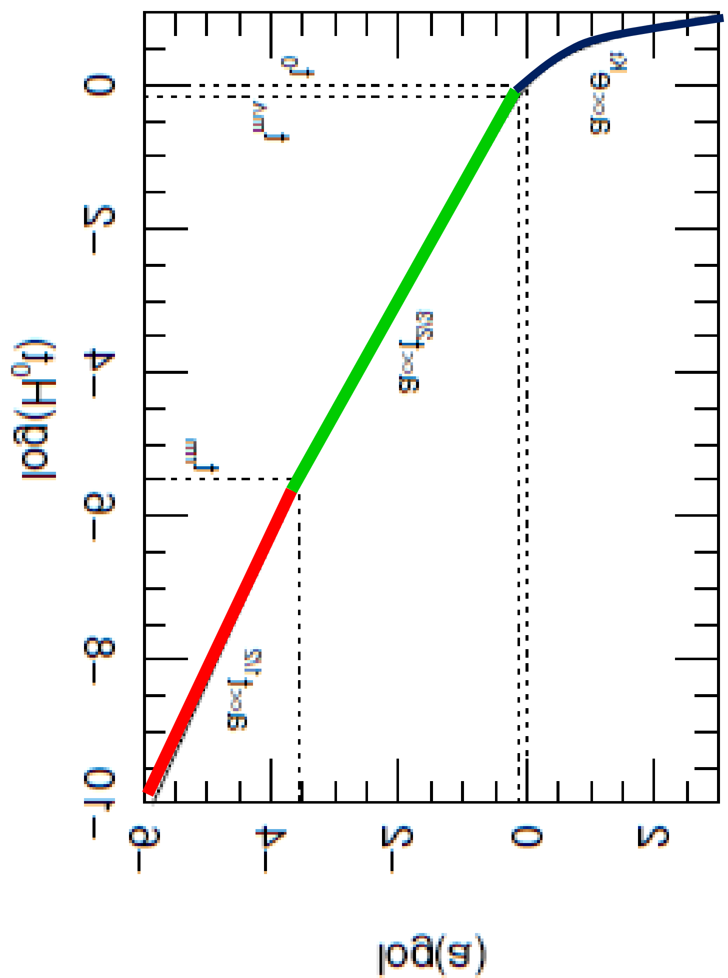
Flat

$\Omega_{\Lambda 0} = 0.7$

$T_0 = 2.725\text{K}$

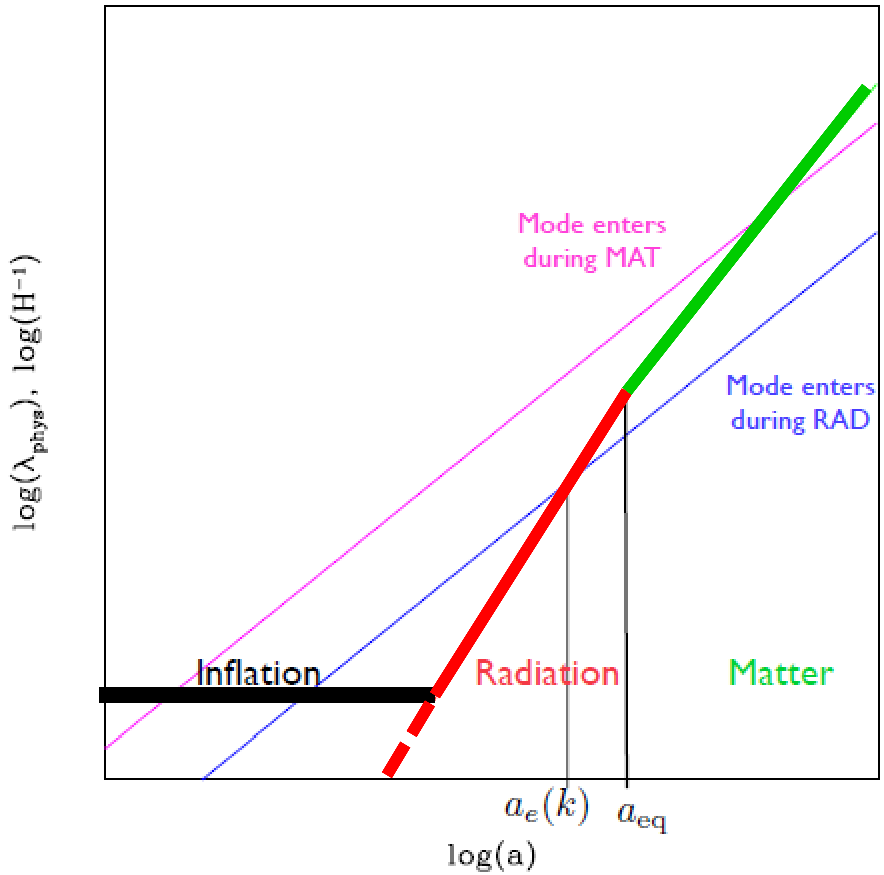
$H_0 = 70$ km/s/Mpc





log(a)
log(H$_0$t)
a∝t$^{1/2}$
a∝t$^{2/3}$
a∝e^{Ht}

Different wavelengths enter horizon at different times



Sub-horizon: Linear theory

- Newtonian analysis:

  $d^2R/dt^2 = - GM/R^2(t) = - (4\pi/3)\, G\rho(t)R(t)\, [1+\delta(t)]$
- M constant means $R^3 \propto \rho^{-1} [1+\delta]^{-1} \propto a^3 [1+\delta]^{-1}$
- I.e., $R \propto a [1+\delta]^{-1/3}$ so $dR/dt \propto HR - d\delta/dt\, (R/3)\, [1+\delta]^{-1}$ and when $|\delta| << 1$ then

  $(d^2R/dt^2)/R = (d^2a/dt^2)/a - (d^2\delta/dt^2)/3 - (2/3)H\, (d\delta/dt)$

  $= - (4\pi/3)\, G\rho(t)\, [1+\delta(t)]$
- Friedmann equation: $(d^2a/dt^2)/a = - (4\pi/3)\, G\rho(t)$ so

  $(d^2\delta/dt^2) + 2H\, (d\delta/dt) = 4\pi\, G\rho(t)\, \delta(t) = (3/2)\, \Omega_m H^2\, \delta(t)$

Linear theory (contd.)

- When radiation dominated ($H = 1/2t$):

$$(d^2\delta/dt^2) + 2H\,(d\delta/dt) = (d^2\delta/dt^2) + (d\delta/dt)/t = 0$$

$$\delta(t) = C_1 + C_2 \ln(t) \quad \text{(weak growth)}$$

- In distant future (H = constant):

$$(d^2\delta/dt^2) + 2H_\Lambda(d\delta/dt) = 0$$

$$\delta(t) = C_1 + C_2 \exp(-2H_\Lambda t)$$

- If flat matter dominated ($H = 2/3t$):

$$\delta(t) = D_+\, t^{2/3} + D_-\, t^{-1} \propto a(t) \quad \text{at late times}$$

- Because linear growth just multiplicative factor, it cannot explain non-Gaussianity at late times

Super-horizon growth

- Start with Friedmann equation when $\kappa=0$:

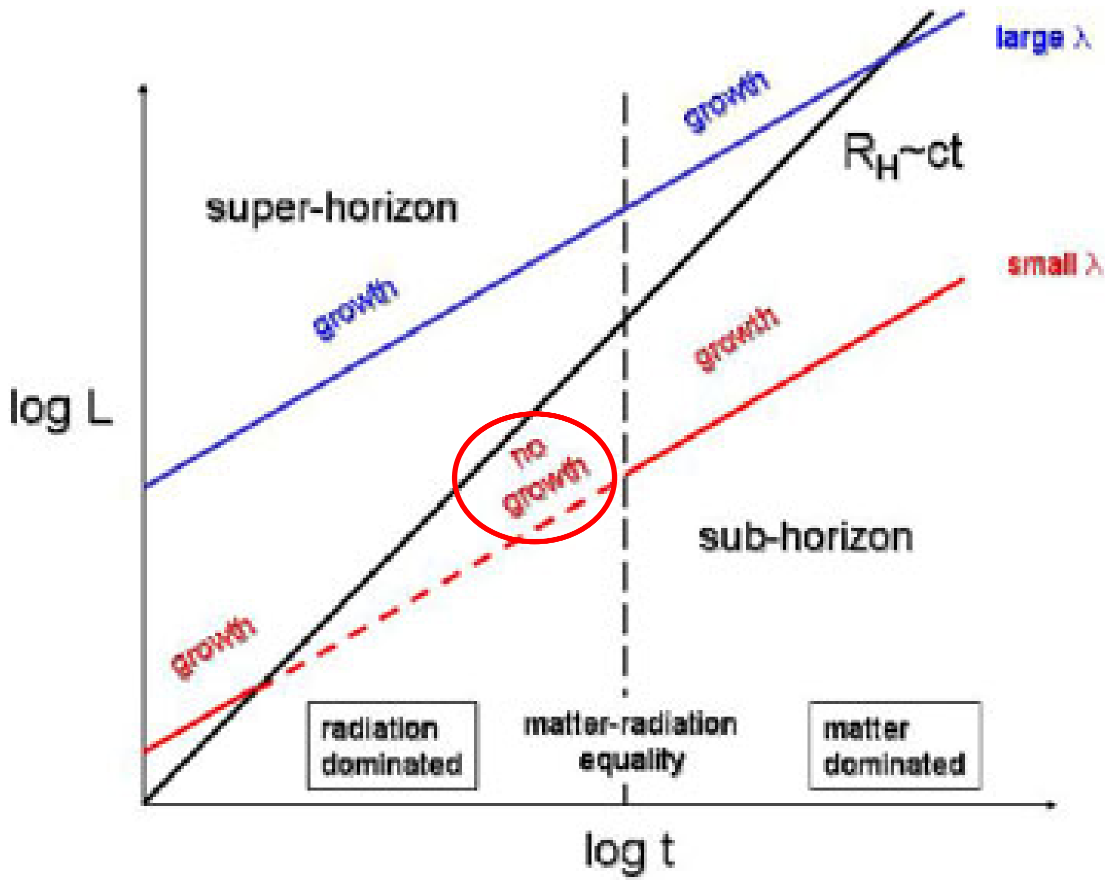
$$H^2 = (8\pi G/3)\ \rho$$

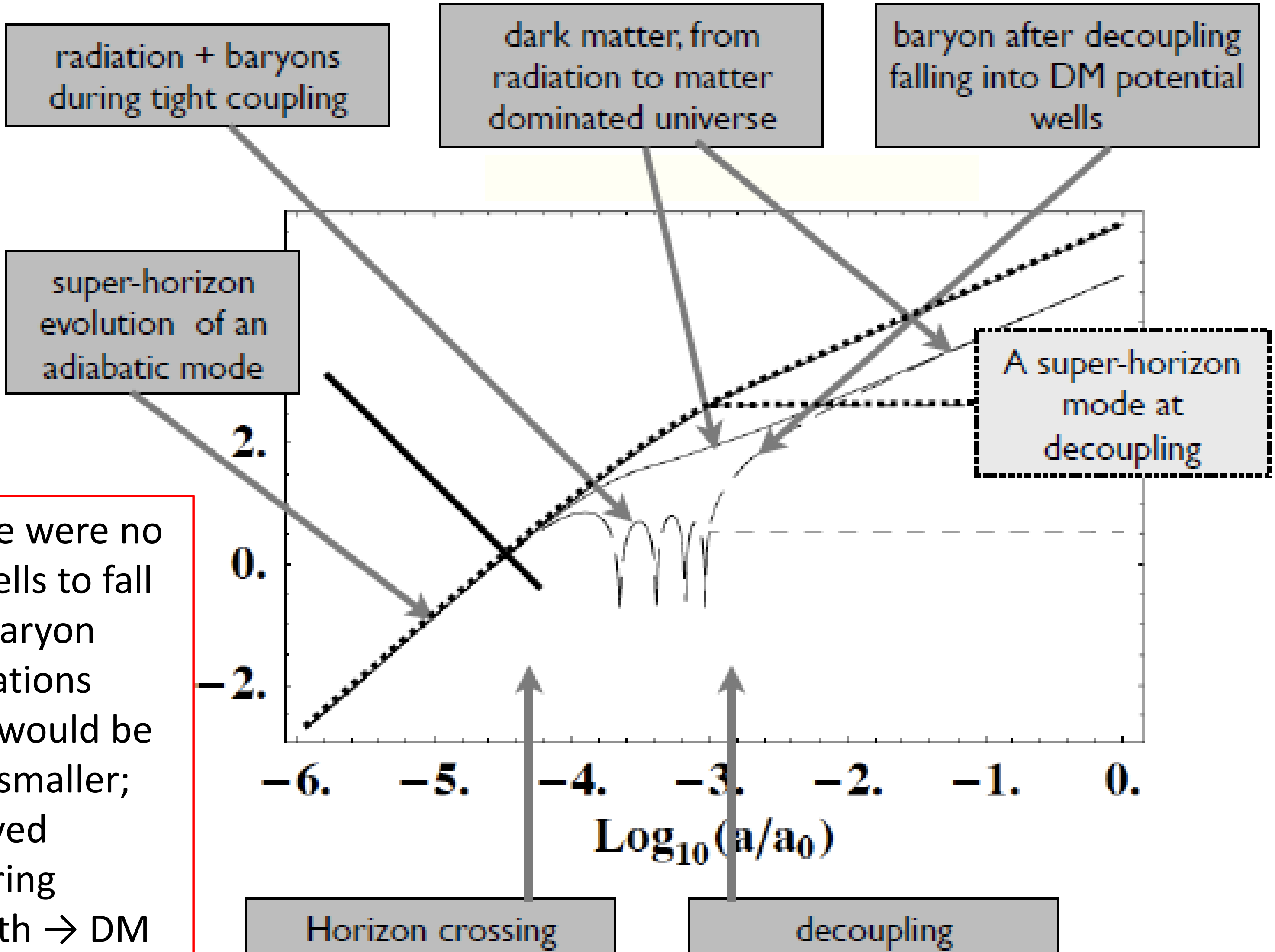
- Now consider a model with same H but slightly higher ρ (so it is a closed universe):

$$H^2 = 8\pi G\rho_1/3 - \kappa/a^2$$

- Then $\delta = (\rho_1 - \rho)/\rho = (\kappa/a^2)/(8\pi G\rho/3)$
- For small δ we have $\delta \propto a$ (matter dominated) but $\delta \propto a^2$ (radiation dominated)

Long and short modes enter horizon at different times, so will grow differently





If there were no DM wells to fall into, baryon fluctuations today would be much smaller; observed clustering strength → DM must exist!

Putting it together

- Consider two modes, λ_1 and $\lambda_2 < \lambda_1$, which entered at $a_1/a_2 = \lambda_1/\lambda_2$ while radiation dominated
- Their amplitudes will be $(a_1/a_2)^2 = (k_2/k_1)^2$ so expect suppression of power $\propto k^{-2}$ at $k>k_{eq}$ (i.e. for the short wavelength modes which entered earlier)
- After entering horizon, dark matter grows only logarithmically until matter domination, after which it grows $\propto a$
- Baryons oscillate (i.e. don't grow) until decoupling, after which they fall into the deeper wells defined by the dark matter

Transfer function is approximately

$T_{CDM}(k) \propto 1/[1+(k/k_{eq})^2]$

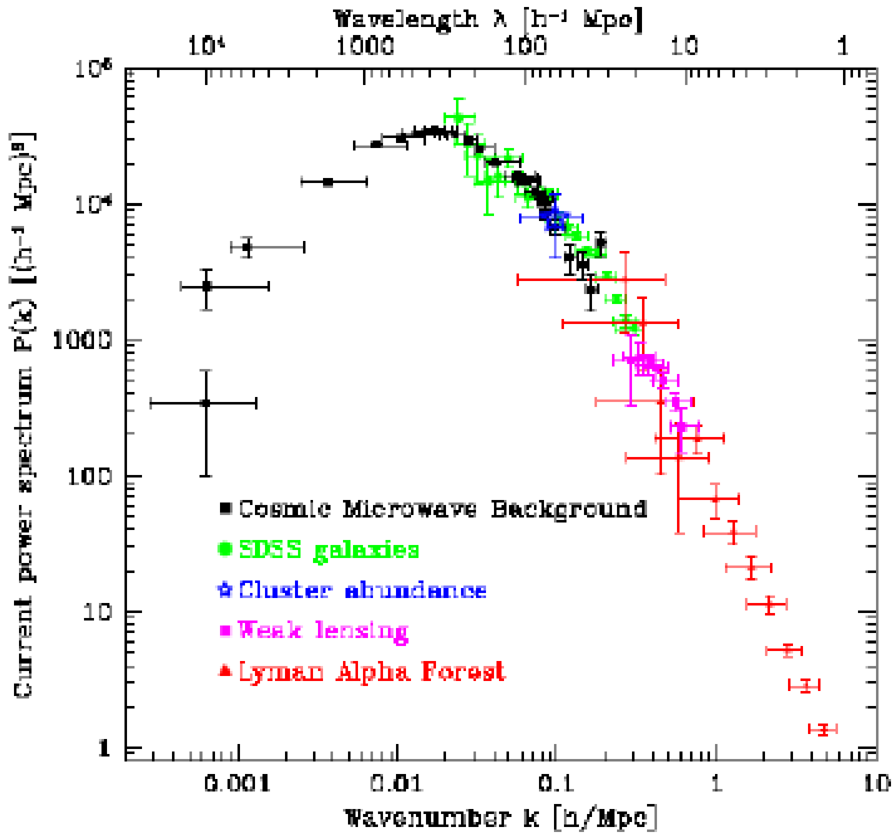
$P(k) \propto k\, T_{CDM}{}^2(k)$

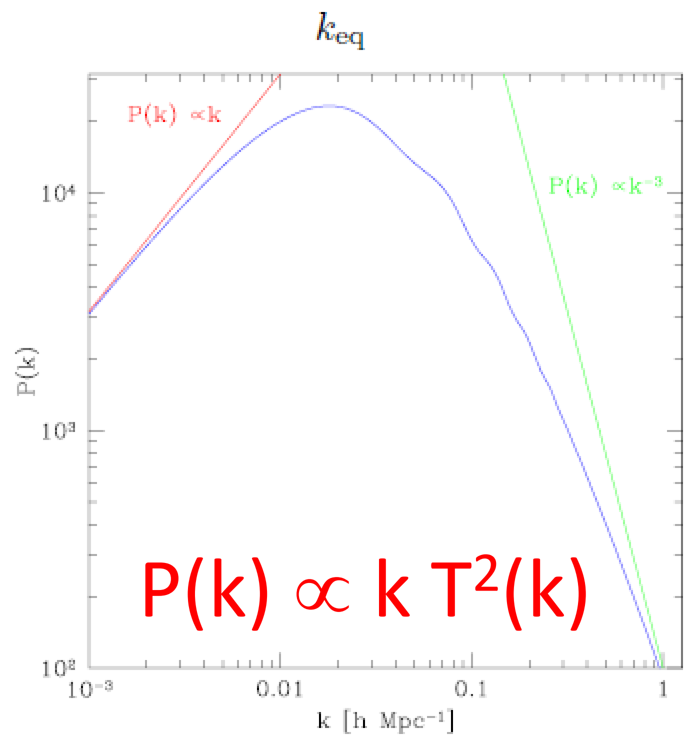
FT of $T_{CDM} = (k_{eq}{}^{-3}/4\pi)\ \exp(-rk_{eq})/rk_{eq}$

so might wish to think of T_{CDM} as describing 'smoothing' on scale R_{eq}

Similarly, sometimes useful to think of P(k) as 'smoothing' of 'white-noise' field to obtain field with correlations

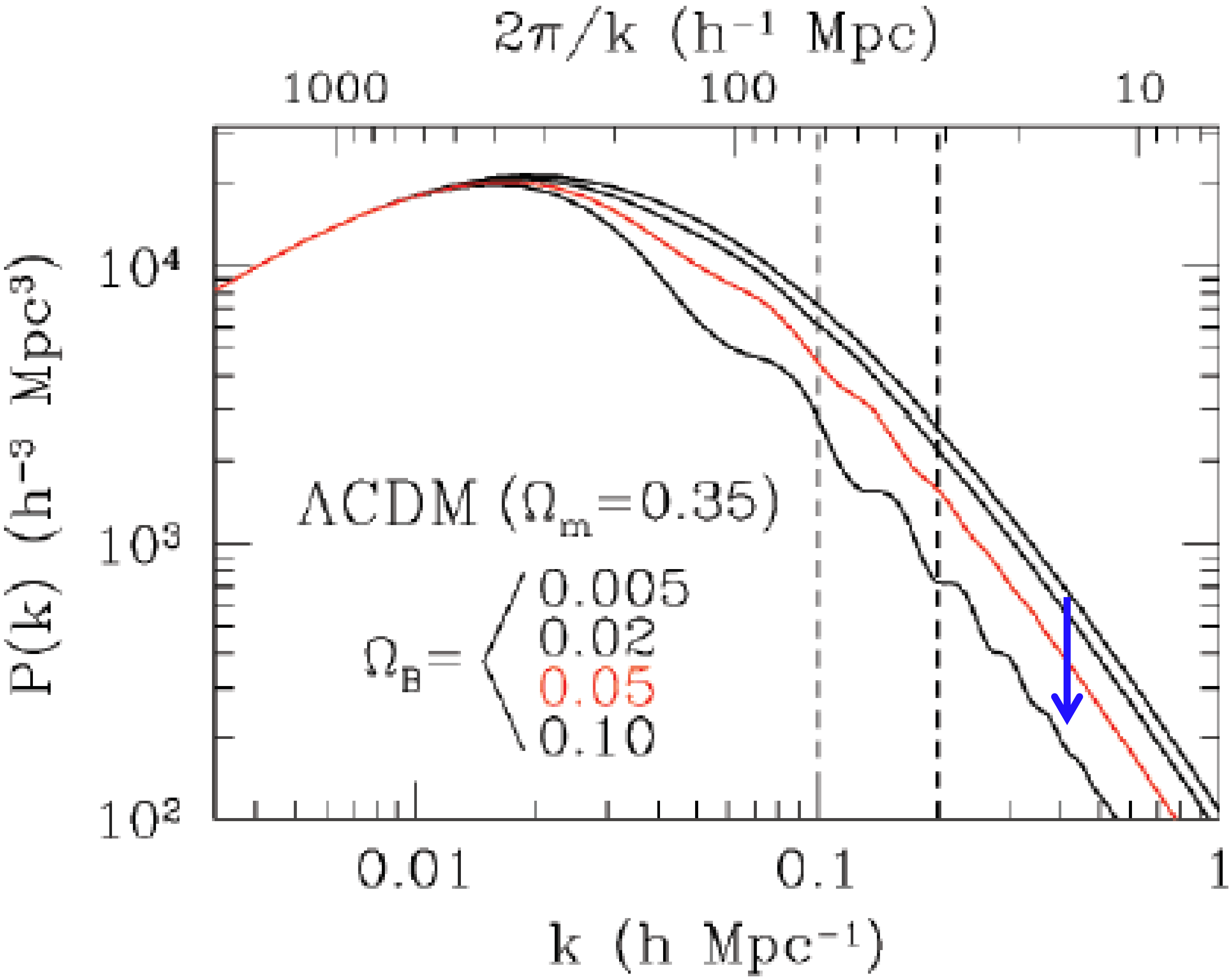
Transfer function:
$T_{CDM}(k) \propto 1/[1+(k/k_{eq})^2]$





$$T_{WDM}(k) \approx T_{CDM}(k)\,[1 + (\alpha k)^2]^{-5}$$

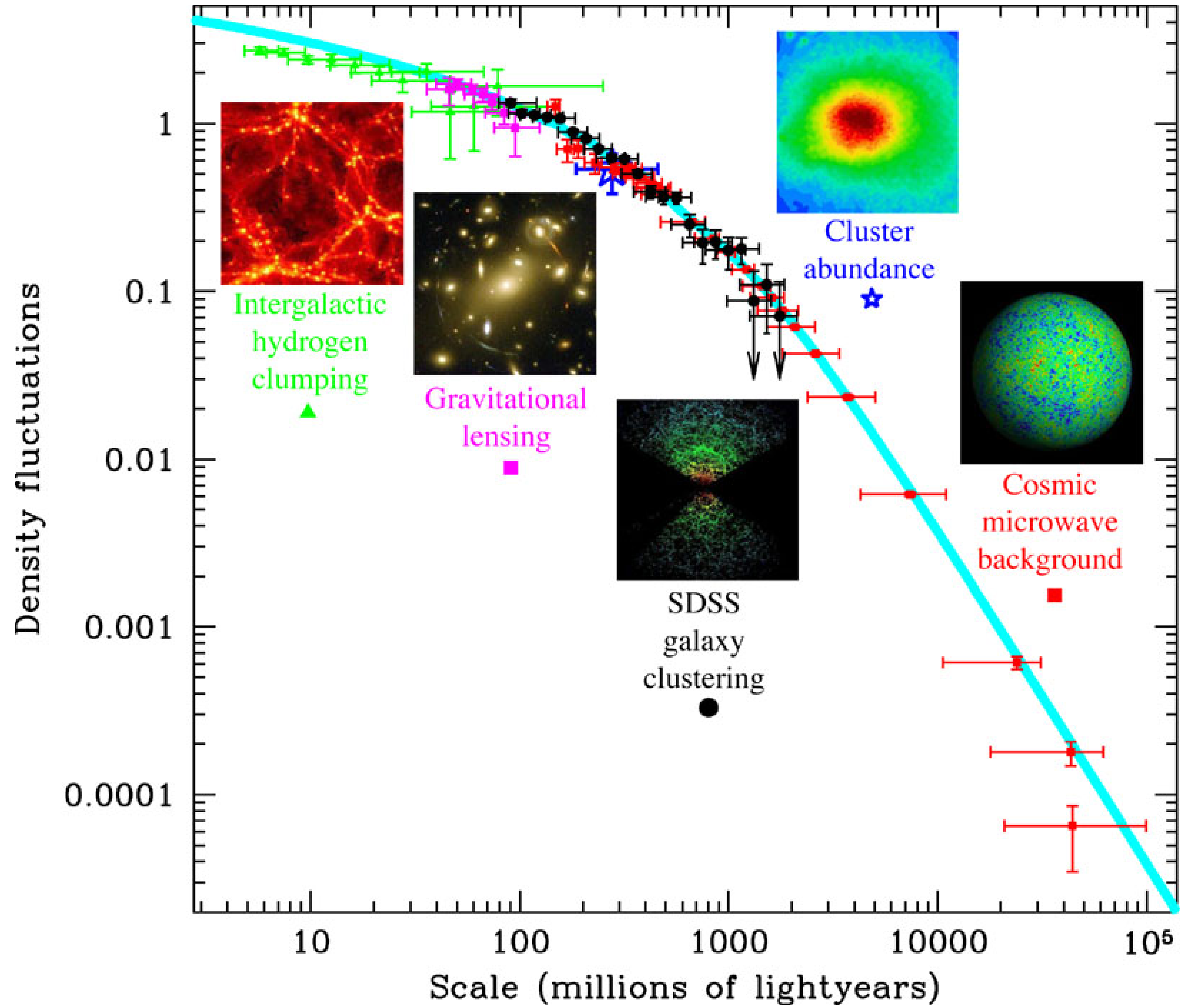
$$\alpha \equiv 0.05 \left(\frac{\Omega_m}{0.4}\right)^{0.15} \left(\frac{h}{0.65}\right)^{1.3} \left(\frac{m_{\rm dm}}{1\,{\rm keV}}\right)^{-1.15} h^{-1}{\rm Mpc}$$



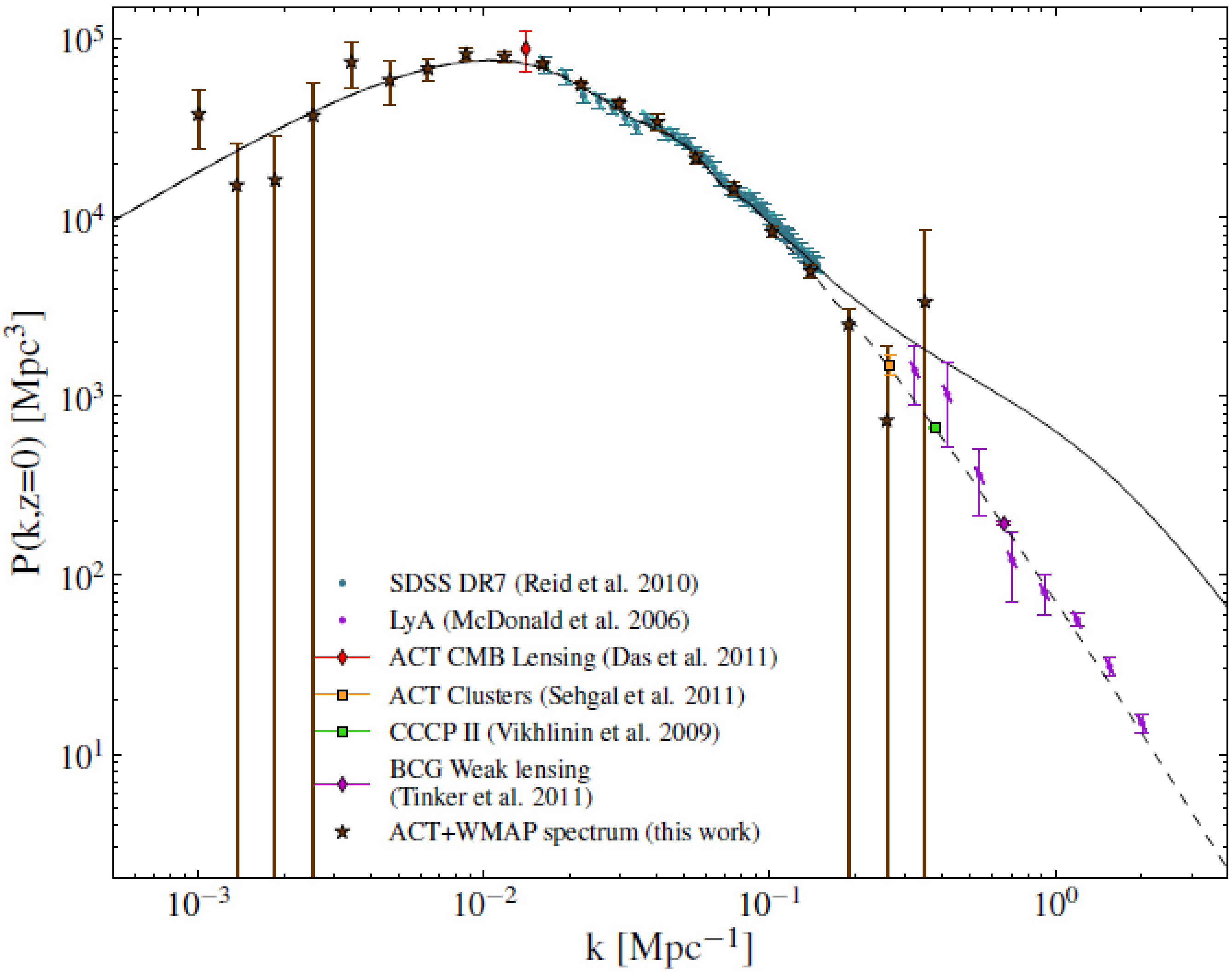
If all matter baryonic, power below 200 Mpc/h is suppressed

Need nonbaryonic gravitating dark matter to explain structure formation

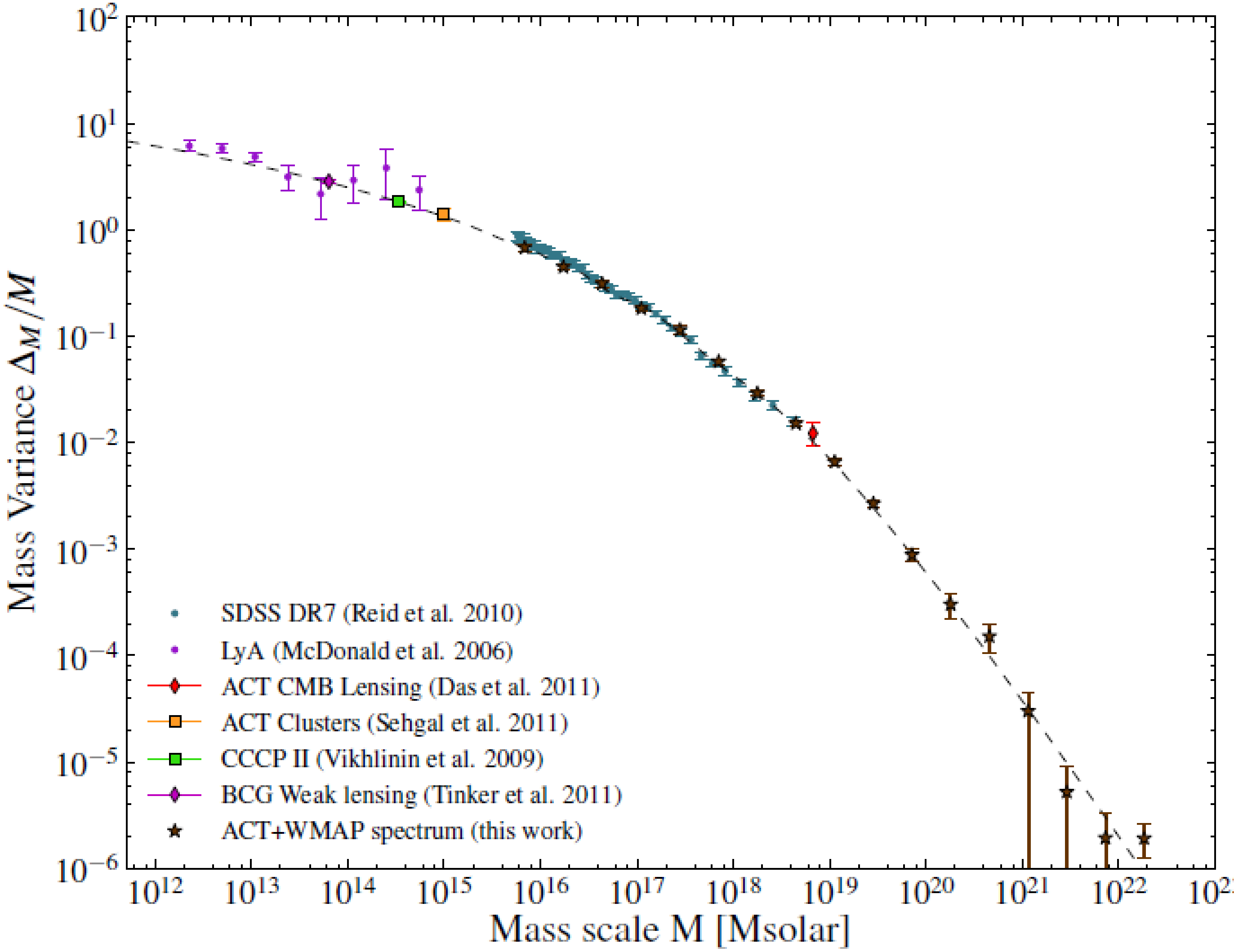
Same, but position- (rather than k-) space



$$\sigma^2(r) = (2\pi)^{-3} \int dk\, 4\pi k^2\, P(k)\, W^2(kr) \qquad W(x) \sim (3/x)\, j_1(x)$$



P(k,z=0) [Mpc^3]
k [Mpc^{-1}]
10^5
10^4
10^3
10^2
10^1
10^{-3}
10^{-2}
10^{-1}
10^0
SDSS DR7 (Reid et al. 2010)
LyA (McDonald et al. 2006)
ACT CMB Lensing (Das et al. 2011)
ACT Clusters (Sehgal et al. 2011)
CCCP II (Vikhlinin et al. 2009)
BCG Weak lensing
(Tinker et al. 2011)
ACT+WMAP spectrum (this work)



Mass Variance Δ_M/M
Mass scale M [Msolar]
SDSS DR7 (Reid et al. 2010)
LyA (McDonald et al. 2006)
ACT CMB Lensing (Das et al. 2011)
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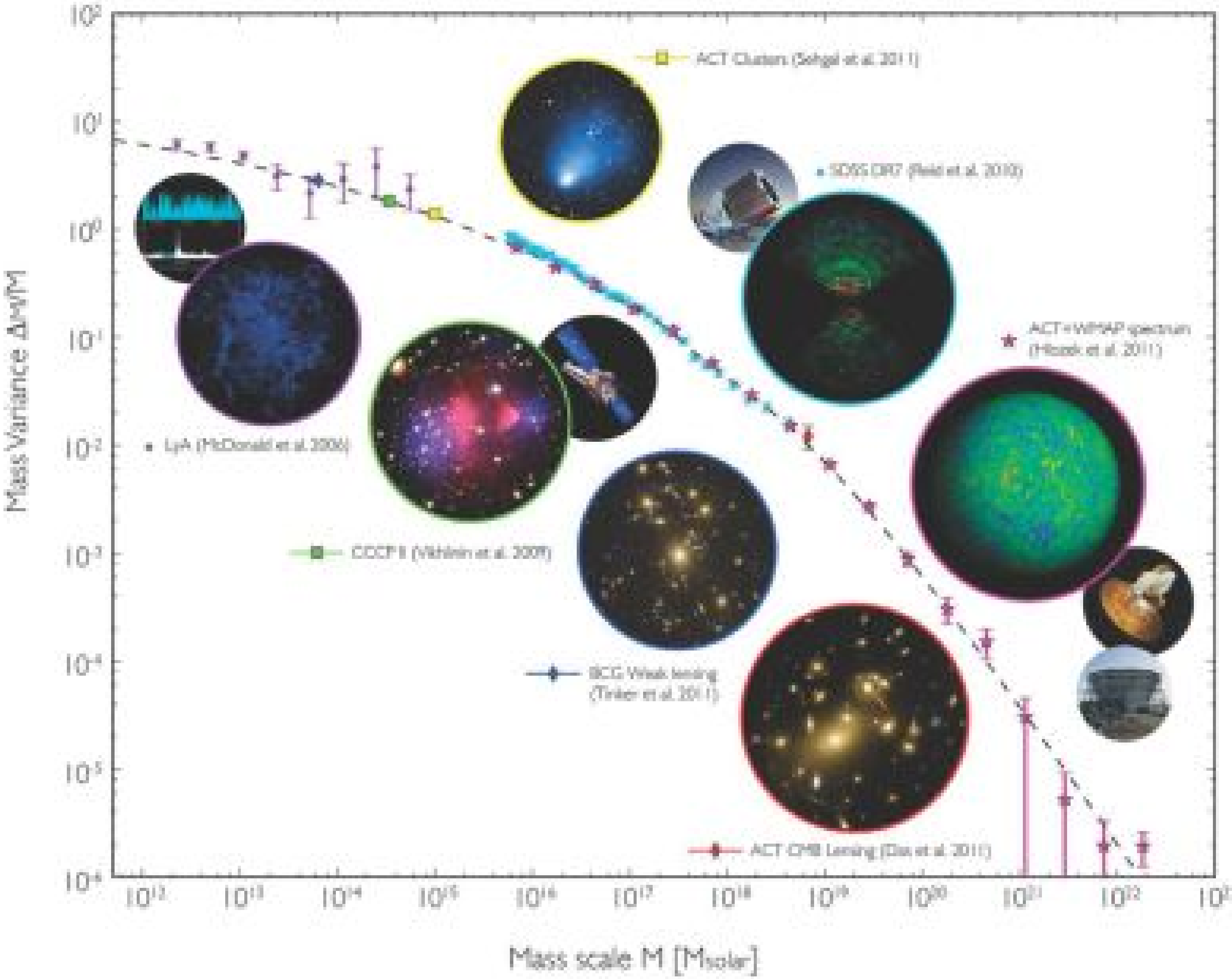

ACT Clusters (Sehgal et al. 2011)
SDSS DR7 (Reid et al. 2010)
ACT+WMAP spectrum (Hlozek et al. 2011)
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ACT CMB Lensing (Das et al. 2011)
Mass Variance ΔM/M
Mass scale M [Msolar]