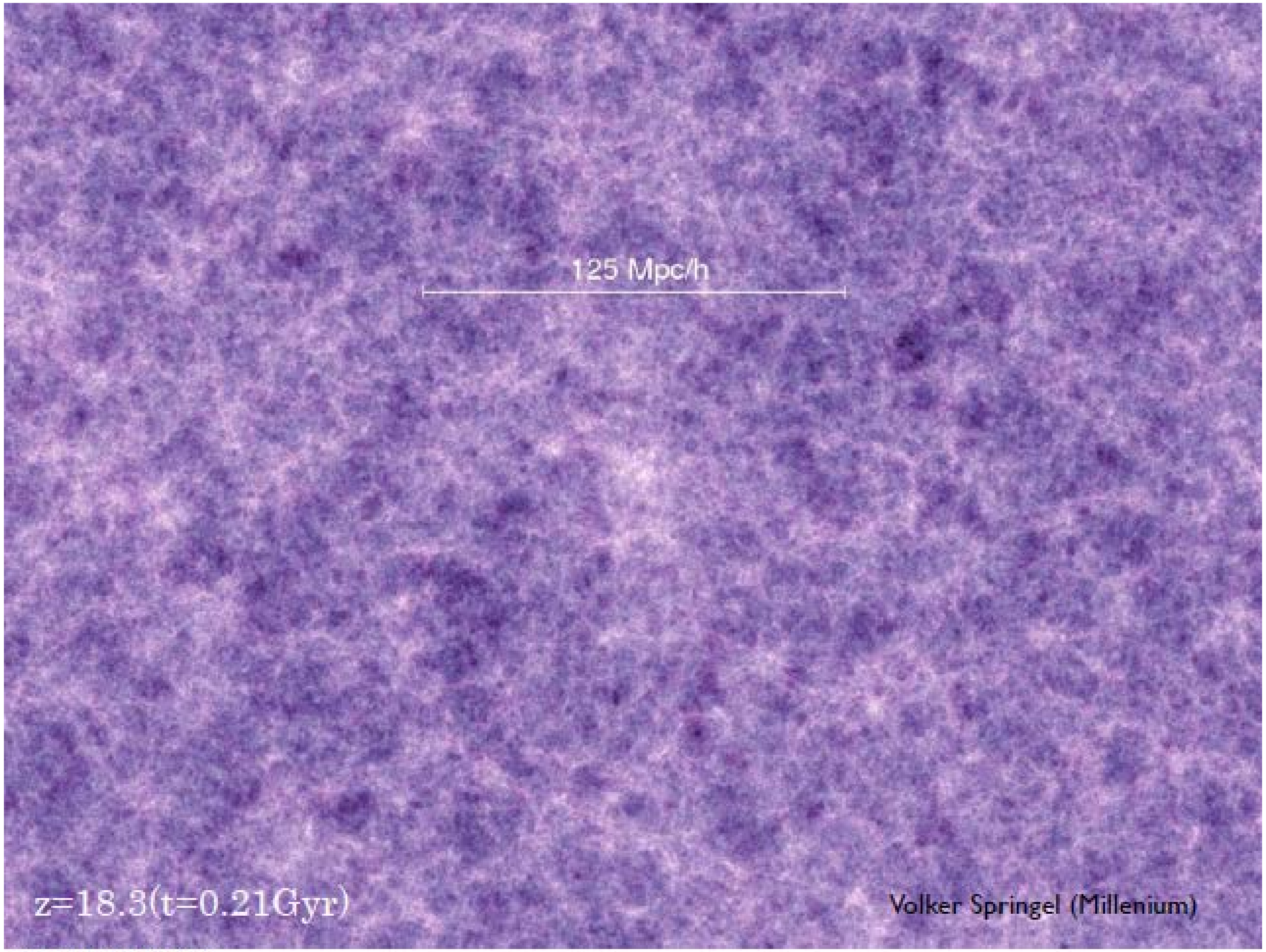
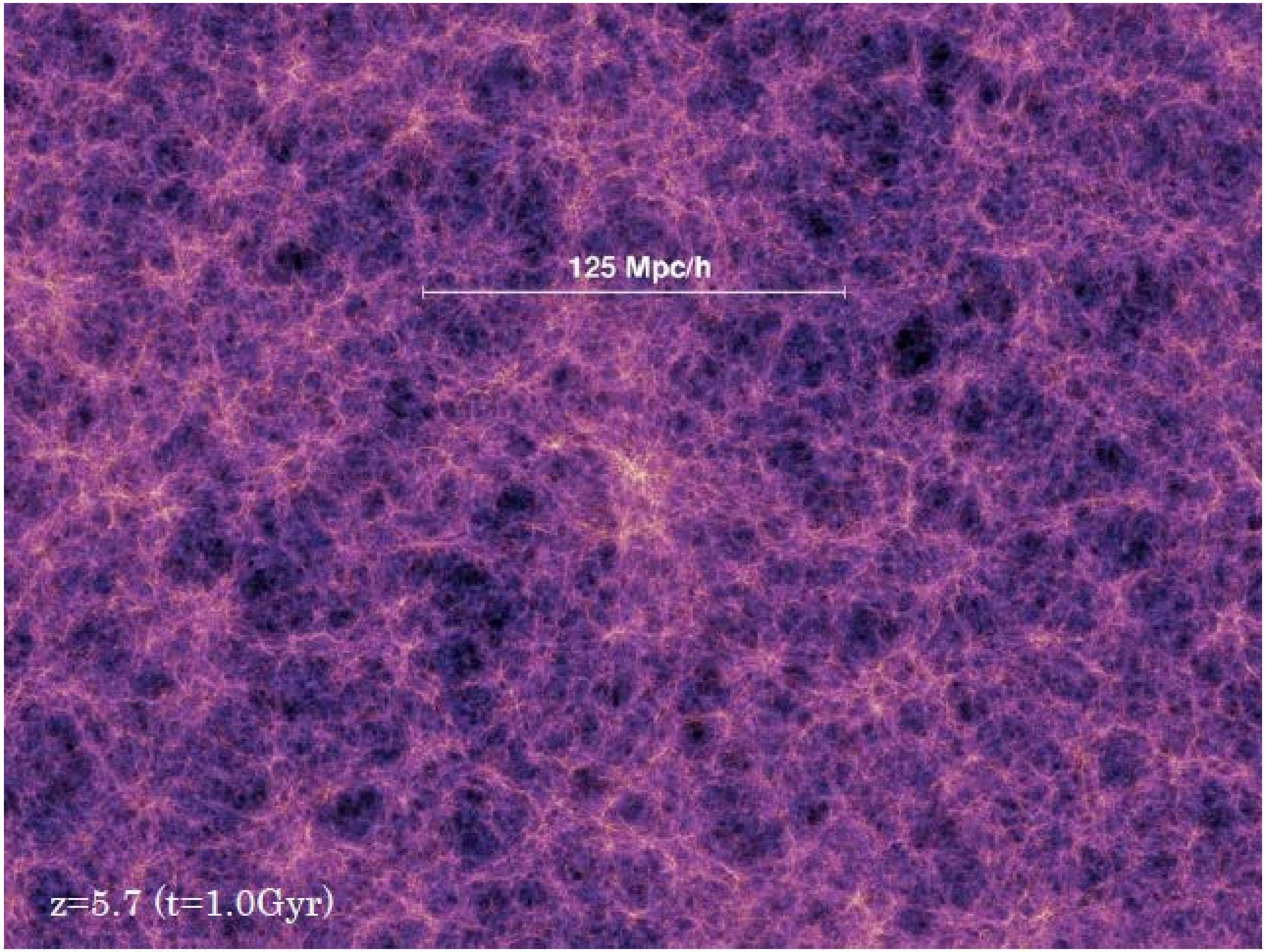
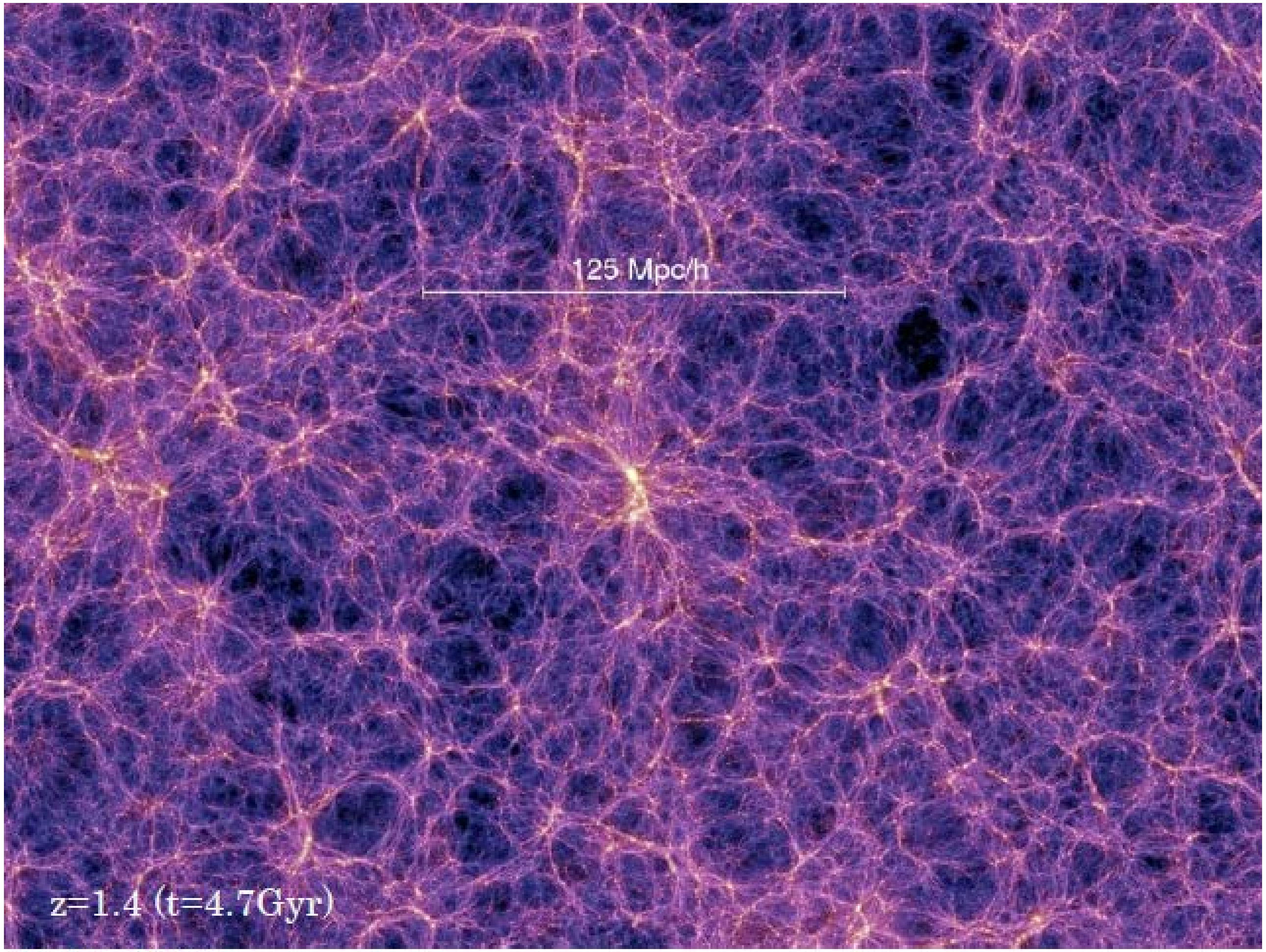
The excursion set approach

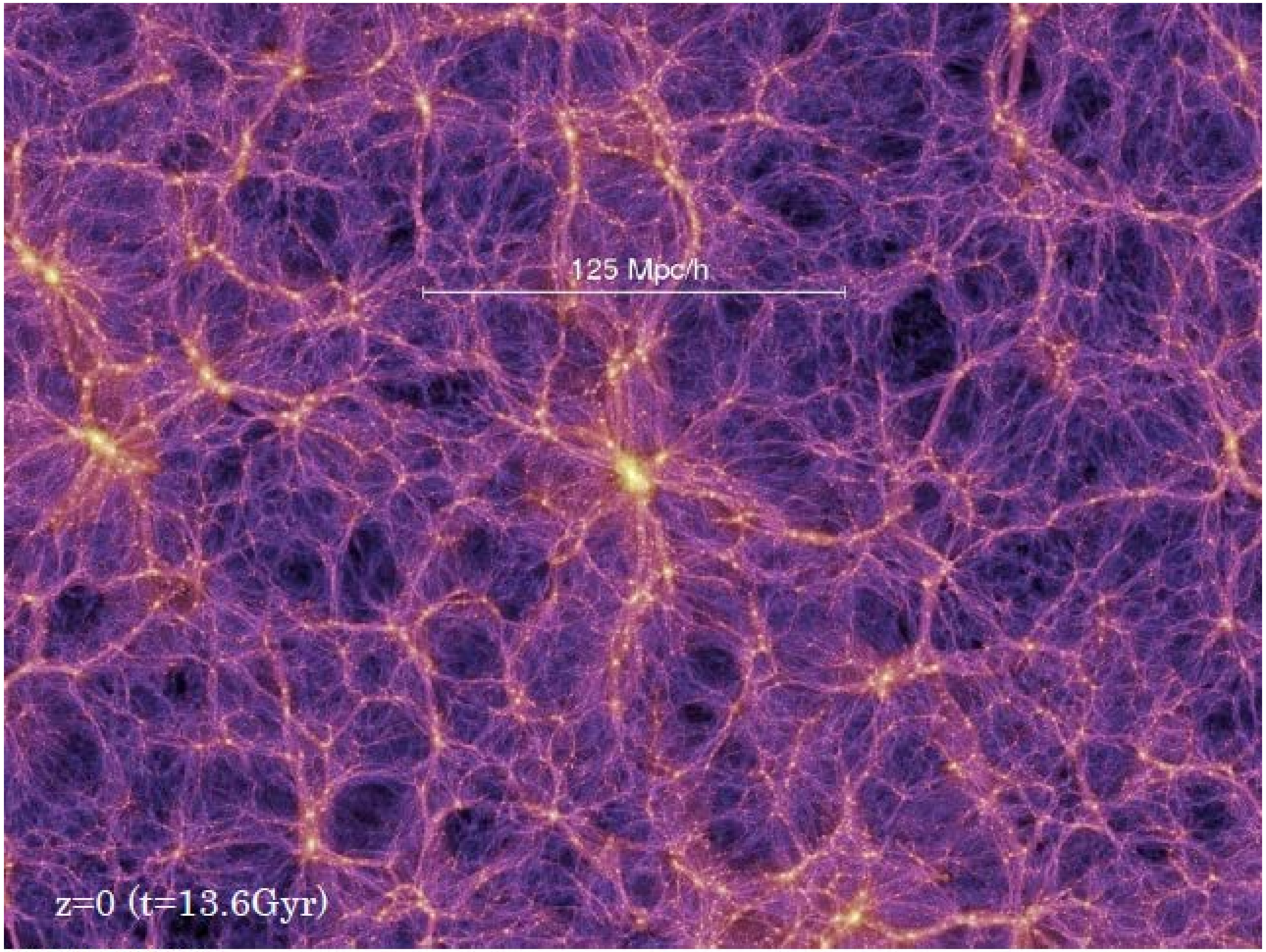
Halo abundances

Halo clustering/bias


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)


125 Mpc/h
z=5.7 (t=1.0Gyr)

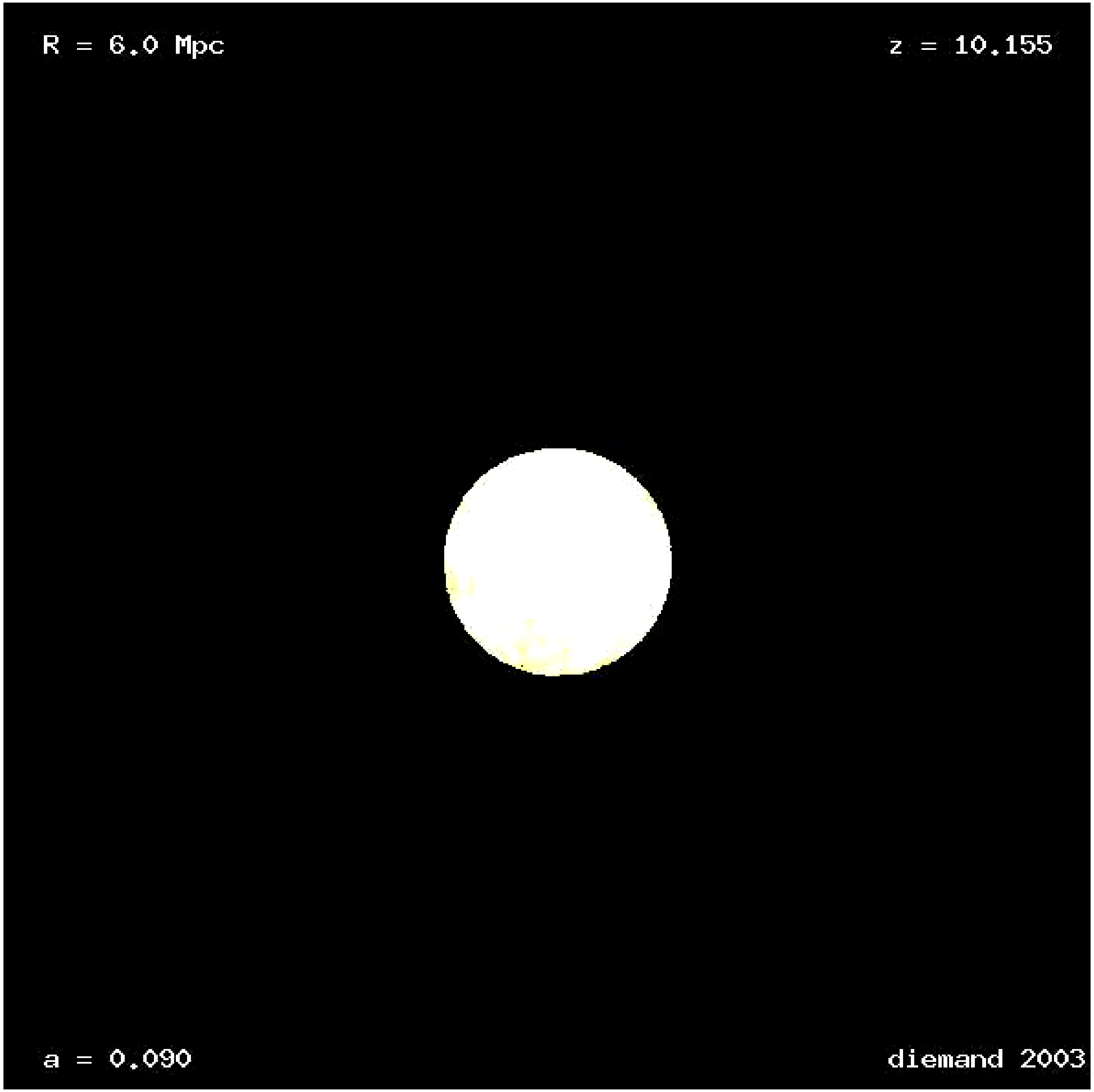

125 Mpc/h
z=1.4 (t=4.7Gyr)


125 Mpc/h
z=0 (t=13.6Gyr)

N-body simulations of

gravitational clustering

in an expanding universe



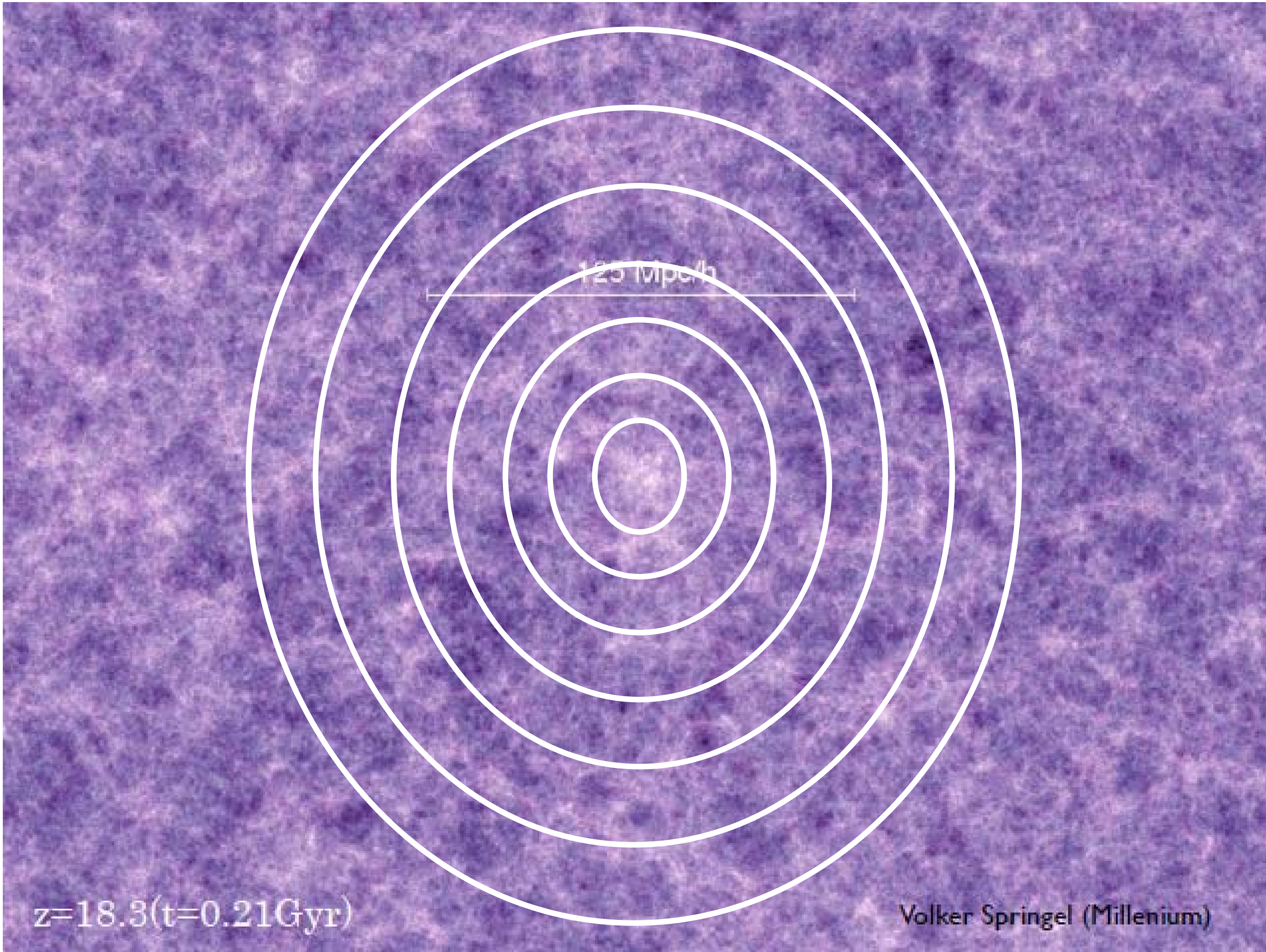
Why study halos?

- Cluster counts contain information about volume and about how gravity won/lost compared to expansion
- Probe geometry and expansion history of Universe, and nature of gravity

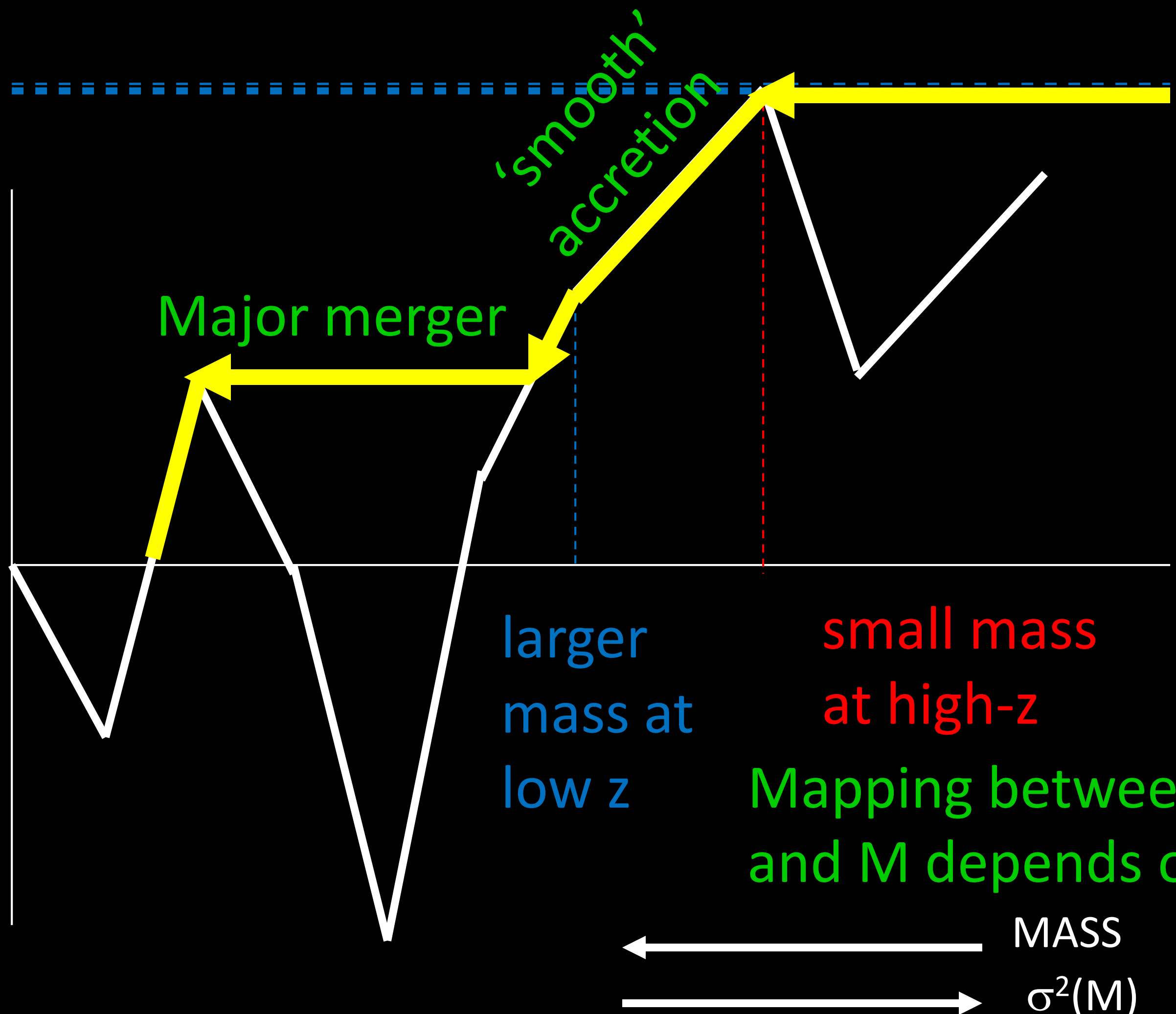
Massive halo = Galaxy cluster

(Simpler than studying galaxies? Less gastrophysics?)


But wait …
We should be doing
this in the INITIAL
fluctuation field!


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)

The excursion set approach

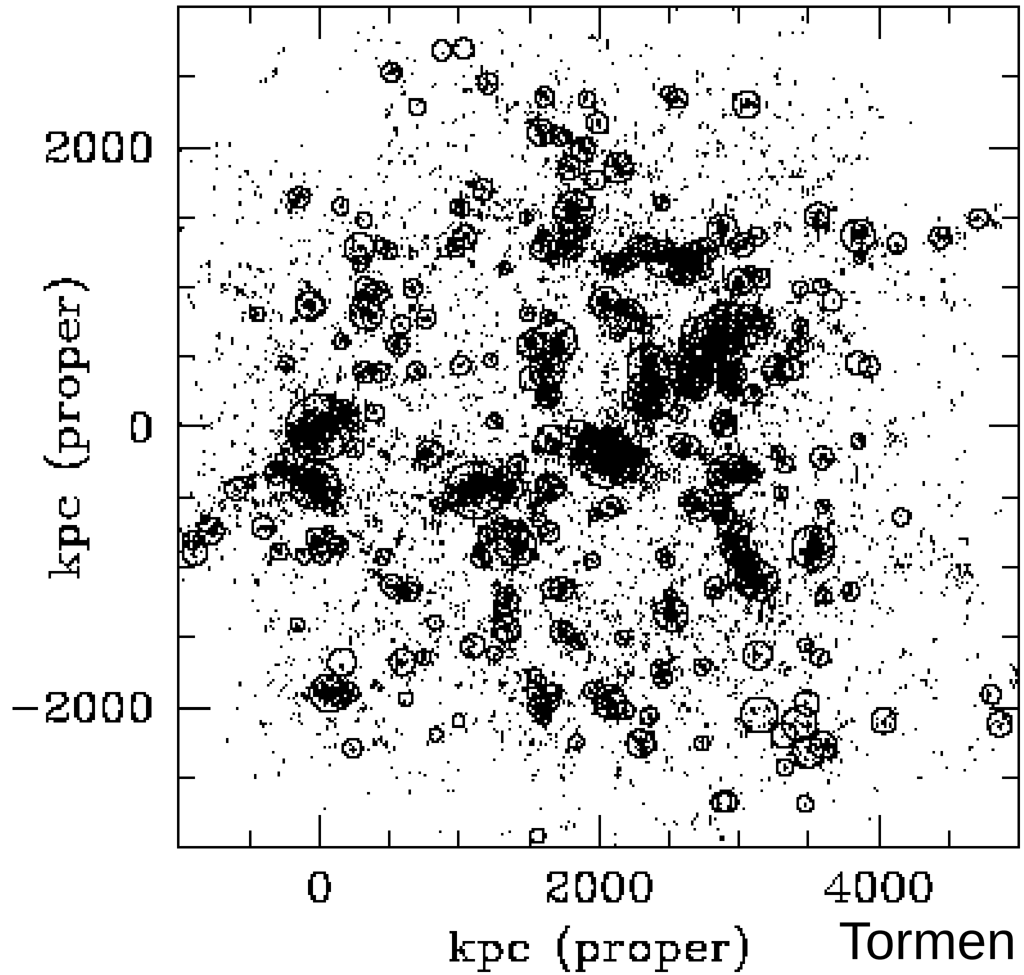


Simplification because…

- Everything local
- Evolution determined by cosmology (competition between gravity and expansion)
- Statistics determined by initial fluctuation field: for Gaussian, specified by initial power-spectrum $P(k)$
- Nearly universal in scaled units: $\delta_c(z)/\sigma(m)$ where $\sigma^2(m) = \langle\delta_m^2\rangle = \int dk/k\ k^3P(k)/2\pi^2\ W^2(kR_m)$ $m \propto R_m^3$
- Fact that only very fat cows are spherical is a detail (*crucial* for precision cosmology); in excursion set approach, mass-dependent barrier height increases with distance along walk

Spherical evolution model

- ‘Collapse’ depends on initial over-density Δ_i; same for all initial sizes
- Critical density depends on cosmology
- Final objects all have same density, whatever their initial sizes
- Collapsed objects called halos are ~ 200× denser than critical (background?!), whatever their mass



Tormen 1997

(Figure shows particles at z~2 which, at z~0, are in a cluster)

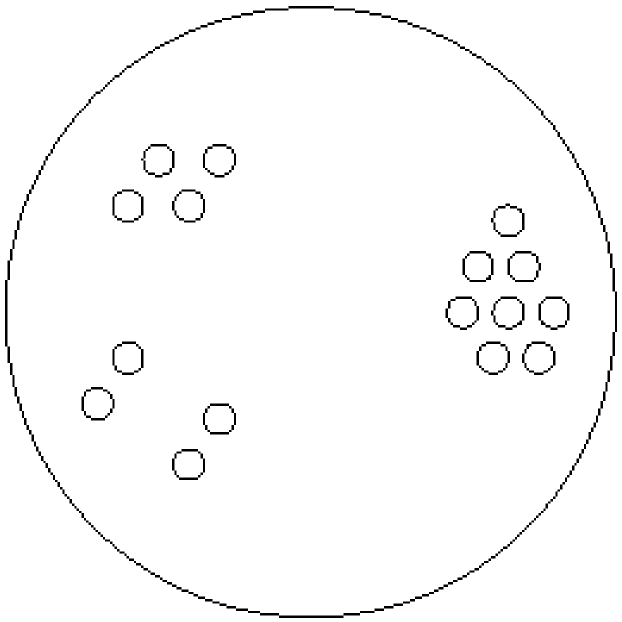


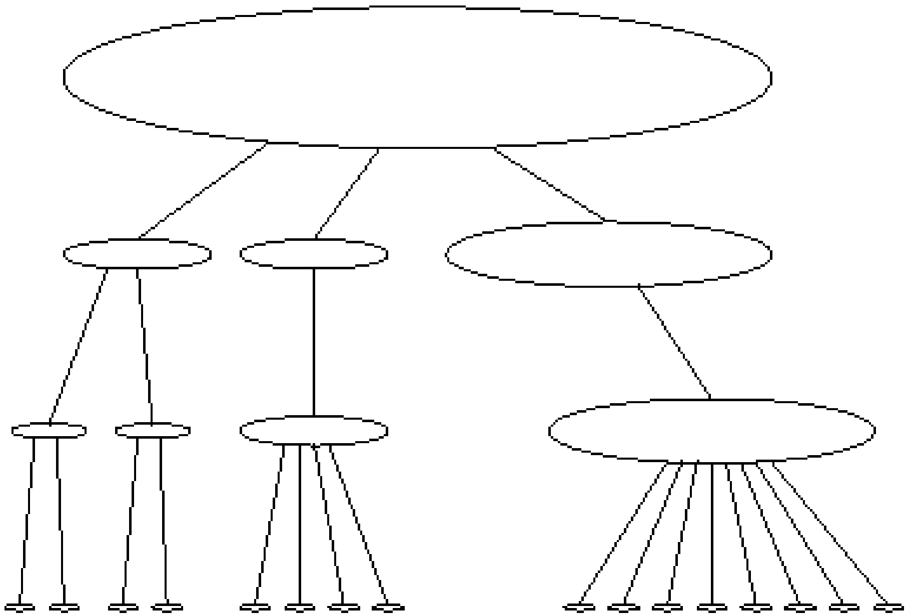




Assume a spherical herd of spherical cows…

Initial spatial distribution within patch (at z~1000)...





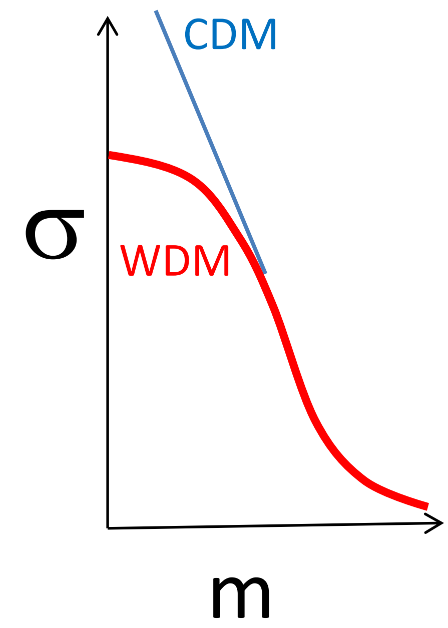
…stochastic (initial conditions Gaussian random field); study ‘forest’ of merger history ‘trees’.

…encodes information about subsequent ‘merger history’ of object

(Mo & White 1996; Sheth 1996)

For WDM …

- At small enough m, $\sigma(m)$ is flat
- Fraction of walks which didn't cross barrier prior to this σ = non-negligible smooth component which was never bound to anything
- f_{smooth} should be larger at high z
- Fewer halos (progenitors) at high z mean less concentrated halos at low z
- f_{smooth} should be larger in voids = voids are 'emptier' (even more so if $\delta_c(m)$ larger at small m)



Spherical evolution mapping …

$(R_{\text{initial}}/R)^3 = \text{Mass}/(\rho_{\text{com}}\text{Volume}) =$

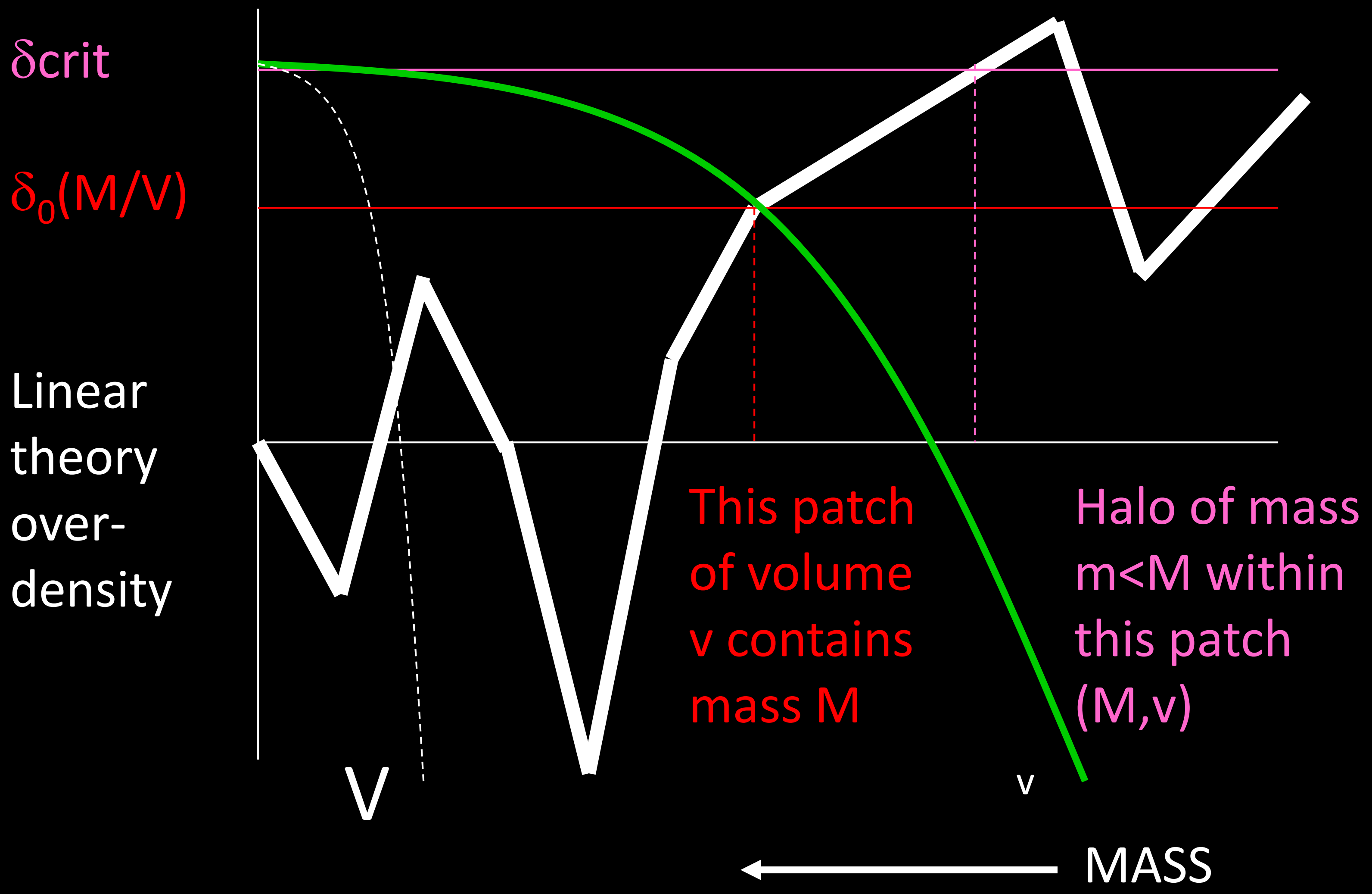
$$1 + \delta \approx (1 - \delta_0/\delta_{sc})^{-\delta sc}$$

… can be inverted:

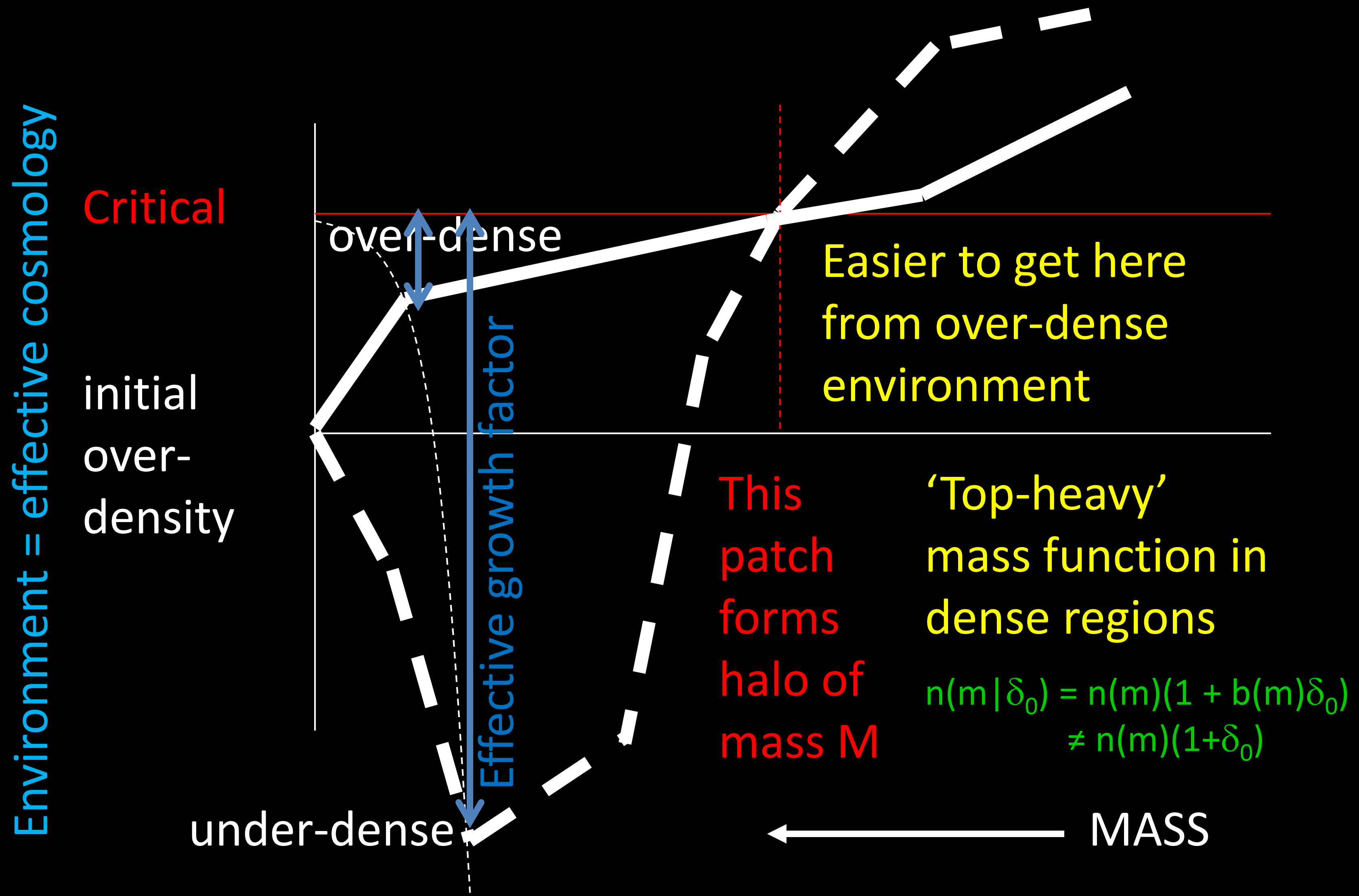
$$(\delta_0/\delta_{sc}) \approx 1 - (M/\rho_{\text{com}}V)^{-1/\delta sc}$$

N.B. For any V, there is a curve $\delta_0(M|V)$.

Moving barriers: The Nonlinear PDF



Correlations with environment



Mon. Not. R. Astron. Soc. 394, 2109–2112 (2009) doi:10.1111/j.1365-2966.2009.14467.x

On the equivalence between the effective cosmology and excursion set treatments of environment

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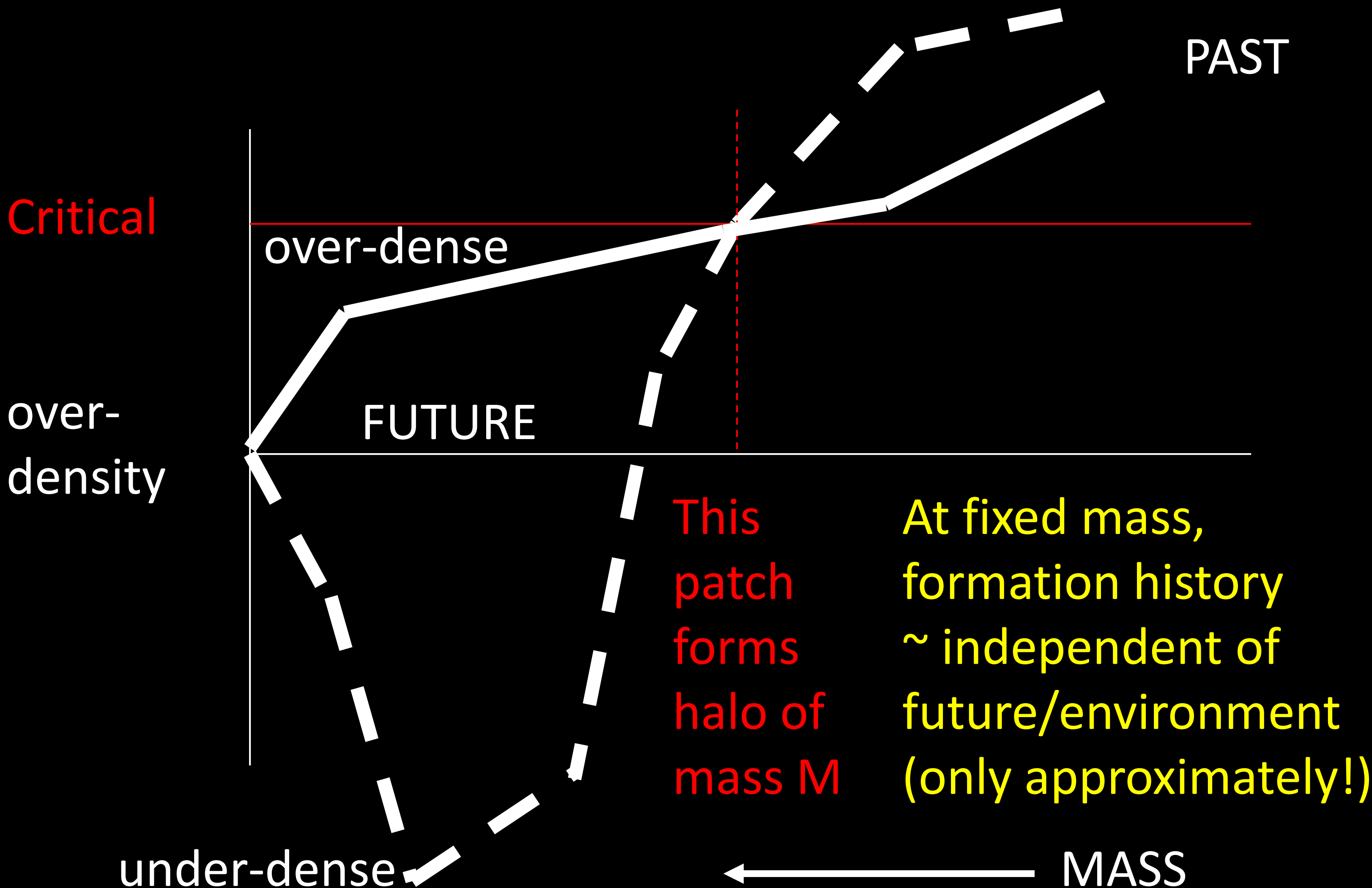
ABSTRACT

In studies of the environmental dependence of structure formation, the large-scale environment is often thought of as providing an effective background cosmology: for example the formation of structure in voids is expected to be just like that in a less dense universe with appropriately modified Hubble and cosmological constants. However, in the excursion set description of structure formation which is commonly used to model this effect, no explicit mention is made of the effective cosmology. Rather, this approach uses the spherical evolution model to compute an effective linear theory growth factor, which is then used to predict the growth and evolution of non-linear structures. We show that these approaches are, in fact, equivalent: a consequence of Birkhoff's theorem. We speculate that this equivalence will not survive in models where the gravitational force law is modified from an inverse square, potentially making the environmental dependence of clustering a good test of such models.

Key words: methods: analytical – dark matter – large-scale structure of Universe.

Correlations with environment



Environmental effects

- In hierarchical models, close connection between evolution and environment (dense region ~ dense universe ~ more evolved)
- Gastrophysics determined by formation history of halo
- Observed correlations with environment test hierarchical galaxy formation models – all environmental effects because massive halos populate densest regions

Assembly bias

- At fixed mass, formation history independent of future/environment *if walks are Markovian* i.e. have uncorrelated steps (White 1996)
- In simulations, at fixed mass, formation history *does* correlate with environment (Sheth & Tormen 2004; Gao et al. 2005; etc.)
- A simple 'Markov Velocities' model captures most of this effect (Musso & Sheth 2014)

Large scale clustering/bias (from the peak-background split)

$$1 + \delta_h(\nu|\delta_0,S_0) = f(\nu|\delta_0,S_0)\,/f(\nu)$$
$$= 1 + b_1(\nu)\delta_0 + \ldots$$

- b(ν) directly from (derivatives of) f(ν) means halo abundances predict halo clustering
- b(ν) increases with ν

→ top-heavy mass function in dense regions:

$$n(m|\delta_0) = n(m)(1 + b(m)\delta_0 + \ldots) \neq n(m)(1+\delta_0)$$

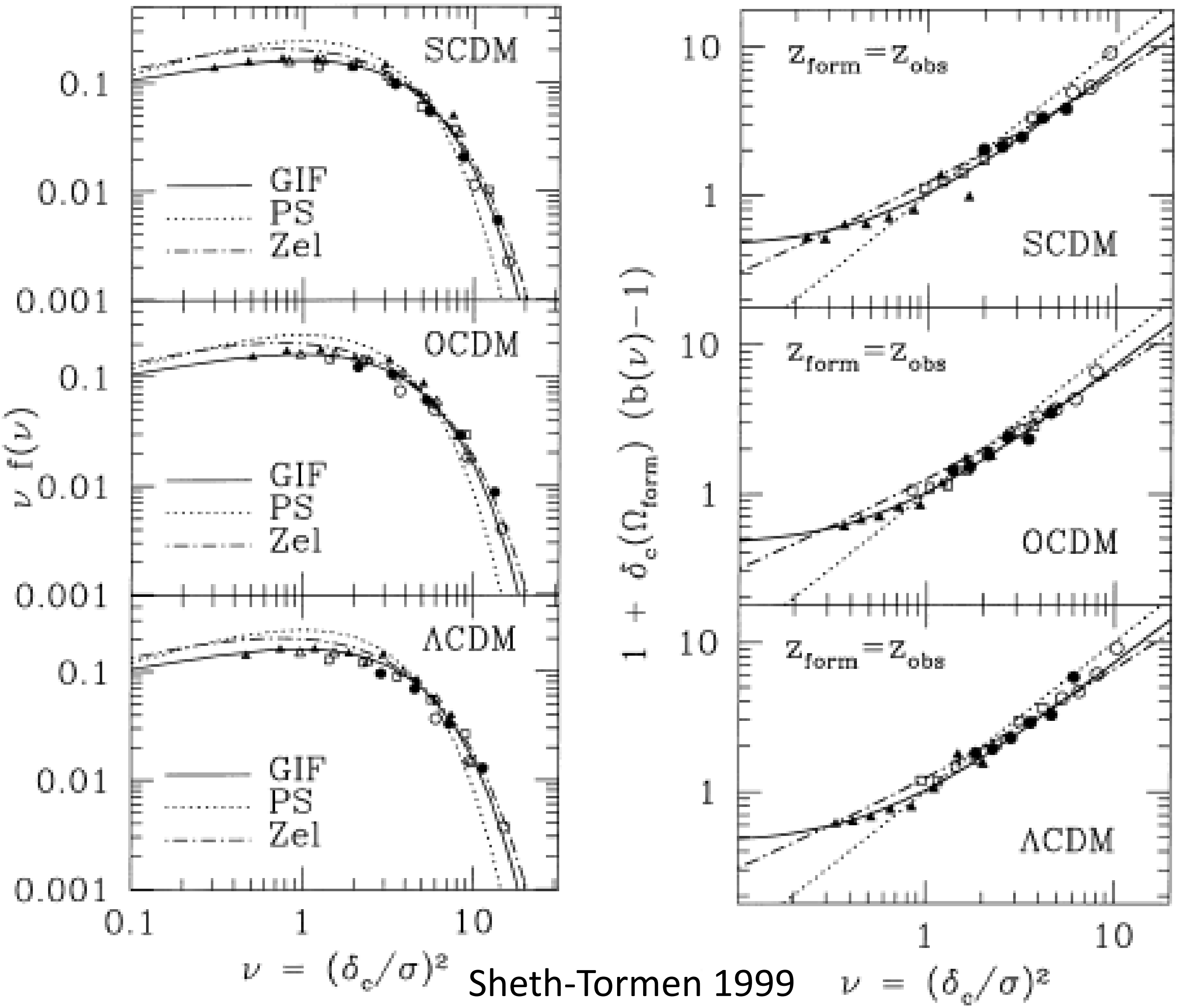
→ massive halos (i.e. larger ν) more clustered:

$$<\delta_h\delta_0> = b_1(\nu) <\delta_0^2> + \ldots$$

(Almost) universal mass function and halo bias

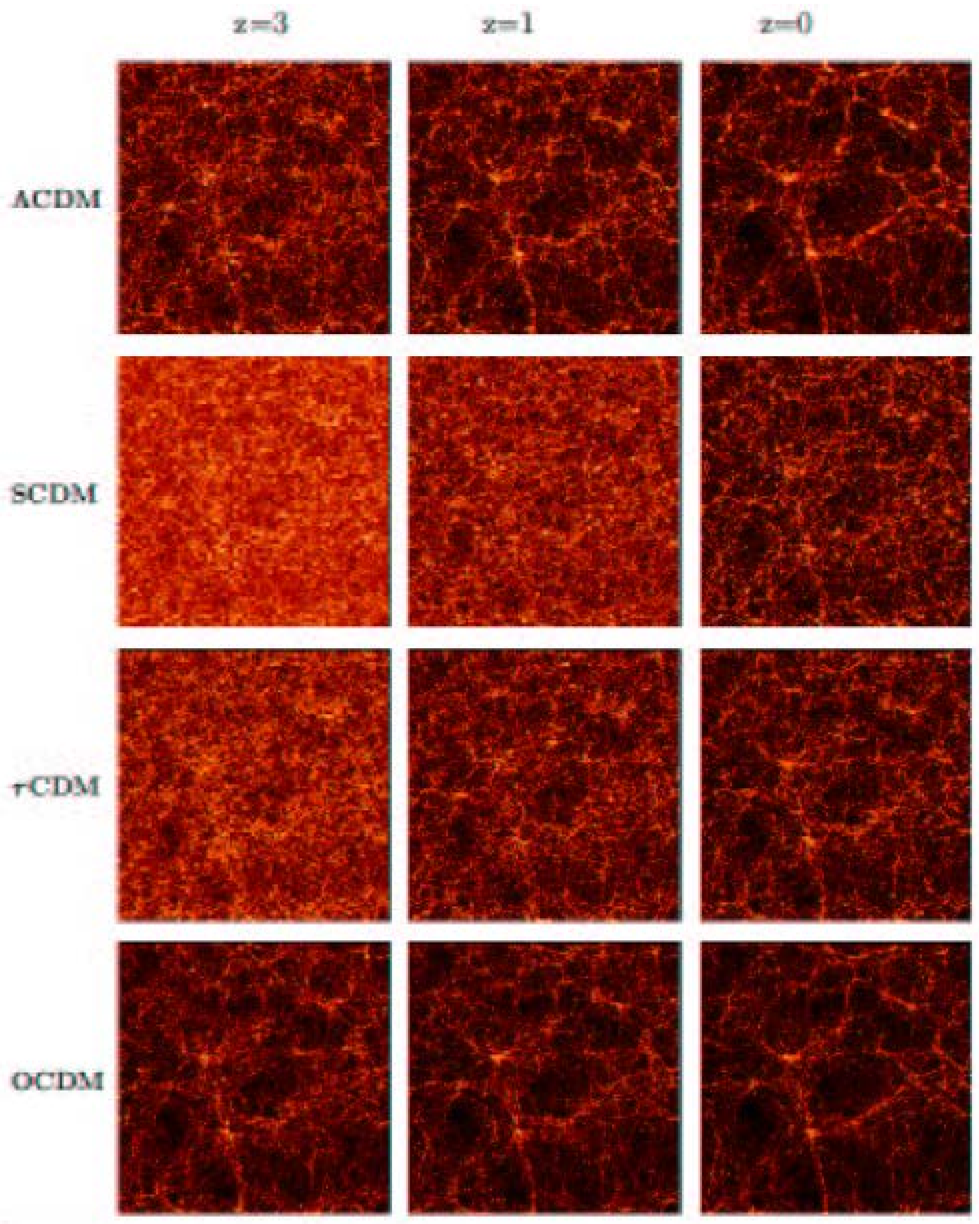
See Paranjape et al (2013) for recent progress in modeling this from first principles

See Castorina et al. (2014) for ν's



Sheth-Tormen 1999

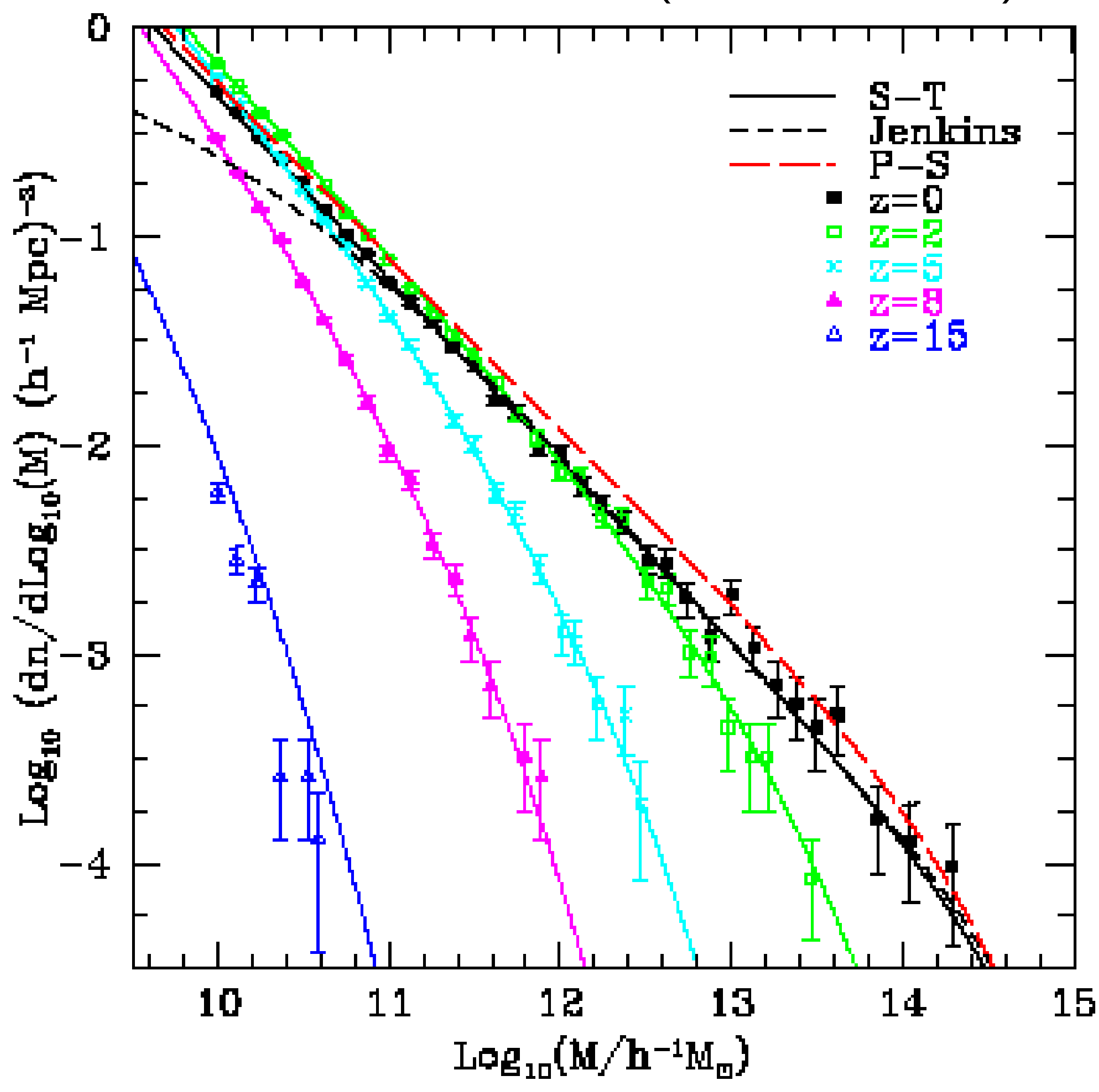
- Structure at a given time, and, more importantly, growth of structure, provides sharp constraints on models



The Halo Mass Function

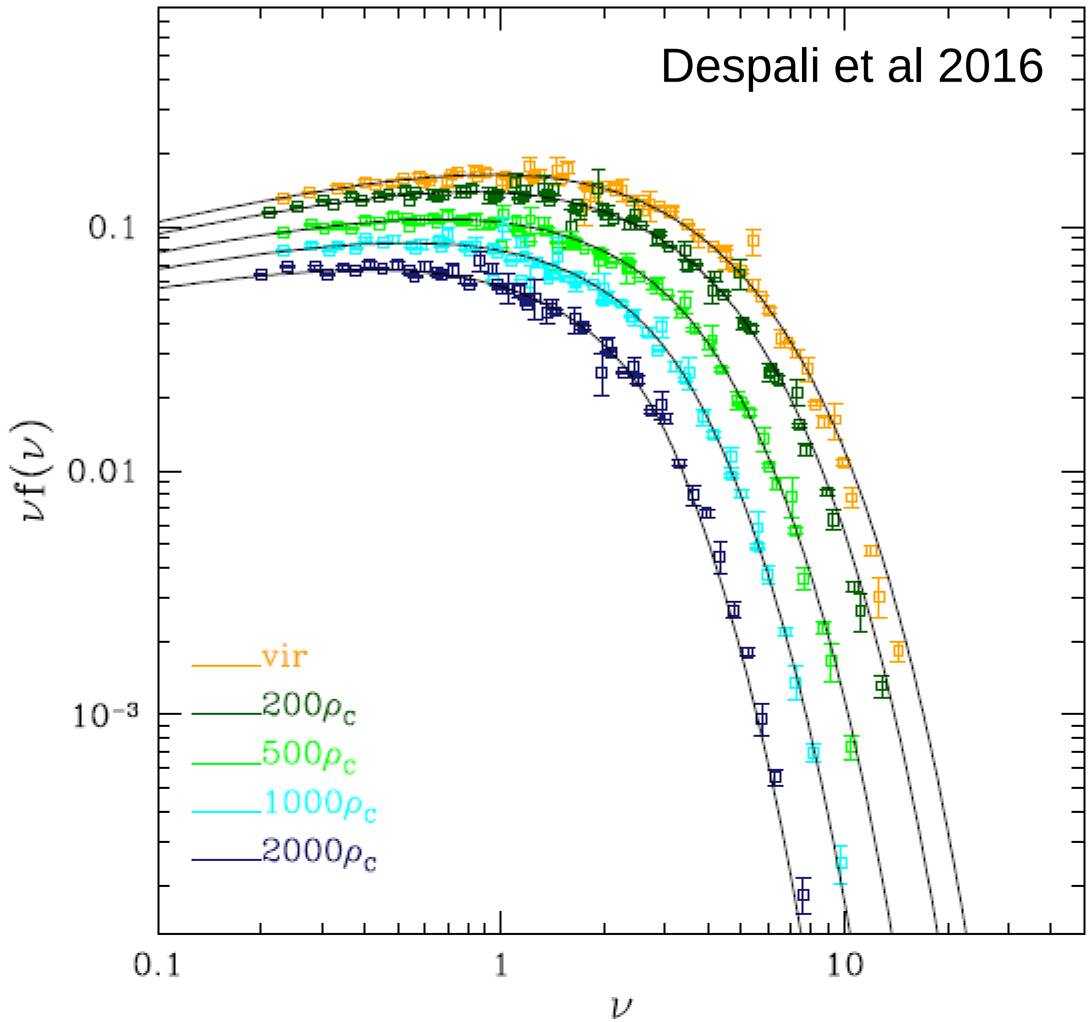
•Small halos collapse/virialize first
•Can also model halo spatial distribution
•Massive halos more strongly clustered

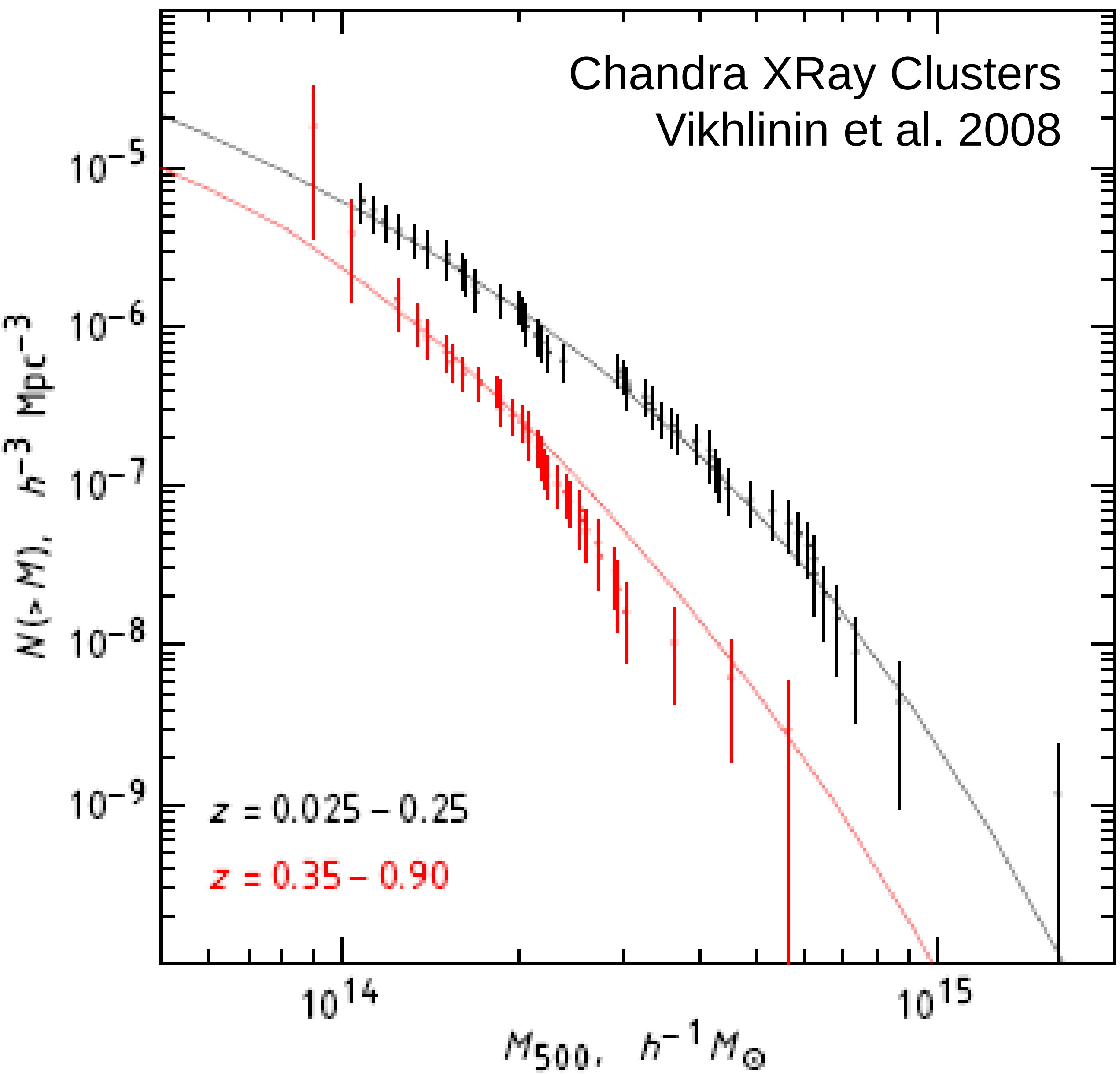
(Reed et al. 2003)



Aside:

Universal mass function + universal profile shape = easy to translate between different halo definitions





Chandra XRay Clusters
Vikhlinin et al. 2008
$z = 0.025 - 0.25$
$z = 0.35 - 0.90$
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{14}
10^{15}
$N(>M), \; h^{-3}\,\mathrm{Mpc}^{-3}$
$M_{500}, \; h^{-1} M_{\odot}$

Study of random walks with correlated steps

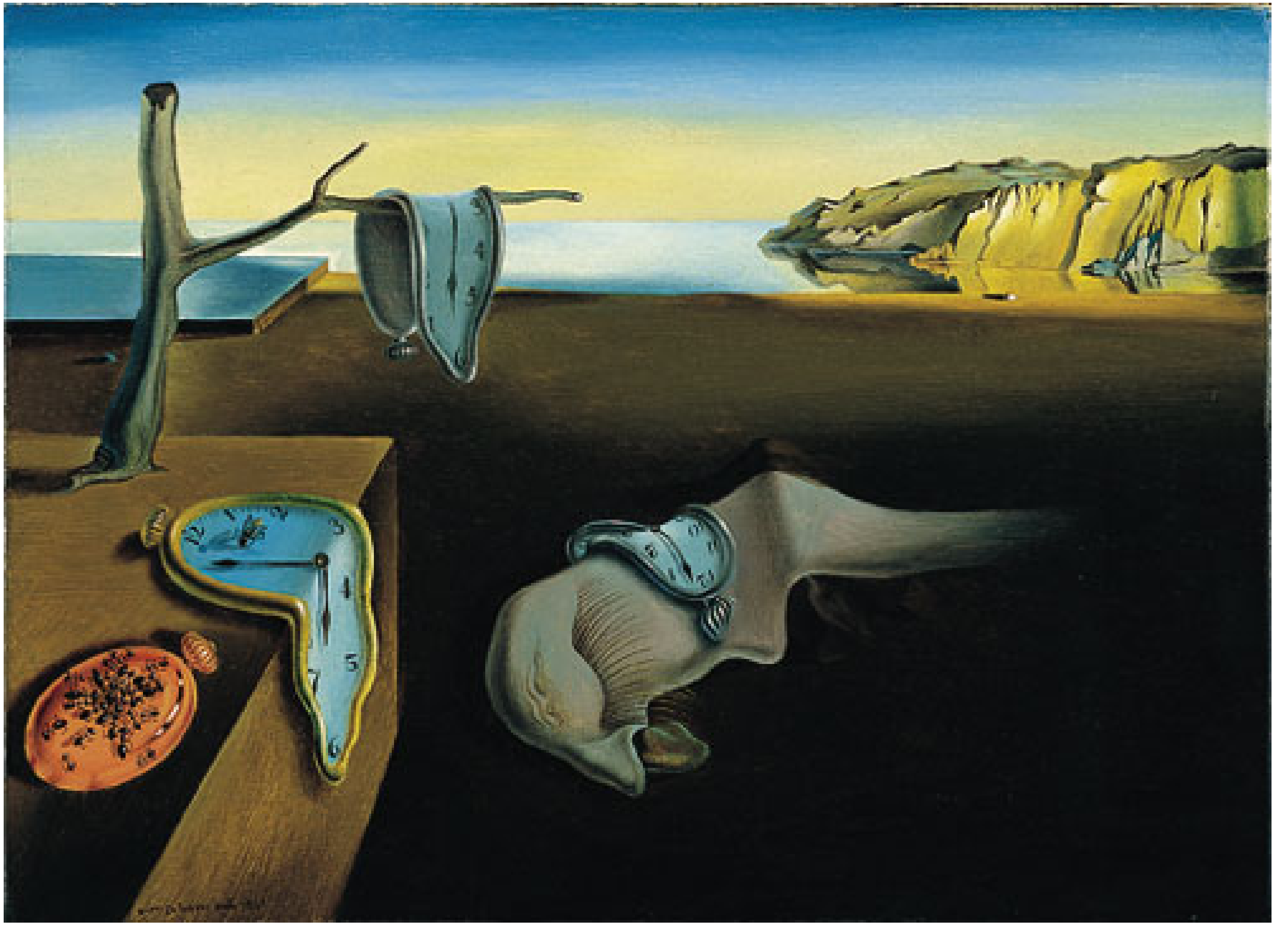
=

Cosmological constraints from large scale structures

Models of halo abundances
and clustering:
Gravity in an expanding universe

Use knowledge of initial conditions
(CMB) to make inferences about
late-time, nonlinear structures

Hierarchical clustering in GR



= the persistence of memory