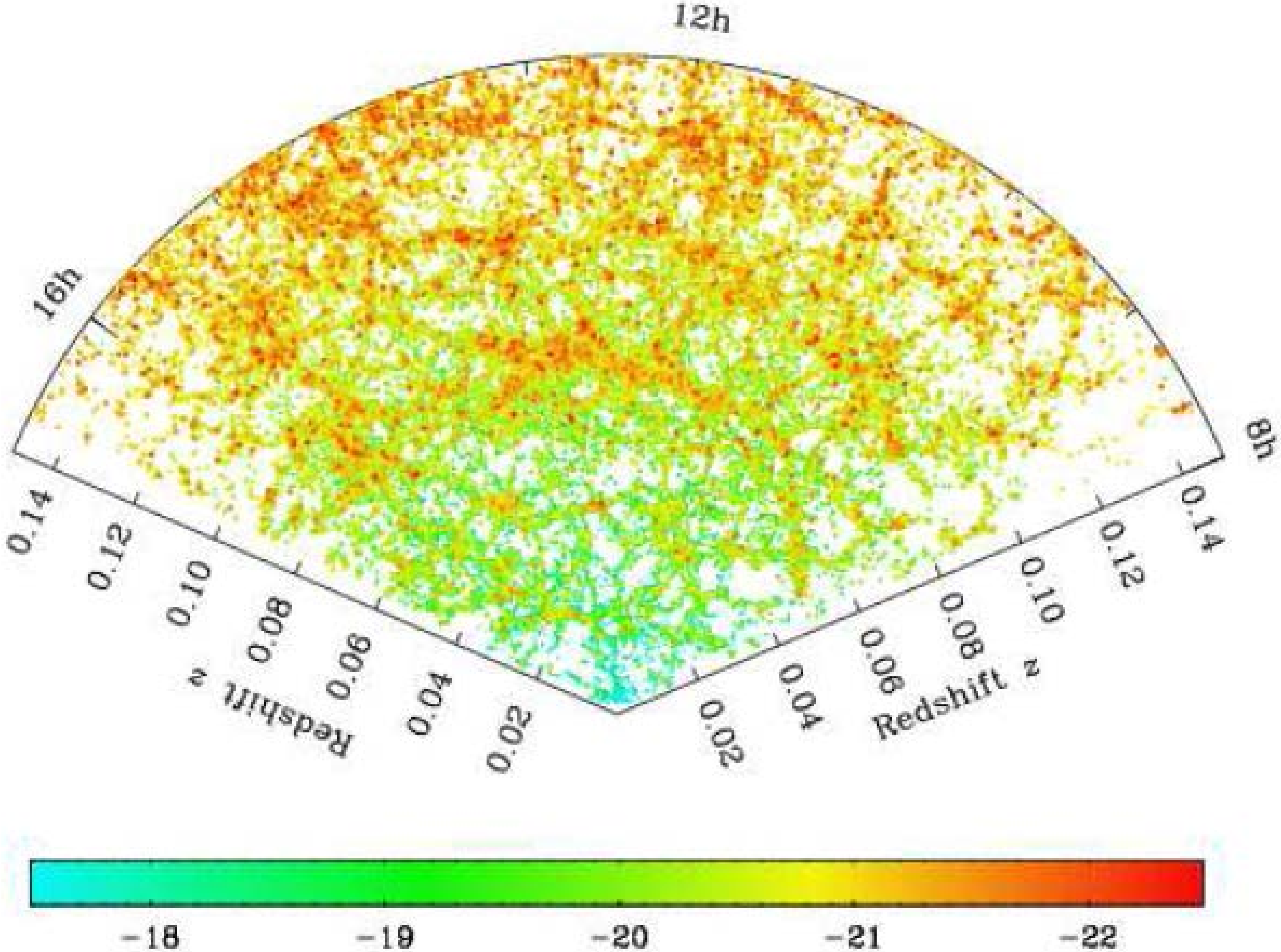
Phenomenology of cosmological structure formation

The halo model: Theory

Halo abundances, clustering, profiles

In practice: HOD, CLF, SHAM

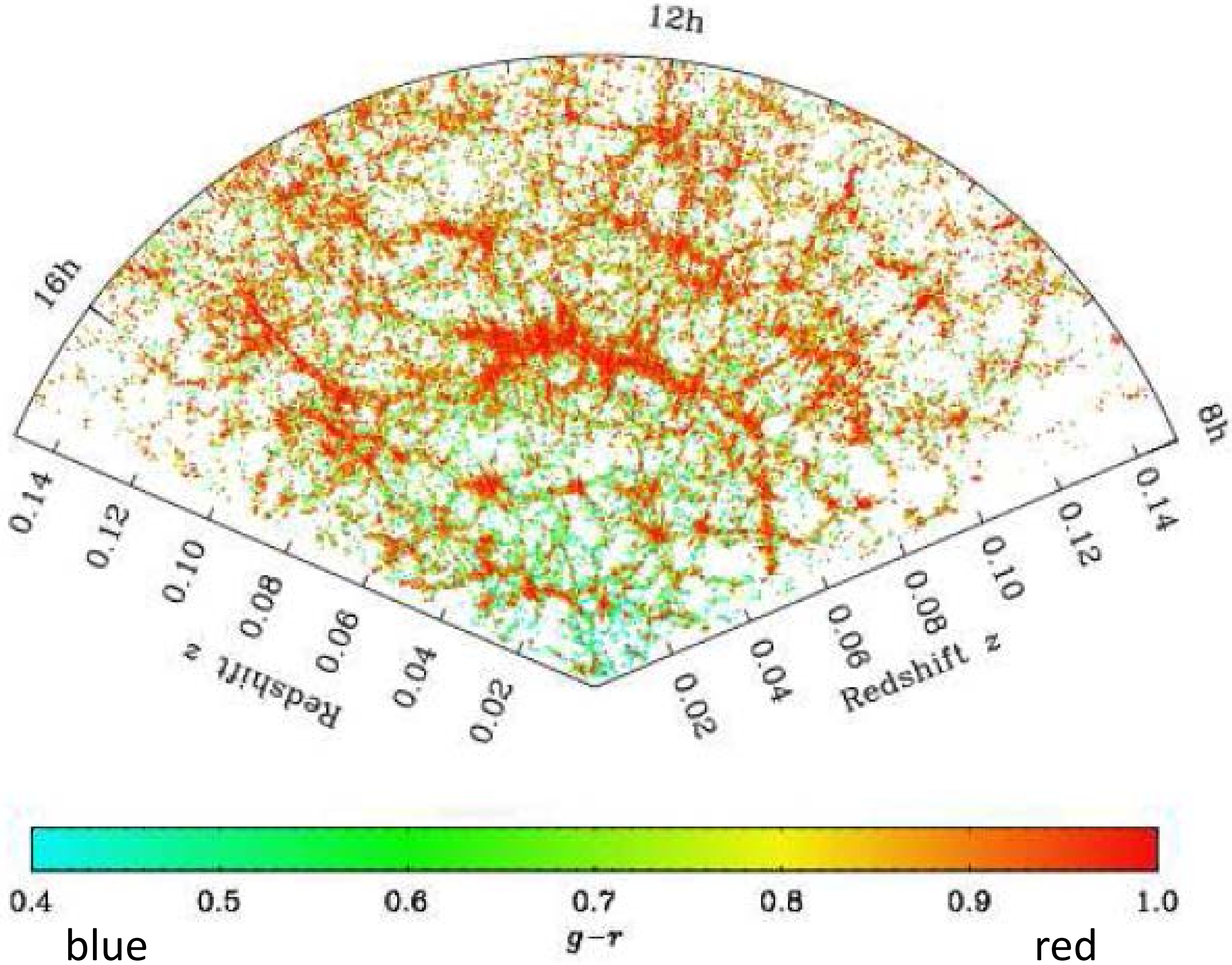
(Assembly bias)



Not luminous

Luminous

Zehavi et al. 2010 (SDSS)


12h
16h
8h
0.14
0.12
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0.08
0.06
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Redshift z
0.02
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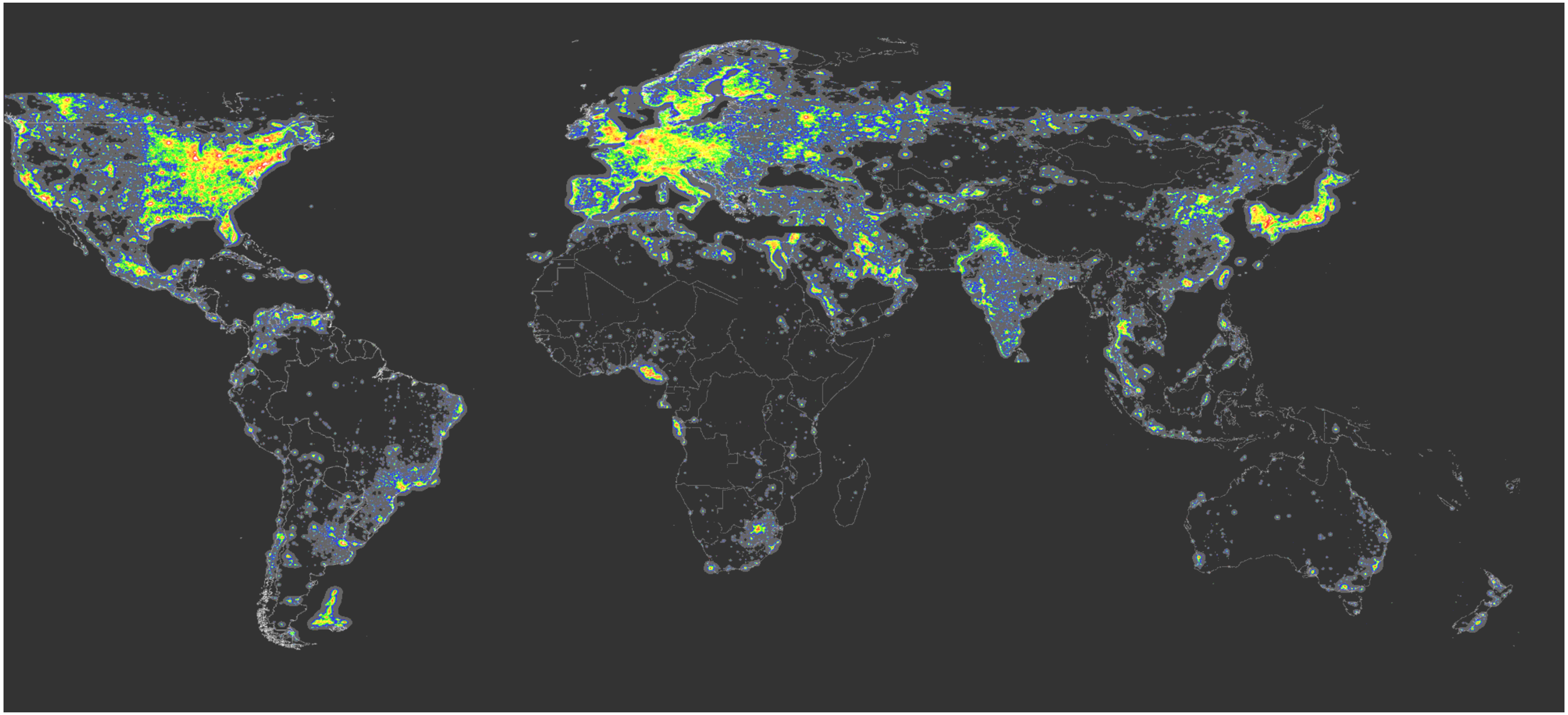
0.06
0.08
0.10
0.12
0.14
Redshift z
0.4
0.5
0.6
0.7
0.8
0.9
1.0
g−r
blue
red

Complication: Light is a biased tracer



Not all galaxies are fair tracers of dark matter;

To use galaxies as probes of underlying dark matter distribution, must understand 'bias'

You can observe a lot just by watching

How to describe different point processes which are all built from the same underlying density field?

THE HALO MODEL

Review in Physics Reports (Cooray & Sheth 2002)

A THEORY OF THE SPATIAL DISTRIBUTION OF GALAXIES*

J. NEYMAN AND E. L. SCOTT
Statistical Laboratory, University of California
Received February 18, 1952

ABSTRACT

A theory of the spatial distribution of galaxies is built, based on the following four main assumptions: (i) galaxies occur only in clusters; (ii) the number of galaxies varies from cluster to cluster, subject to a probabilistic law; (iii) the distribution of galaxies within a cluster is also subject to a probabilistic law; and (iv) the distribution of cluster centers in space is subject to a probabilistic law described as quasi-uniform. The main result obtained is the joint probability generating function $G_{N_1, N_2}(t_1, t_2)$ of numbers N_1 and N_2 of galaxies visible on photographs from two arbitrarily placed regions ω_1 and ω_2, taken with fixed limiting magnitudes m_1 and m_2, respectively. The theory ignores the possibility of light-absorbing clouds. The function $G_{N_1, N_2}(t_1, t_2)$ is expressed in terms of four functions left unspecified, which govern the details of the structure contemplated. Methods are indicated whereby approximations to these functions can be obtained and whereby the general validity of the hypotheses can be tested.

Center-satellite process requires knowledge of how

1) halo abundance; 2) halo clustering; 3) halo profiles;

4) number of galaxies per halo; all depend on halo mass (+ ...)

(Revived, then discarded in 1970s by Peebles, McClelland & Silk)

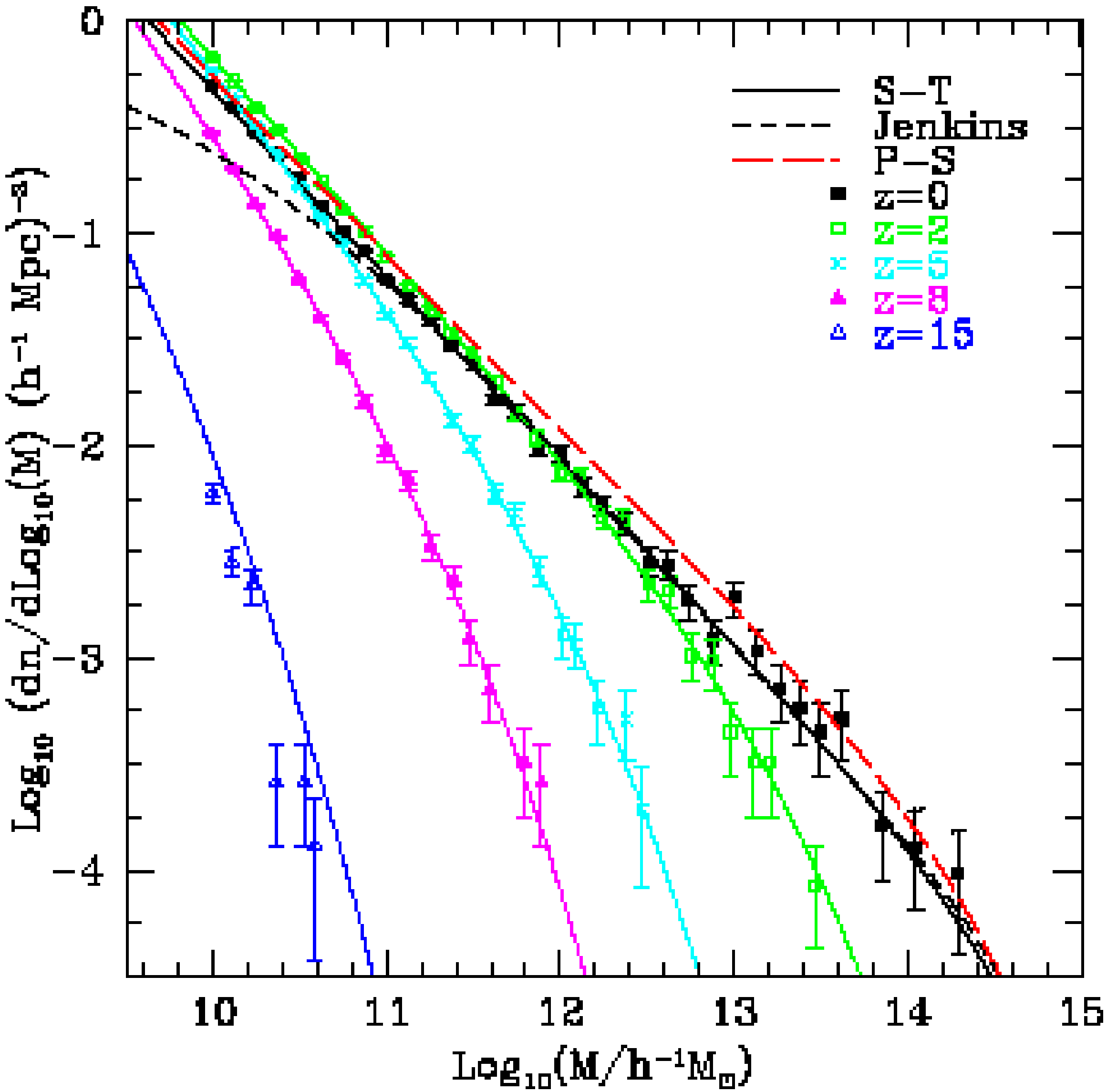
Halo-
model
≈

Circles in
circles

The Halo Mass Function

- •Small halos collapse/virialize first
- •Can also model halo spatial distribution
- •Massive halos more strongly clustered

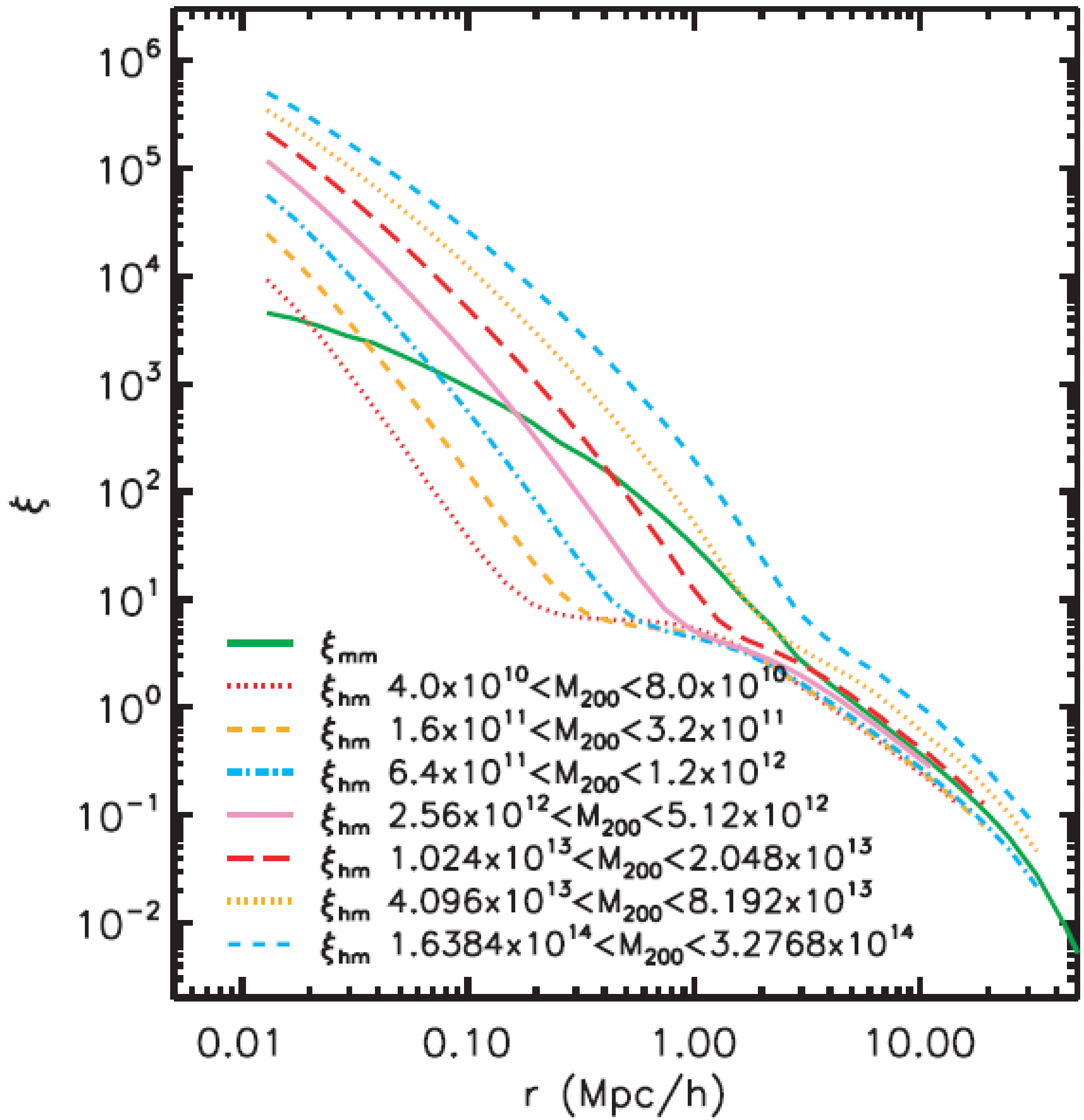
(Reed et al. 2003)



• Can also measure/model halo spatial distribution (and its evolution)

• On large scales, linear bias $\xi_{hm}(r) = b\,\xi_{mm}(r)$ is good approximation

• At any given time, massive halos are more strongly clustered



Close connection between abundance and spatial distribution (bias):

- Let δ_R denote δ on scale R
- A halo of mass M forms from a patch where $\delta_R > \delta_c$, $\delta_{R+dR} < \delta_c$, …
- Abundance of halos of mass M from

  $p(\delta_R > \delta_c, \delta_{R+dR} < \delta_c, \ldots)$
- Bias related to $p(\delta > \delta_c, \delta_{R+dR} < \delta_c, \ldots | \Delta \text{ on } R_\Delta)$
  - Namely, write this as Taylor series in Δ; linear term in expansion is linear bias factor.

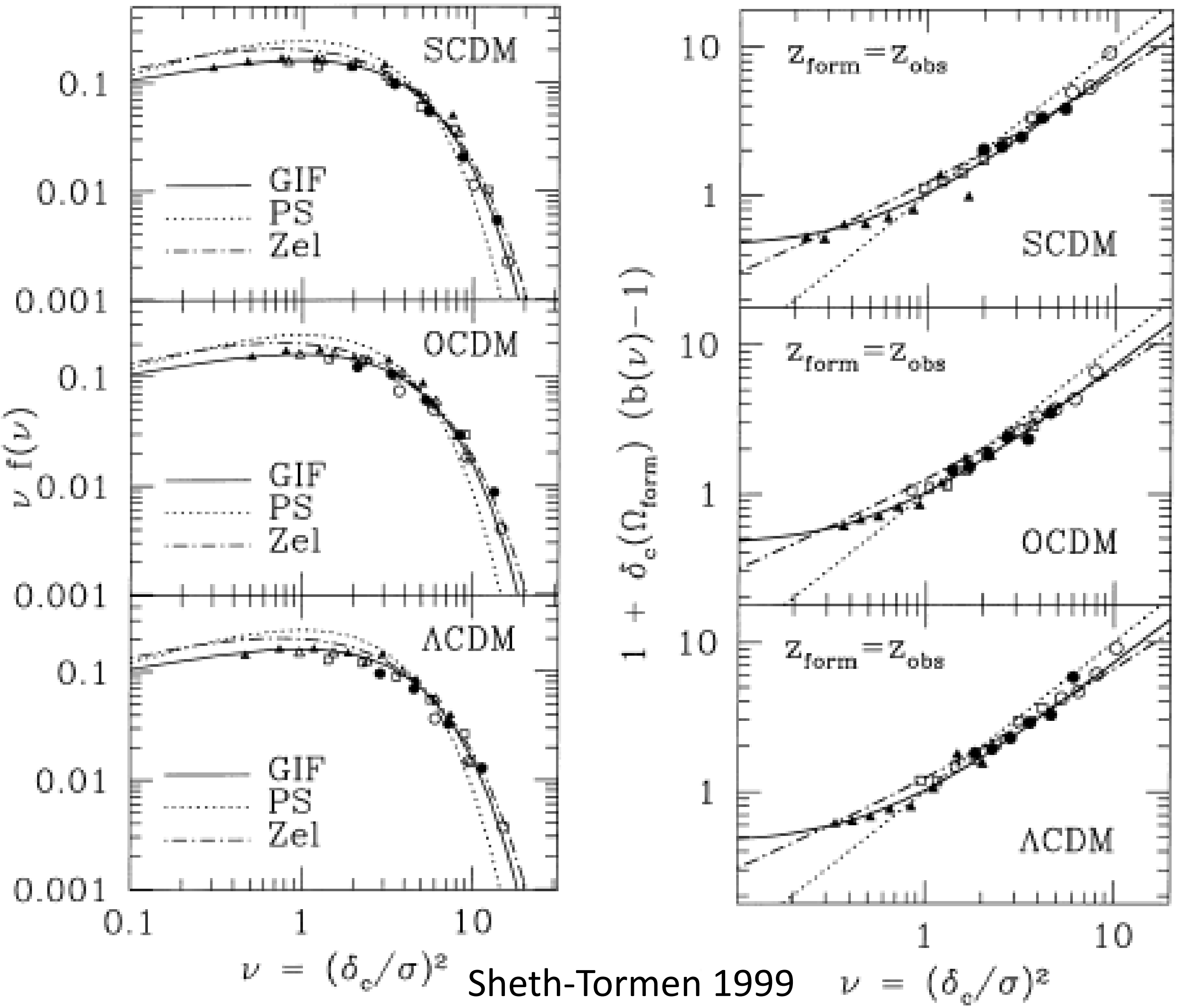
(Almost) universal mass function

$(m/\rho)\,(dn/d\ln\nu) = \nu f(\nu) = A\,[1 + (q\nu)^{-p}]\,\mathrm{sqrt}(q\nu/2\pi)\,\exp(-q\nu/2)$

where all $\nu = (\delta_c/\sigma)^2$ and A ensures integral over all ν is unity

and halo bias

$b(\nu) = 1 + d\ln f/d\delta_c$

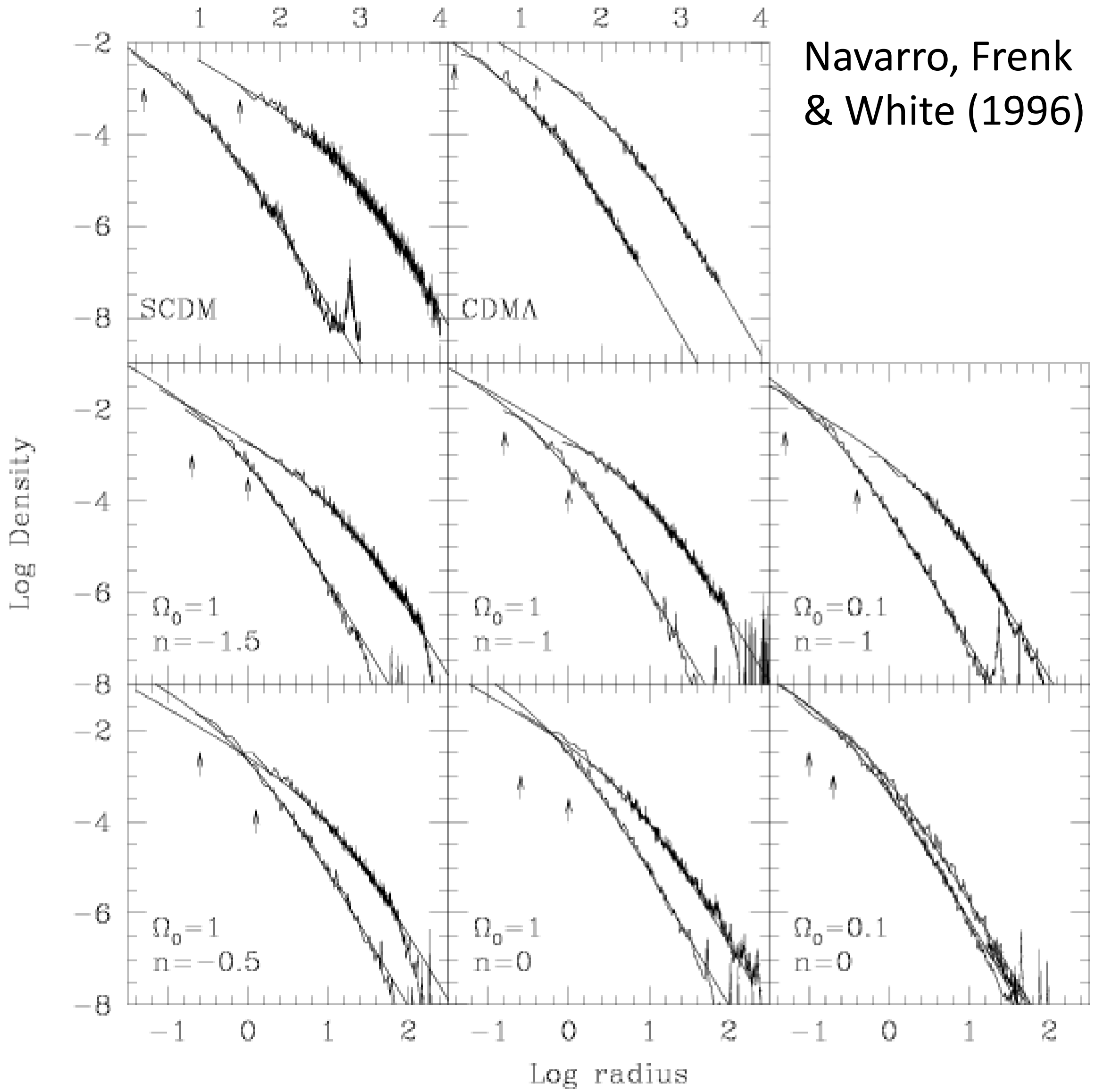


Sheth-Tormen 1999

Universal Halo Profiles

$\rho(r) = 4\rho_s/(r/r_s)/(1+ r/r_s)^2$

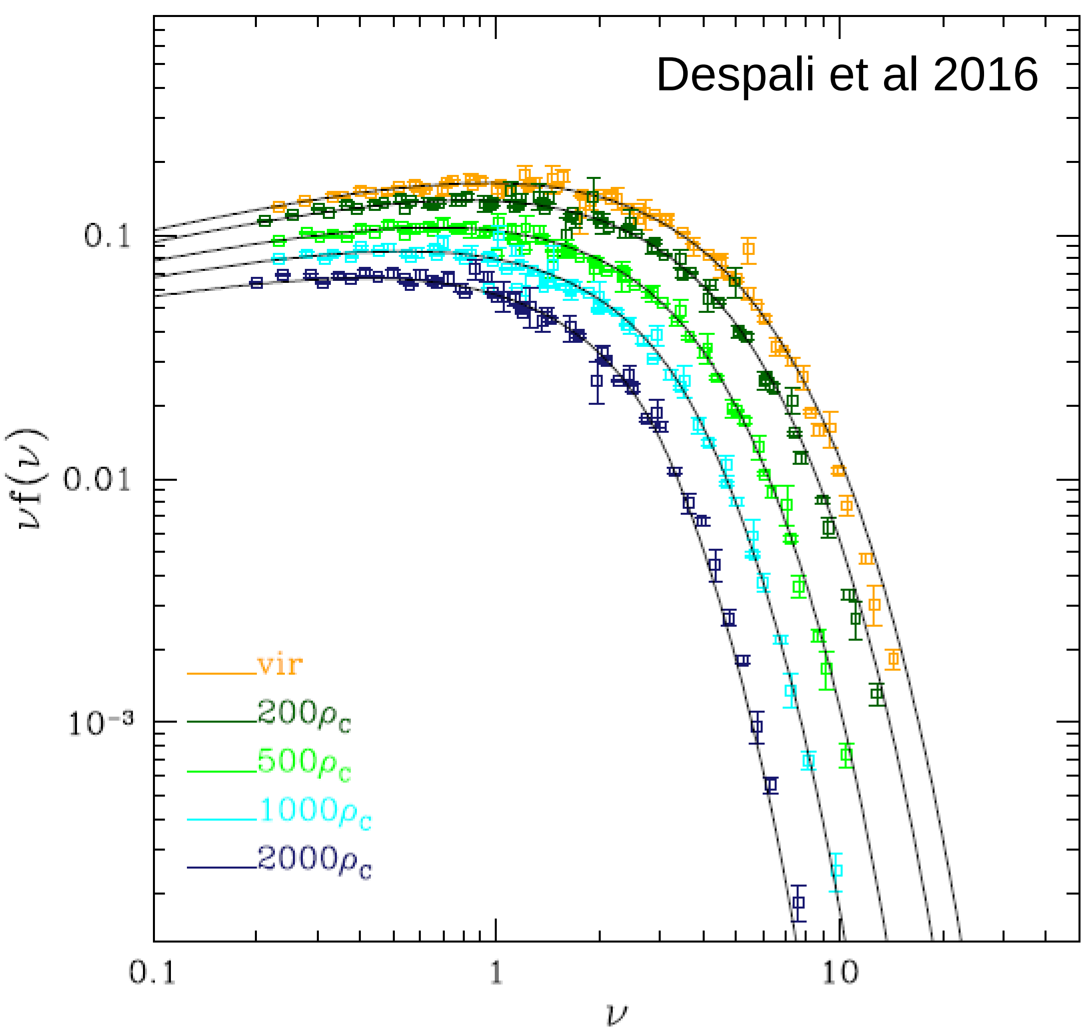
- Not quite isothermal
- Scale radius r_s depend on halo mass, formation time
- Massive halos less concentrated (partially built-in from GRF initial conditions)
- Distribution of shapes (axis-ratios) known (Jing & Suto 2001)



Navarro, Frenk & White (1996)

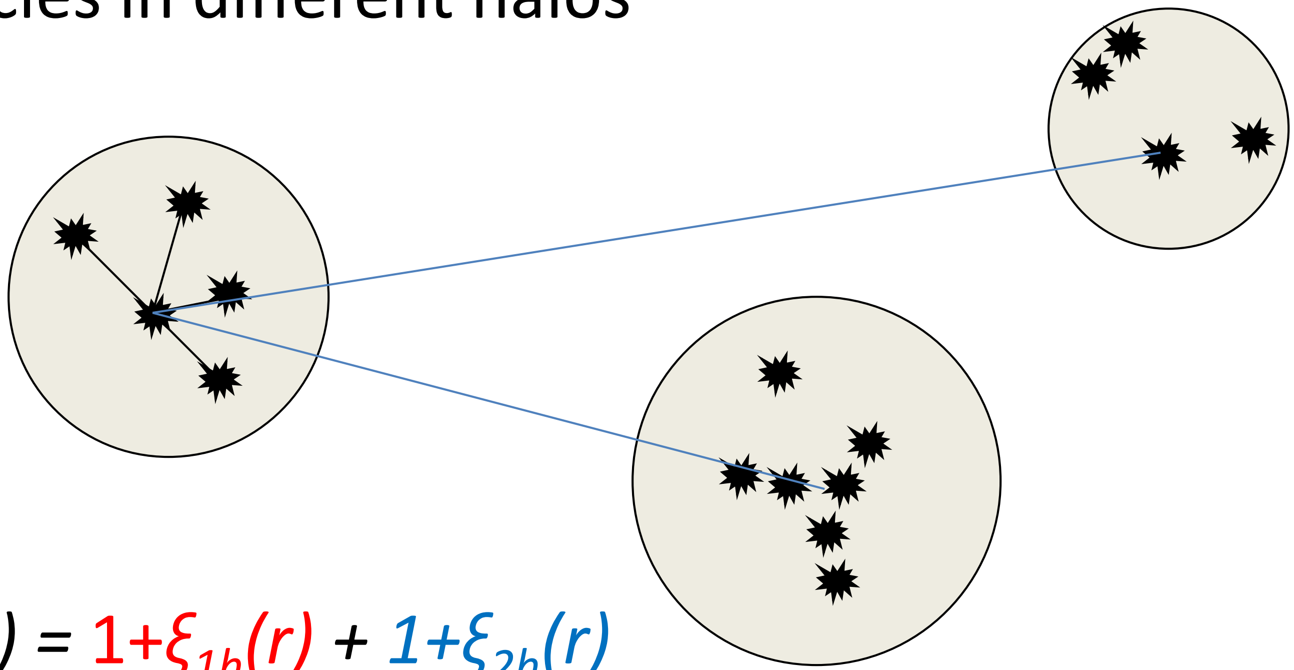
Aside:

Universal mass function + universal profile shape = easy to translate between different halo definitions



The halo-model of clustering

- Two types of pairs: both particles in same halo, or particles in different halos



- $1+\xi(r) = 1+\xi_{1h}(r) + 1+\xi_{2h}(r)$
- *All* physics can be decomposed similarly: 'nonlinear' effects from within halo, 'linear' from outside

The dark-matter correlation function

$$\xi_{dm}(r) = 1+\xi_{1h}(r) + \xi_{2h}(r)$$

- $1+\xi_{1h}(r) \sim \int dm\, n(m)\, m^2\, \xi_{dm}(r|m)/\rho^2$
- $n(m)$: comoving number density of m-halos
- Comoving mass density: $\rho = \int dm\, n(m)\, m$
- $\xi_{dm}(r|m)$: fraction of total pairs, m^2, in an *m*-halo which have separation *r;* depends on (convolution of) density profile within *m*-halos
- This term only matters on scales smaller than the virial radius of a typical M_* halo (~ Mpc)
  - Need not know spatial distribution of halos!

$$\xi_{dm}(r) = 1+\xi_{1h}(r) + \xi_{2h}(r)$$

- $\xi_{2h}(r) \approx \int dm_1 \, \frac{m_1 n(m_1)}{\rho} \int dm_2 \, \frac{m_2 n(m_2)}{\rho} \, \xi_{2h}(r|m_1,m_2)$

- Two-halo term dominates on large scales, where peak-background split estimate of halo clustering should be accurate: $\delta_h \sim b(m)\,\delta_{dm}$
- $\xi_{2h}(r|m_1,m_2) \sim \langle\delta_h^2\rangle \sim b(m_1)b(m_2)\,\langle\delta_{dm}^2\rangle$
- $\xi_{2h}(r) \approx [\int dm\ mn(m)\ b(m)/\rho]^2\ \xi_{dm}(r)$
- On large scales, linear theory is accurate: $\xi_{dm}(r) \approx \xi_{Lin}(r)$ so $\xi_{2h}(r) \approx b_{eff}^2\ \xi_{Lin}(r)$

Dark matter power spectrum

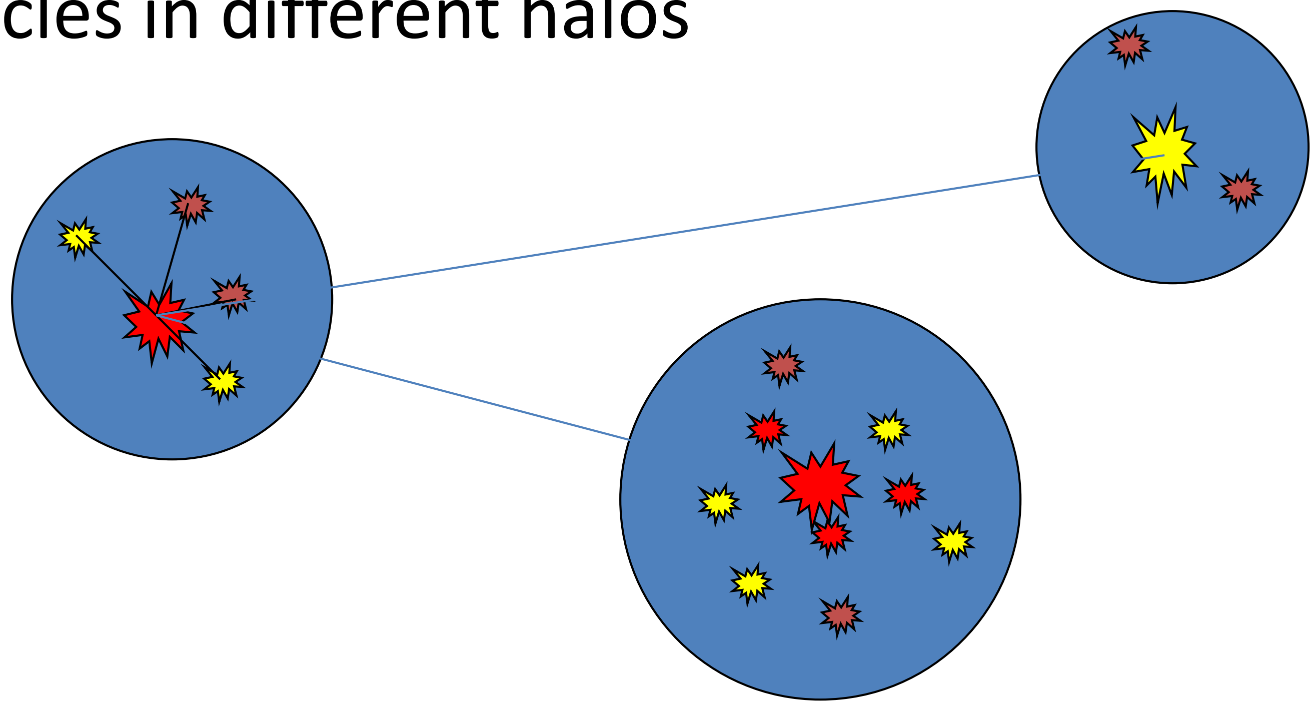
- Convolutions in real space are products in k-space, so $P(k)$ is easier than $\xi_{1h}(r)$

$$P(k) = P_{1h}(k) + P_{2h}(k)$$

- $P_{1h}(k) = \int dm\, n(m)\, m^2\, |u_{dm}(k|m)|^2/\rho^2$
- $P_{2h}(k) \approx [\int dm\, n(m)\, b(m)\, m\, u_{dm}(k|m)/\rho]^2\, P_{dm}(k)$

The halo-model of galaxy clustering

- Two types of particles: central + ‘satellite’
- Two types of pairs: both particles in same halo, or particles in different halos



- $1+\xi_{obs}(r) = 1+\xi_{1h}(r) + 1+\xi_{2h}(r)$

  $n_t(n_t-1)[1+\xi_{1h}(r)] = 2n_c n_s[1+\xi_{cs}(r)] + n_s(n_s-1)[1+\xi_{ss}(r)]$

The halo-model of galaxy clustering

- Write as sum of two components:
  - $1+\xi_{1gal}(r) = \int dm\, n(m)\, g_2(m)\, \xi_{dm}(m|r)/\rho_{gal}^2$
  - $\xi_{2gal}(r) \approx [\int dm\, n(m)\, g_1(m)\, b(m)/\rho_{gal}]^2\, \xi_{dm}(r)$
  - $\rho_{gal} = \int dm\, n(m)\, g_1(m)$: number density of galaxies
  - $\xi_{dm}(m|r)$: fraction of pairs in m-halos at separation *r*
- Think of mean number of galaxies, $g_1(m) = \langle N|m \rangle$, as a weight applied to each dark matter halo
  - And $g_2(m) = \langle N(N-1)|m \rangle$ is mean number of distinct pairs
  - Galaxies 'biased' if $g_1(m)$ not proportional to m, …, $g_n(m)$ not proportional to m^n (Jing, Mo & Boerner 1998; Benson et al. 2000; Peacock & Smith 2000; Seljak 2000; Scoccimarro et al. 2001)
  - Central + Poisson satellites model (see later) works well
- Similarly, Y_{SZ} or T_X are just a weight applied to halos, so same formalism can model cluster clustering

Power spectrum

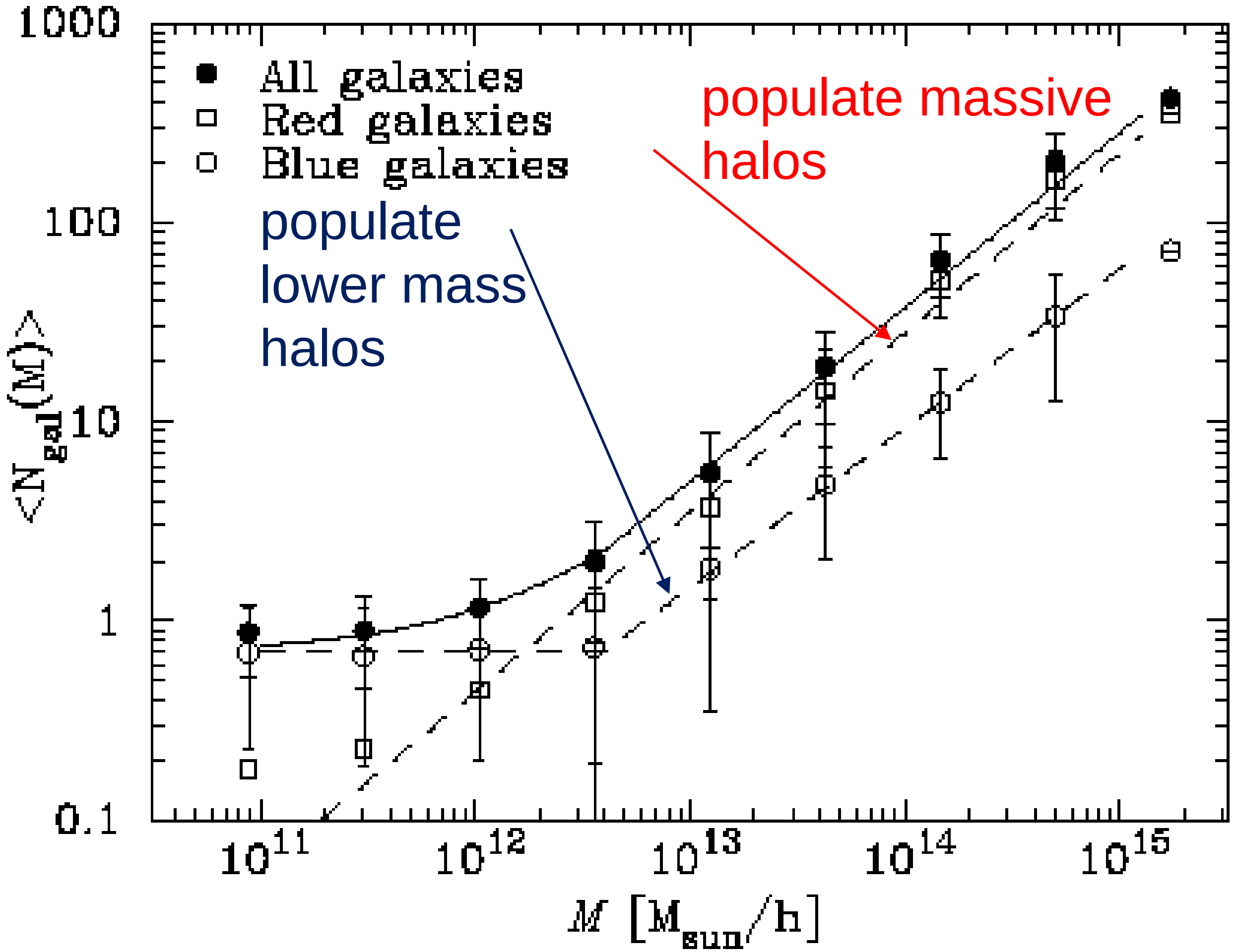
- Convolutions in real space are products in k-space, so P(k) is easier than ξ(r):

$$P(k) = P_{1h}(k) + P_{2h}(k)$$

- $P_{1h}(k) = \int dm\ n(m)\ g_2(m)\ |u_{dm}(k|m)|^2/\rho^2$
- $P_{2h}(k) \approx [\int dm\ n(m)\ b(m)\ g_1(m)\ u_{dm}(k|m)/\rho]^2\ P_{dm}(k)$

- Galaxies 'biased' if $g_n(m)$ not proportional to m^n

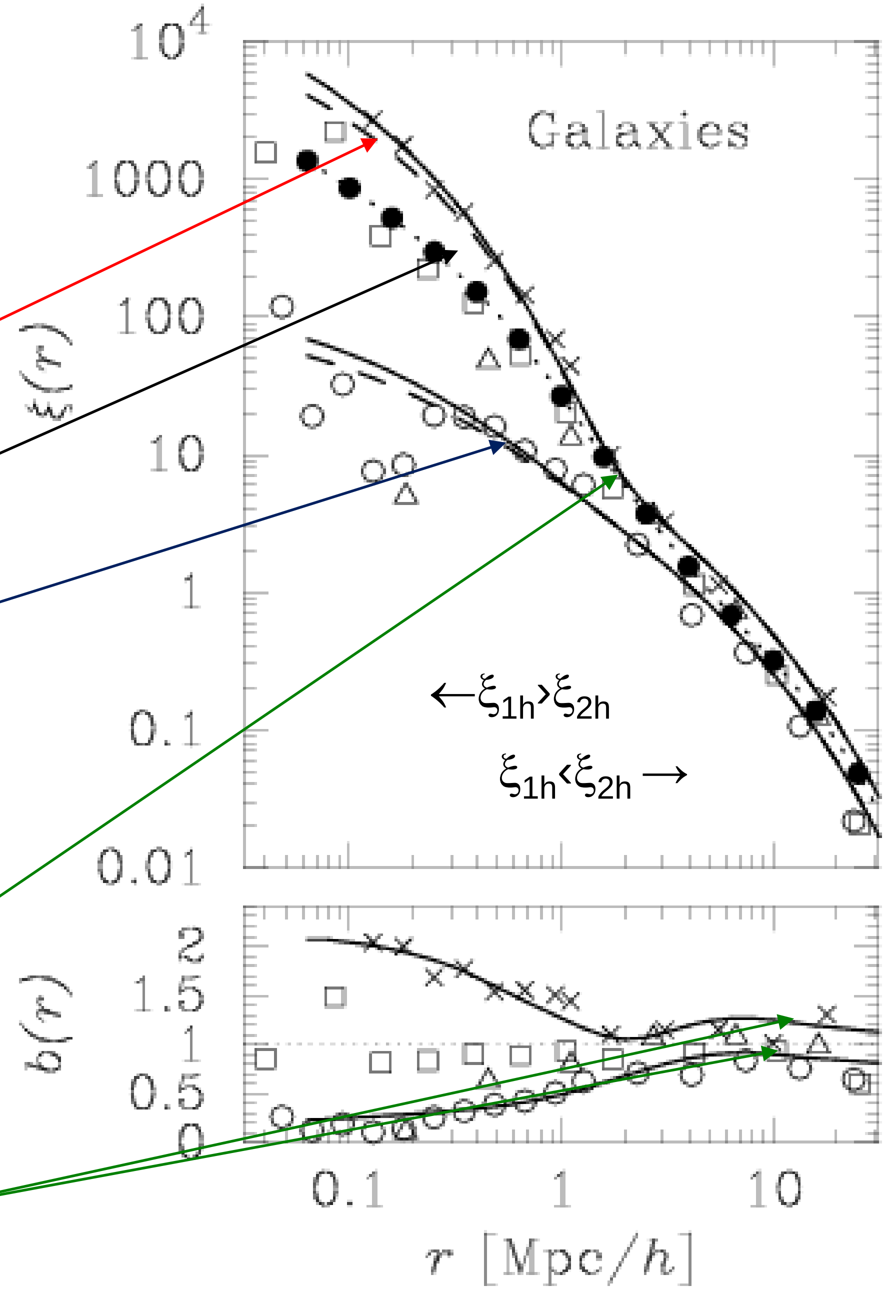
Type-dependent clustering: Why?



Spatial distribution within halos second order effect (on >100 kpc)

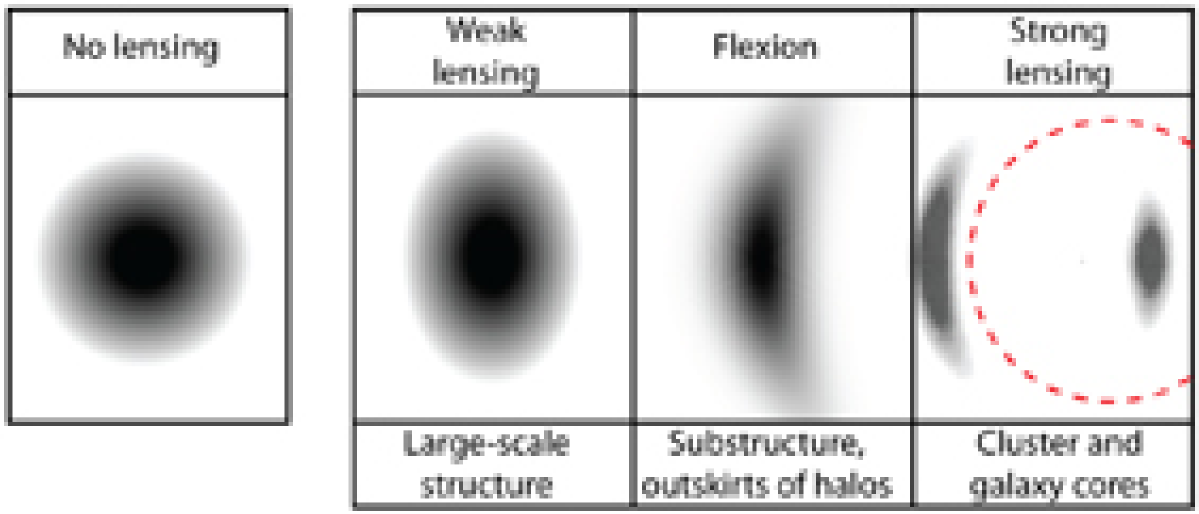
Comparison with simulations: OK!

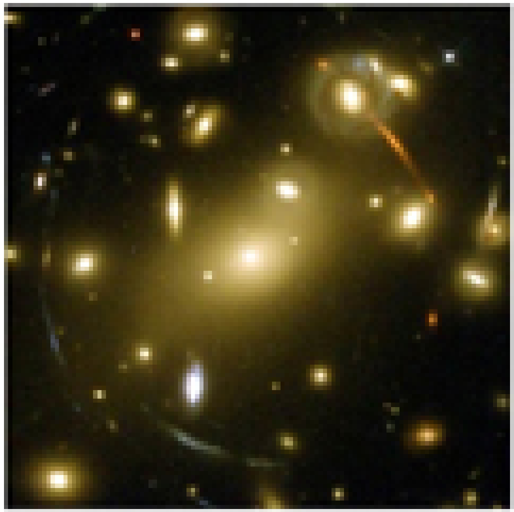
- Halo model calculation of $\xi(r)$
- Red galaxies
- Dark matter
- Blue galaxies
- Note inflection at scale of transition from 1halo term to 2-halo term (~ virial radius)
- Bias constant at large *r*

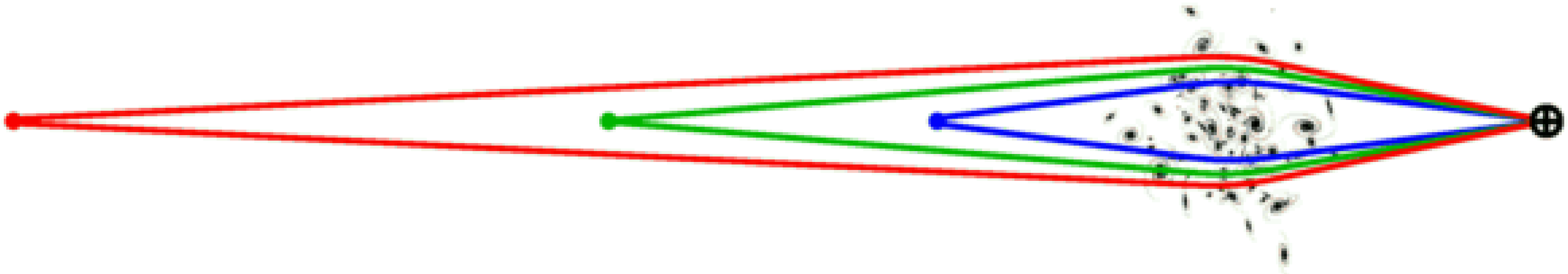


Cosmology from Gravitational Lensing

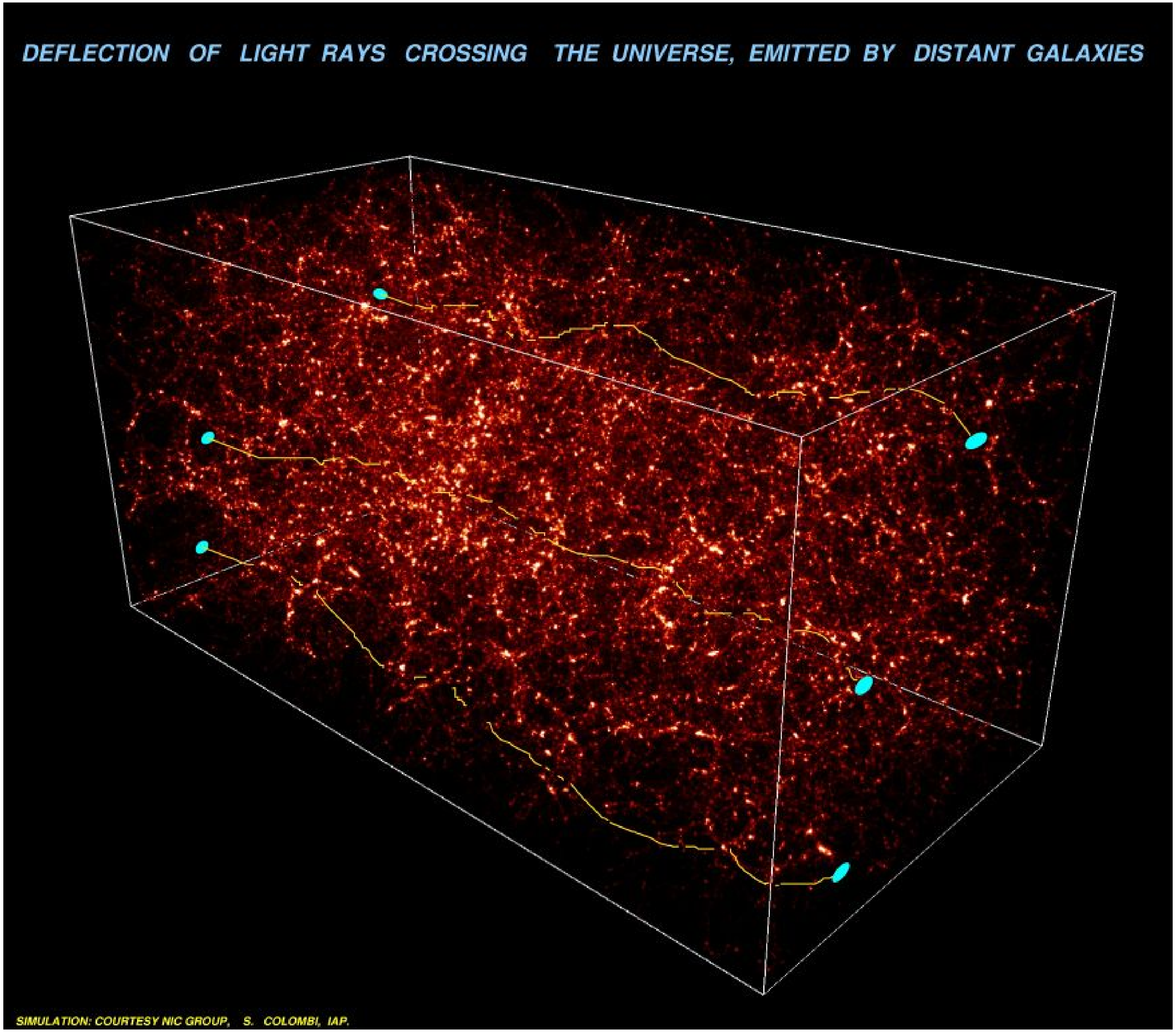
Volume as function of redshift
Growth of fluctuations with time







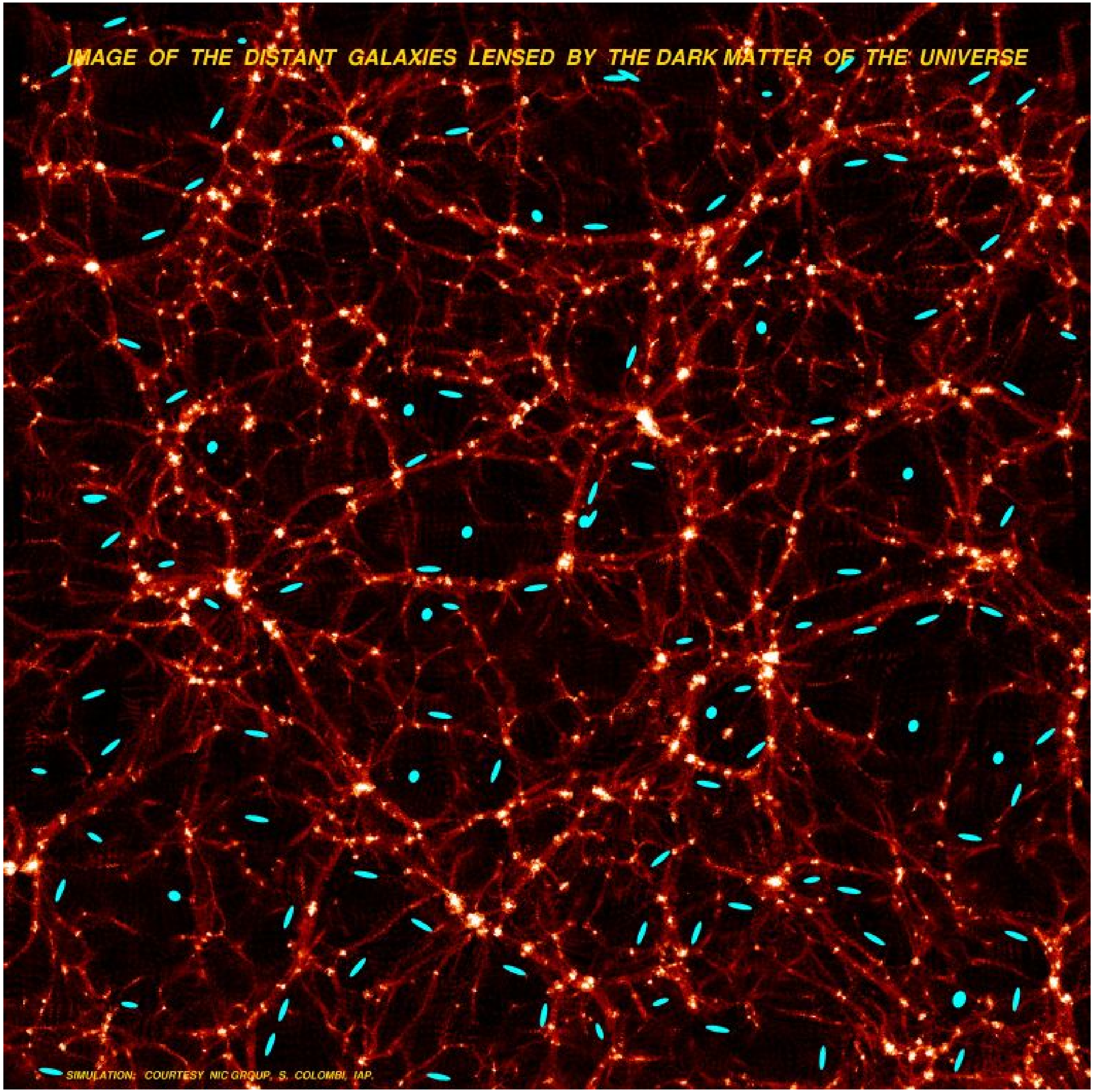
- Focal length strong function of cluster-centric distance; highly distorted images possible
- Strong lensing if source lies close to lens-observer axis; weaker effects if impact parameter large
- Strong lensing: Cosmology from distribution of image separations, magnification ratios, time delays; but these are rare events, so require large dataset
- Weak lensing: Cosmology from correlations (shapes or magnifications); small signal requires large dataset



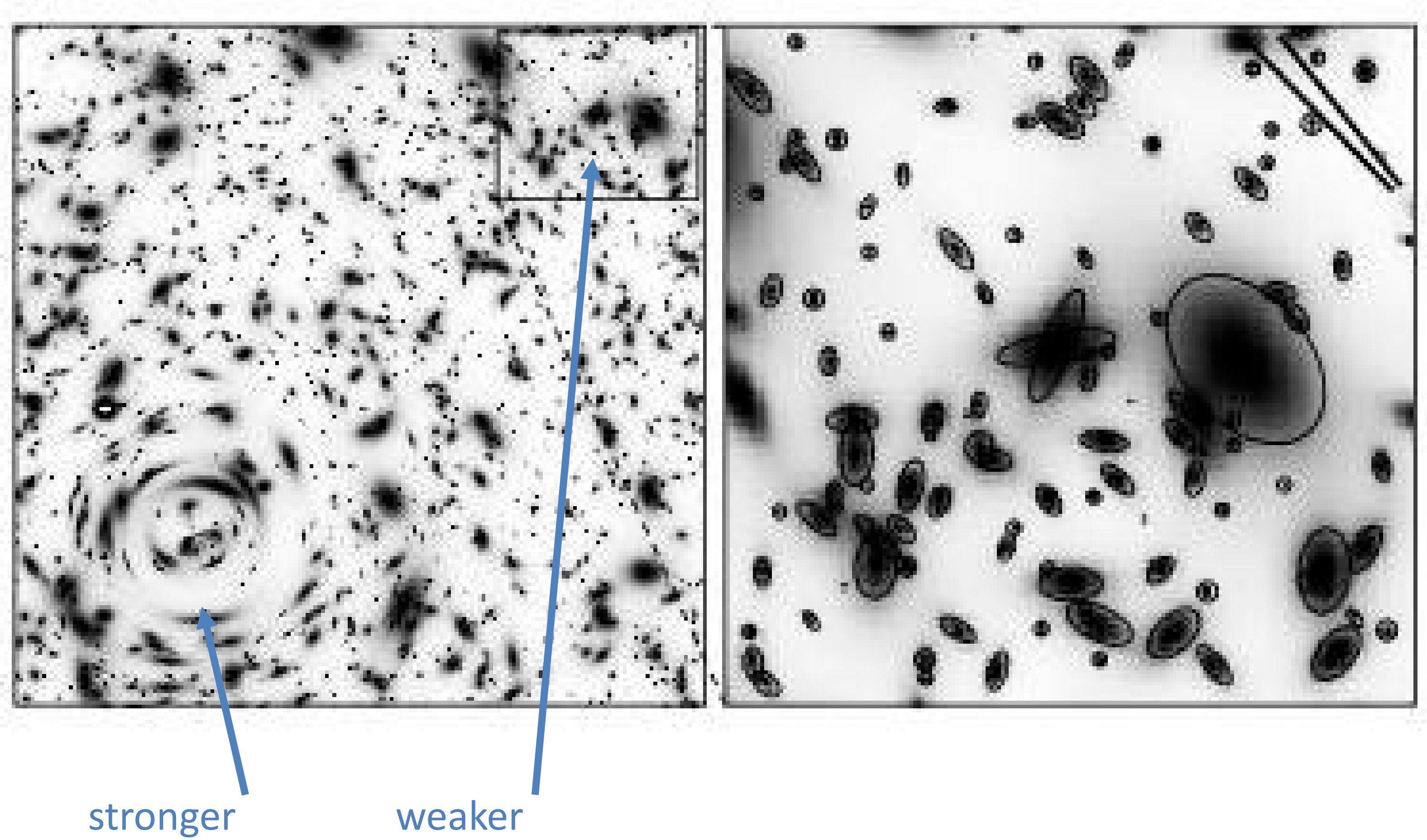
Lensing provides a measure of dark matter along line of sight

Weak lensing: Image distortions correlated with dark matter distribution

E.g., lensed image ellipticities aligned parallel to filaments, tangential to knots (clusters)



The shear power of lensing



Cosmology from measurements of correlated shapes; better constraints if finer bins in source or lens positions possible

Galaxy-lensing power spectrum

$$P(k) = P_{1h}(k) + P_{2h}(k)$$

- $P_{1h}(k) = \int dm\, n(m)\, mu(k|m)\, g_1(m) u_g(k|m)/n_g\rho$
- $P_{2h}(k) \approx [\int dm\, n(m)\, b(m)\, m\, u(k|m)/\rho]$

  $\times\, [\int dm\, n(m)\, b(m)\, g_1(m)\, u_g(k|m)/n_g]\, P_{dm}(k)$

The other half of phase-space: Velocities

Just as statistics can be split into two regimes, so too can the physics: linear + nonlinear

Non-Maxwellian Velocities?

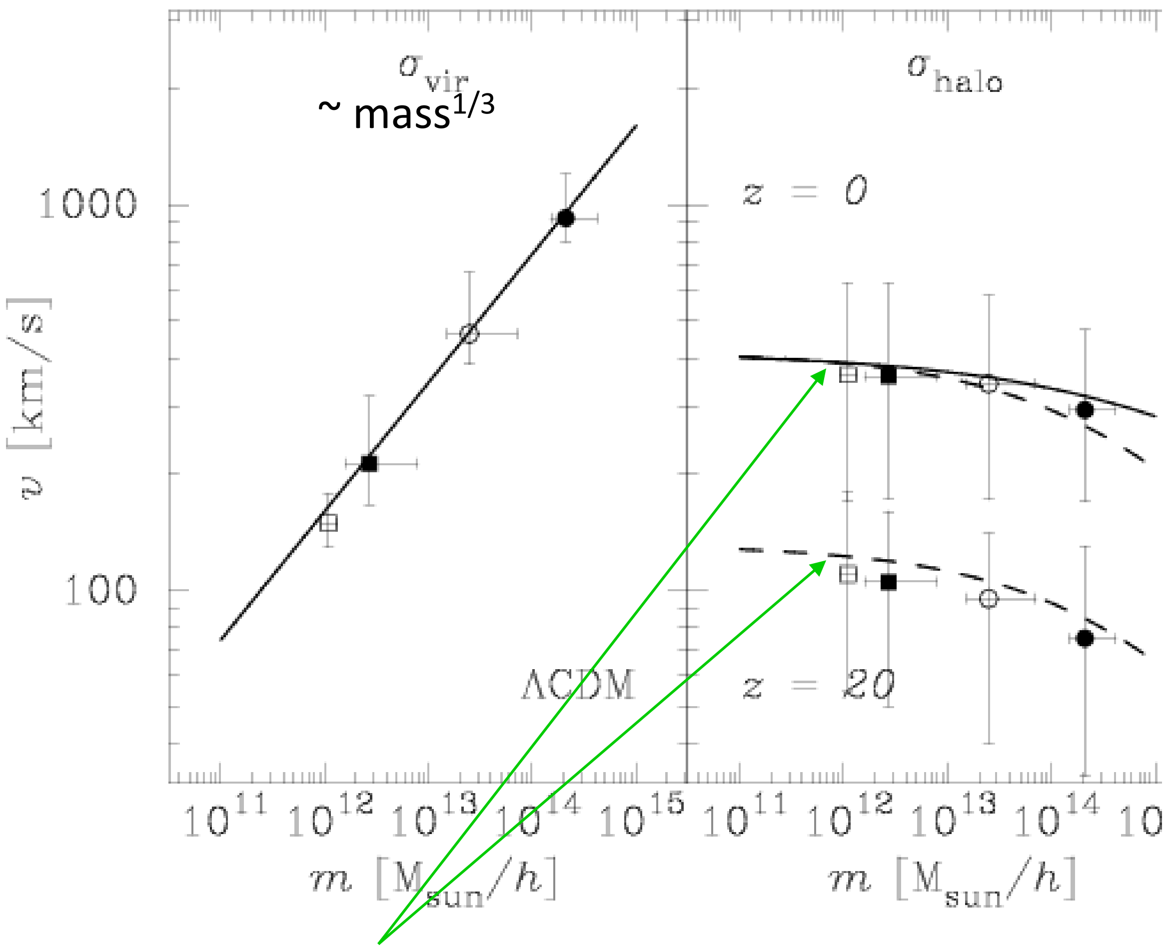
- $v = v_{vir} + v_{halo}$
- Maxwellian/Gaussian velocity within halo (dispersion depends on parent halo mass, because $v^2 \sim GM/r_{\rm vir} \sim M^{2/3}$)

  + Gaussian velocity of parent halo (from linear theory ≈ independent of *m*)
- Hence, at fixed *m*, distribution of v is convolution of two Gaussians, i.e.,

p(v|*m*) is Gaussian, with dispersion

$$\sigma_{\rm vir}^2(m) + \sigma_{\rm Lin}^2 = (m/m_*)^{2/3}\sigma_{\rm vir}^2(m_*) + \sigma_{\rm Lin}^2$$

Two contributions to velocities



- Virial motions (i.e., nonlinear theory terms) dominate for particles in massive halos
- Halo motions (linear theory) dominate for particles in low mass halos

Growth rate of halo motions ~ consistent with linear theory; Zeldovich should be good approximation for halo motions

Exponential tails are generic

- $p(\mathrm{v}) = \int dm\, mn(m)\, \mathrm{G}(\mathrm{v}|m)$

  $\mathcal{F}(t) = \int d\mathrm{v}\, e^{i\mathrm{v}t}\, p(\mathrm{v}) = \int dm\, n(m)m\, \mathrm{e}^{-t^2\sigma_{\mathrm{vir}}^2(m)/2}\, \mathrm{e}^{-t^2\sigma_{\mathrm{Lin}}^2/2}$

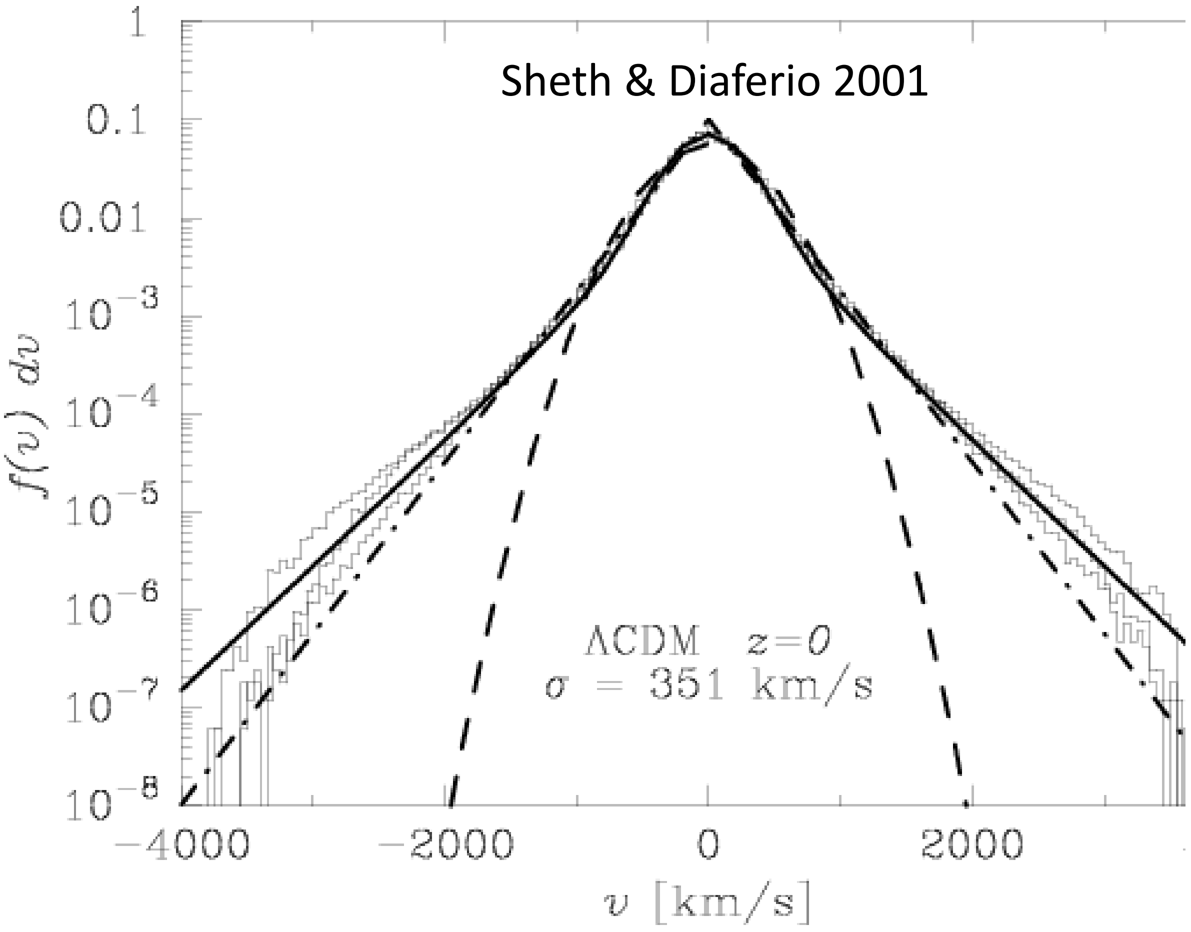
- For $P(k) \sim k^{-1}$, mass function $n(m) \sim$ power-law times $\exp[-(m/m_*)^{2/3}/2]$, so integral is:

  $\mathcal{F}(t) = \mathrm{e}^{-t^2\sigma_{\mathrm{Lin}}^2/2}\, [1 + t^2\sigma_{\mathrm{vir}}^2(m_*)]^{-1/2}$

- Fourier transform is product of Gaussian and FT of K_0 Bessel function, so $p(\mathrm{v})$ is convolution of $\mathrm{G}(\mathrm{v})$ with $\mathrm{K}_0(\mathrm{v})$

- Since $\sigma_{\mathrm{vir}}(m_*) \sim \sigma_{\mathrm{Lin}}$, $p(\mathrm{v}) \sim$ Gaussian at $|\mathrm{v}| < \sigma_{\mathrm{Lin}}$ but exponential-like tails extend to large v

Comparison with simulations



Gaussian core with exponential tails as expected

Redshift space power spectrum

$$P_s(k) = P_{1h}(k) + P_{2h}(k)$$

$$u_s(k|m) = u(k|m)\, e^{-k^2\mu^2\sigma^2_{vir}(m)/2}$$

- $P_{1h}(k) = (1 + f\mu^2)^2 \int dm\, n(m)\, g_2(m)\, |u_s(k|m)|^2/n_g^2$
- $P_{2h}(k) \approx [\int dm\, n(m)\, (b(m) + f\mu^2)\, g_1(m)\, u_s(k|m)/n_g]^2 \times P_{dm}(k)$

Halo Model: HOD, CLF, SHAM

- Goal is to infer p(N|m) from measurements of abundance and clustering
  - Abundance constrains $\langle N|m \rangle = g_1(m)$
  - 1-halo term of n-pt clustering constrains $g_n(m)$
- HOD uses abundance and 2pt statistics to constrain p(N|m) from different samples (Zehavi et al. 2011; Skibba et al. 2014)
- CLF now does too, to constrain $\phi(L|m)$ (Lu et al. 2014)
  - Since $\langle N(>L)|m \rangle = \phi(>L|m)$, HOD~CLF but with different systematics
- SHAM (Klypin+ 1999; Sheth-Jain 2003; Conroy+ 2006) uses abundance only, but gets 2pt stats quite well anyway (Moster et al. 2013)
  - Problematic for samples where relation to halo mass is not monotonic (e.g., color selected samples)

Halo model in practice:
Central + Poisson satellites

- In this model we want to place one galaxy close to (at!) the halo center, and the others with an ~NFW profile around it. So, if we define $u_s(m|k) = u(k|m)\, e^{-k^2\mu^2\sigma_{vir}(m)^2/2}$ then we can write this model, with z-space distortions, as (real space is σ_{vir}=0 and f=0):
- $g_1(m)\, u(k|m)$

  $\rightarrow f_{cen}(m)\, [1 + \langle N_{sat}|m\rangle\, u_s(k|m)]\, (1 + f\mu^2)$

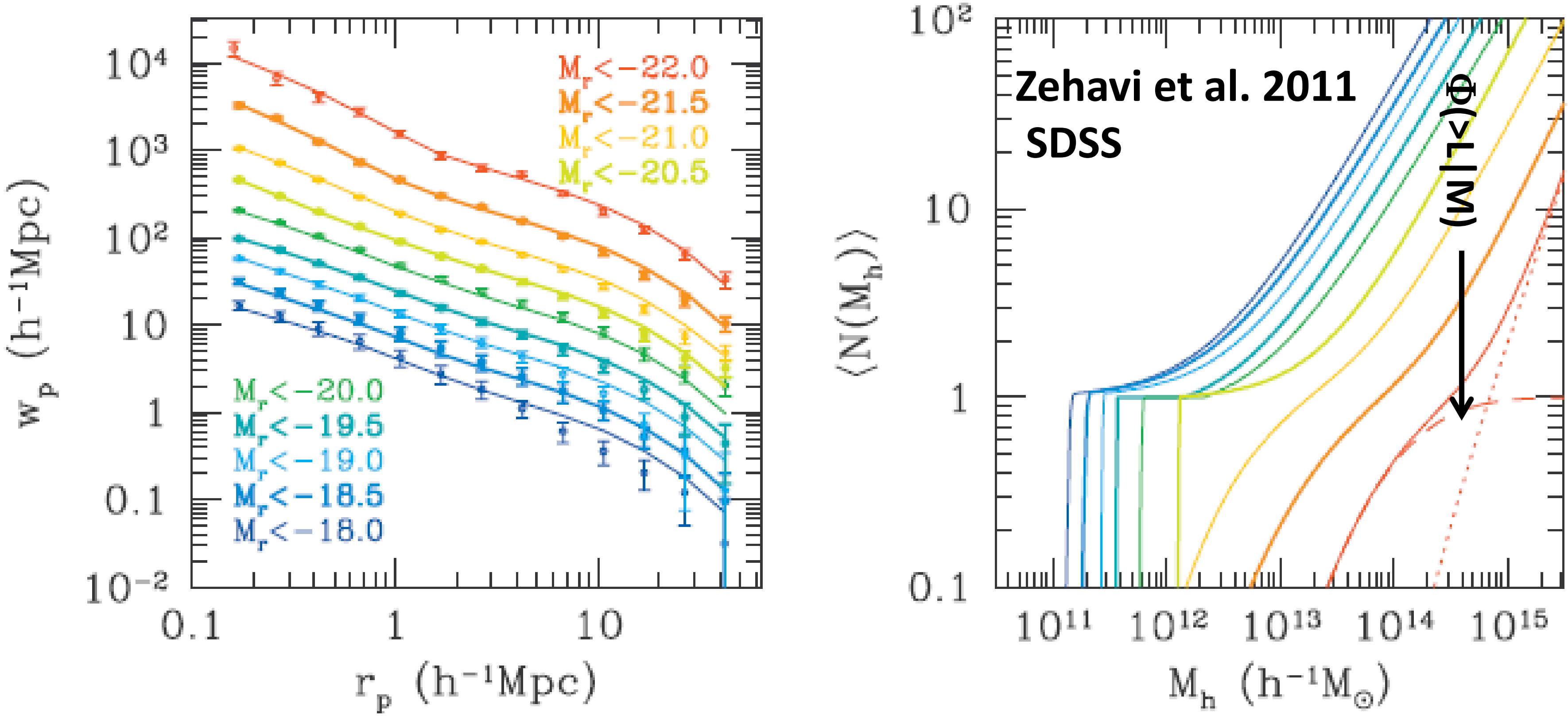
  – (1 instead of u, because the central galaxy is at center, so the relevant 'density profile' is a delta function)
- $g_2(m)\, u^2(k|m)$

  $\rightarrow f_{cen}(m)\, [2\langle N_{sat}|m\rangle\, u_s(k|m) + \langle N_{sat}(N_{sat} - 1)|m\rangle\, u_s^2(k|m)]\, (1 + f\mu^2)^2$

  $= f_{cen}(m)\, [2\langle N_{sat}|m\rangle\, u_s(k|m) + \langle N_{sat}|m\rangle^2\, u_s^2(k|m)]\, (1 + f\mu^2)^2$

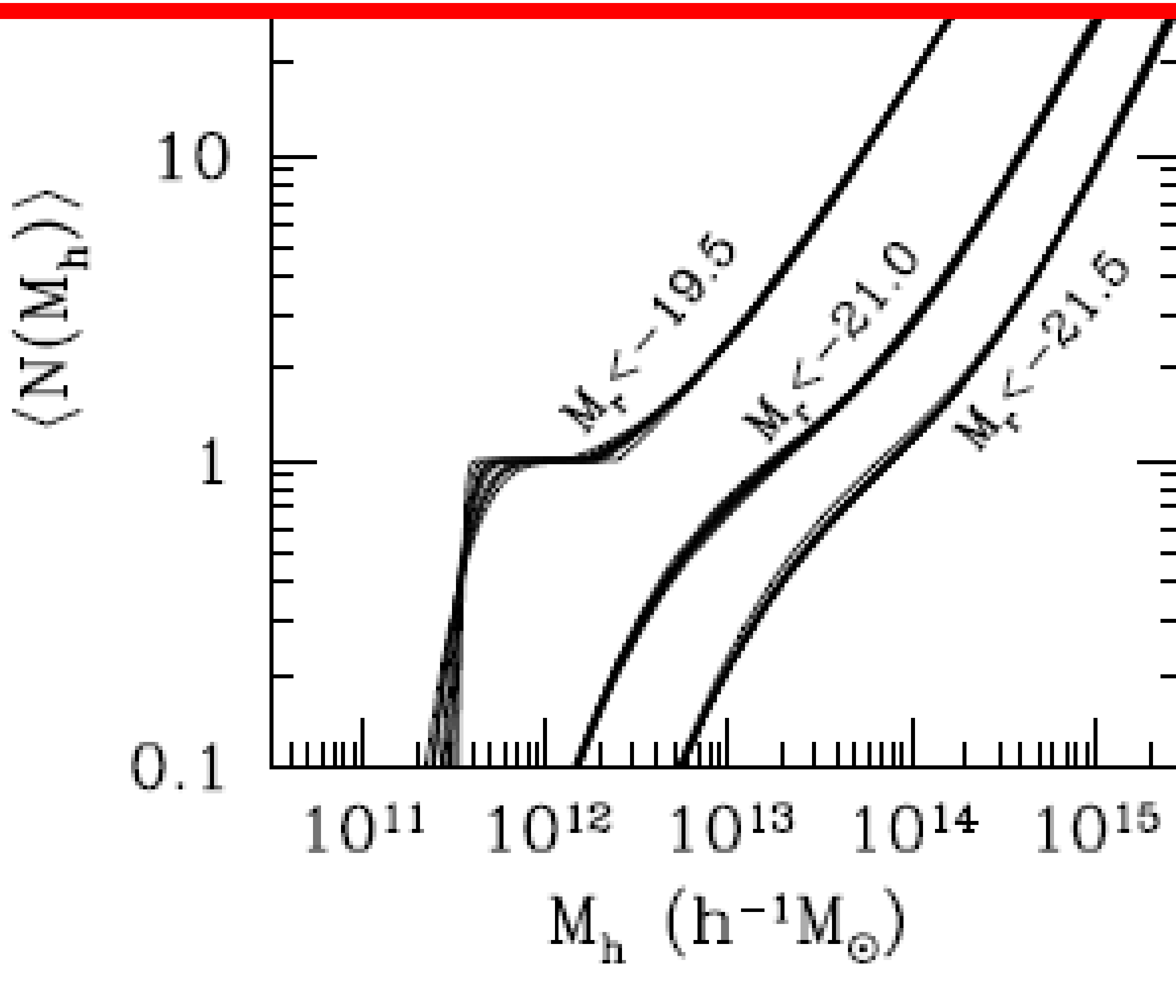
  cen-sat pairs sat-sat pairs

Luminosity dependence of clustering



$$<N_{gal}|m> = f_{cen}(m)\ [1 + <N_{sat}|m>]$$

$$\langle N(M_h)\rangle = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\log M_h - \log M_{\min}}{\sigma_{\log M}}\right)\right]\left[1+\left(\frac{M_h - M_0}{M_1'}\right)^{\alpha}\right], \tag{6}$$



Bells and whistles (which matter for CDM→WDM)

- Mass-concentration and scatter
  - Different profiles for red vs blue
- Distribution of halo shapes
  - Correlation of shapes with surrounding large scale structure
  - Projection effects matter for conc-m relation!
- Substructure = galaxies? Correlations with concentration/formation, time/environment
  - Correlation of substructure with large scale structure

This is a very active field

Nobody goes there anymore –
it’s too crowded

- In early days Halo Model was touted by some as being the end of SAMs; SAMs argued Assembly bias was end of Halo Model
- Increased complexity means SHAM, MEAN not far from SAM (though still simpler)

You should always go to other people’s funerals; otherwise they won’t go to yours.

Halo Model based approaches attractive because they interpret observations in language which is easy to relate to simulations, semi-analytic models

Increased complexity is blurring difference between SHAMs and SAMs

Observational and Assembly biases matter!

Halo Model based approaches attractive because they interpret observations in language which is easy to relate to simulations, semi-analytic models

Increased complexity is blurring difference between SHAMs and SAMs

Observational and Assembly biases matter!

You had better know
where you're going,
or you might not get there

You can observe a lot just by watching

Halo model works well because galaxies small compared to spaces between them

(no halo model yet of Ly-α forest)

Halo Model is simplistic …

- Nonlinear physics on small scales from virial theorem
- Linear perturbation theory on scales larger than virial radius (exploits 20 years of hard work between 1970-1990)
- Halo mass is more efficient language (than e.g., dark matter density) for describing nonlinear field

…but quite accurate!

Useful for cosmology and galaxy formation from Large Scale Structure Sky Surveys

- Baryon Acoustic Oscillations
- Cluster counts and clustering
- Weak gravitational lensing
- Redshift space distortions
- (Supernovae IA)

- Your name here!