Thermodynamics of Work in Open Quantum Systems (and some other stuff…)

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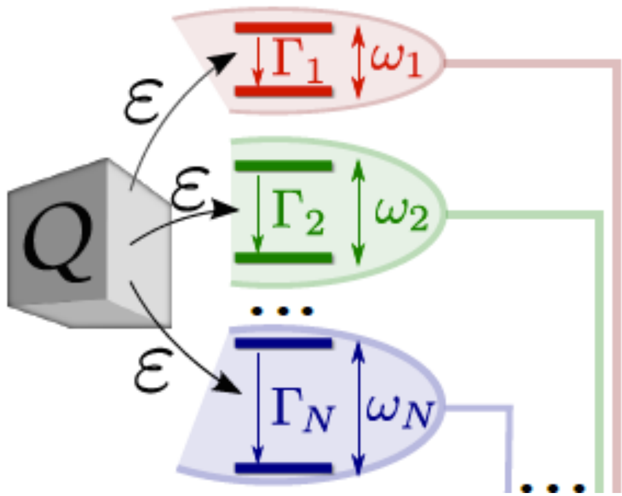

[S. Suomela *et al.*, Phys. Rev. B **90**, 094304 (2014); Phys. Rev. E **91**, 022126 (2015);
S. Suomela, J. Pekola, T.A-N., unpublished]

Tehran April 15th, 2015


Otaniemi campus
Tapio's office
The campus is located in **Otaniemi Technology Hub** in Espoo city, 10 km from the center of Helsinki.
Designed by Alvar Aalto in 1960's.
A unique combination of education, study and business densely packed into a small area by the sea.
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Aalto University (HUT until 2010)

A merger of three leading Finnish universities in January 2010

University of Art and Design Helsinki
founded 1871

Helsinki School of Economics
founded 1911

Helsinki University of Technology
founded 1849

The first Foundation University in Finland, with an independent board
Initial capital of the foundation 700 M€, now 1 000 M€

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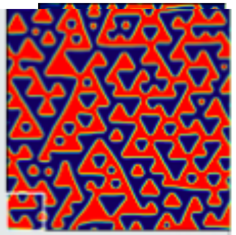
Multiscale Statistical Physics group of Tapio Ala-Nissilä: Between Quantum and Classical, and Nano and Macro



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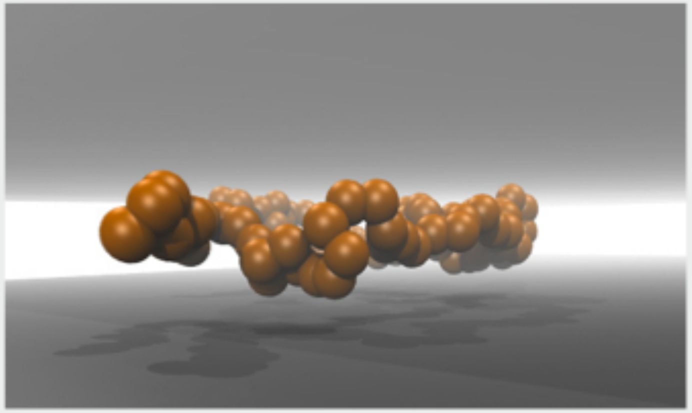
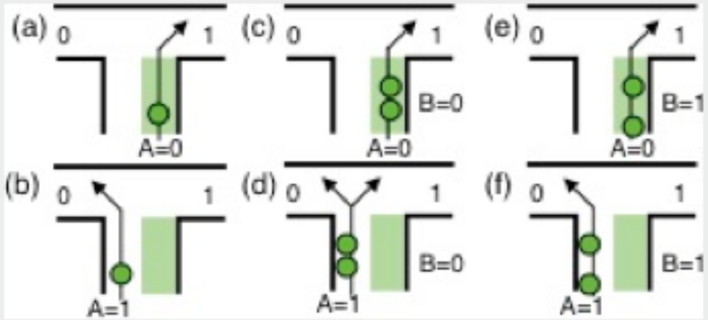
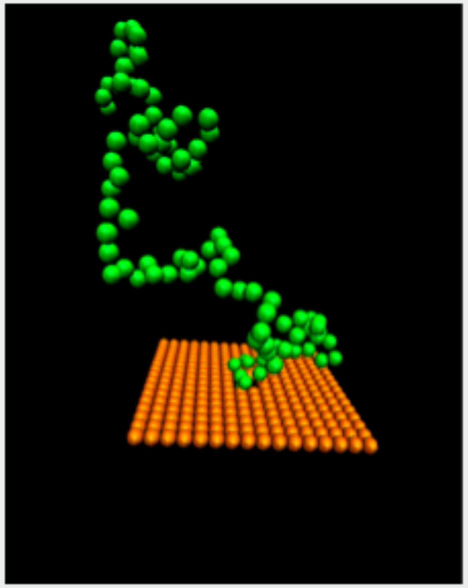


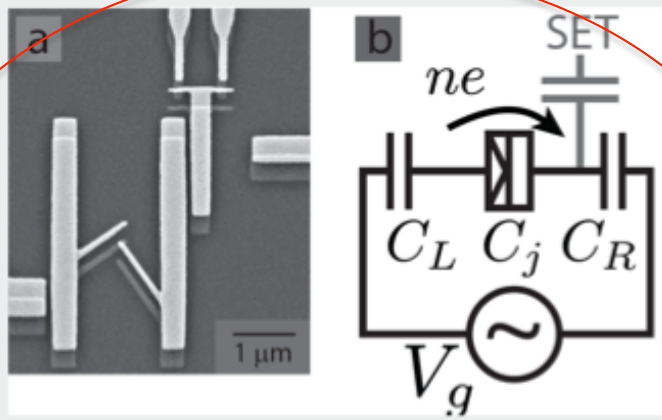
Micro and Nanofluidic systems



"Colloidal Computer"



Polymer translocation through nanopores



$$\langle e^{-\beta(W-\Delta F)}\rangle = 1\,?$$

Fluctuations in small systems

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Nanofluid heat transfer measurements

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International Journal of Heat and Mass Transfer 61 (2013) 439–448

Contents lists available at SciVerse ScienceDirect

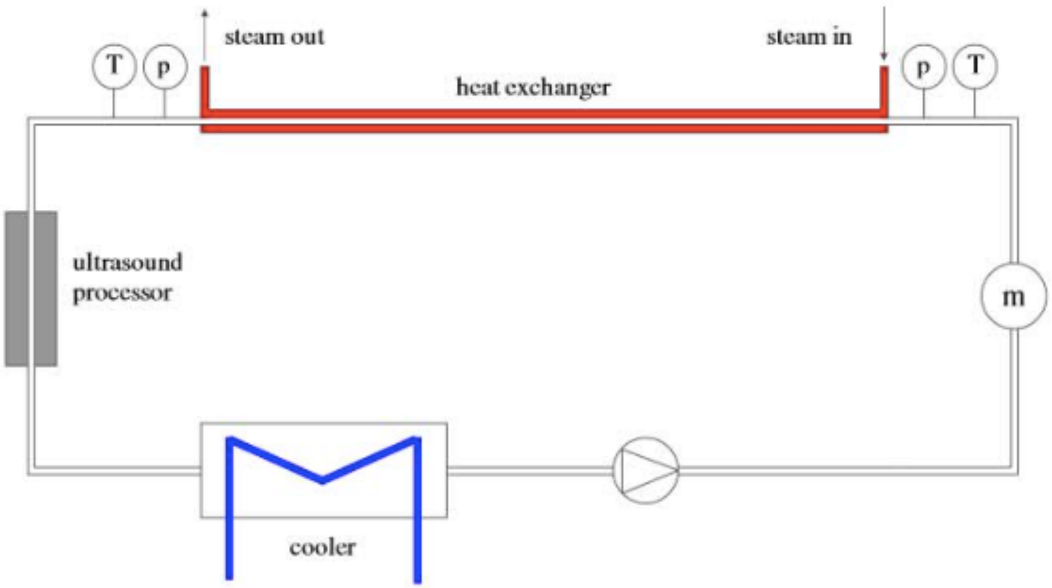
International Journal of Heat and Mass Transfer

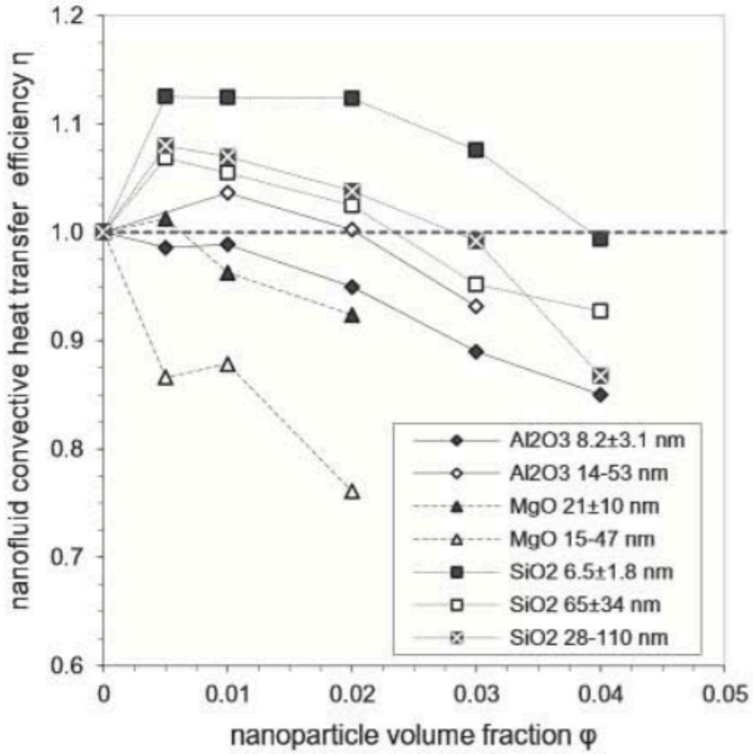
journal homepage: www.elsevier.com/locate/ijhmt

ELSEVIER

Influence of particle size and shape on turbulent heat transfer characteristics and pressure losses in water-based nanofluids

Arttu Meriläinen [a,*], Ari Seppälä [a], Kari Saari [a], Jani Seitsonen [b], Janne Ruokolainen [b], Sakari Puisto [c], Niko Rostedt [c], Tapio Ala-Nissila [d,e]





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Nanofluid heat transfer measurements

International Journal of Heat and Mass Transfer 81 (2015) 246–251

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Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

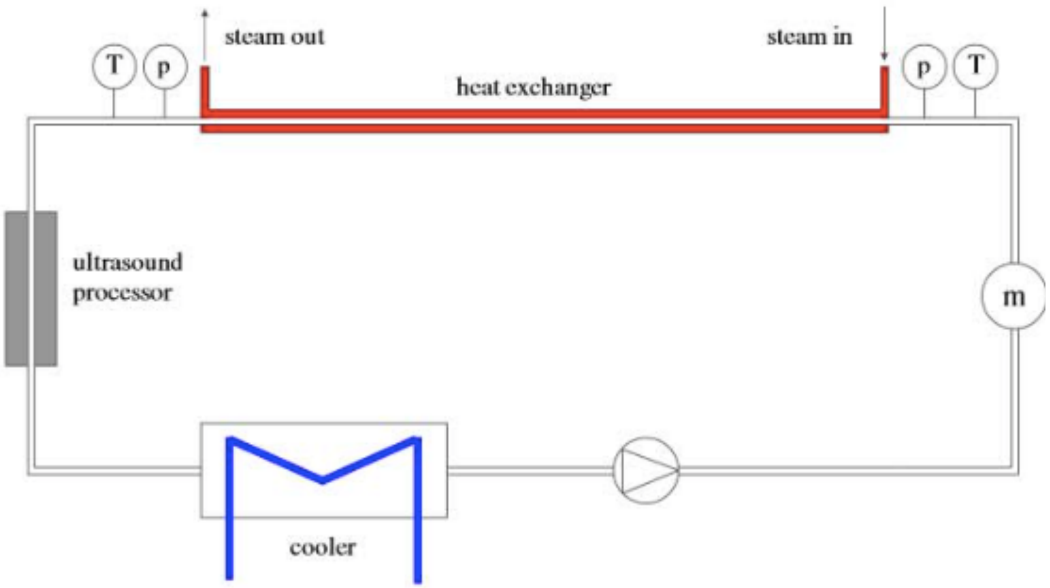
journal homepage: www.elsevier.com/locate/ijhmt

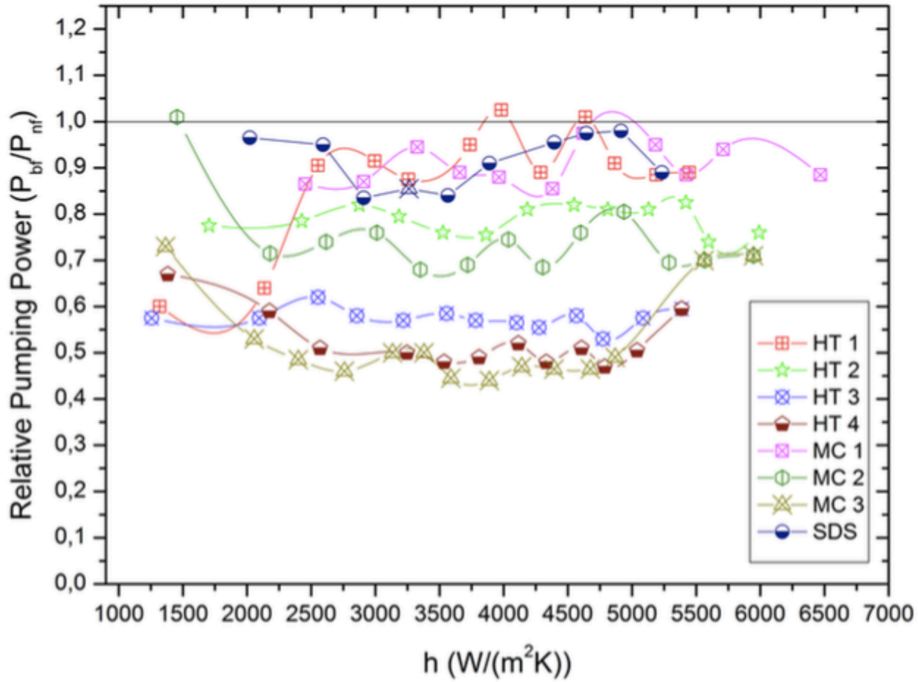
ELSEVIER

Turbulent heat transfer characteristics in a circular tube and thermal properties of n-decane-in-water nanoemulsion fluids and micelles-in-water fluids

CrossMark

Sampo Saarinen [a], Salla Puupponen [a], Arttu Meriläinen [b], Aliakbar Joneidi [a], Ari Seppälä [a,*], Kari Saari [a], Tapio Ala-Nissila [c,d]





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Miracles happen at nanoscale!

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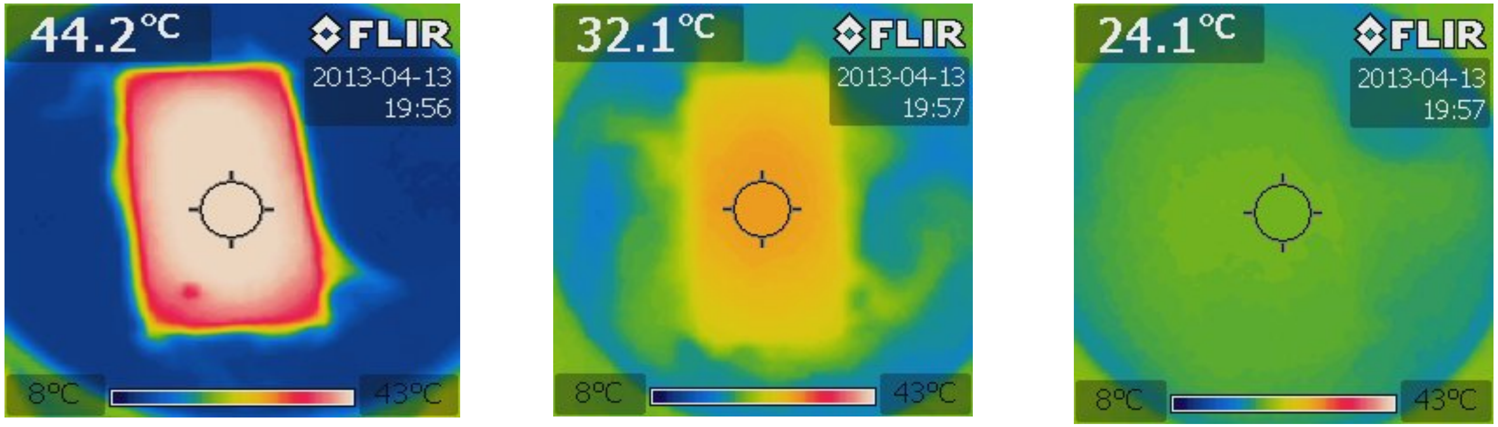
Arrow of time



Suppose the piece is a cube of aluminium, edge length x

If x = 5 cm => $P_R / P_L = 10^{10^{10}}$

If x = 5 nm => $P_R / P_L = 4.5$

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Stochastic thermodynamics

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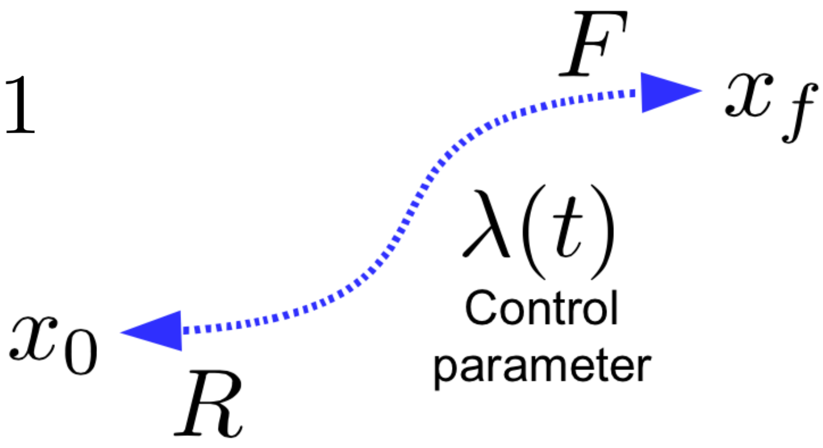
Thermodynamic quantities (heat, energy, entropy) become *fluctuating stochastic quantities,* which in many (*but not in all*) cases obey *fluctuation relations*

Jarzynski (1997): $\langle e^{-\beta W_d} \rangle = 1$

Crooks FT (1998):

$$\frac{P_F(-W_d)}{P_R(W_d)} = e^{-\beta W_d}$$



Seifert (2005): $\langle e^{-\Delta S_T} \rangle = 1$ (generally true)

etc.

$$\boxed{W_d = W - \Delta F}$$

"dissipative work"

Derivation of Fluctuation relations

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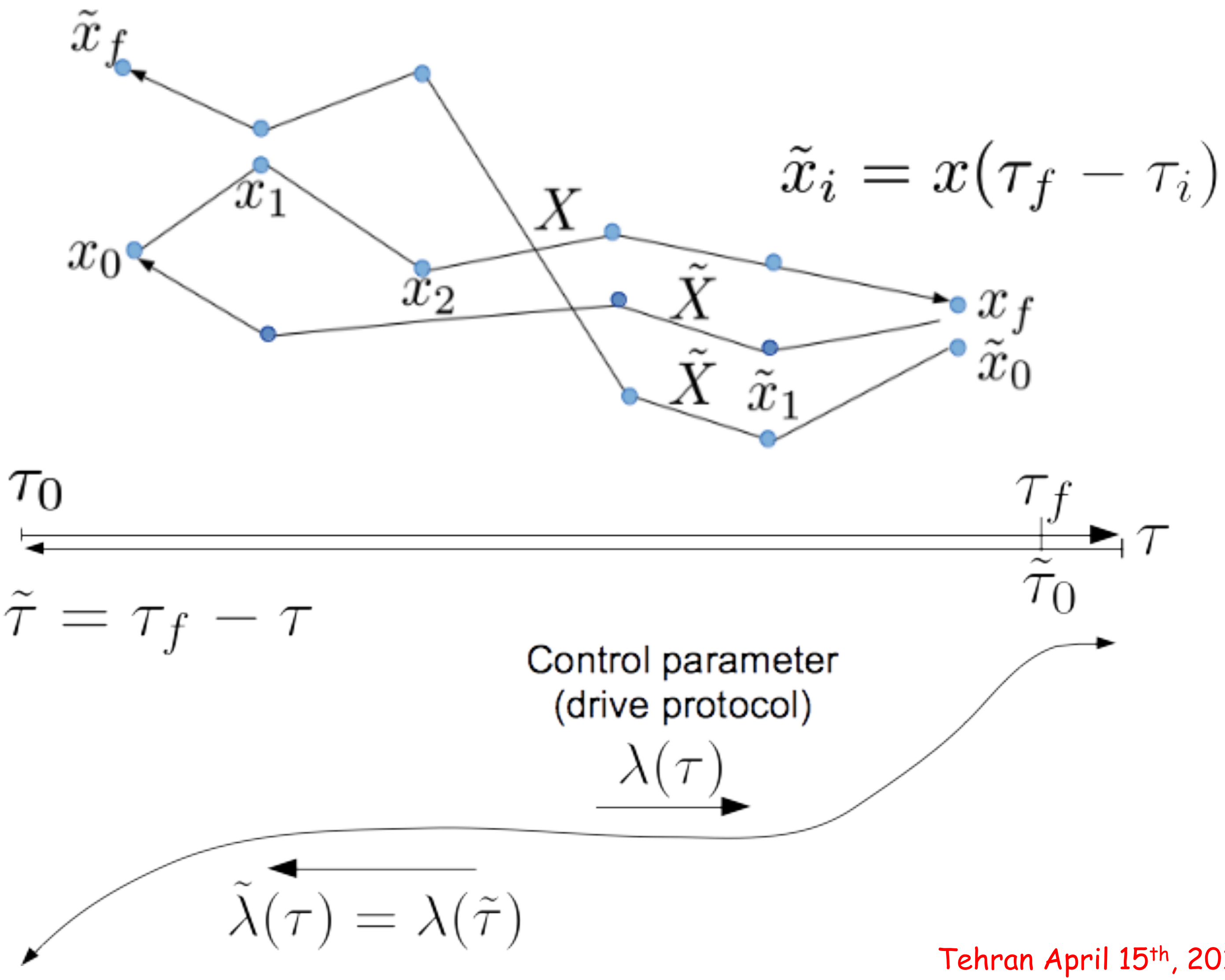
Stochastic forward and reverse paths X and $\tilde{X}$ for a driven (open) system with states x_i

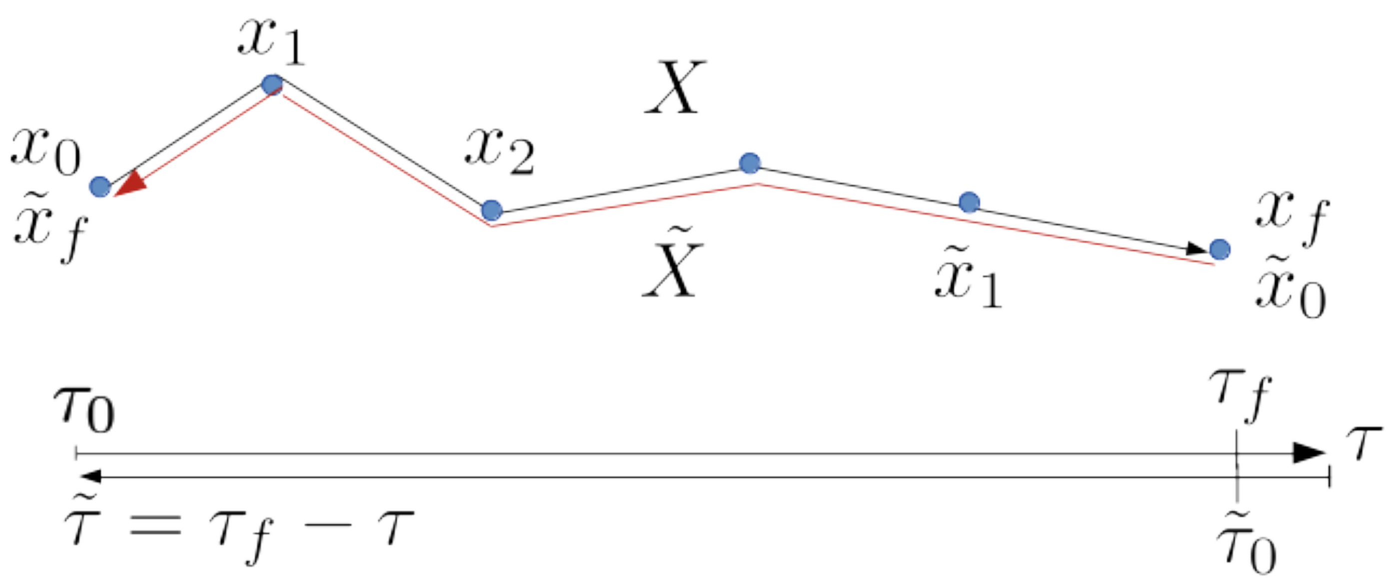
System states
$x_i = x(\tau_i)$



Derivation of Fluctuation relations

Path probabilities with transition rates

$$W_{x_j,x_{j-1}}(\lambda_j)$$



Forward path $P[X|x_0] = C[X]\prod_{j=1}^{N} W_{x_j,x_{j-1}}(\lambda_j)$

Time-reversed path $P[\tilde{X}|\tilde{x}_0] = C[X]\prod_{j=1}^{N} W_{x_{j-1},x_j}(\lambda_j)$

Derivation of Fluctuation relations

The total entropy change can be written as (Seifert (2005))

$$(*)\Delta S_T[X] = \ln \frac{P[x_0]P[X|x_0]}{P[\tilde{x}_0]P[\tilde{X}|\tilde{x}_0]}$$

Derivation of Fluctuation relations

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This can be split as

$$\Delta S_T[X] = \Delta S_S[X] + \Delta S_R[X] = \ln \frac{P[x_0]}{P[\tilde{x}_0]} + \ln \frac{P[X|x_0]}{P[\tilde{X}|\tilde{x}_0]}$$

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Derivation of Fluctuation relations

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$$(*)\Delta S_T[X] = \ln \frac{P[x_0]P[X|x_0]}{P[\tilde{x}_0]P[\tilde{X}|\tilde{x}_0]} \qquad (**)\langle e^{-\Delta S_T[X]}\rangle = 1$$

Mathematical identity
Conservation of probability

This can be split as

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Derivation of Fluctuation relations

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Assuming "local" detailed balance and Boltzmann distribution

$$\frac{W_{x_j,x_{j-1}}(\lambda_j)}{W_{x_{j-1},x_j}(\lambda_j)} = e^{-\beta(E_{x_j}-E_{x_{j-1}})(\lambda_j)}$$

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Derivation of Fluctuation relations

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Using (local) detailed balance & canonical equilibrium:

$$\Delta S_T[X] = \beta \Delta E[X] - \beta \Delta F - \beta Q[X] = \beta(W[X] - \Delta F)$$

The total entropy expression $(*) \Delta S_T[X] = \ln \frac{P[x_0]P[X|x_0]}{P[\tilde{x}_0]P[\tilde{X}|\tilde{x}_0]}$ becomes the Crooks FT

$$\frac{P_F(-W_d)}{P_R(W_d)} = e^{-\beta W_d}$$

The Seifert integral FT $(**) \langle e^{-\Delta S_T[X]} \rangle = 1$ becomes the Jarzynski eqn.

$$\langle e^{-\beta W_d} \rangle = 1$$

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Work in Quantum Systems

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Problem: There is no (unique) Hermitian "work" operator $\hat{W}$ (there is only the energy operator $\hat{H}$)

Solution(?): Several possible definitions of quantum work have been proposed:

M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. **81**, 1665 (2009)
P. Solinas, D.V. Averin, and J.P. Pekola, Phys. Rev. B **87**, 060508 (2013)
J.M. Horowitz and J.M.R. Parrondo, New J. Phys. **15**, 085028 (2013)
S. Suomela, P. Solinas, J.P. Pekola, J. Ankerhold and T. Ala-Nissila, Phys. Rev. B **90**, 094304 (2014)

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Work in unitary quantum system

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Assume unitary time evolution with

$$\hat{U}(t) = T_{>} e^{-\frac{i}{\hbar}\int_{t_A}^{t_B} dt' \, \hat{H}(t')}$$

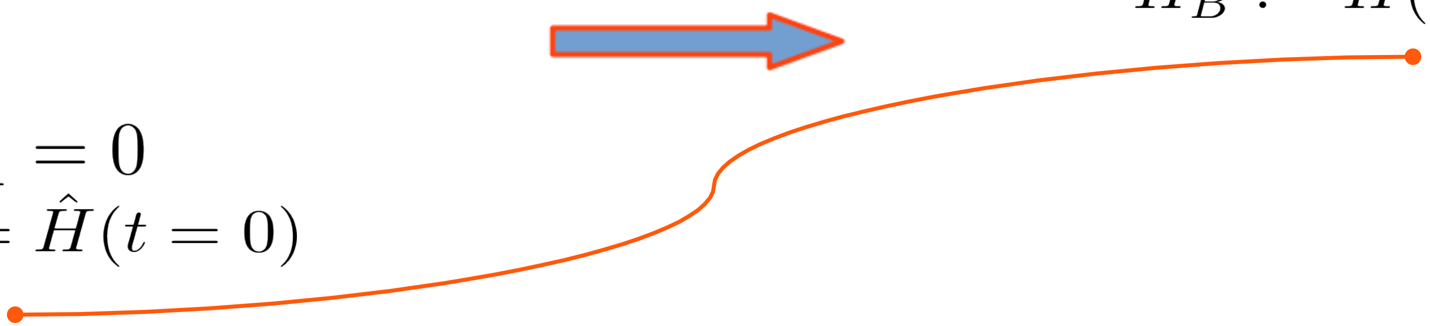
$$\lambda(t)$$

$$t = t_B$$

$$\hat{H}_B := \hat{H}(t = t_B)$$

$$t_A = 0$$

$$\hat{H}_A := \hat{H}(t = 0)$$



$$\hat{H}_A|\psi_n^{(A)}\rangle = E_n^{(A)}|\psi_n^{(A)}\rangle \longrightarrow \hat{U}(t_B)|\psi_n^{(A)}\rangle$$

Work in unitary quantum system

$$t = t_B$$

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$$\lambda(t)$$

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$$\hat{H}_A := \hat{H}(t = 0)$$

$$\hat{H}_A|\psi_n^{(A)}\rangle = E_n^{(A)}|\psi_n^{(A)}\rangle \longrightarrow \hat{U}(t_B)|\psi_n^{(A)}\rangle$$

The probability to be in state $|\psi_n^{(A)}\rangle$

$$p_n^{(A)} = \frac{e^{-\beta E_n^{(A)}}}{\mathcal{Z}_A} = e^{-\beta(F_A - E_n^{(A)})}$$

$$F_A = -k_B T \ln \mathcal{Z}_A$$

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Work in unitary quantum system

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$$t = t_B$$
$$\hat{H}_B := \hat{H}(t = t_B)$$

$$\lambda(t)$$

$$t_A = 0$$
$$\hat{H}_A := \hat{H}(t = 0)$$

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$$F_A = -k_B T \ln \mathcal{Z}_A$$

The probability to be in state $|\psi_m^{(B)}\rangle$ with energy $E_m^{(B)}$

$$p_{nm} = |\langle\psi_m^{(B)}|\hat{U}(t_B)|\psi_n^{(A)}\rangle|^2$$

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Work in unitary quantum system

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- The work done on the system is given by

$$W = E_m^{(B)} - E_n^{(A)},$$

since the system is isolated

- The Jarzynski average

$$\begin{aligned}
\langle e^{-\beta W} \rangle &= \sum_n p_n^{(A)} \sum_m p_{mn} e^{-\beta W} \\
&= e^{-\beta F_A} \sum_{n,m} e^{-\beta E_n^{(A)}} |\langle \psi_m^{(B)} | \hat{U}(t_B) | \psi_n^{(A)} \rangle|^2 e^{-\beta (E_m^{(B)} - E_n^{(A)})} \\
&= e^{-\beta F_A} \sum_{n,m} \langle \psi_m^{(B)} | \hat{U}(t_B) | \psi_n^{(A)} \rangle \langle \psi_n^{(A)} | \hat{U}(t_B) | \psi_m^{(B)} \rangle e^{-\beta E_m^{(B)}} \\
&= e^{\beta F_A} \sum_m e^{-\beta E_m^{(B)}} = e^{\beta F_A} \times e^{-\beta F_B} = e^{-\beta \Delta F}
\end{aligned}$$

Work in unitary quantum system

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$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

- Jarzynski equation is recovered in the *two-point measurement protocol* (TMP) in the case where the dynamics is unitary

- Similar to the classical case, if the system is *not* isolated during the drive, there are problems with the Jarzynski equation...

Work in open quantum systems

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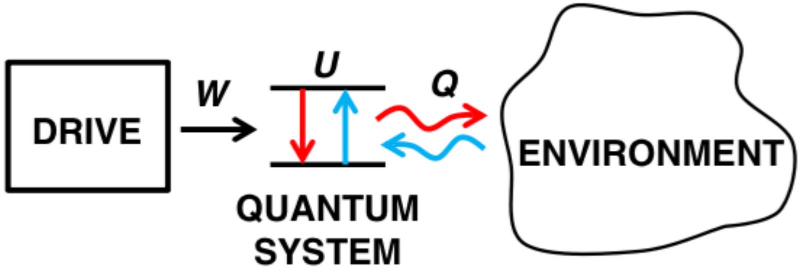
- Consider an open quantum system driven by a classical protocol $\lambda(t)$. The total Hamiltonian

$$\hat{H}(t) = \hat{H}_S(t) + \hat{H}_B + \hat{H}_C$$

system bath system-bath coupling



[Hekking & Pekola PRL (2013)]

where

$$\hat{H}_S(t) = \hat{H}_0 + \hat{V}(t) = \hat{H}_0 + \lambda(t)\hat{V}_0$$

[Quantum drive considered in J. Salmilehto *et al.*, PRE **89**, 052128 (2014)]

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Work in open quantum systems

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- To consider work on an O.Q.S., define the *Power Operator* [Solinas *et al.* PRB (2013)]:

$$\hat{P}(t) := \frac{\partial \hat{H}}{\partial \lambda}\dot{\lambda} = \frac{\partial \hat{H}}{\partial t}$$

- The average power is given by

$$\langle \hat{P}(t) \rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{P}(t)]$$

and the work $\langle W \rangle := \int_{t_A}^{t_B} \langle \hat{P}(t) \rangle dt$

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Work in open quantum systems

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- If the dynamics can be described by a master equation with dissipation operator $\hat{\mathcal{L}}(\rho)$, then

$$\langle W\rangle = \langle \hat{H}(t_B)\rangle - \langle \hat{H}(t_A)\rangle - \int_{t_A}^{t_B} dt\,\mathrm{Tr}[\hat{\mathcal{L}}(\rho)\hat{H}(t)]$$

vanishes for isolated systems (“heat” term)

Power operator approach gives work different from TMP even for closed systems when at coherent superposition of eigenstates

Stochastic quantum dynamics

- The classical stochastic dynamics can be generalized to the quantum case by considering instantaneous transitions induced by (weak) coupling to the environment

- For open systems, the *Lindblad master equation* contains coupling terms with the environment

- For single realizations: Stochastic evolution

  [Horowitz & Parrondo, New J. Phys. (2013)]

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Stochastic quantum dynamics

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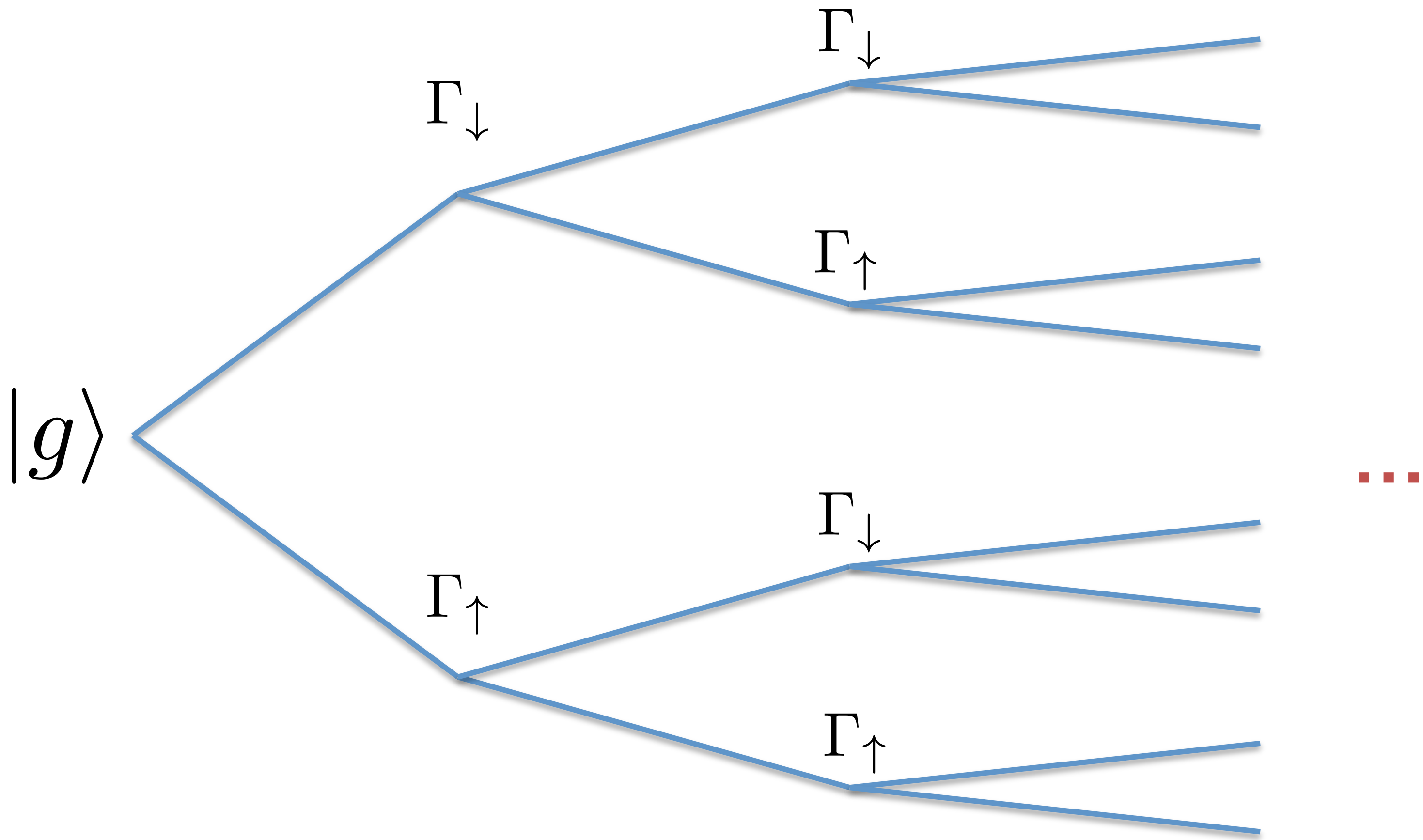

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Stochastic quantum dynamics

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- The Lindblad master equation:

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar}\left[\hat{H}_S(t),\hat{\rho}(t)\right] + \sum_m \hat{C}_m(t)\hat{\rho}(t)\hat{C}_m^\dagger(t) - \frac{1}{2}\sum_m\left(\hat{C}_m^\dagger(t)\hat{C}_m(t)\hat{\rho}(t) + \hat{\rho}(t)\hat{C}_m^\dagger(t)\hat{C}_m(t)\right)$$

Main assumptions: Weak coupling to the heat bath, memory-free heat bath, and neglecting fast oscillations

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Stochastic quantum dynamics

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Stochastic quantum dynamics

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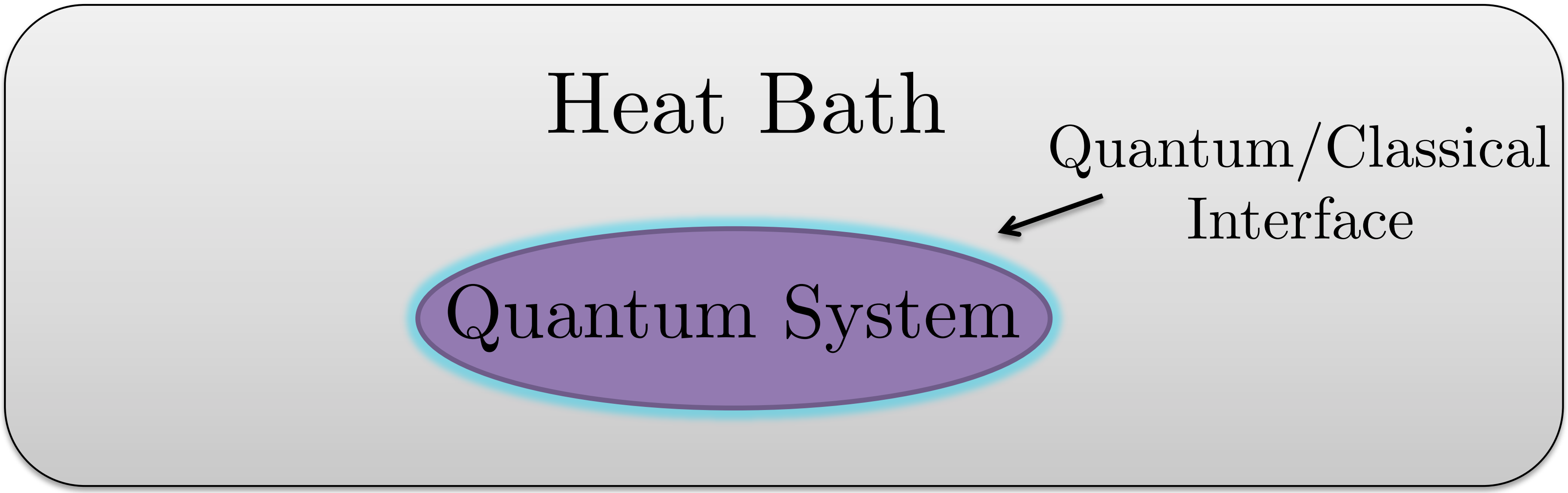
$$-\frac{1}{2}\sum_m \left(\hat{C}_m^\dagger(t)\hat{C}_m(t)\hat{\rho}(t) + \hat{\rho}(t)\hat{C}_m^\dagger(t)\hat{C}_m(t)\right)$$

Main assumptions: Weak coupling to the heat bath, memory-free heat bath, and neglecting fast oscillations

Stochastic quantum dynamics

- Quantum jump (Monte Carlo wave function) method: Stochastic unraveling of the Lindblad equation
- Semi-Classical: Bath acts as a classical measurer



[Mølmer, Castin & Dalibard, JOSA B (1993); Badescu, Ying & TA-N, PRL (2001); Hekking & Pekola, PRL (2013)]

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1. Derivation of Integral Fluctuation Theorem for stochastic quantum dynamics

[S. Suomela, J. Salmilehto, I.G. Savenko, T.A-N & M. Möttönen, Phys. Rev. E **91**, 022126 (2015)]

Tehran April 15th, 2015

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Stochastic quantum dynamics: IFT

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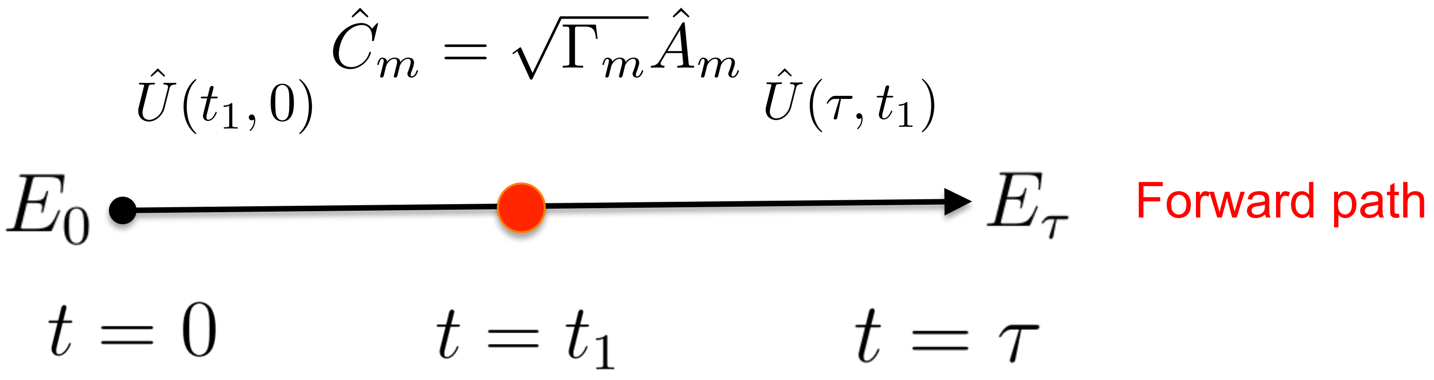

Between jumps the time evolution is given by

$$\hat{U}(t_{j+1}, t_j) = \mathcal{T}_{\leftarrow} e^{-\frac{i}{\hbar}\int_{t_j}^{t_{j+1}}\left(\hat{H}_S(t) - \frac{i\hbar}{2}\sum_m \hat{C}_m^\dagger \hat{C}_m\right)dt}$$

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Stochastic quantum dynamics: IFT

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$$\hat{C}_m = \sqrt{\Gamma_m}\hat{A}_m$$

$\hat{U}(t_1, 0)$ $\hat{U}(\tau, t_1)$

E_0 → E_τ Forward path

$t = 0$ $t = t_1$ $t = \tau$

$$\hat{C}'_m = \sqrt{\Gamma'_m}\hat{A}^\dagger_m$$

$\hat{U}^\dagger(t_1, 0)$ $\hat{U}^\dagger(\tau, t_1)$

E_0 ← E_τ Reverse path (‘)

$t = 0$ $t = t_1$ $t = \tau$

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Stochastic quantum dynamics: IFT

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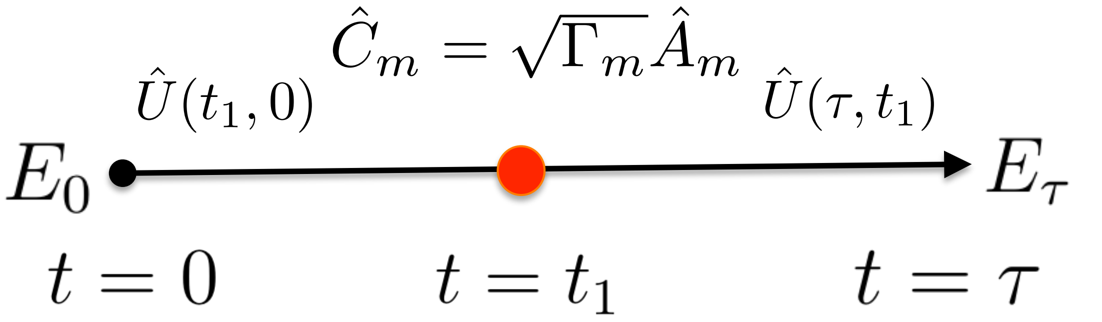

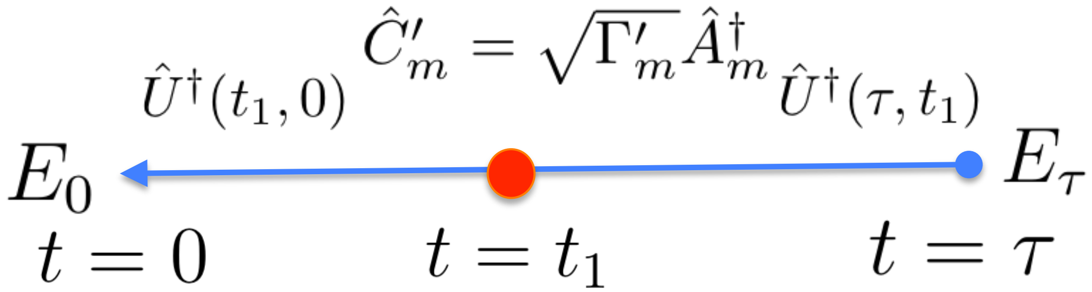

If $\sum_m \Gamma_m \hat{A}^\dagger_m \hat{A}_m = \sum_m \Gamma'_m \hat{A}_m \hat{A}^\dagger_m$

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Stochastic quantum dynamics: IFT

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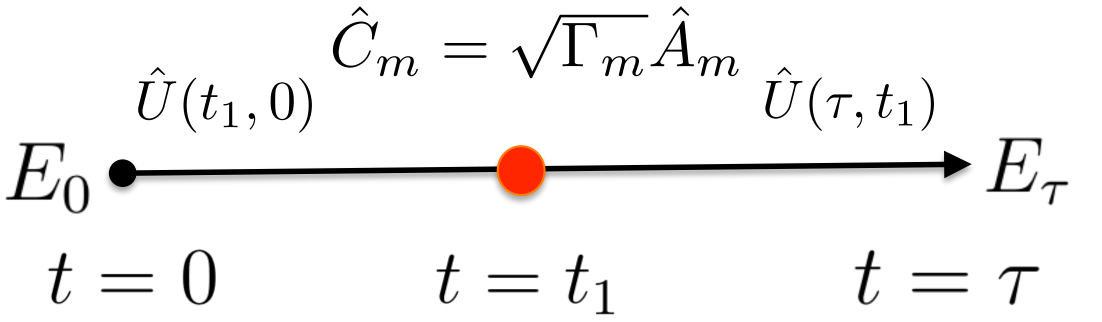

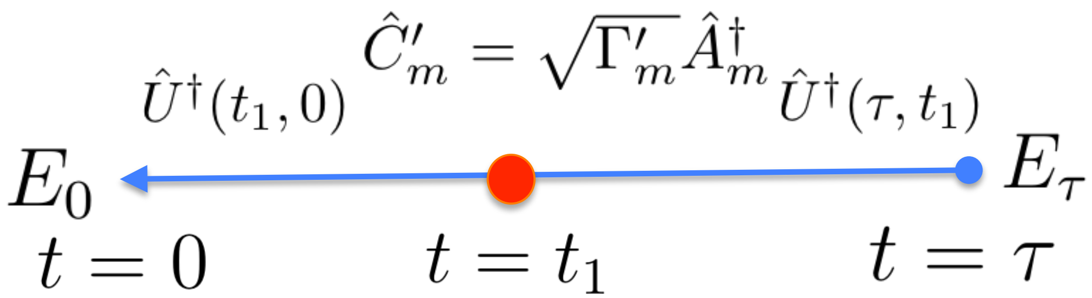

If $\sum_m \Gamma_m \hat{A}^\dagger_m \hat{A}_m = \sum_m \Gamma'_m \hat{A}_m \hat{A}^\dagger_m$

$$\boxed{\left\langle \frac{Pr'(E(\tau)=E_\tau)}{Pr(E(t_0)=E_0)} \prod_{j=1}^{N} \frac{\Gamma'_{m_j}(t_j)}{\Gamma_{m_j}(t_j)} \right\rangle = 1}$$

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Stochastic quantum dynamics: IFT

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$$\left\langle \frac{Pr'(E(\tau)=E_\tau)}{Pr(E(t_0)=E_0)} \prod_{j=1}^{N} \frac{\Gamma'_{m_j}(t_j)}{\Gamma_{m_j}(t_j)} \right\rangle = 1$$

Assuming Boltzmann distribution *and* that the noise source follows detailed balance s.t. the ratio of the transition rate gives dissipated heat, gives the Jarzynski eq.

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

2. Derivation of generating functional and moments of work

[S. Suomela, P. Solinas, J.P. Pekola, J. Ankerhold & T.A-N,
Phys. Rev. B **90**, 094304 (2014)]

Generating functional within TMP

$$G(u) = \mathrm{Tr}\left\{ \mathcal{T}_{\rightarrow} \exp\left(\int_0^{\tau} dt \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(iu)^n}{n!2^n} \hat{C}_n^H(t) \right) \times \mathcal{T}_{\leftarrow} \exp\left(\int_0^{\tau} dt \sum_{n=1}^{\infty} \frac{(iu)^n}{n!2^n} \hat{C}_n^H(t) \right) \bar{\hat{\rho}}_0 \right\}$$

where $\hat{C}_1(\tau) = \hat{P}(\tau) = \partial_\tau \hat{H}(\tau),$

$$\hat{C}_2(\tau) = \left[\hat{H}(\tau), \partial_\tau \hat{H}(\tau) \right],$$

etc.

[Generalization of Esposito, Harbola, Mukamel, RMP (2009)]

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Generating functional within TMP: Moments of work

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$$\langle W \rangle = \int_0^\tau dt_1 \langle \hat{P}^H(t_1) \rangle,$$

$$\langle W^2 \rangle = 2 \int_0^\tau dt_1 \int_0^{t_1} dt_2 \mathrm{Re} \left\{ \langle \hat{P}^H(t_1) \hat{P}^H(t_2) \rangle \right\},$$

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Generating functional within TMP: Moments of work

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$$\langle W^3\rangle_0 = 3\int_0^\tau dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \mathrm{Re}\left\{\langle \hat{P}^H(t_1)\hat{P}^H(t_2)\hat{P}^H(t_3)\rangle + \langle \hat{P}^H(t_3)\hat{P}^H(t_1)\hat{P}^H(t_2)\rangle\right\}, \quad \text{if } [\hat{H}(t), \partial_t \hat{H}(t)] = 0$$

$$\langle W^3\rangle = \langle W^3\rangle_0 + \frac{1}{4}\int_0^\tau dt \langle \hat{C}_3^H(t)\rangle + \frac{3}{2}\int_0^\tau dt_1 \int_0^{t_1} dt_2 \mathrm{Re}\left\{\langle \hat{C}_1^H(t_1)\hat{C}_2^H(t_2)\rangle\right\},$$

...

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Moments of work: Weakly driven & coupled 2-level OQS

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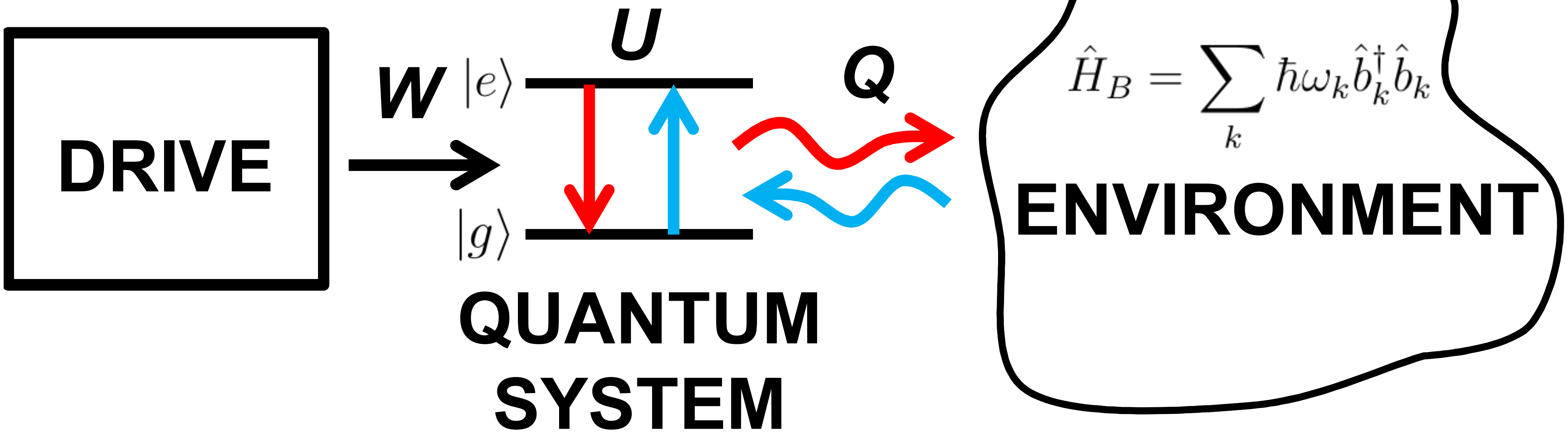
$$\hat{H}_C = \sum_k (\hat{a}^\dagger + \hat{a}) \otimes (g_k \hat{b}_k^\dagger + g_k^* \hat{b}_k)$$

$$\hat{V}(t) = \lambda(t)(\hat{a}^\dagger + \hat{a})$$

$$\lambda(t) = \lambda_0 \sin(\omega_0 t)$$

$$\hat{H}_0 = \hbar\omega_0 \hat{a}^\dagger \hat{a}$$



[F.W.J. Hekking and J.P. Pekola, PRL **111**, 093602 (2013)]

Moments of work: Weakly driven & coupled 2-level OQS

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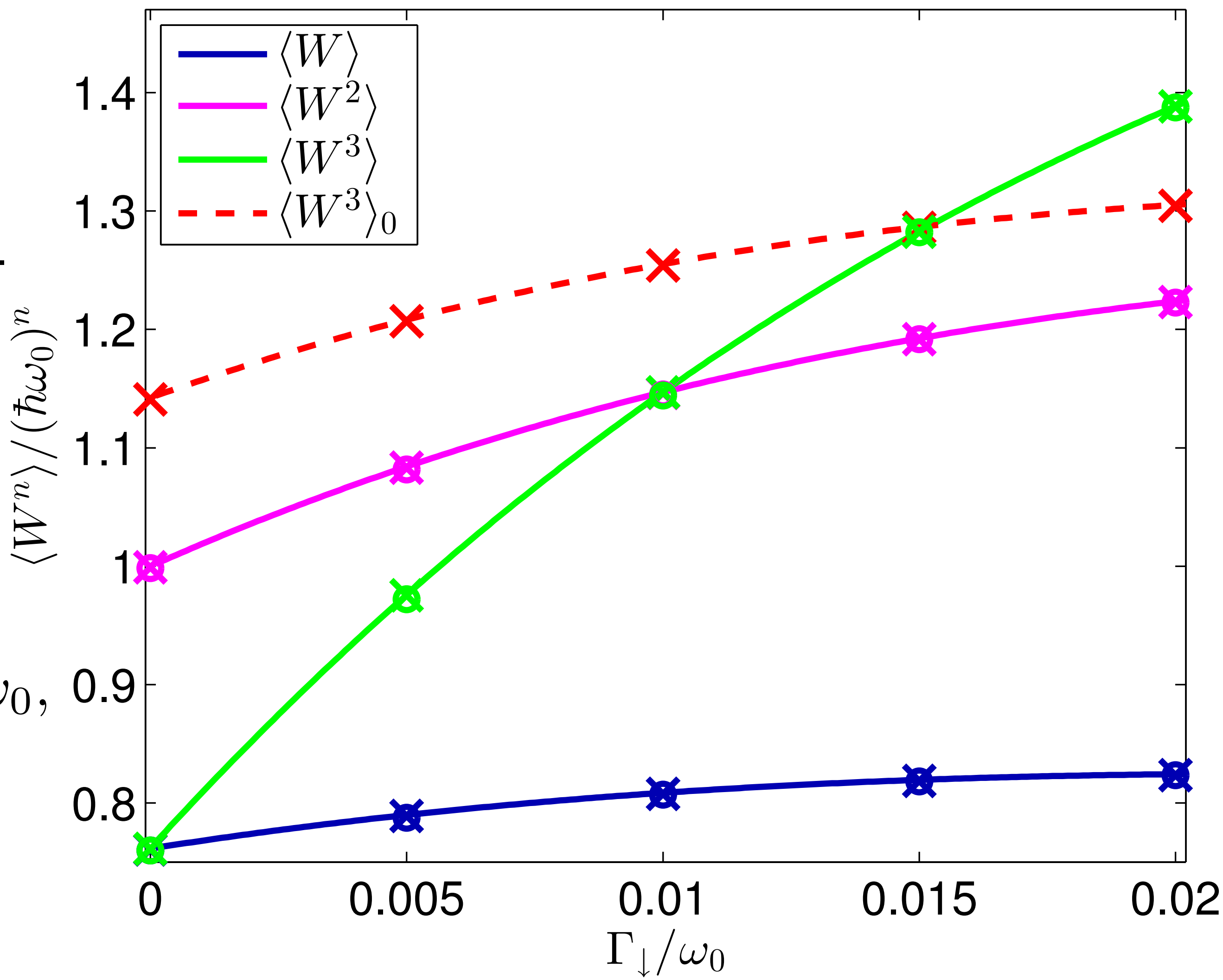
- ● Quantum jump
- ✖ Numerical Master Eqn.



$\lambda_0 = 0.05\hbar\omega_0,$

$\omega_0\tau = 20\pi$

3. Nearly adiabatically driven OQS

[S. Suomela, J. Salmilehto, I.G. Savenko, T.A-N & M. Möttönen, Phys. Rev. E **91**, 022126 (2015)]

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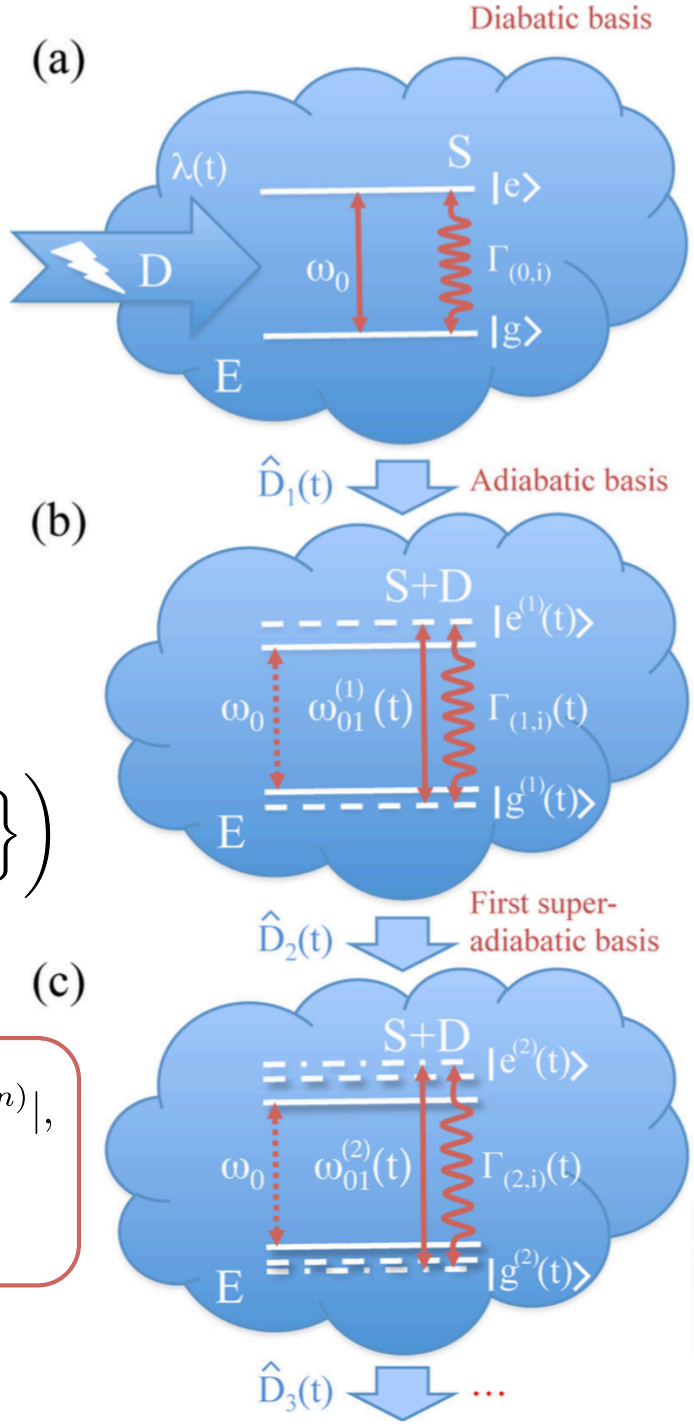
- Adiabatic renormalization of the time-dependent basis gives the Lindblad equation (Samu Suomela's talk on Thu.)

$$\dot{\hat{\rho}}_S = -\frac{i}{\hbar}\left[\hat{H}_S, \hat{\rho}_S\right] + \sum_{i=0}^{2}\left(\hat{L}_{(n,i)}\hat{\rho}_S\hat{L}^\dagger_{(n,i)} - \frac{1}{2}\left\{\hat{L}^\dagger_{(n,i)}\hat{L}_{(n,i)}, \hat{\rho}_S\right\}\right)$$

$$\hat{L}_{(n,0)} = \sqrt{\Gamma_{(n,0)}}|g^{(n)}\rangle\langle e^{(n)}|, \quad \hat{L}_{(n,1)} = \sqrt{\Gamma_{(n,1)}}|e^{(n)}\rangle\langle g^{(n)}|,$$
$$\hat{L}_{(n,2)} = \sqrt{\Gamma_{(n,2)}}(|e^{(n)}\rangle\langle e^{(n)}| - |g^{(n)}\rangle\langle g^{(n)}|)$$



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Adiabatic renormalization

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The adiabatic renormalization gives the following master equation:

$$\dot{\hat{\rho}}_S = -\frac{i}{\hbar}\left[\hat{H}_S, \hat{\rho}_S\right] + \sum_{i=0}^{2}\left(\hat{L}_{(n,i)}\hat{\rho}_S\hat{L}^{\dagger}_{(n,i)} - \frac{1}{2}\left\{\hat{L}^{\dagger}_{(n,i)}\hat{L}_{(n,i)}, \hat{\rho}_S\right\}\right)$$

$$\hat{L}_{(n,0)} = \sqrt{\Gamma_{(n,0)}}|g^{(n)}\rangle\langle e^{(n)}|, \qquad \hat{L}_{(n,1)} = \sqrt{\Gamma_{(n,1)}}|e^{(n)}\rangle\langle g^{(n)}|,$$

$$\hat{L}_{(n,2)} = \sqrt{\Gamma_{(n,2)}}(|e^{(n)}\rangle\langle e^{(n)}| - |g^{(n)}\rangle\langle g^{(n)}|)$$

Adiabatic renormalization

- Recall the IFT: $\left\langle \frac{Pr'(E(\tau)=E_\tau)}{Pr(E(t_0)=E_0)} \prod_{j=1}^{N} \frac{\Gamma'_{m_j}(t_j)}{\Gamma_{m_j}(t_j)} \right\rangle = 1$

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Adiabatic renormalization

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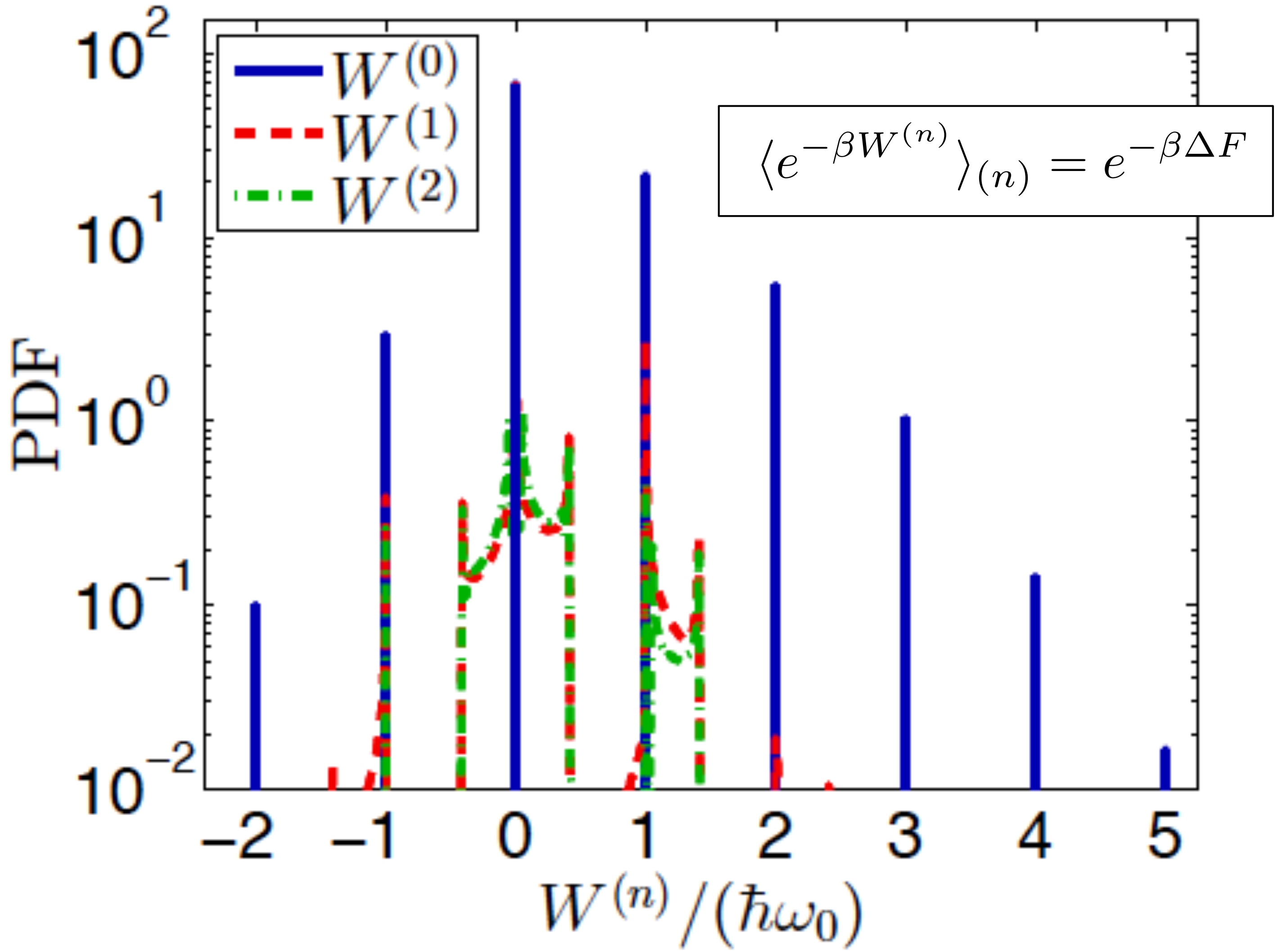
- Recall the IFT: $\left\langle \frac{Pr'(E(\tau)=E_\tau)}{Pr(E(t_0)=E_0)} \prod_{j=1}^{N} \frac{\Gamma'_{m_j}(t_j)}{\Gamma_{m_j}(t_j)} \right\rangle = 1$

$$\langle e^{-\beta W^{(n)}} \rangle_{(n)} = e^{-\beta \Delta F}$$

Adiabatic renormalization



Single Electron Box (SEB)

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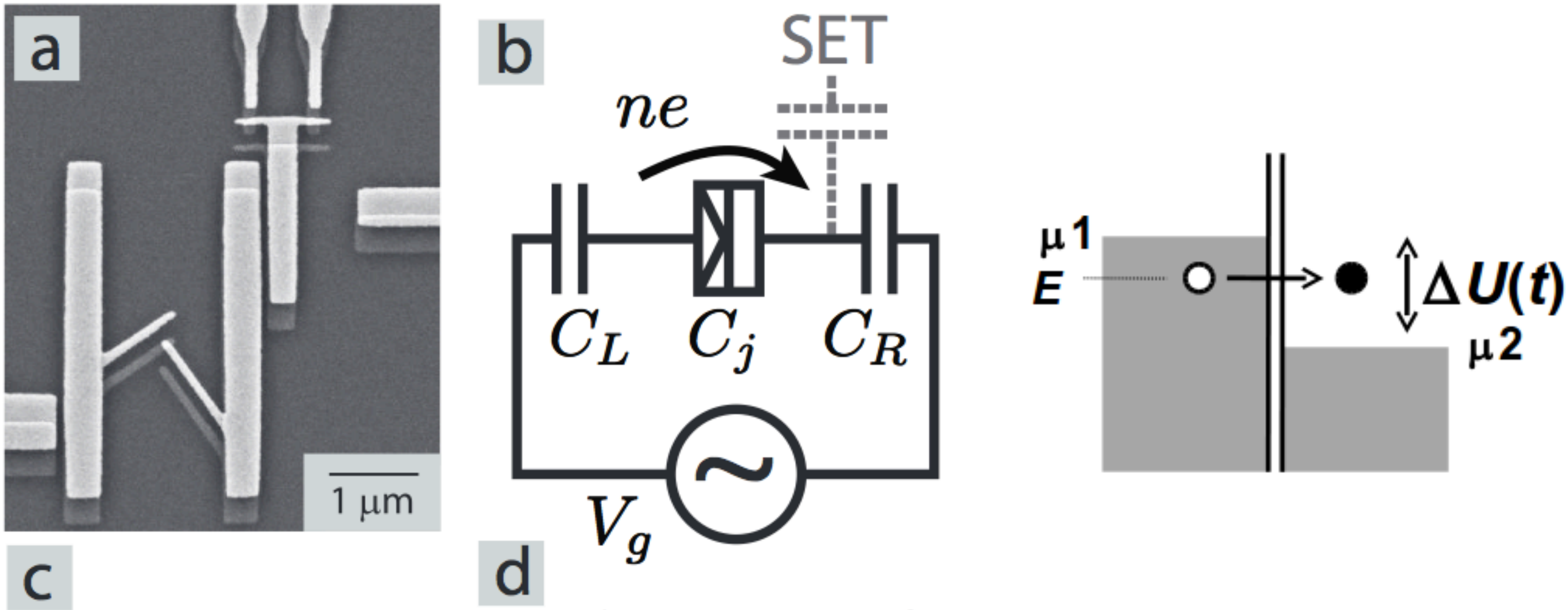

Electrons are driven one by one from lead to island and *stochastically* undergo transitions

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Single Electron Box (SEB)

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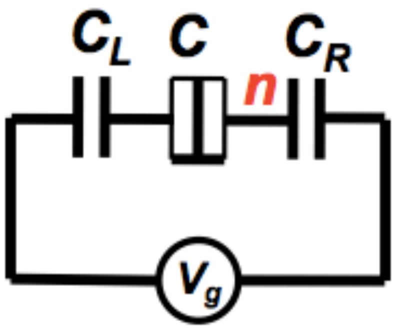
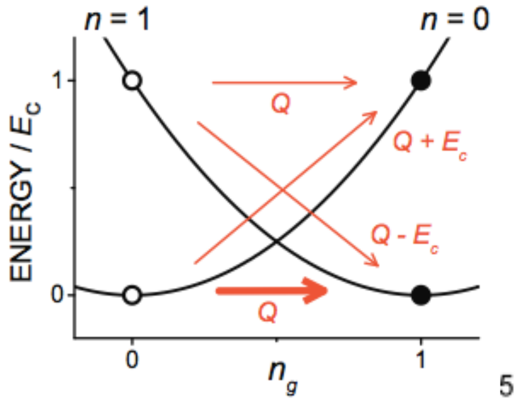
Dissipation in single-electron transitions

SEB





$$n_g = C_g V_g / e$$

$$H = E_C (n - n_g)^2$$

Heat generated in a tunneling event *i*:

$$Q_i = \pm 2 E_C (n_{g,i} - 1/2)$$

Total heat generated in a process:

$$Q = \sum_i Q_i$$

Work in a process:

$$W = Q + \Delta U$$

Change in internal (charging) energy

[D. Averin and J. Pekola, EPL **96**, 67004 (2011)]

SEB in Equilibrium: Experimental Results

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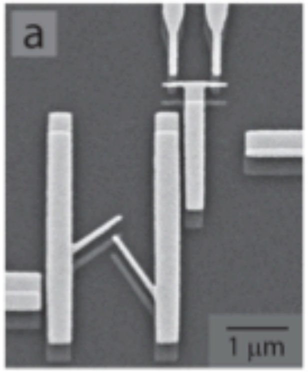
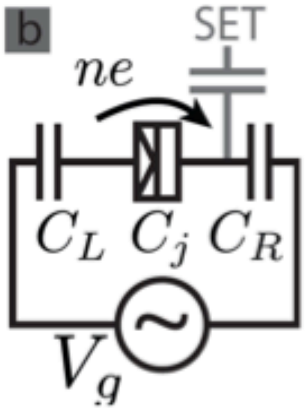
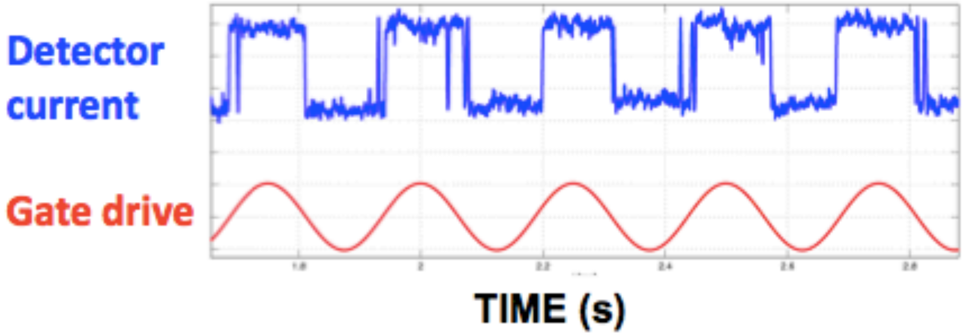
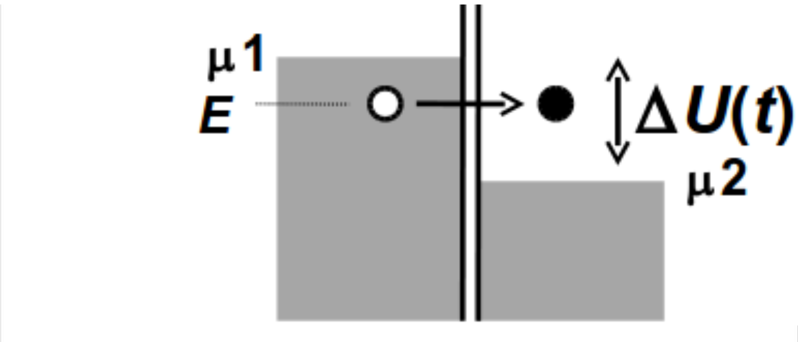
[O.-P. Saira *et al.*, PRL **109**, 180601 (2012); J.V. Koski *et al.*, Nat. Phys. **9**, 644 (2013)]

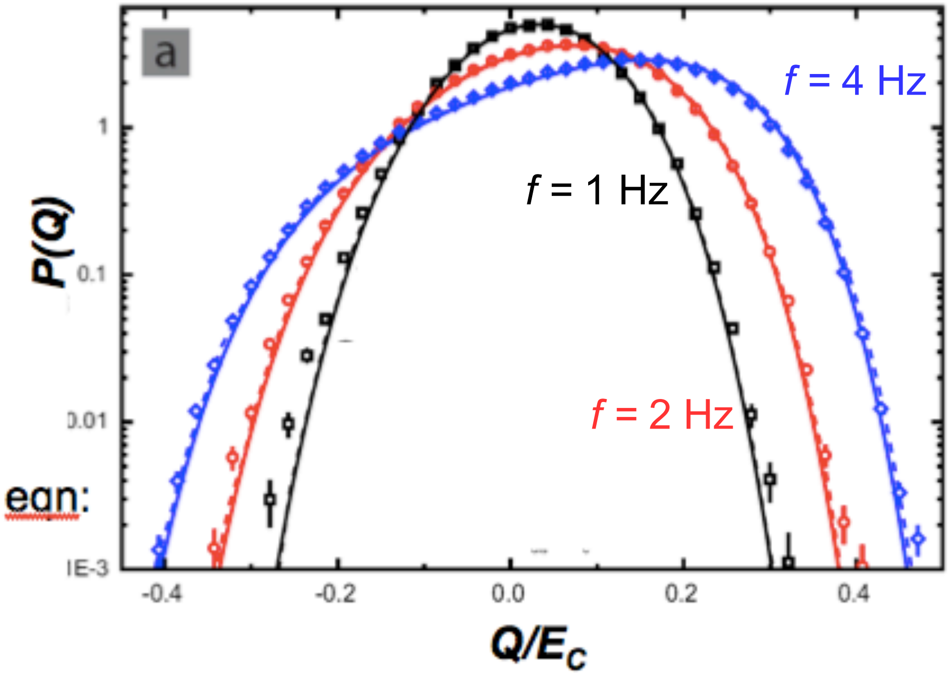






$$\Gamma^+(t)/\Gamma^-(t) = e^{\beta\Delta U}$$





Distributions satisfy the Jarzynski eqn:

$$\boxed{\langle e^{-\beta(W-\Delta F)}\rangle = 1.03 \pm 0.03}$$

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SEB Overheating by Transitions

[J.P. Pekola, A. Kutvonen, T.A-N., JSTAT P02033 (2013)]

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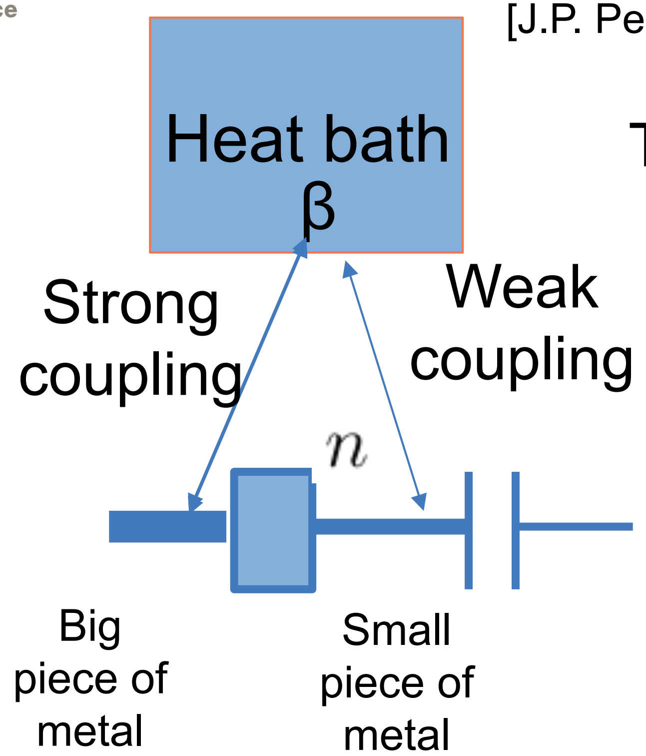


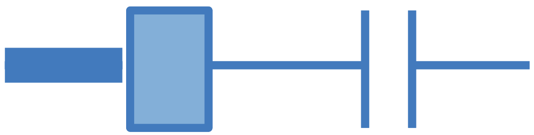
Transition rates

$$\Gamma^{\pm}(t) = \frac{G_T}{e^2} \int f_L(\beta_L, E)[1 - f_I(\beta_I, E \pm \Delta U(t))]dE$$

Electron-electron relaxation is the fastest timescale in the problem

Depends on the energy of the tunneling electron

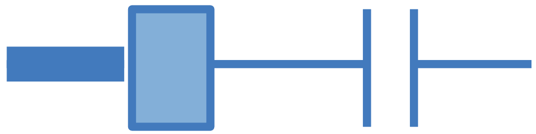
$f_L(E,\beta)$ $f_I(E,\beta)$



Initial equilibrium

Tunneling event perturbs the system

$f_L(E,\beta)$ $f_I(E,\beta+\Delta\beta)$



Non-equilibrium

$$\frac{\Gamma^+(t)}{\Gamma^-(t)} = e^{\beta \Delta U(t)}[1 - \frac{1}{2}\langle Q\rangle \Delta\beta + \cdots]$$

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Analytic and Numerical Results for Overheated SEB

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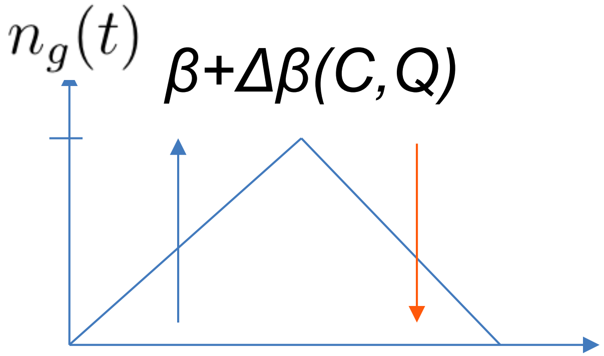
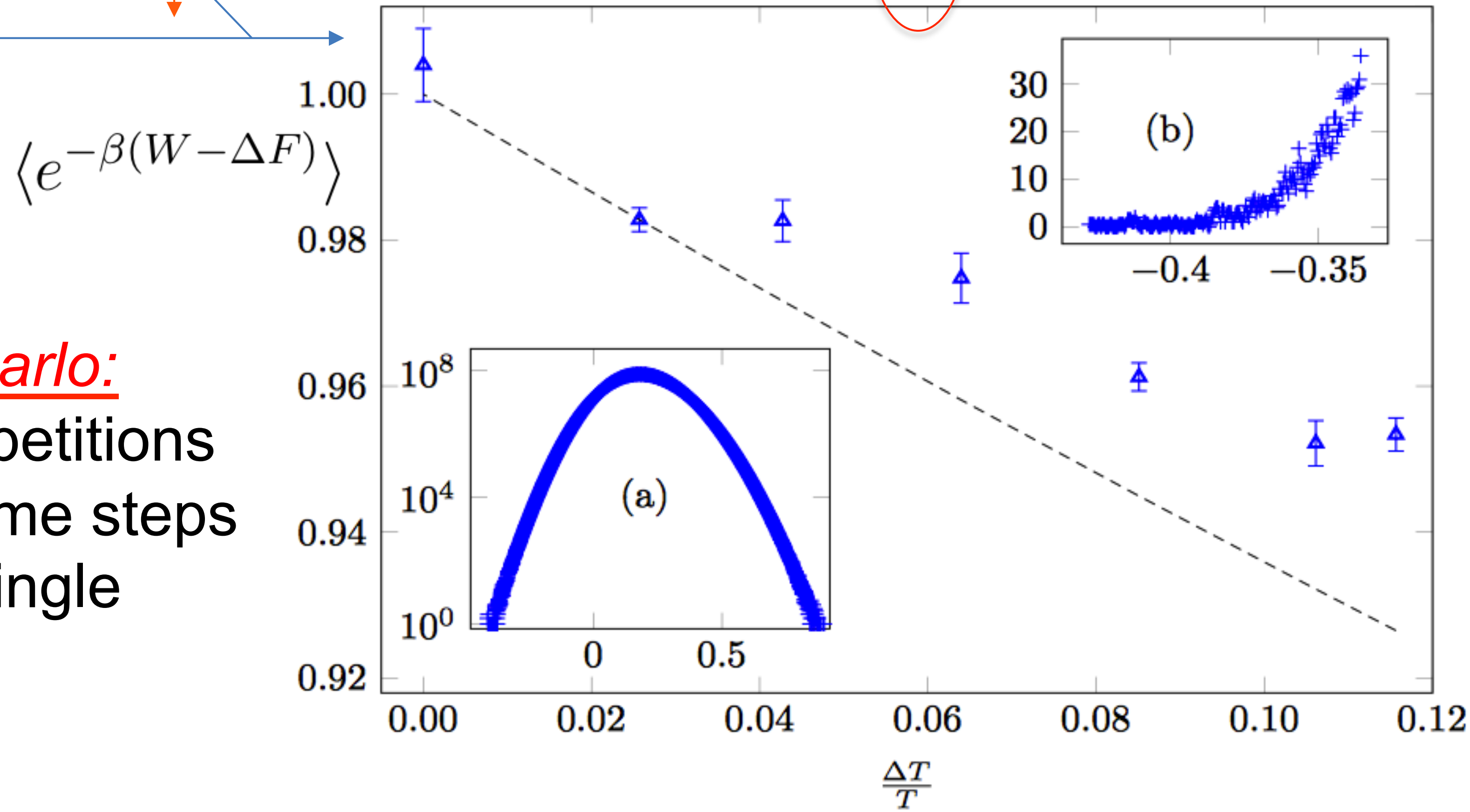
Analytically assume single (double) jump trajectory, expand tunneling rates in *Δβ*:

$$\langle e^{-\beta(W-\Delta F)}\rangle = 1 - \frac{k_B}{4C}\beta^2\langle Q\rangle^2 + \mathcal{O}[(\Delta\beta)^2]$$





Monte Carlo:

- 10^{11} repetitions
- 8000 time steps
- 95 % single jumps

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Summary and Thanks

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For driven, open quantum systems it's possible to derive fluctuation relations for work in the TMP (Lindblad) setting

The power operator naturally appears in the (exact) expressions for the moments of the work distribution

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Paolo Solinas (Genova)
Joachim Ankerhold (Ulm)

Juha Salmilehto (Yale)
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Tehran April 15th, 2015