



Quantum Cryptography with structured photons

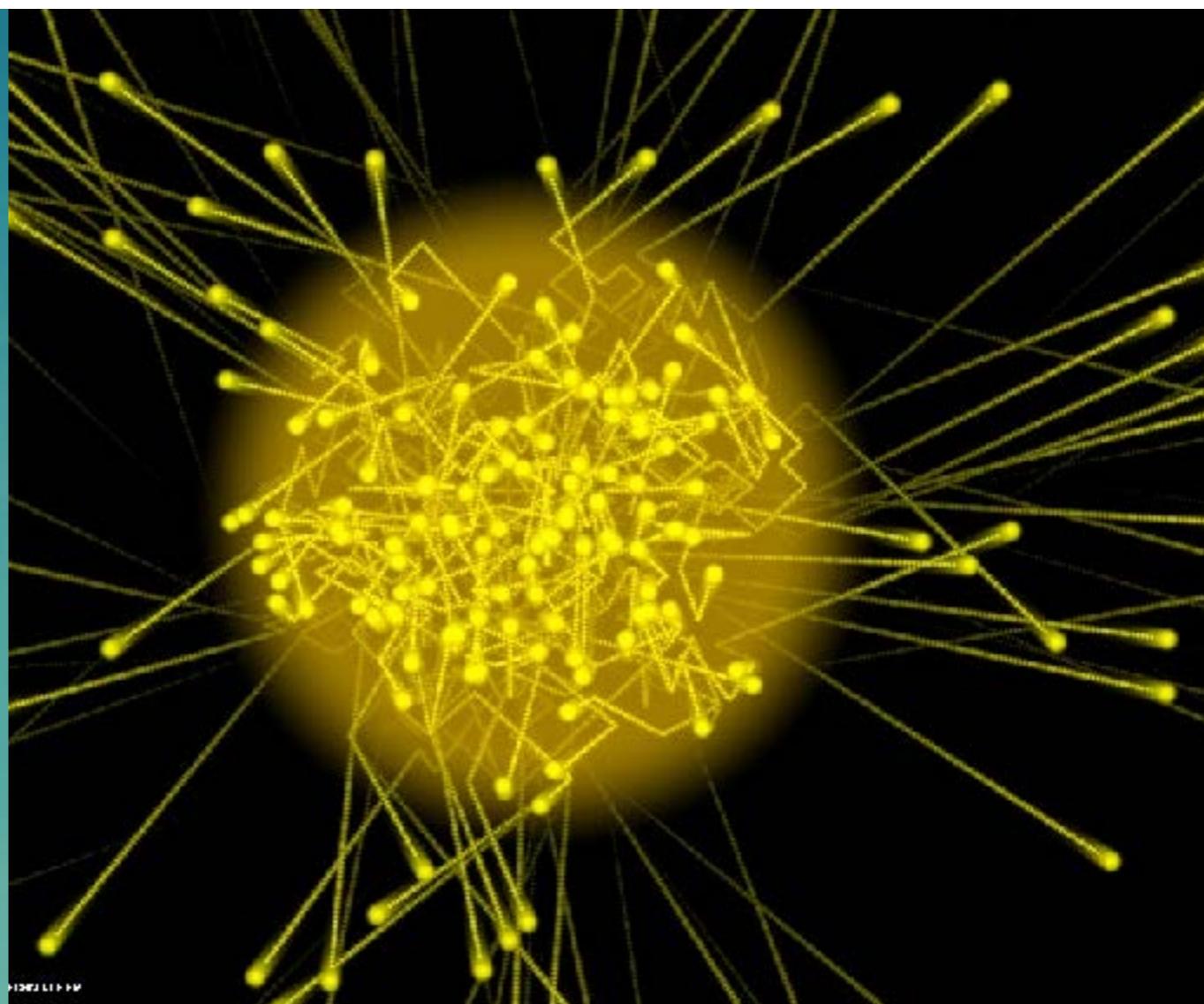
PRÉSENTÉ PAR : EBRAHIM KARIMI | PRESENTED BY: EBRAHIM KARIMI

Photon: The Quanta of Light

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Photon: The Quanta of Light

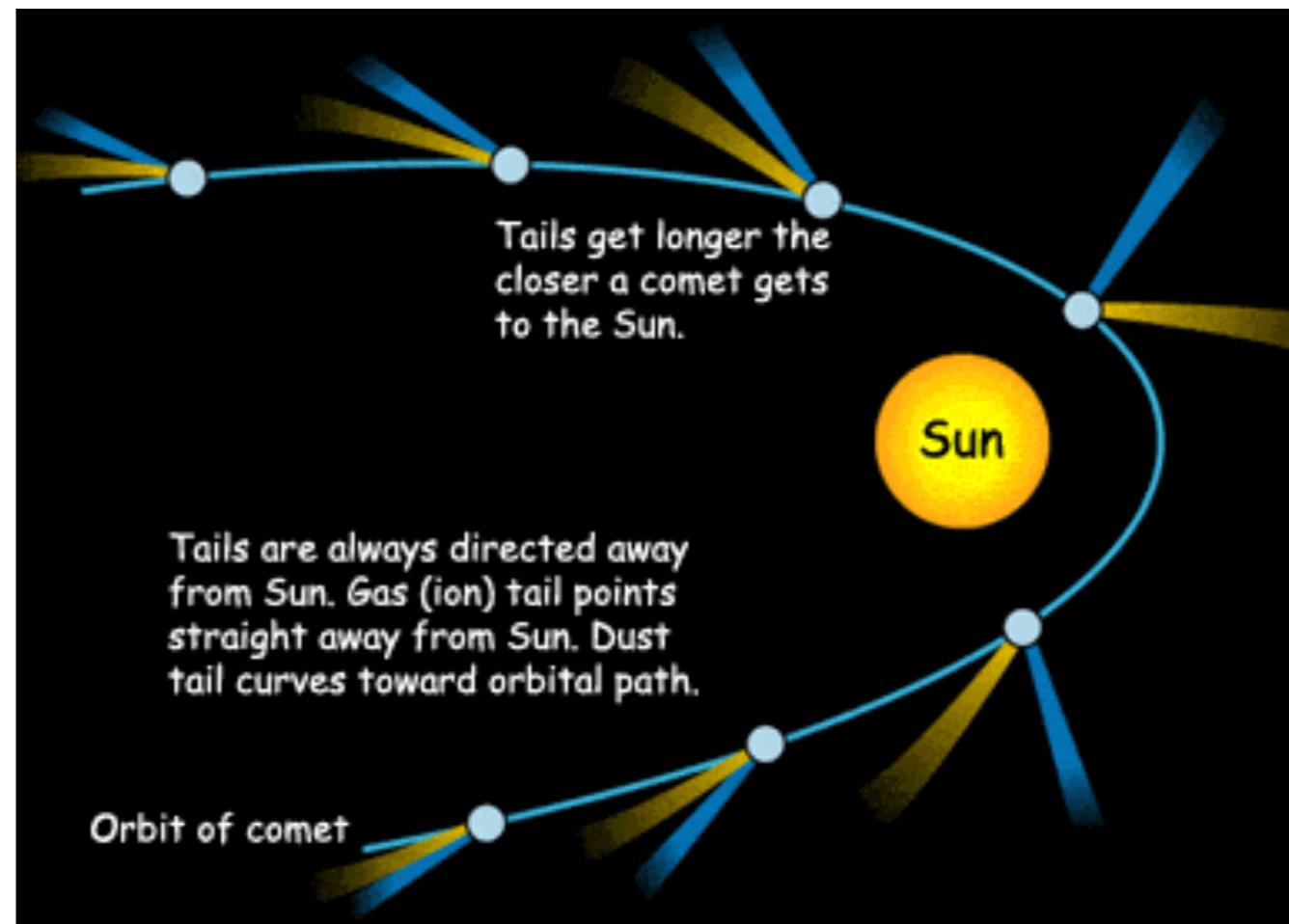


Photon: Linear Momentum

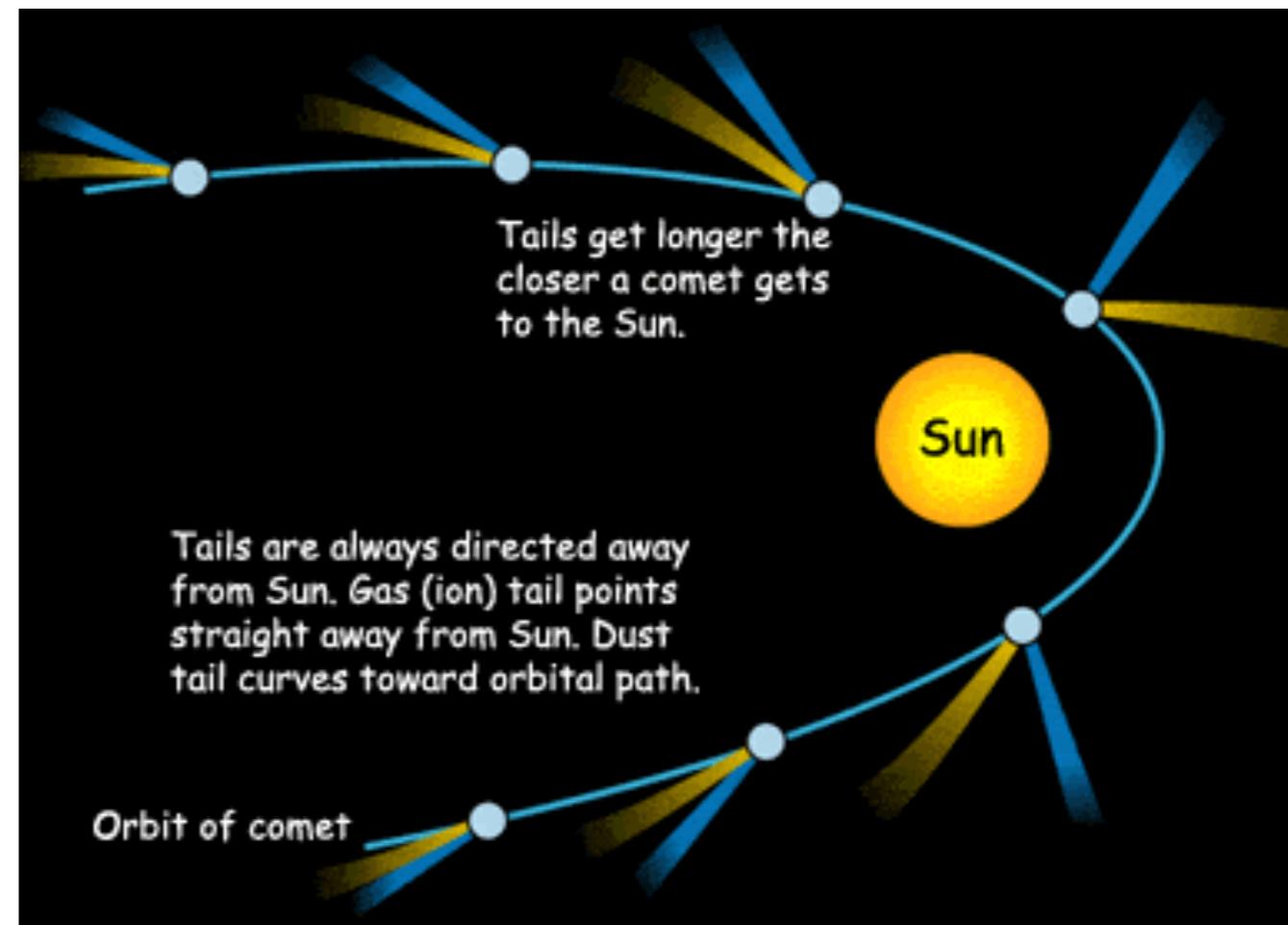
Photon: Linear Momentum



Photon: Linear Momentum



Photon: Linear Momentum



$$\tilde{p} \propto (\tilde{E}^* \times \tilde{B})$$

$$\tilde{p} = \hbar \tilde{k}$$

Photon: Optical Angular Momentum

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$$\tilde{j} = \frac{1}{2\mu_0} \left(\tilde{r} \times (\tilde{E}^* \times \tilde{B}) \right)$$

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Angular momentum decomposition

$$\tilde{J} = \tilde{S} + \tilde{L}$$

$$S_i = \frac{1}{2\mu_0\omega} \int d^3r E_j^* (-i\epsilon_{i,j,k}) E_k$$

$$L_i = \frac{1}{2\mu_0\omega} \int d^3r E_j^* (-i\tilde{r} \times \tilde{\nabla})_i E_j$$

Photon: Optical Angular Momentum

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Spin term

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OAM term

$$L_i = \frac{1}{2\mu_0\omega} \int d^3r E_j^* (-i\tilde{r} \times \tilde{\nabla})_i E_j$$

Photon: Optical Angular Momentum

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Longitudinal component of angular momentum

$$S_i = \frac{1}{2\mu_0\omega} \int d^3r E_j^* (-i\epsilon_{i,j,k}) E_k$$

$$\hat{S}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$L_i = \frac{1}{2\mu_0\omega} \int d^3r E_j^* (-i\tilde{r} \times \tilde{\nabla})_i E_j$$

$$\hat{L}_z = -i\partial_\phi$$

Photon: Optical Angular Momentum

Longitudinal component of angular momentum

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$$\hat{S}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_z = \hbar \int d\sigma_\perp (|E_L|^2 - |E_R|^2)$$

$$L_i = \frac{1}{2\mu_0\omega} \int d^3r E_j^* (-i \tilde{r} \times \nabla)_i E_j$$

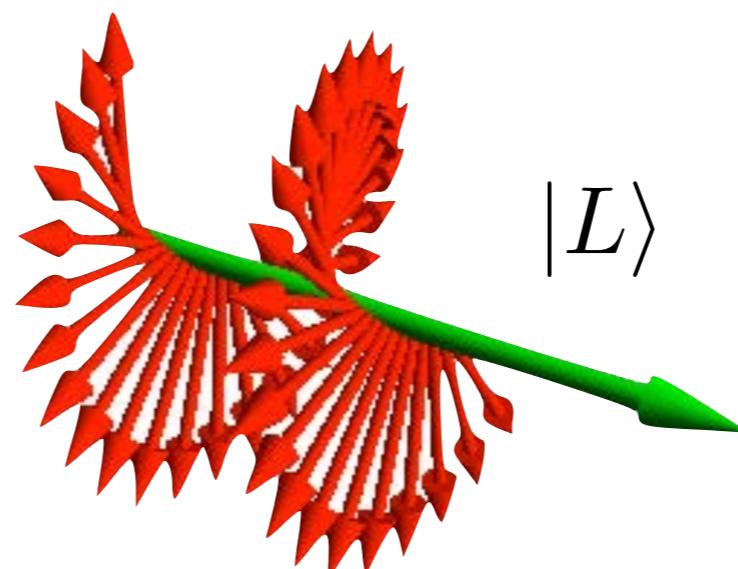
$$\hat{L}_z = -i\partial_\phi$$

$$L_z = \hbar \int d\sigma_\perp (E_L^* \partial_\phi E_L + E_R^* \partial_\phi E_R)$$

Photon: Spin Angular Momentum

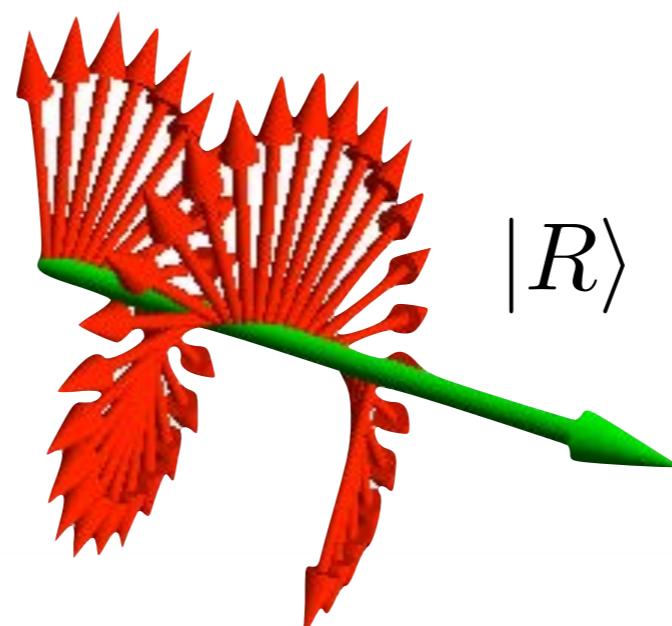
- Circular polarisations:
(Left and Right)
- Spin angular momentum
may take two values of

vectorial picture



angular momentum

$$J_z = +\hbar$$



$$J_z = -\hbar$$



$|L\rangle$ Left-handed circular polarisation
 $|R\rangle$ Right-handed circular polarisation

Photon: Optical Angular Momentum

- Optical field expression in the cylindrical coordinate is

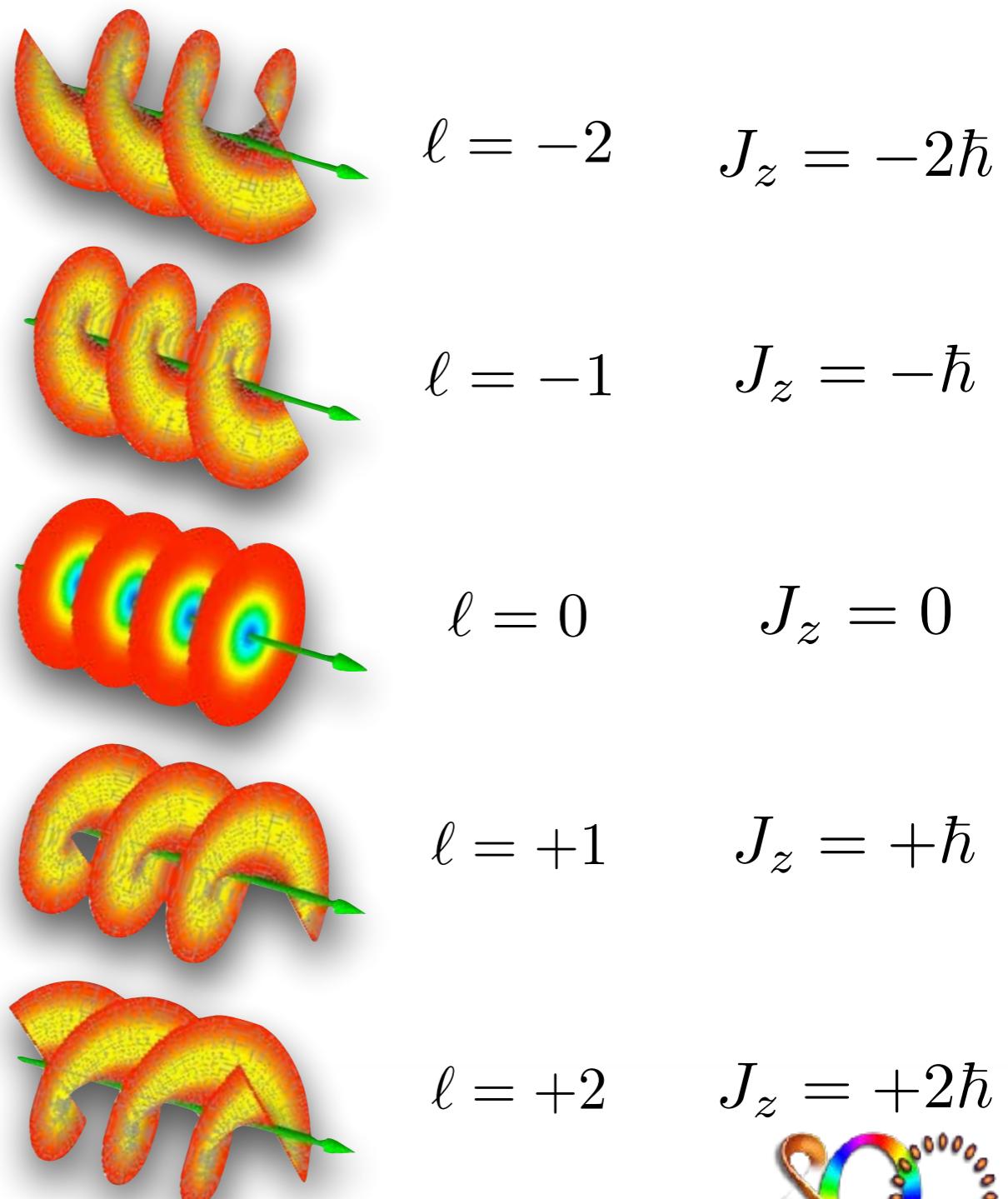
$$\hat{L}_z = -i\partial_\phi$$

$$E(r, \varphi, z, t) = E(r, z)e^{i\ell\varphi} e^{i(kz - \omega t)}$$

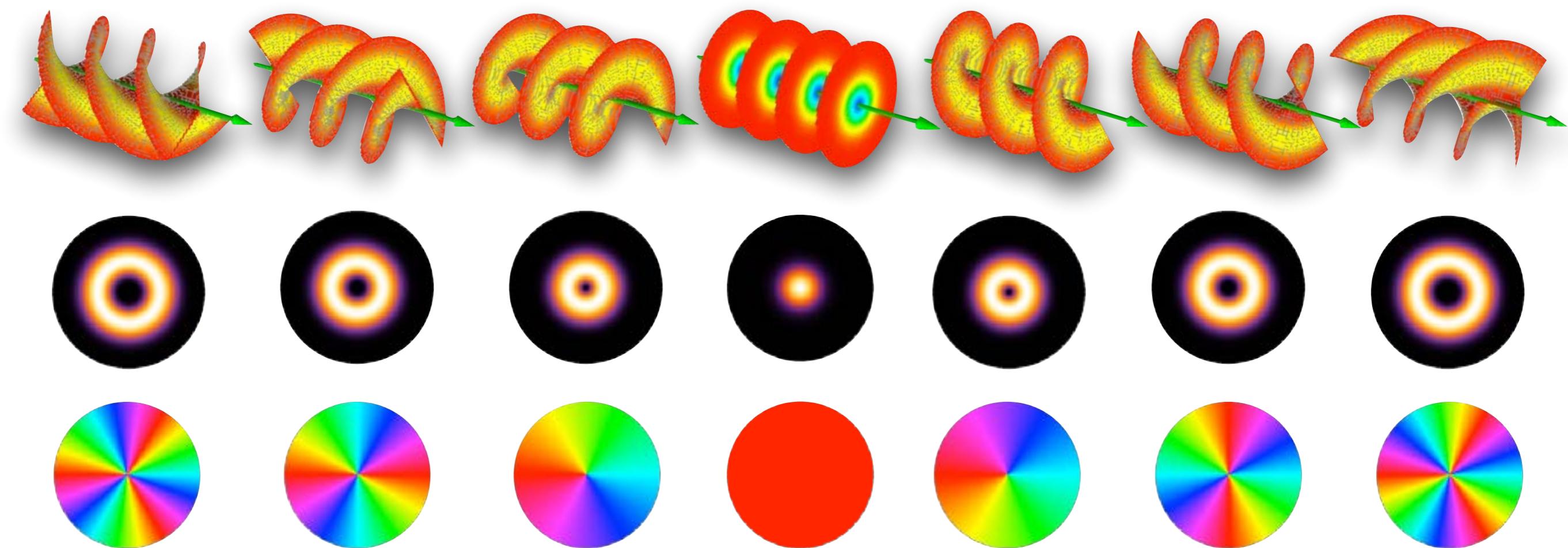
$$J_z = \ell\hbar$$

In the contrast, orbital angular momentum may take any of the infinite values $\ell = \dots -2, -1, 0, 1, 2, \dots$

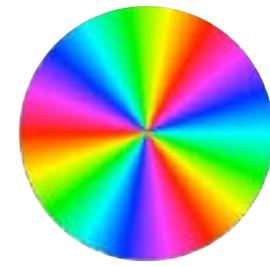
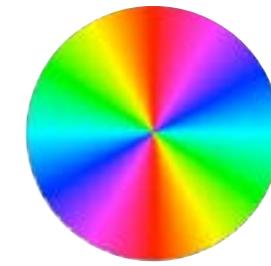
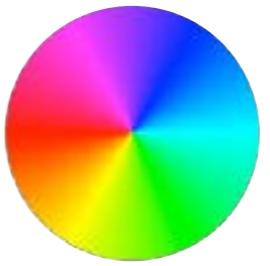
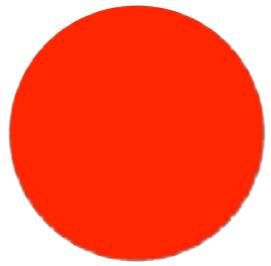
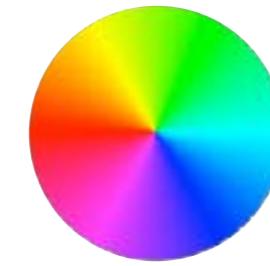
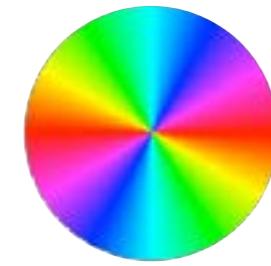
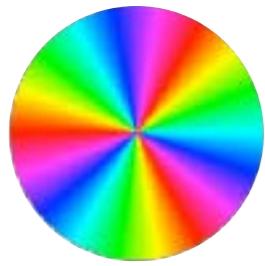
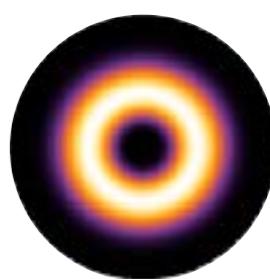
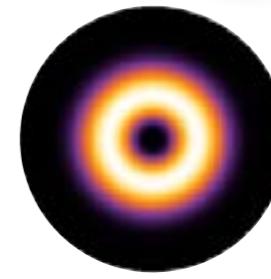
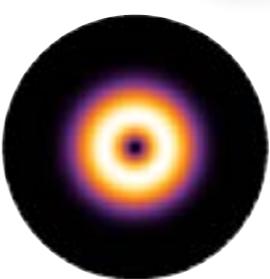
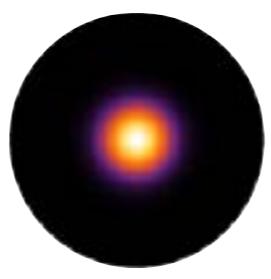
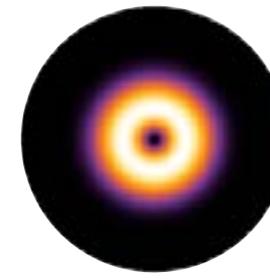
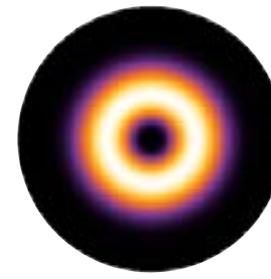
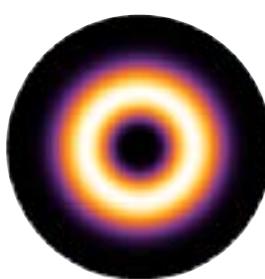
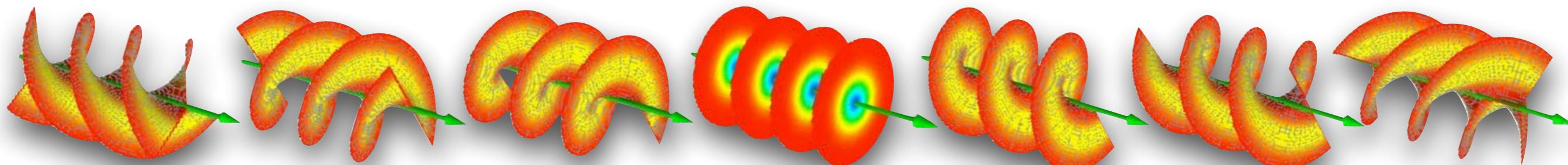
Laguerre-Gaussian modes are one set of paraxial wave mode which carry the OAM.



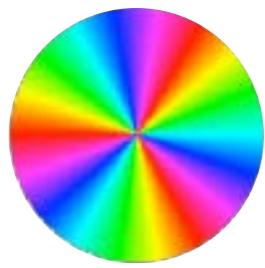
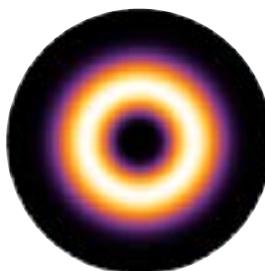
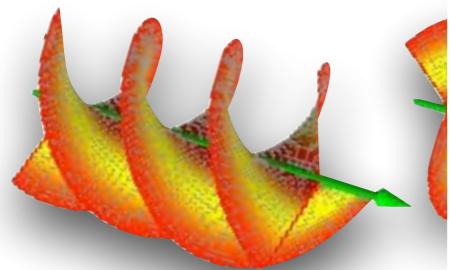
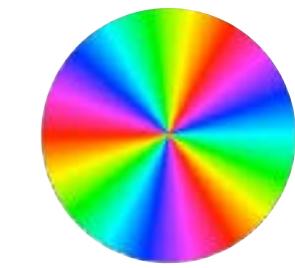
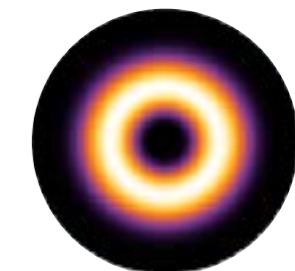
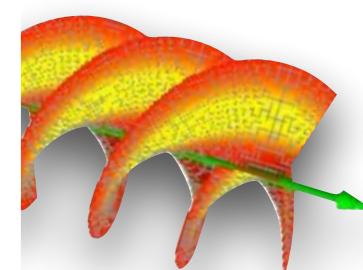
Photon: Optical Angular Momentum

 $\ell = -3$ $\ell = -2$ $\ell = -1$ $\ell = 0$ $\ell = +1$ $\ell = +2$ $\ell = +3$ 

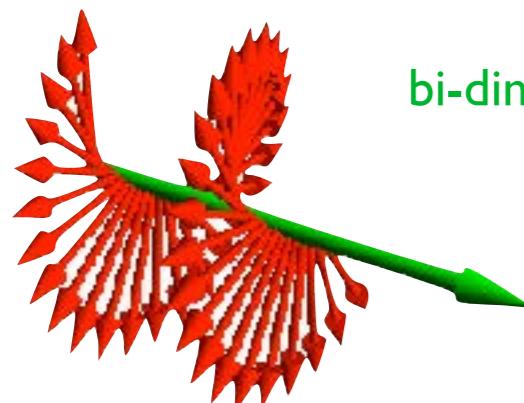
Photon: Optical Angular Momentum

 $\ell = -3$ $\ell = -2$ $\ell = -1$ $\ell = 0$ $\ell = +1$ $\ell = +2$ $\ell = +3$  $| -3 \rangle$ $| -2 \rangle$ $| -1 \rangle$ $| 0 \rangle$ $| +1 \rangle$ $| +2 \rangle$ $| +3 \rangle$

Photon: Optical Angular Momentum

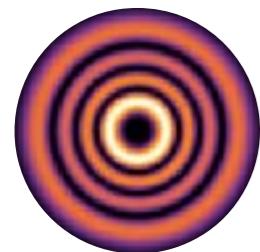
 $\ell = -3$  $|-3\rangle$ $|-2\rangle$ $|-1\rangle$ $|0\rangle$ $|+1\rangle$ $|+2\rangle$ $|+3\rangle$ $\ell = +3$ 

Photon: Degrees of Freedom



bi-dimensional variable

$|\pi\rangle$ Polarisation (Spin)

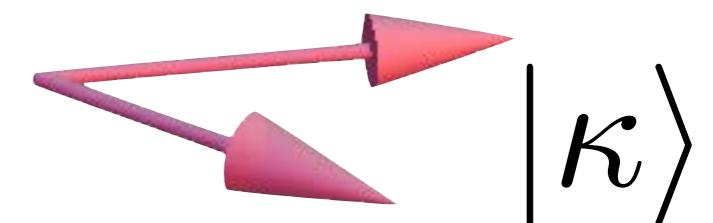


infinite dimension (discrete)

$|p\rangle$ Radial mode (p number)

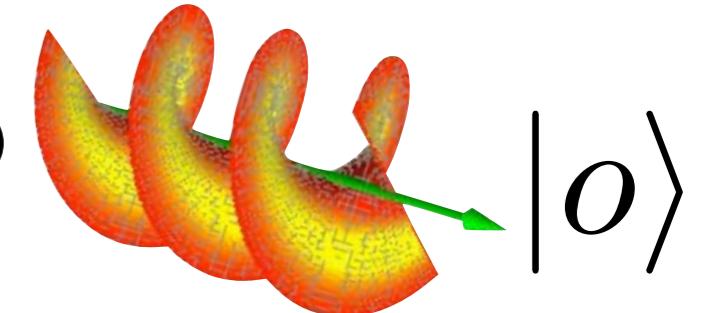
Linear momentum (k -vector)

continues variable



$|\kappa\rangle$

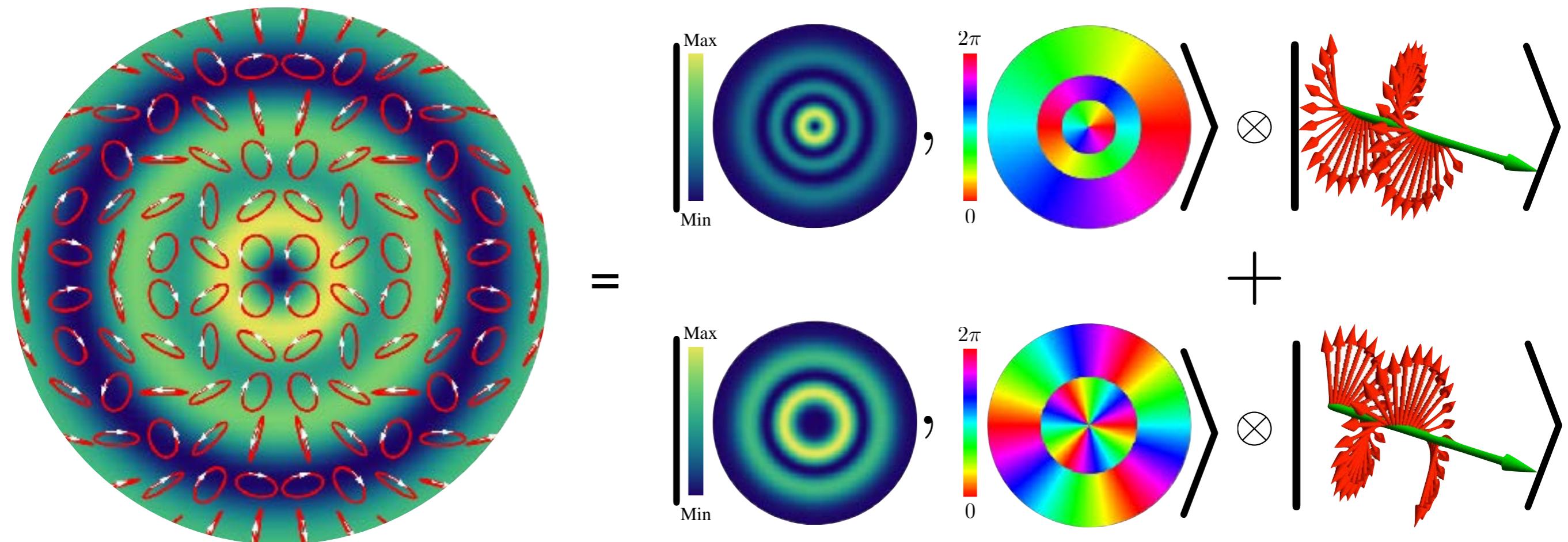
Orbital angular momentum (OAM)



infinite dimension (discrete)

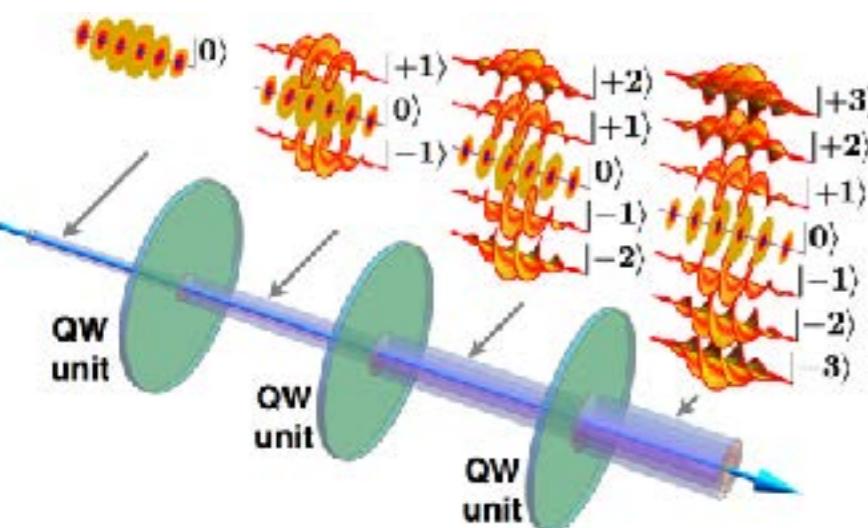


Structural Photons

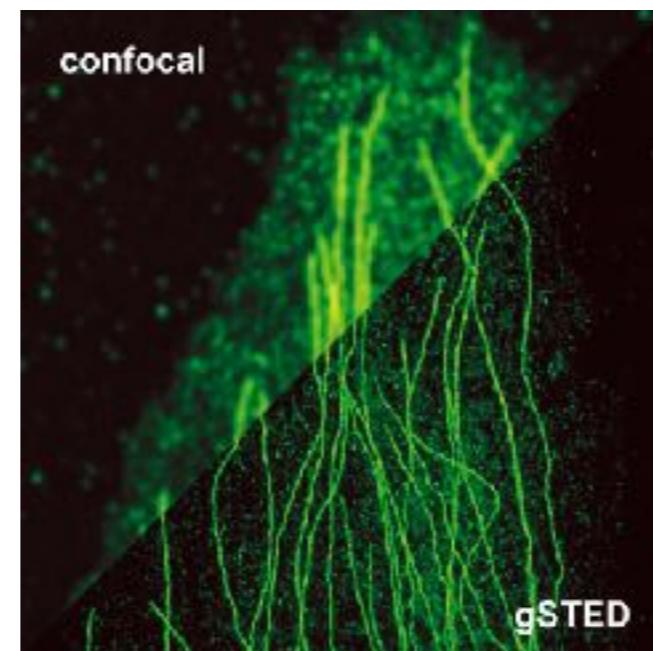


Application of Structural Photons

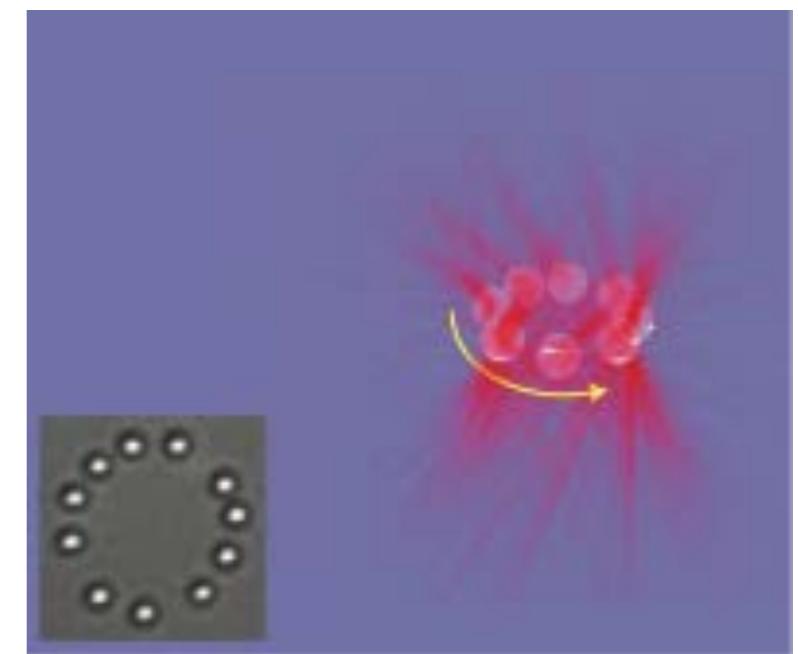
Quantum Computation



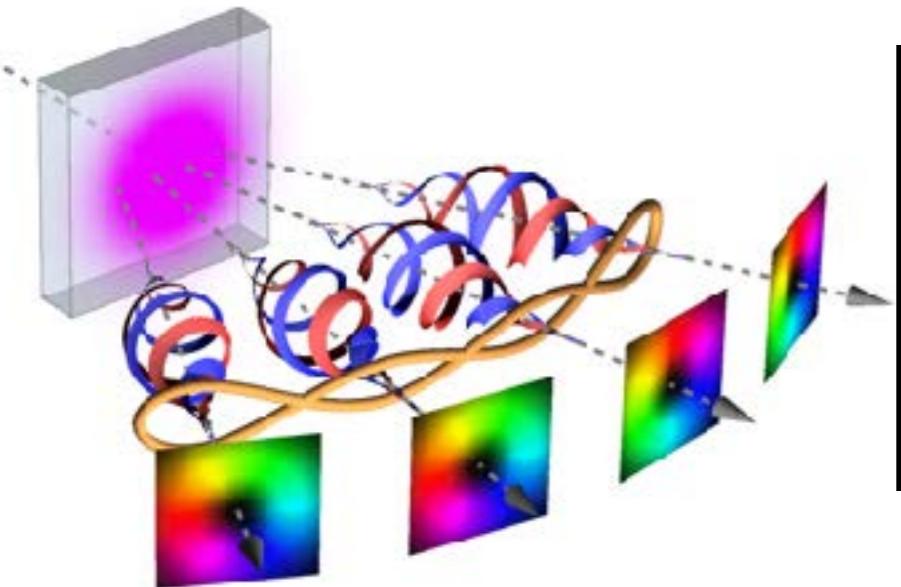
Optical Microscopy



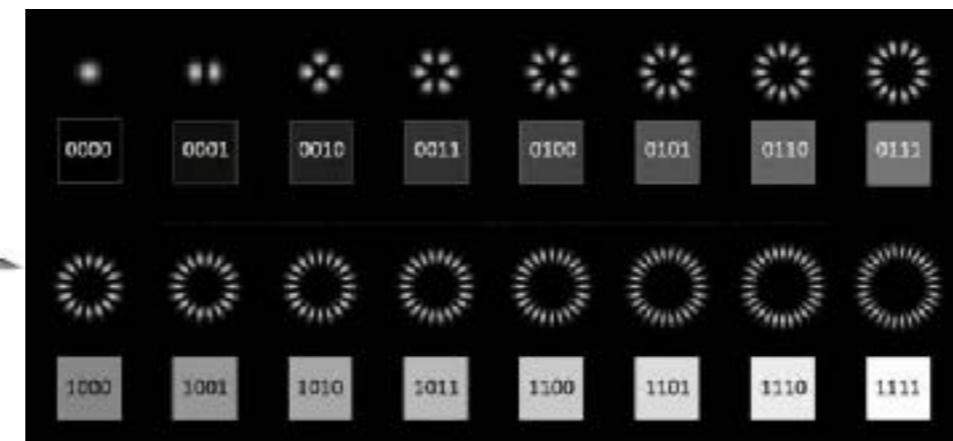
Optical Manipulation



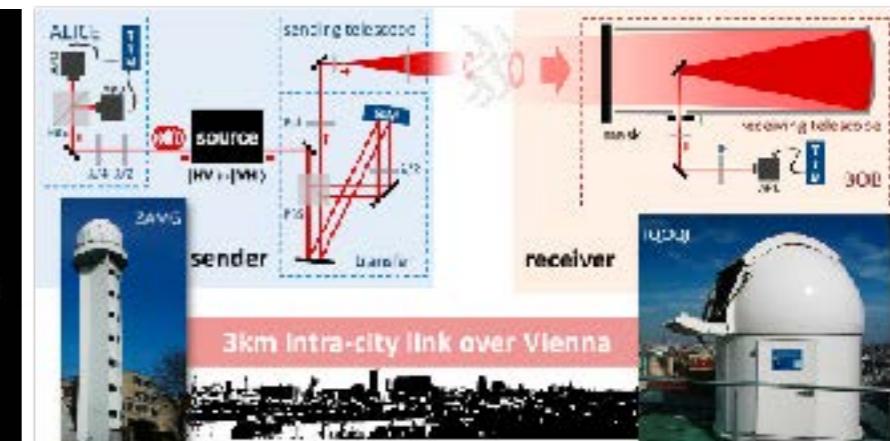
High-dimensional entanglement



Classical communication



Quantum Cryptography



The importance of Orbital Angular Momentum

The importance of Orbital Angular Momentum

Let us send letter M

The importance of Orbital Angular Momentum

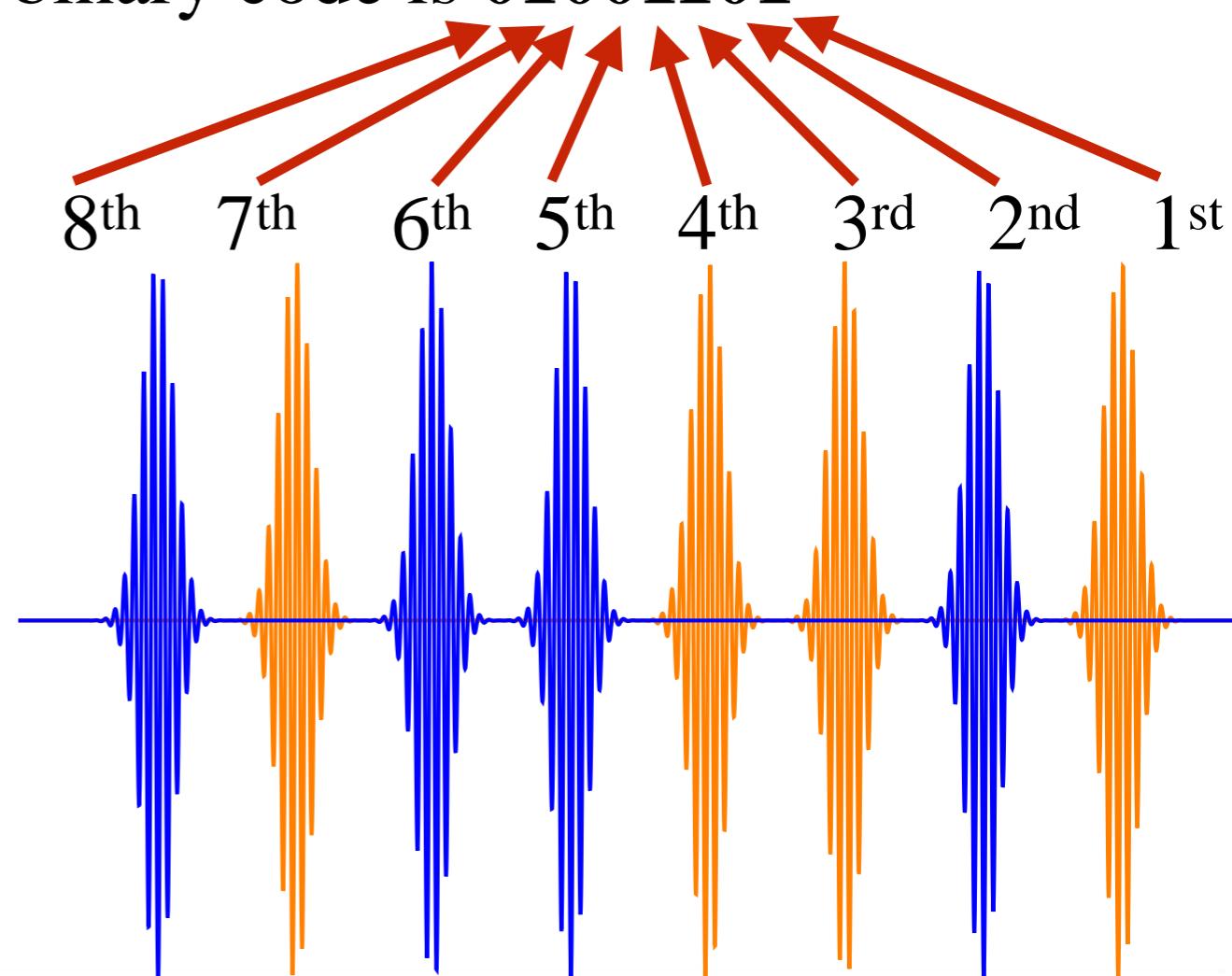
Let us send letter **M**

Letter **M** in the binary code is **01001101**

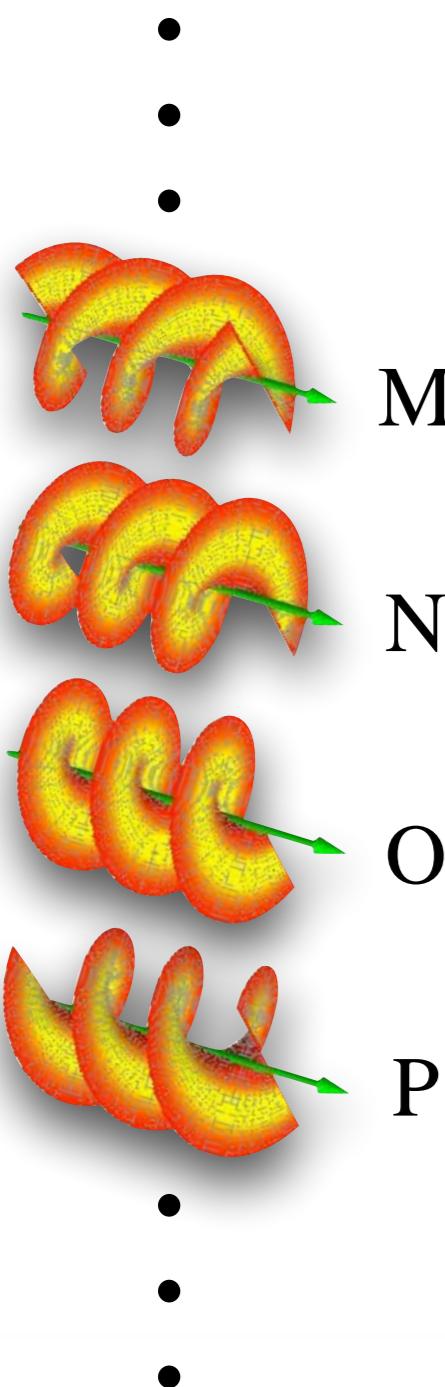
The importance of Orbital Angular Momentum

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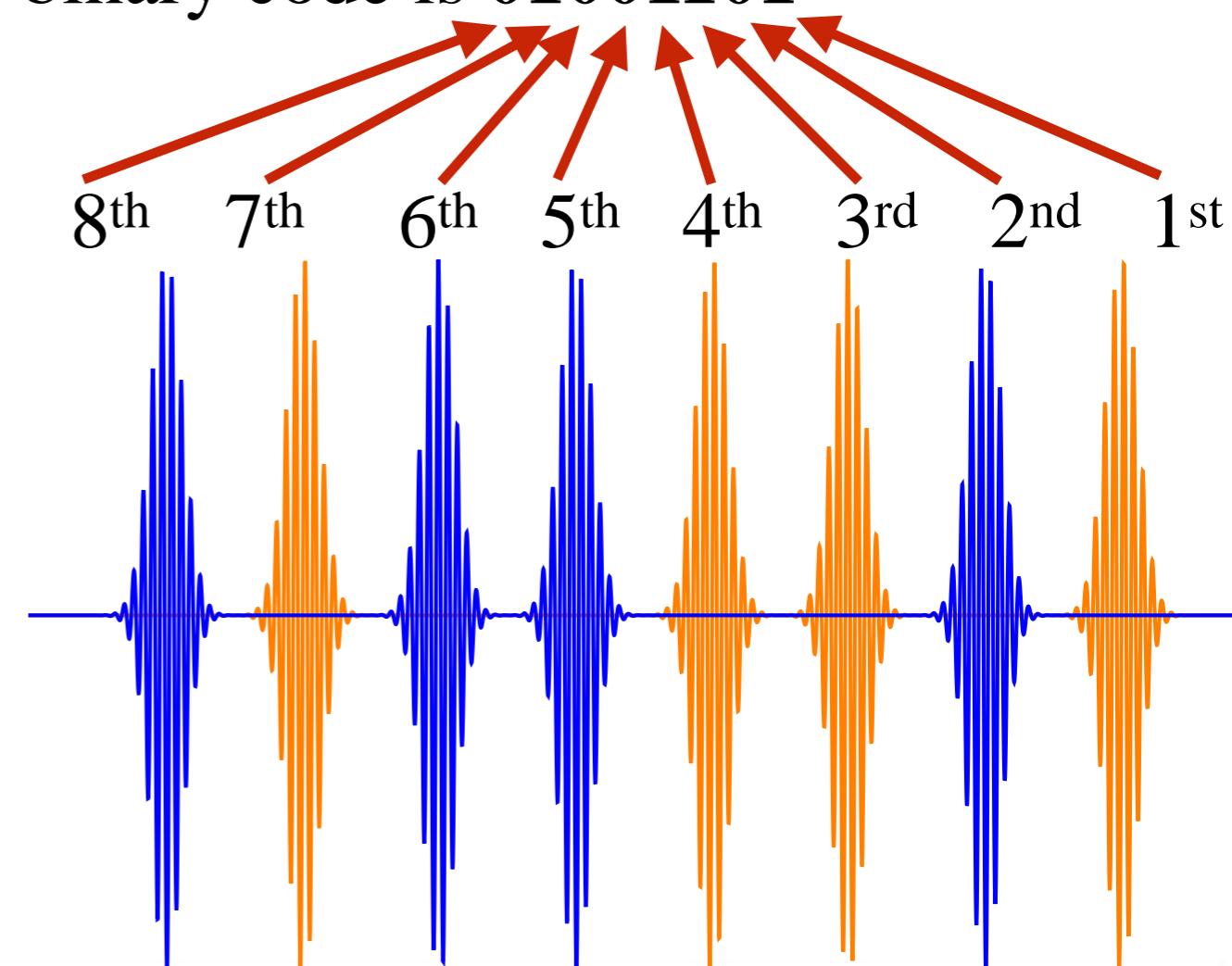


The importance of Orbital Angular Momentum



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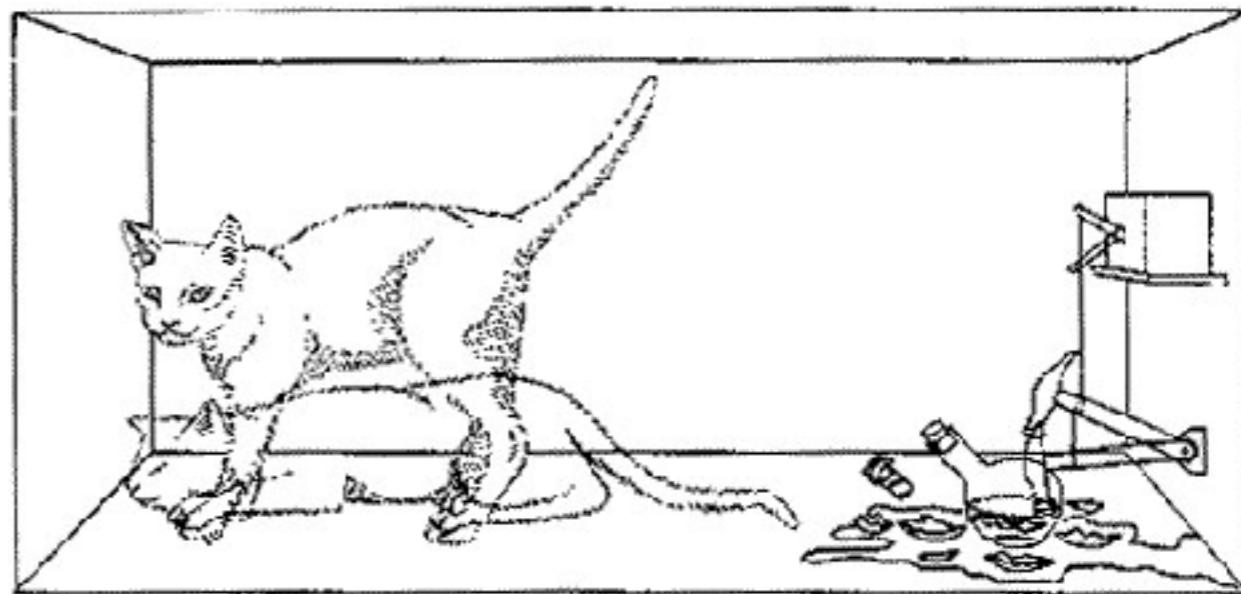


Quantum Cryptography

Quantum World

Quantum World

THERE IS NO DEFINITE REALITY

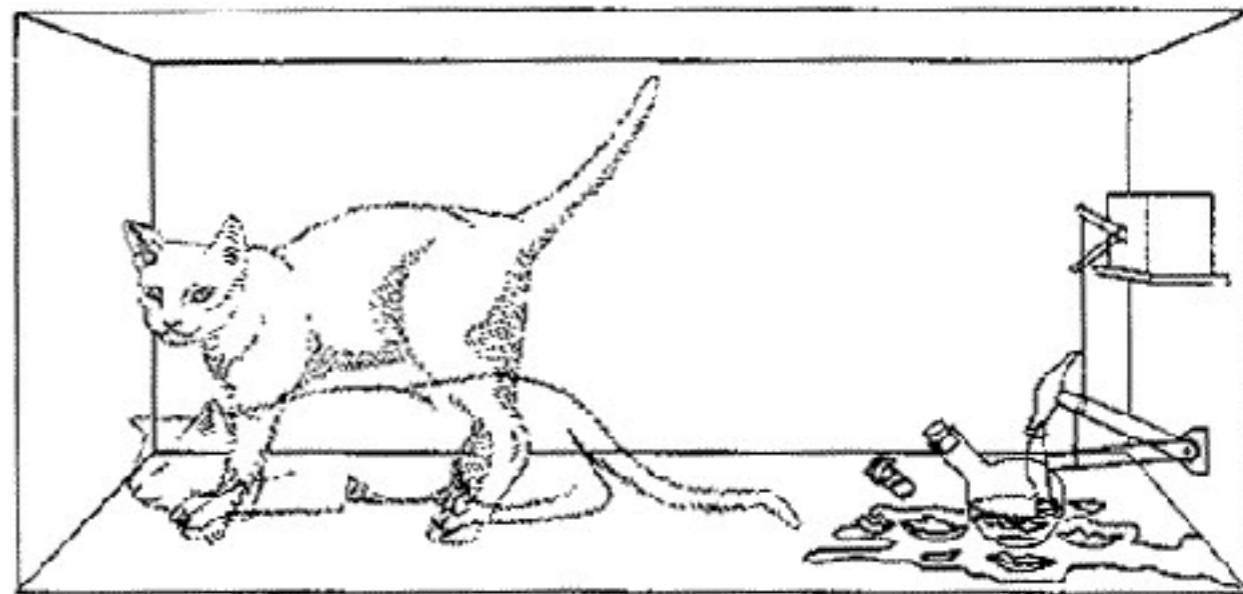


Quantum World

CONJUGATE QUANTITIES CANNOT
BE MEASURED SIMULTANEOUSLY



THERE IS NO DEFINITE REALITY



Quantum World

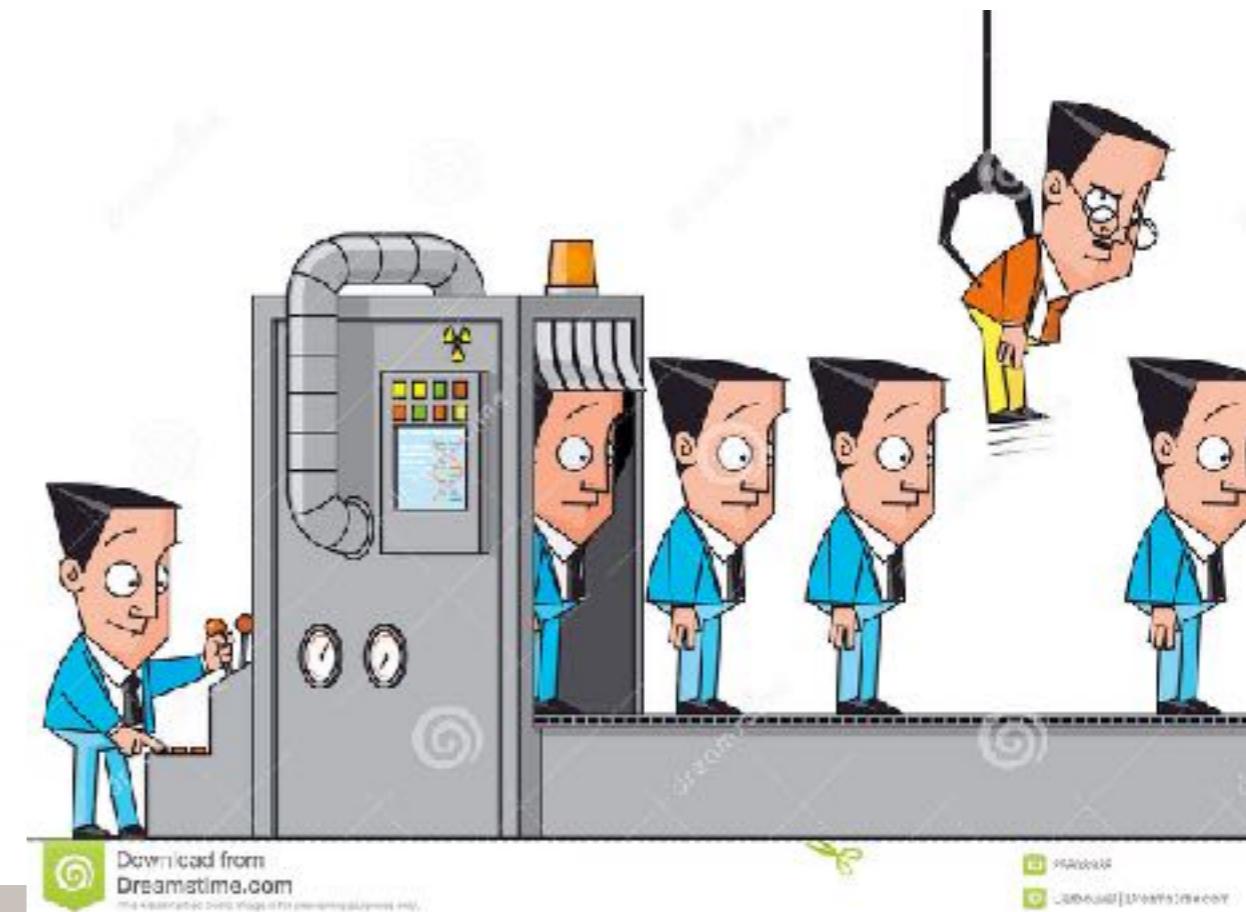
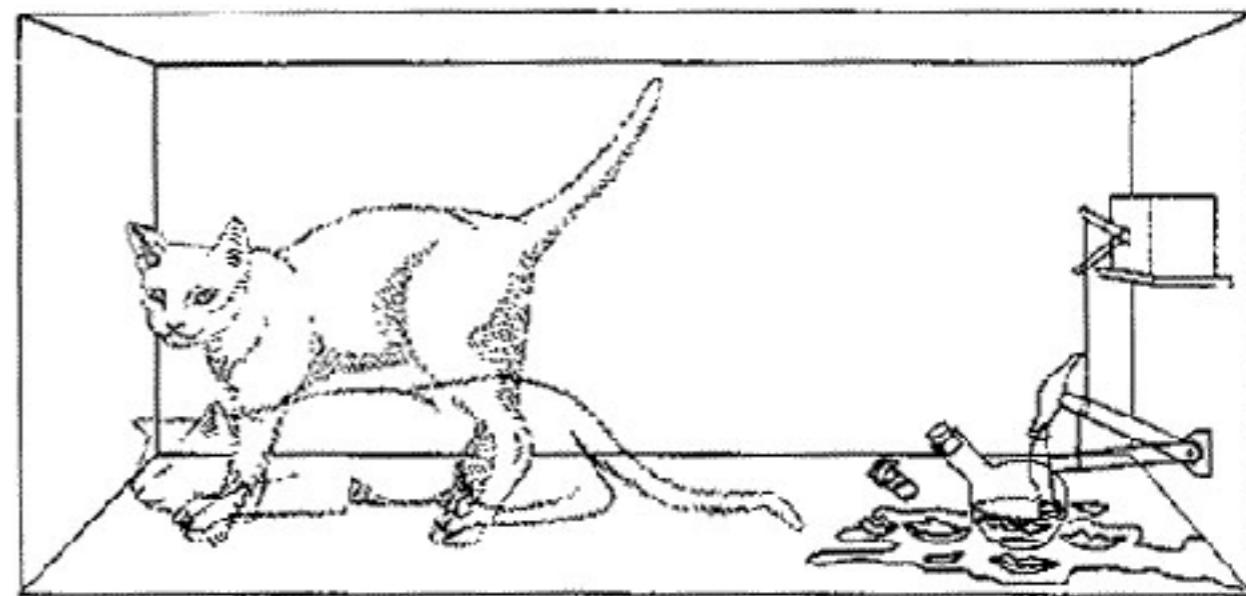
CONJUGATE QUANTITIES CANNOT
BE MEASURED SIMULTANEOUSLY



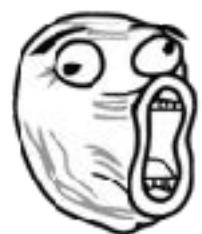
QUANTUM INFORMATION CANNOT BE
CLONED WITHOUT INTRODUCING ERRORS

sqogroup.ca

THERE IS NO DEFINITE REALITY



Cryptography



Alice



Bob

Cryptography



Alice

PUBLIC CHANNEL



Bob

Cryptography



Alice

PUBLIC CHANNEL

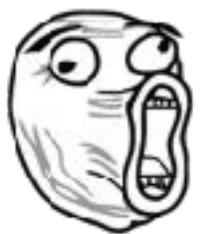


Eve



Bob

Cryptography



Alice



PUBLIC CHANNEL

wsqi jsv xli kpsvmiw sj xlmw asvph; erh wsqi
wmkl jsv xli tvstlix'w tevehmwi xs gsqi;
el, xeoи xli gewl, erh pix xli gvihamx ks,
rsv liih xli vyqfpi sj e hmwxerx hvyq!

Eve

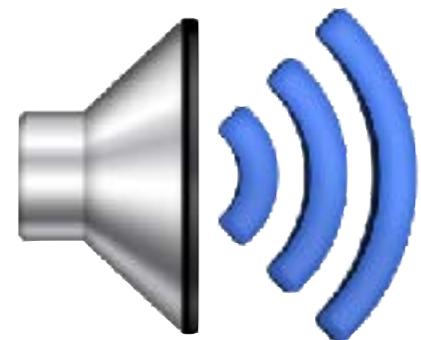


Bob

Cryptography



Alice



PUBLIC CHANNEL

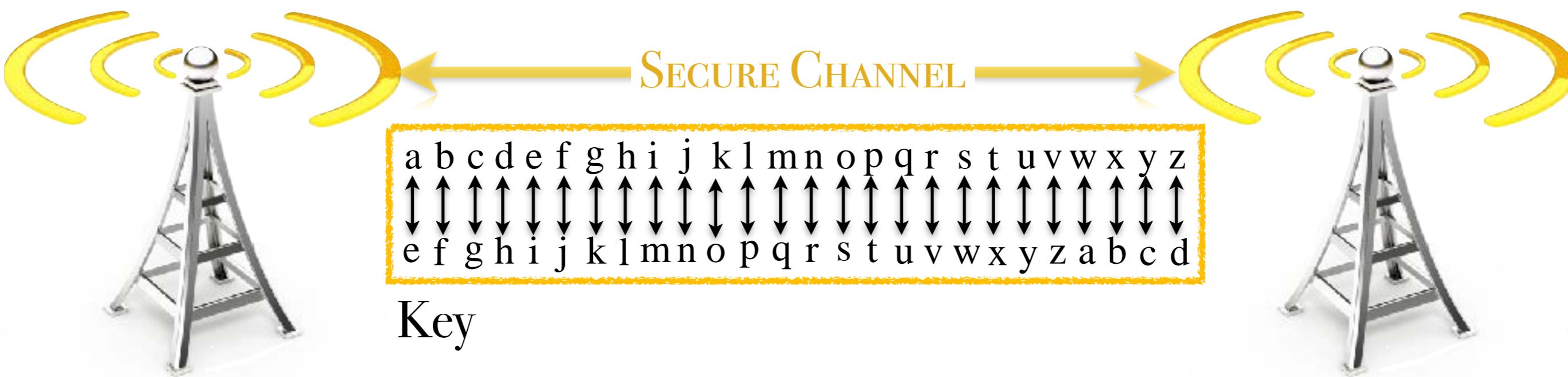


Eve



Bob

wsqi jsv xli kpsvmiw sj xlmw asvph; erh wsqi
wmkl jsv xli tvstlix'w tevehmwi xs gsqi;
el, xeoи xli gewl, erh pix xli gvihamx ks,
rsv liih xli vyqfpi sj e hmwxerx hvyq!



Cryptography



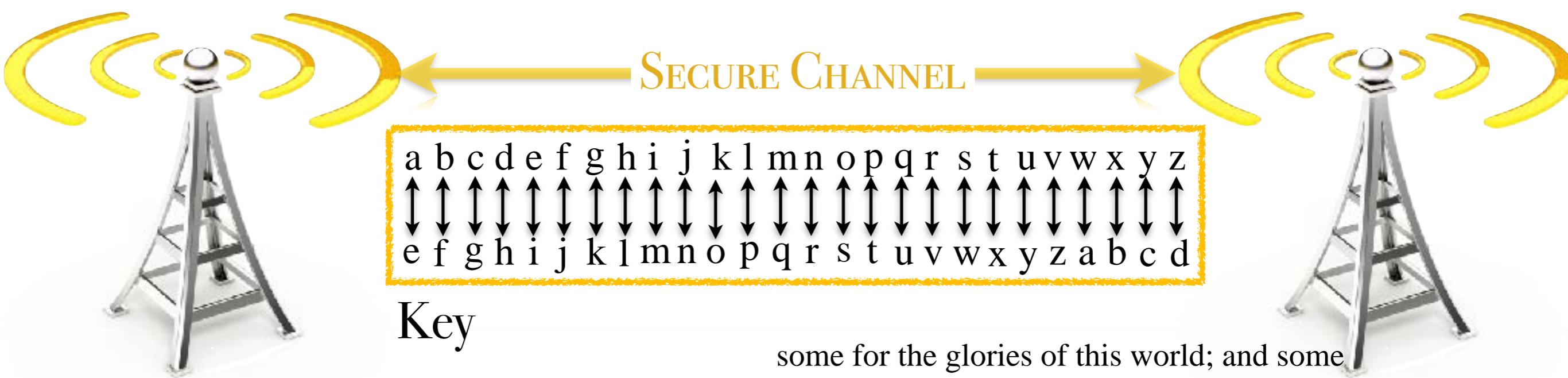
Alice

PUBLIC CHANNEL



Bob

wsqi jsv xli kpsvmiw sj xlmw asvph; erh wsqi
wmkl jsv xli tvstlix'w tevehmwi xs gsqi;
el, xeoи xli gewl, erh pix xli gvihamx ks,
rsv liih xli vyqfpi sj e hmwxerx hvyq!



Key

some for the glories of this world; and some
sigh for the prophet's paradise to come;
ah, take the cash, and let the credit go,
nor heed the rumble of a distant drum!

Quantum Cryptography



Alice



Bob

CONJUGATE QUANTITIES

$$\left\{ \begin{matrix} 0 \\ 1 \end{matrix}, \text{ } \begin{matrix} 0 \\ 1 \end{matrix} \right\} \circ \left\{ \begin{matrix} 0 \\ 1 \end{matrix}, \text{ } \begin{matrix} 0 \\ 1 \end{matrix} \right\} + \text{SQ}$$

Quantum Cryptography

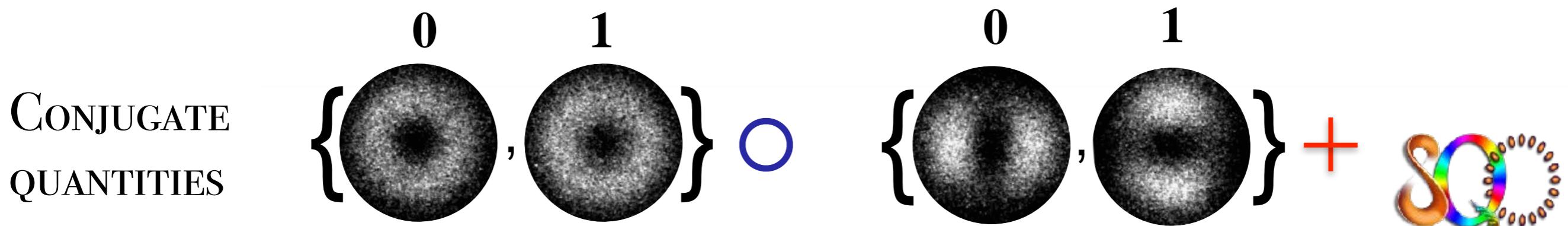


Alice

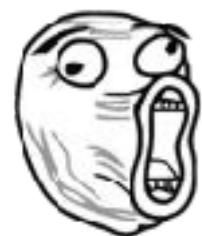
SECURE CHANNEL



Bob



Quantum Cryptography



Alice

SECURE CHANNEL



Bob

ALICE + O + + + O O + + O
0 0 0 1 0 1 1 1 1 0

CONJUGATE
QUANTITIES

$\{0, 1\}$ \circ $\{0, 1\}$ + SQ

Quantum Cryptography



Alice

SECURE CHANNEL



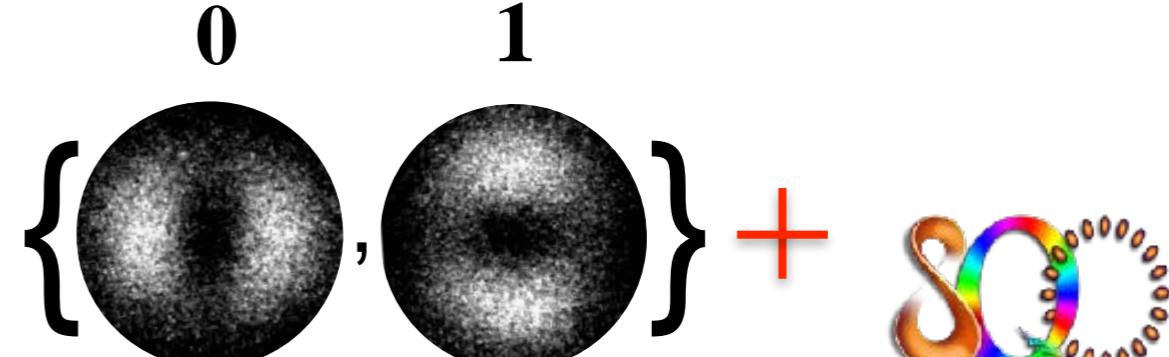
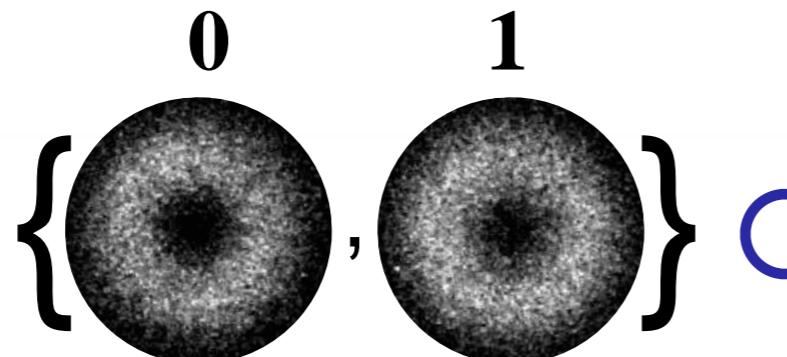
Bob

ALICE + O + + + O O + + O

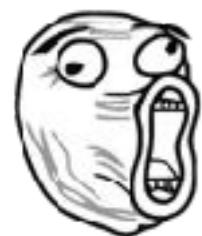
0 0 0 1 0 1 1 1 0

O O + O O + O O + + BOB

CONJUGATE
QUANTITIES



Quantum Cryptography

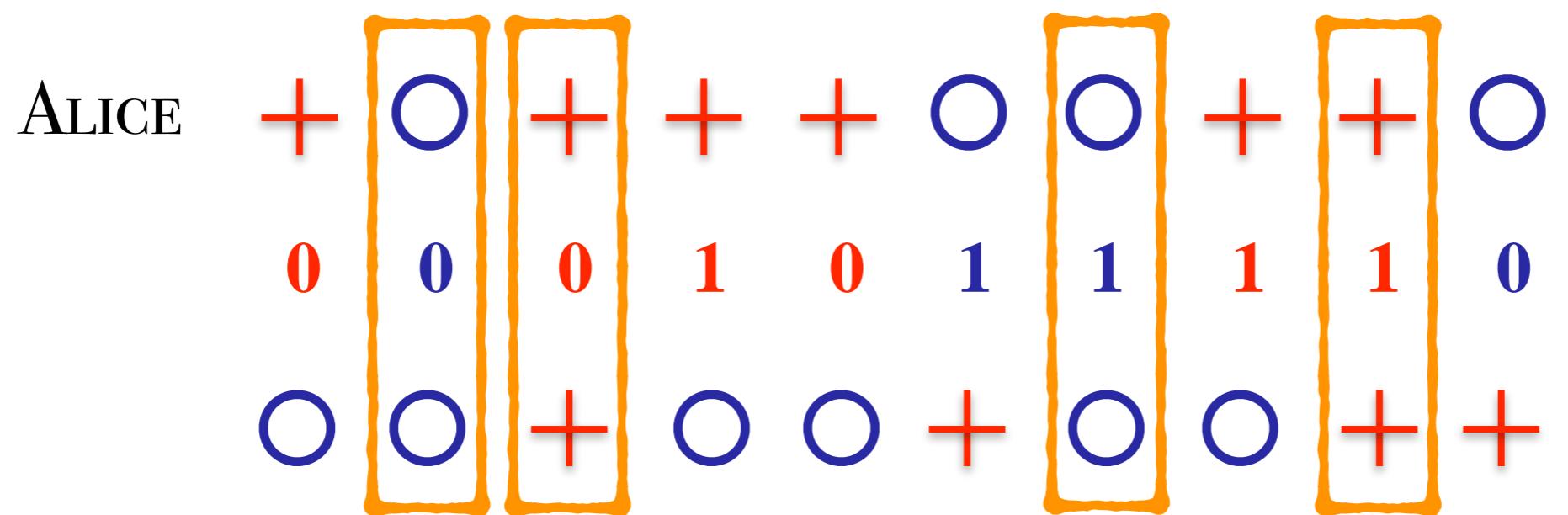


Alice

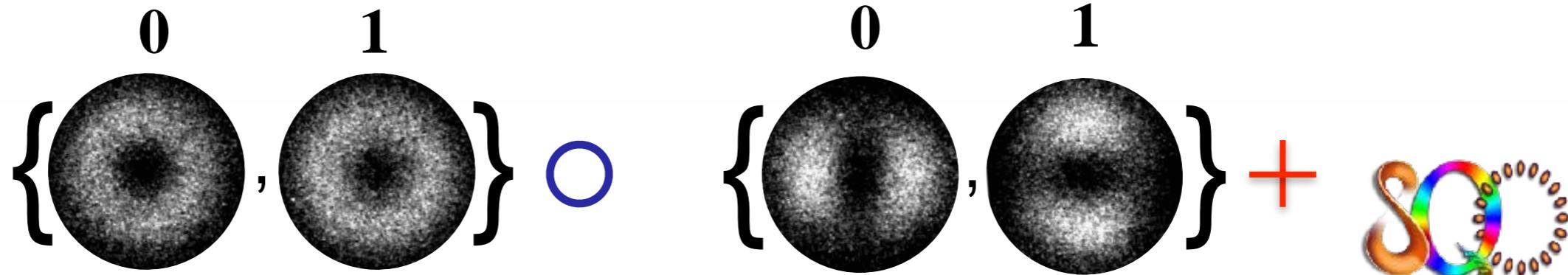
SECURE CHANNEL



Bob



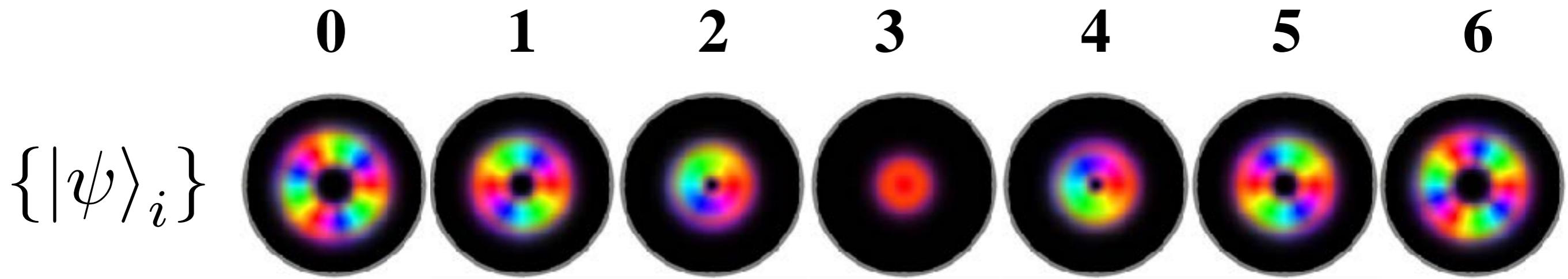
CONJUGATE
QUANTITIES



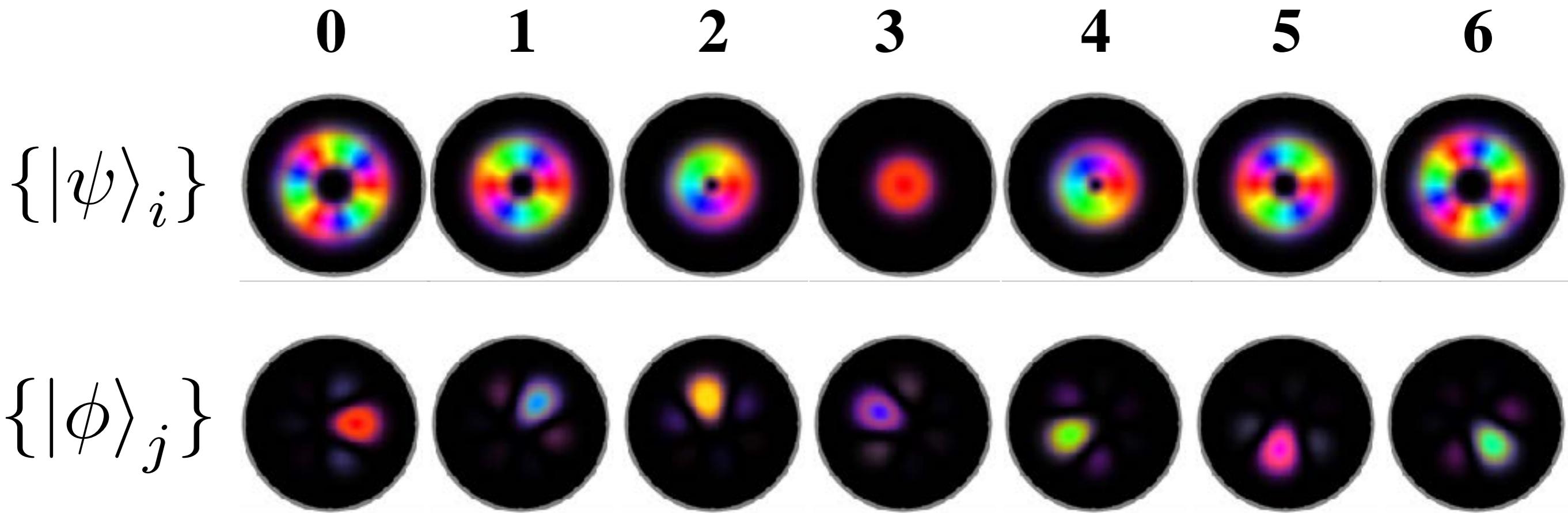
High-Dimensional QKD

0 1 2 3 4 5 6

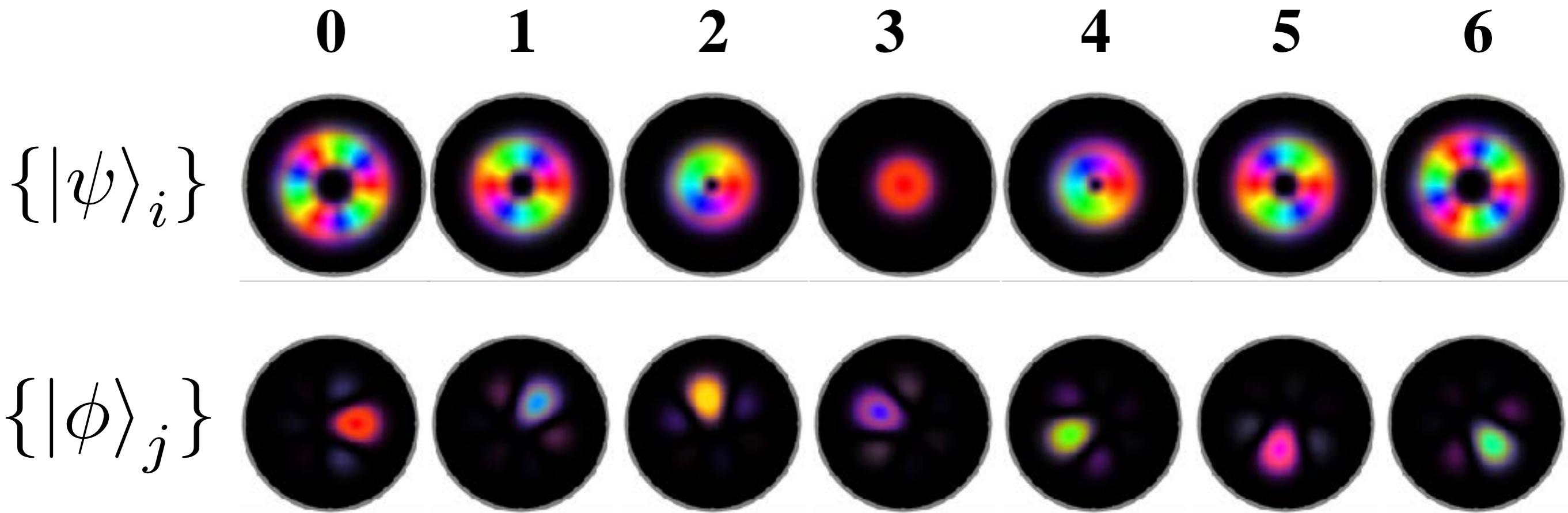
High-Dimensional QKD



High-Dimensional QKD



High-Dimensional QKD



$$|\langle \psi | \phi \rangle|^2 = \frac{1}{7}$$

sqogroup.ca

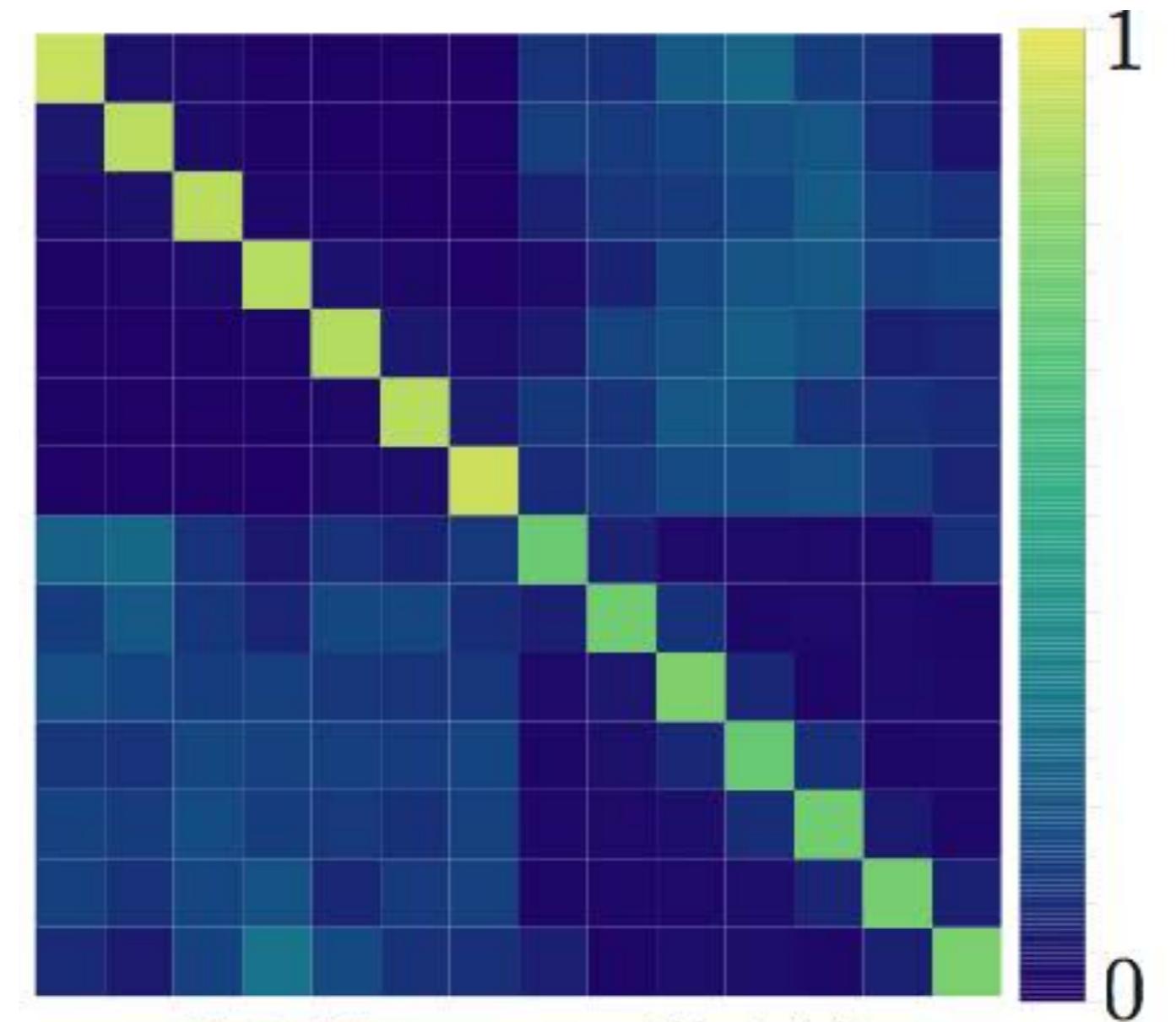


Experimental results

$\{|\psi\rangle_i\}$

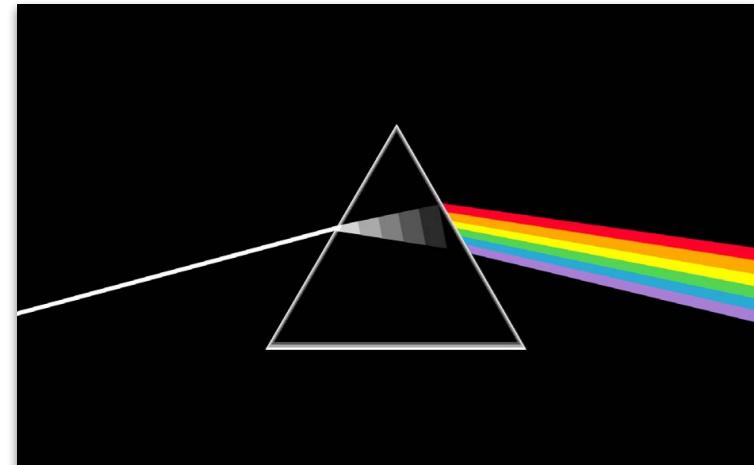
ALICE

$\{|\phi\rangle_j\}$



Encryption results

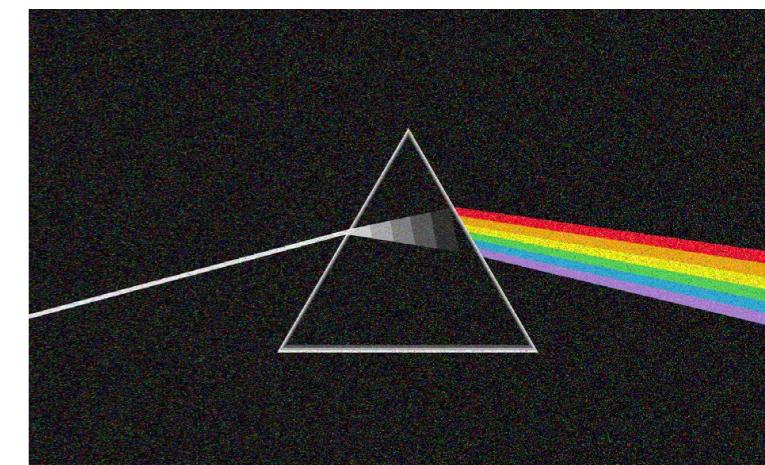
Sender (Alice)



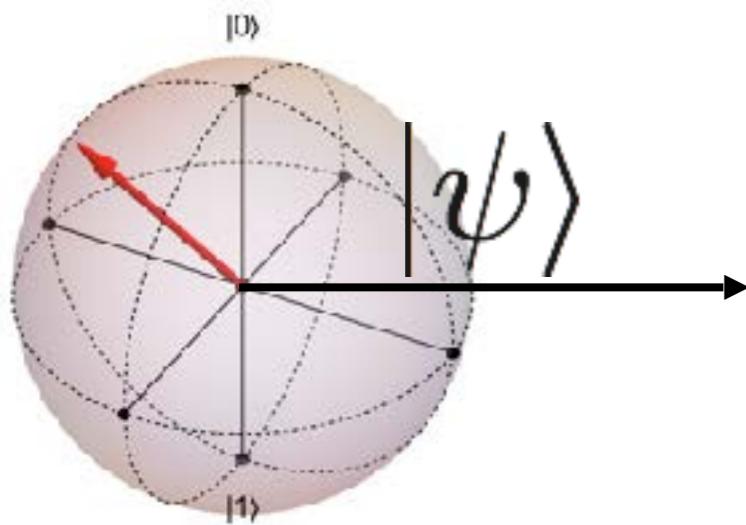
Encrypted message



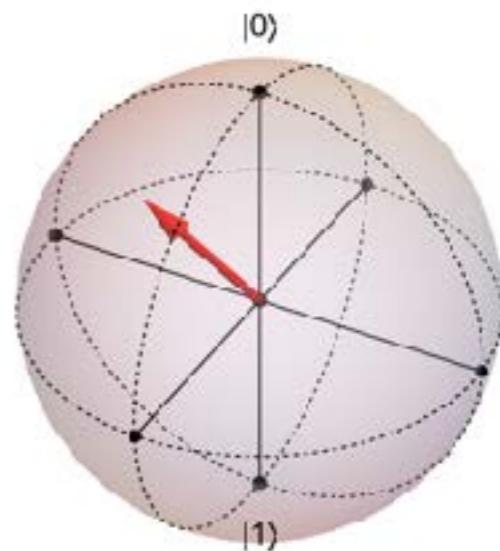
Receiver (Bob)



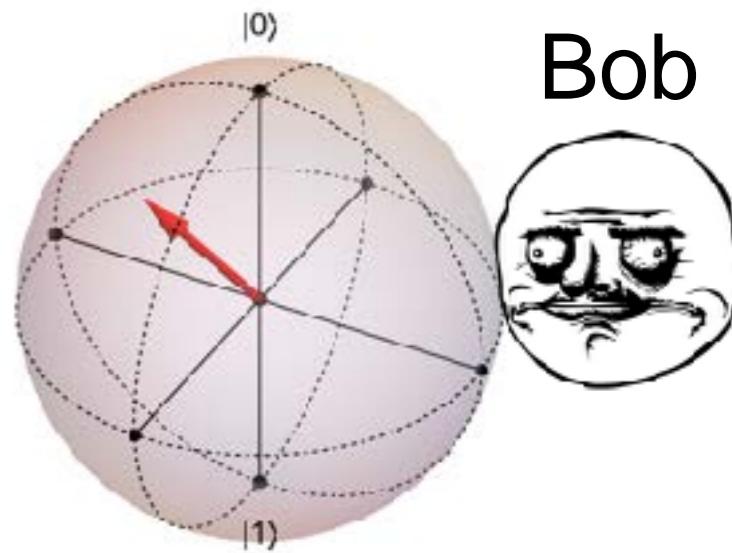
Cloning Attack



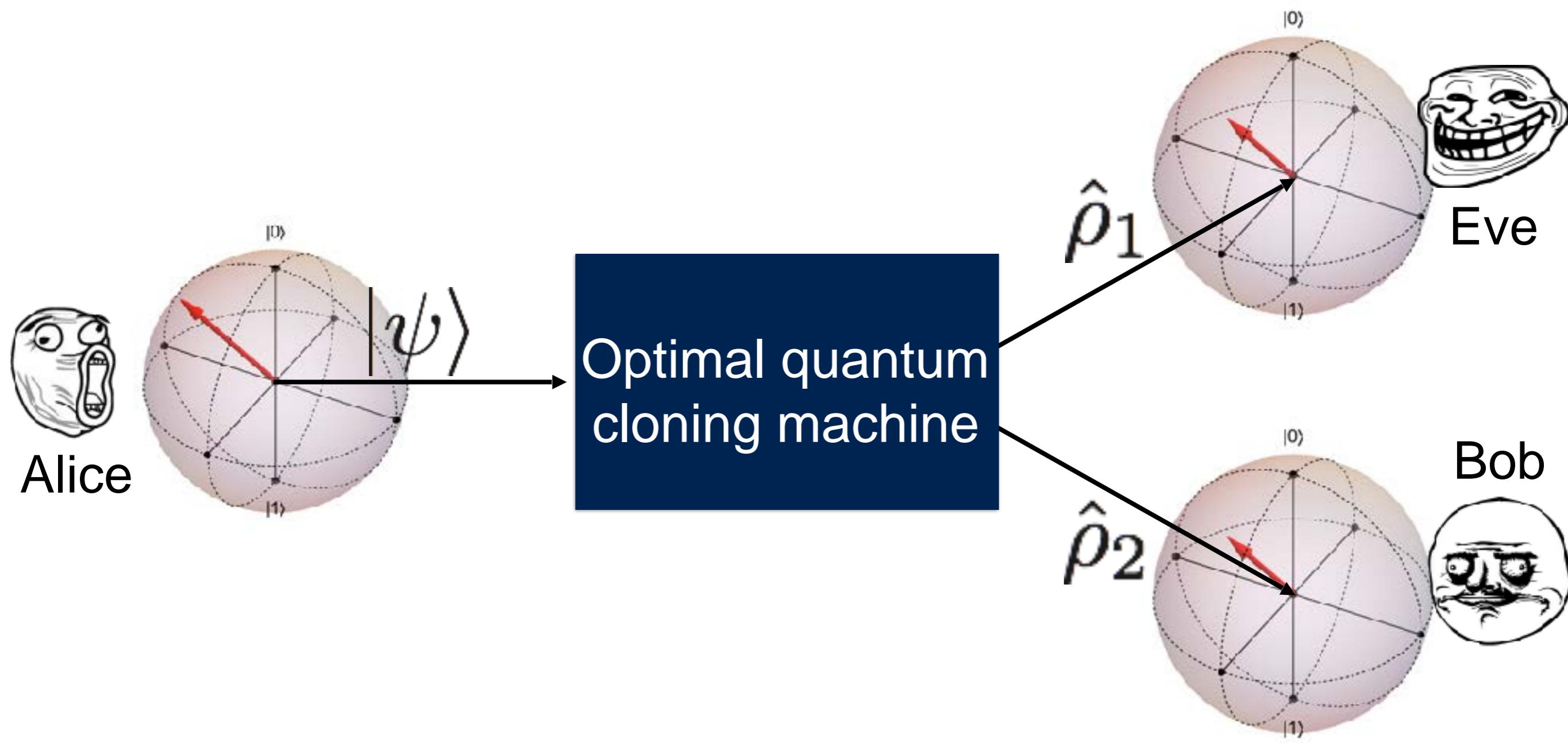
Alice



Bob

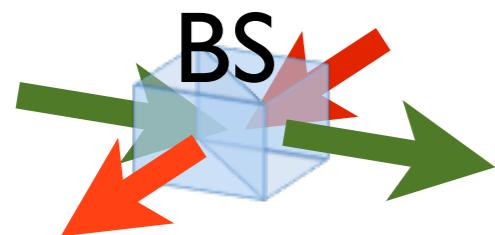


Cloning Attack



Optimal Cloning Attack: Hong-Ou-Mandel

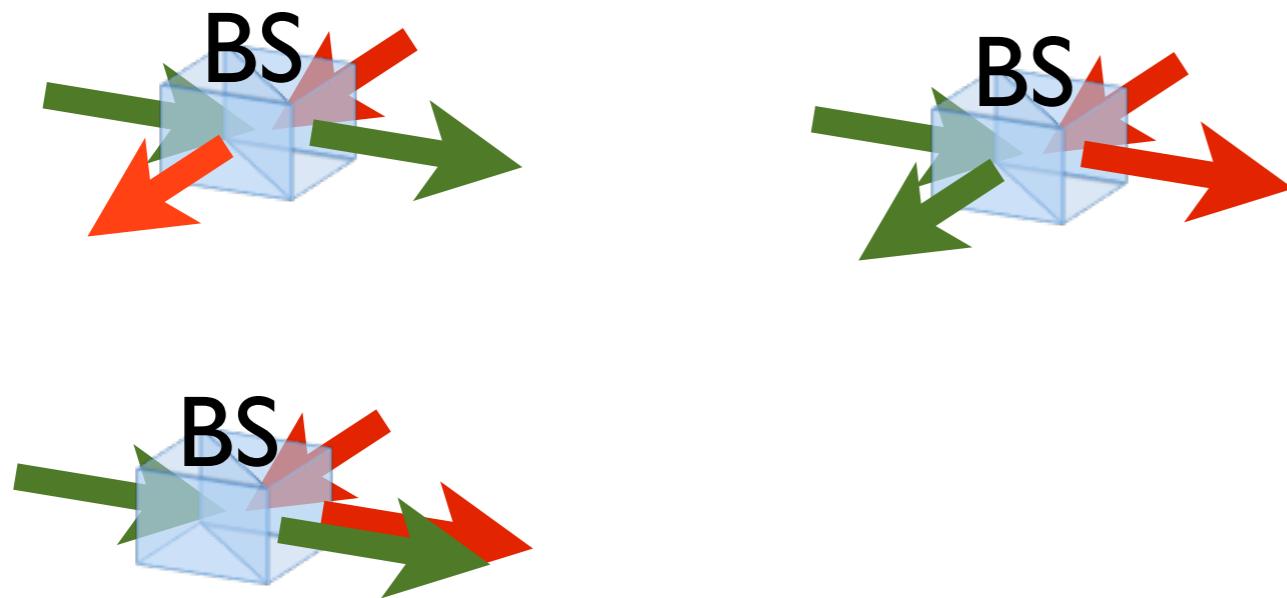
Optimal Cloning Attack: Hong-Ou-Mandel



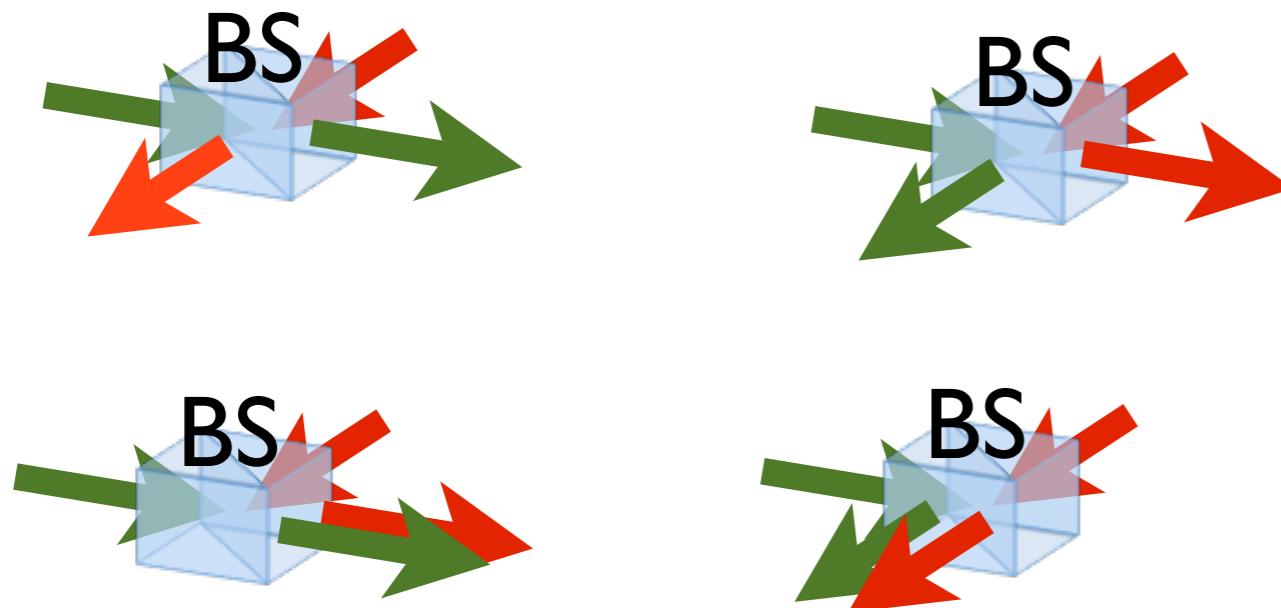
Optimal Cloning Attack: Hong-Ou-Mandel



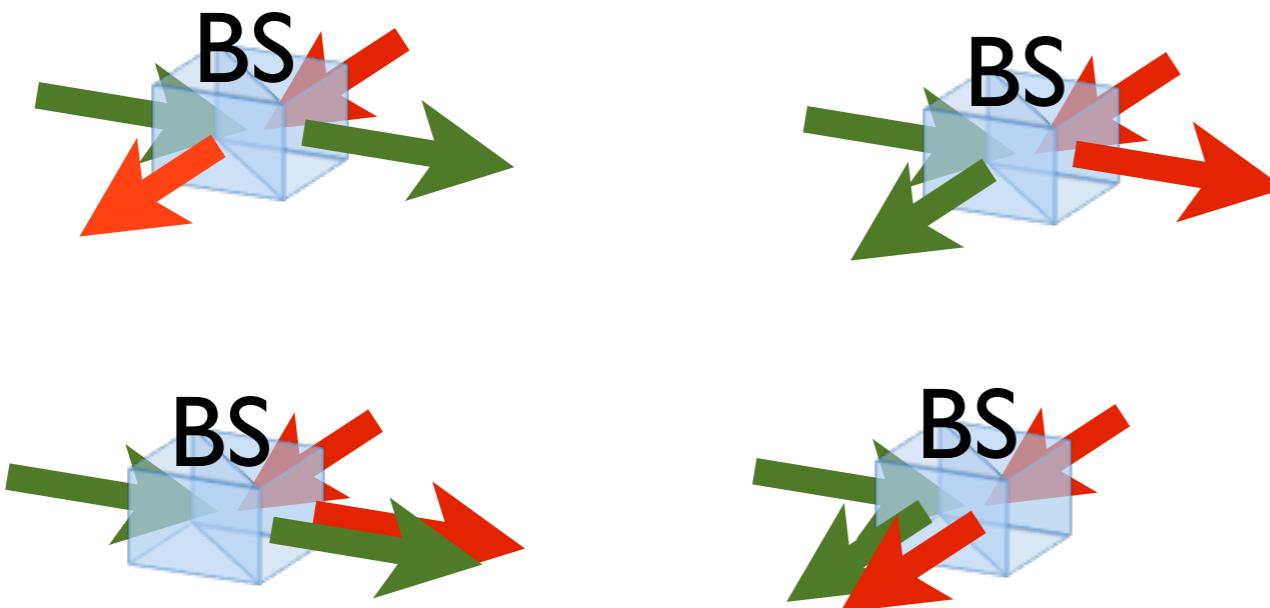
Optimal Cloning Attack: Hong-Ou-Mandel



Optimal Cloning Attack: Hong-Ou-Mandel

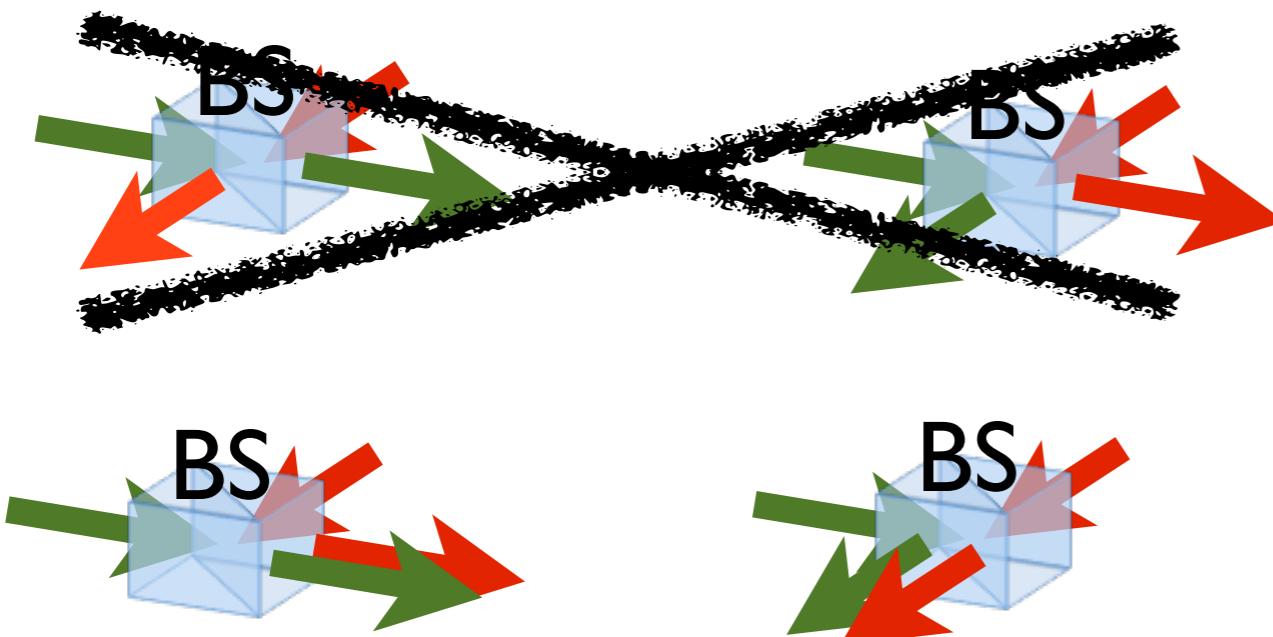


Optimal Cloning Attack: Hong-Ou-Mandel



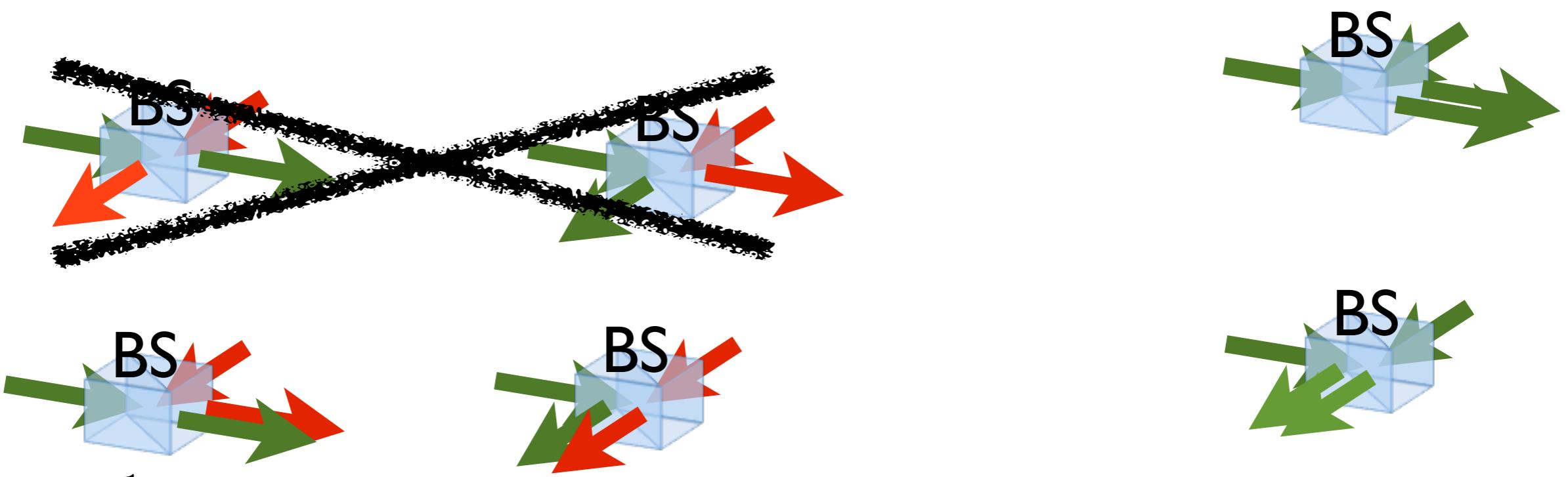
$$|\psi\rangle_{12,out} = \frac{1}{2} (i|c\rangle_1|c\rangle_2 + |c\rangle_1|d\rangle_2 - |d\rangle_1|c\rangle_2 + i|d\rangle_1|d\rangle_2)$$

Optimal Cloning Attack: Hong-Ou-Mandel



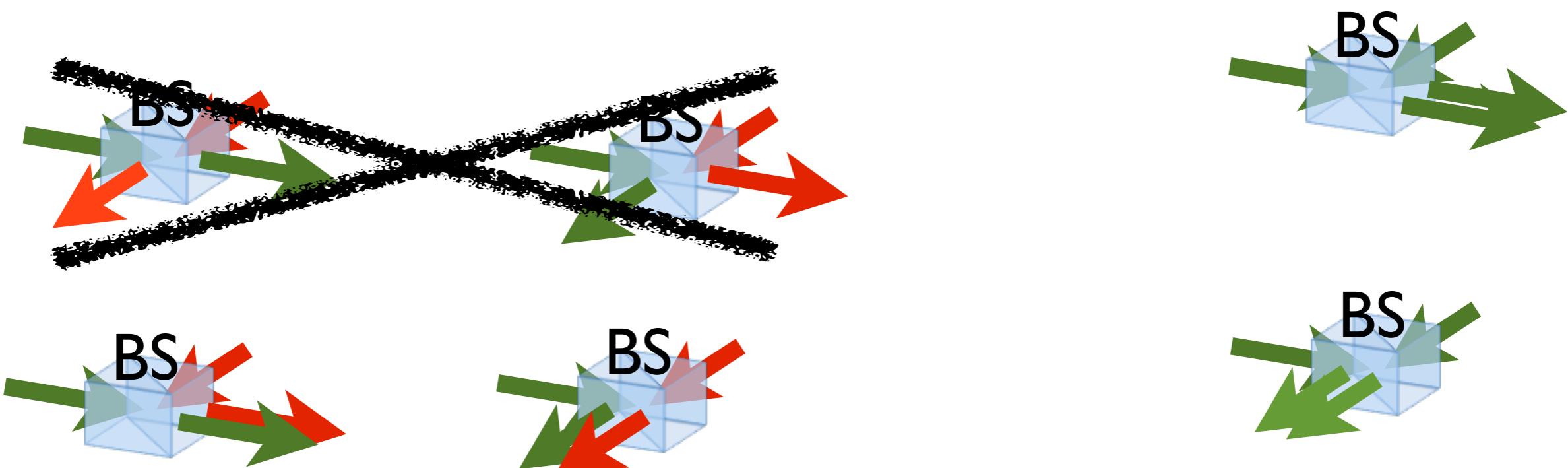
$$|\psi\rangle_{12,out} = \frac{1}{2} (i|c\rangle_1|c\rangle_2 + |c\rangle_1|d\rangle_2 - |d\rangle_1|c\rangle_2 + i|d\rangle_1|d\rangle_2)$$

Optimal Cloning Attack: Hong-Ou-Mandel



$$|\psi\rangle_{12,out} = \frac{1}{2} (i|c\rangle_1|c\rangle_2 + |c\rangle_1|d\rangle_2 - |d\rangle_1|c\rangle_2 + i|d\rangle_1|d\rangle_2)$$

Optimal Cloning Attack: Hong-Ou-Mandel

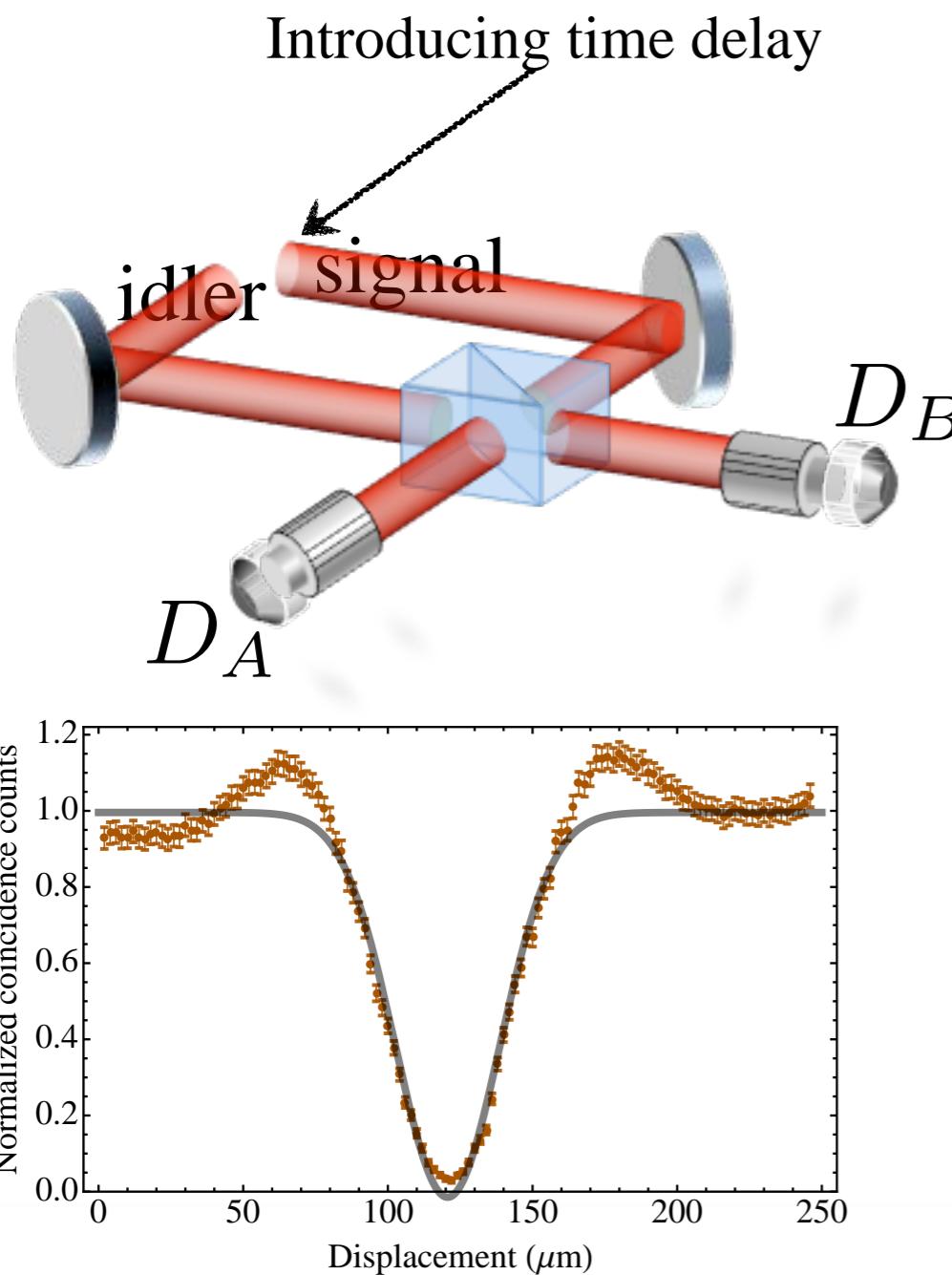


$$|\psi\rangle_{12,out} = \frac{1}{2} (i|c\rangle_1|c\rangle_2 + |c\rangle_1|d\rangle_2 - |d\rangle_1|c\rangle_2 + i|d\rangle_1|d\rangle_2)$$

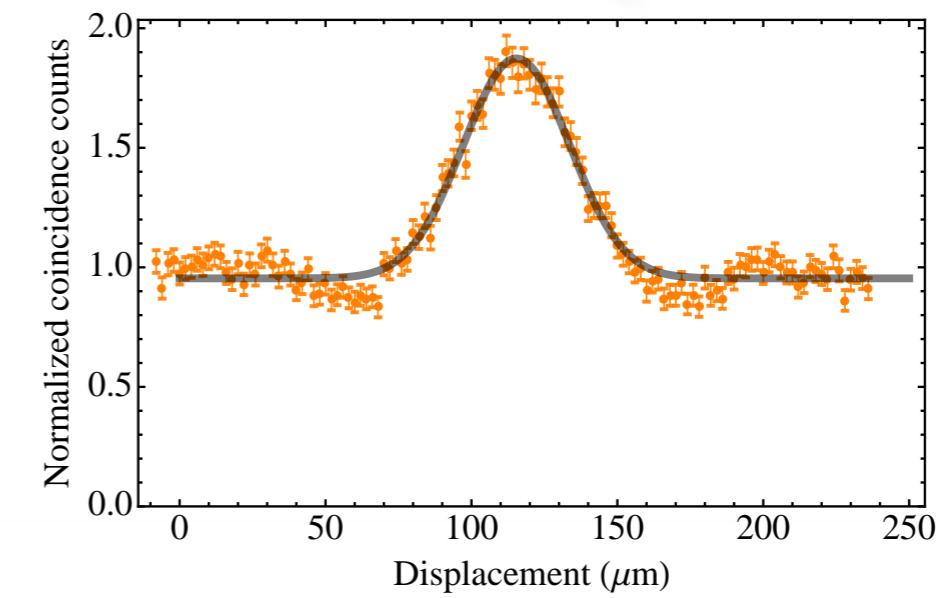
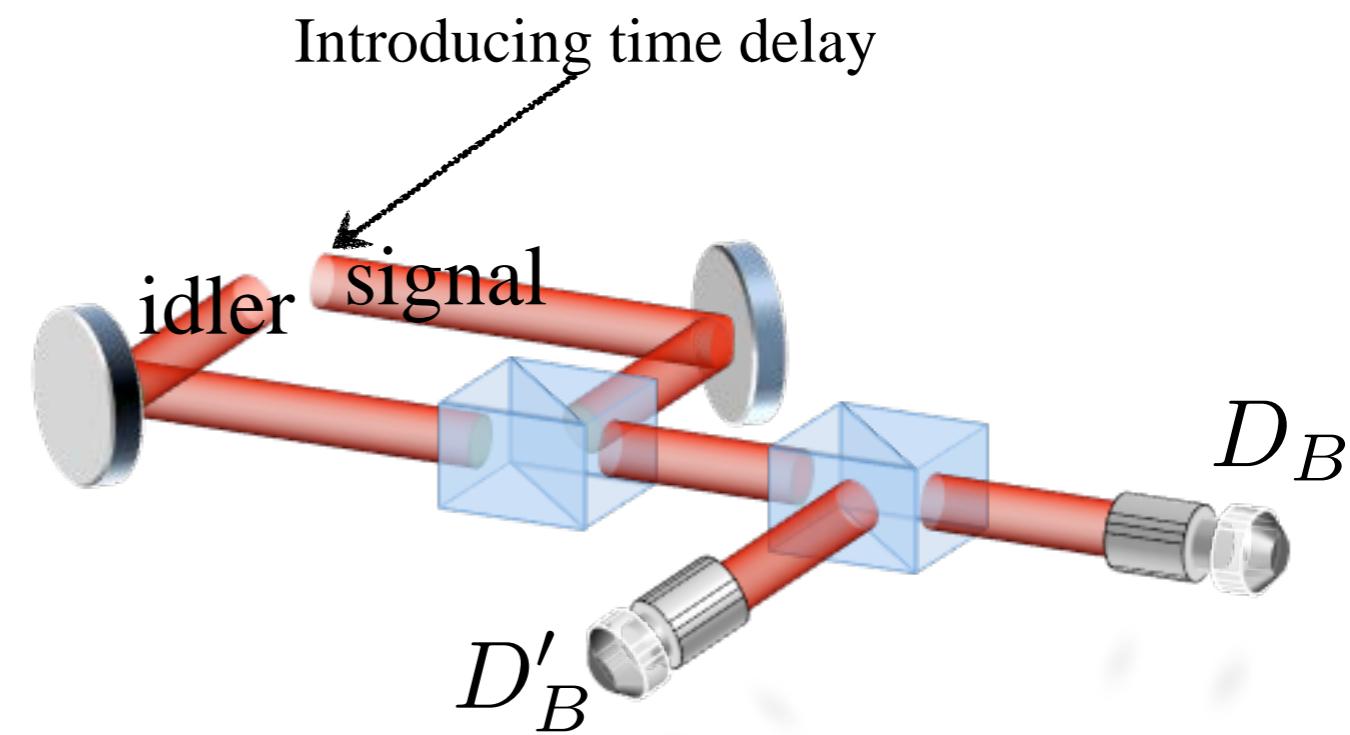
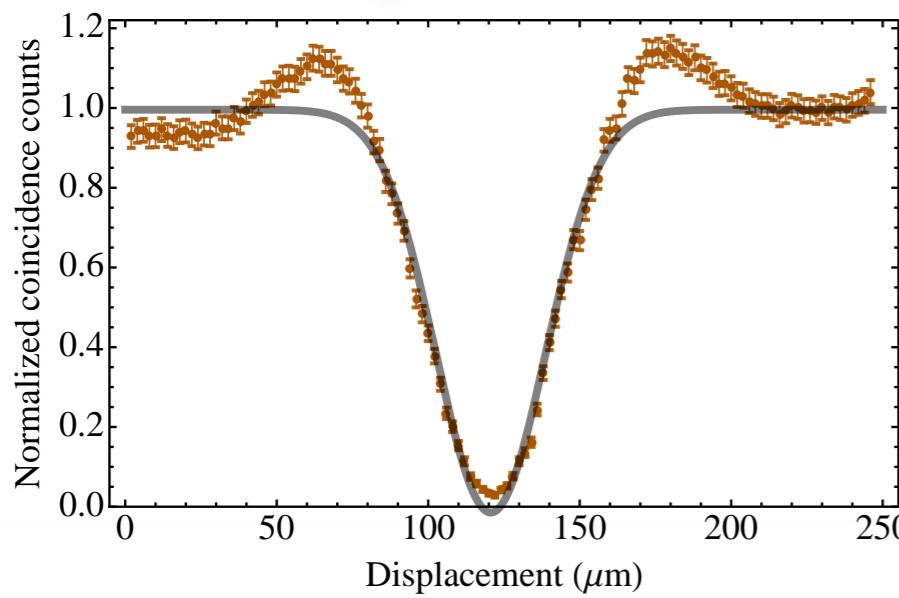
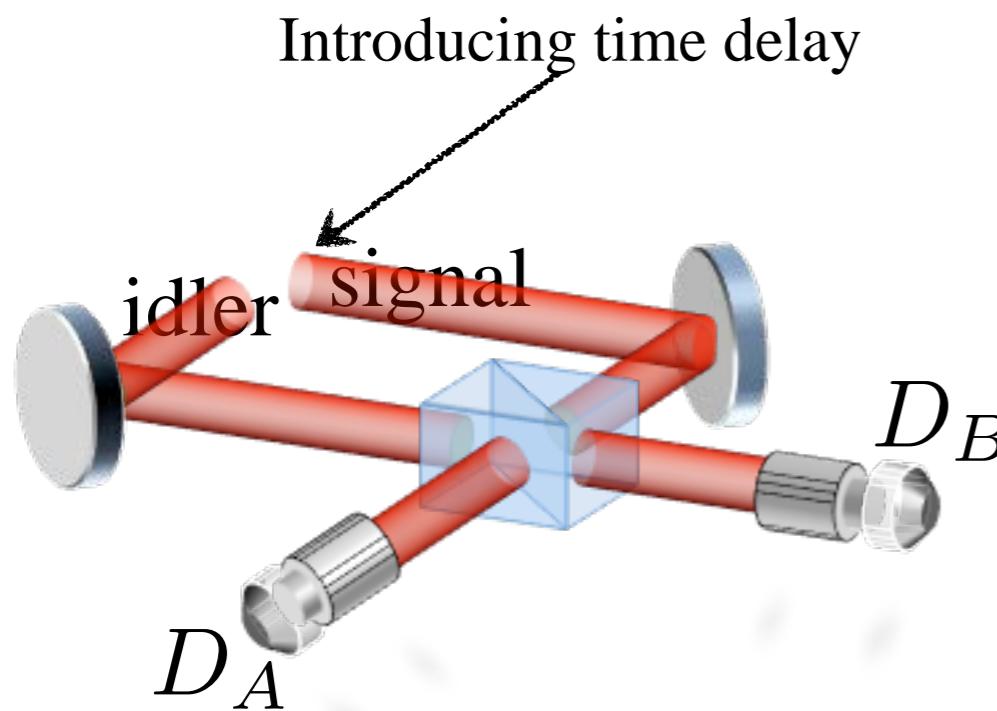
$$|\psi\rangle_{out} = \frac{1}{\sqrt{2}} (|c\rangle^2 + |d\rangle^2)$$

Optimal Cloning Attack: Hong-Ou-Mandel

Optimal Cloning Attack: Hong-Ou-Mandel

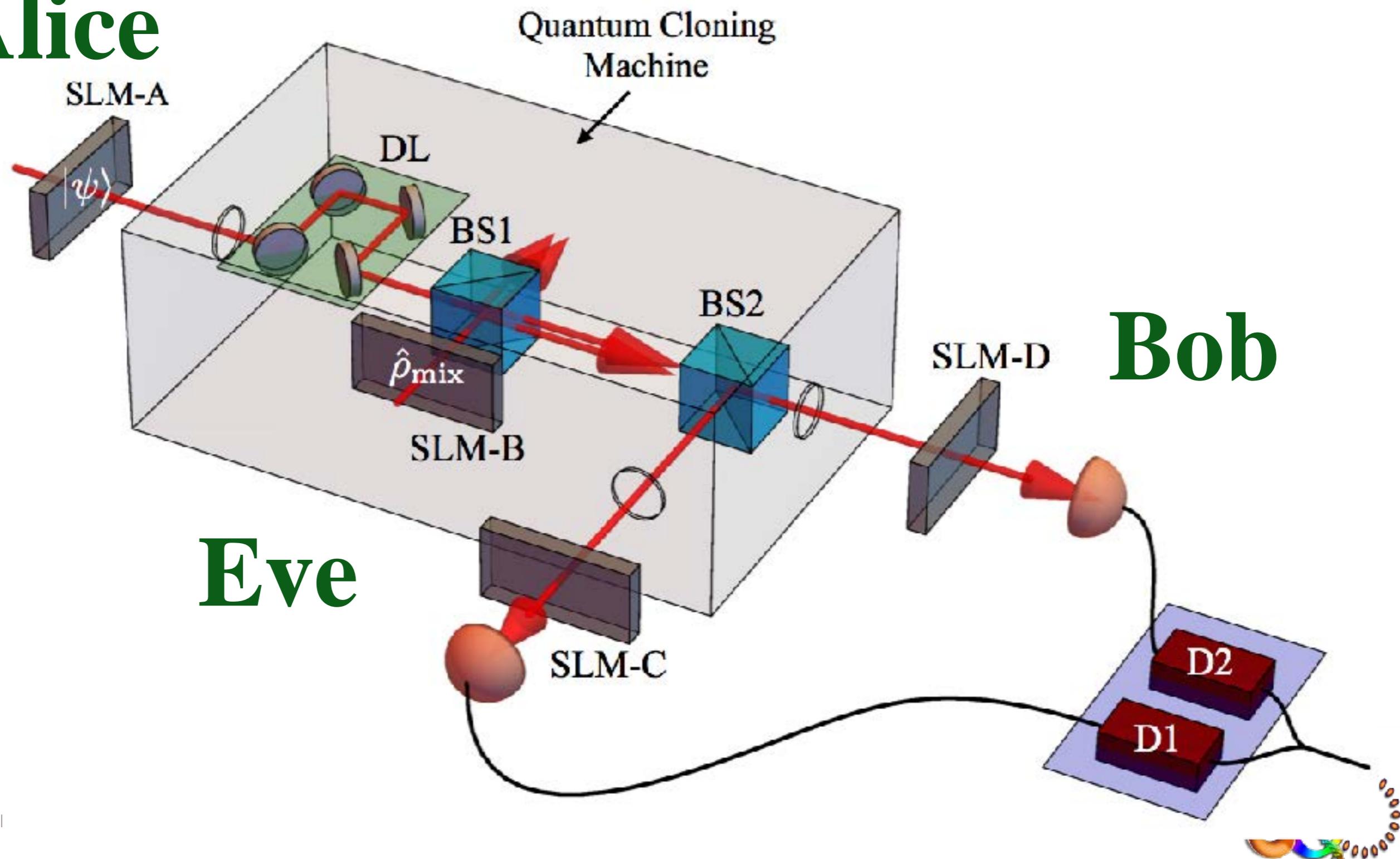


Optimal Cloning Attack: Hong-Ou-Mandel



Optimal Cloning Attack: Experimental Setup

Alice



Bob

Eve

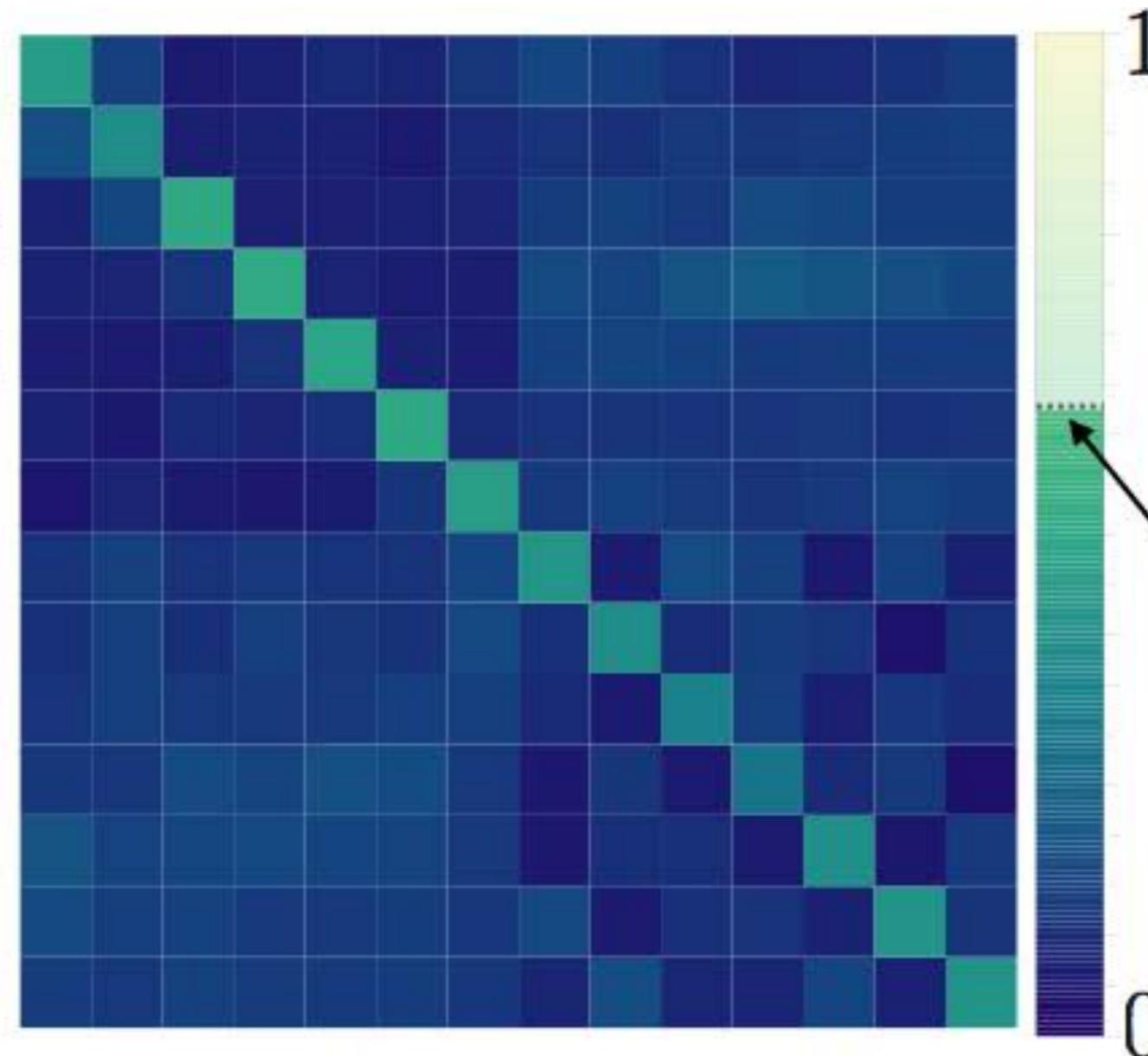
Optimal Cloning Attack: Experimental Setup

Optimal Cloning Attack: Experimental Results

$\{|\psi\rangle_i\}$

ALICE

$\{|\phi\rangle_j\}$



$\{|\psi\rangle_i\}$

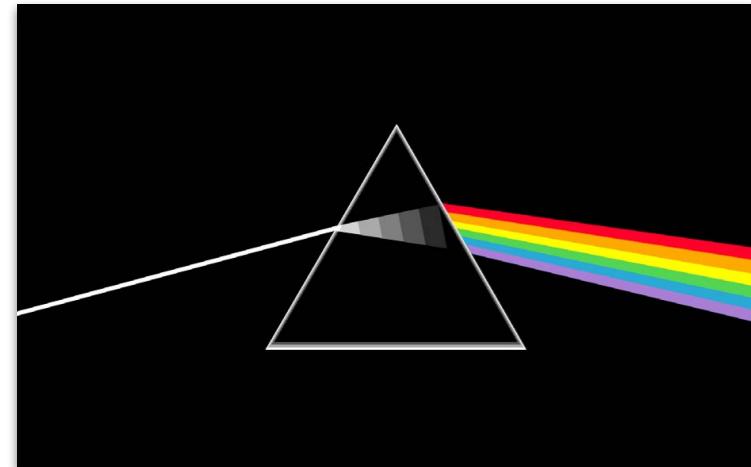
$\{|\phi\rangle_j\}$

BOB

Optimal cloning
attack

Secure Channel under Cloning Attack

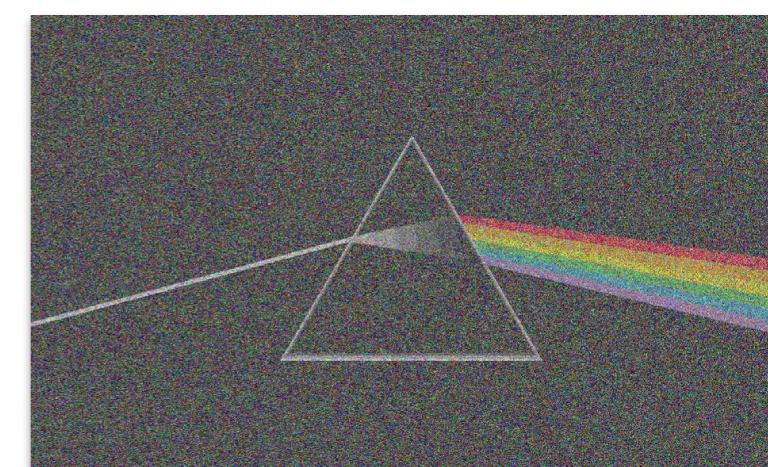
Sender (Alice)



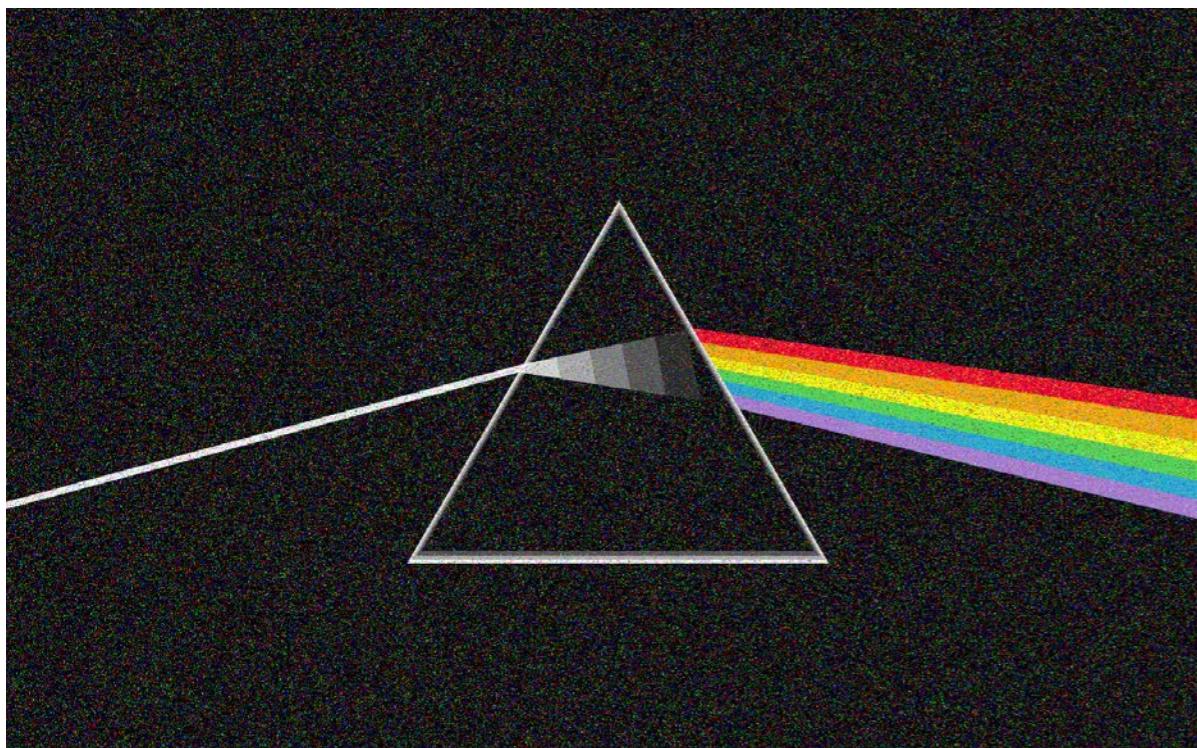
Encrypted message



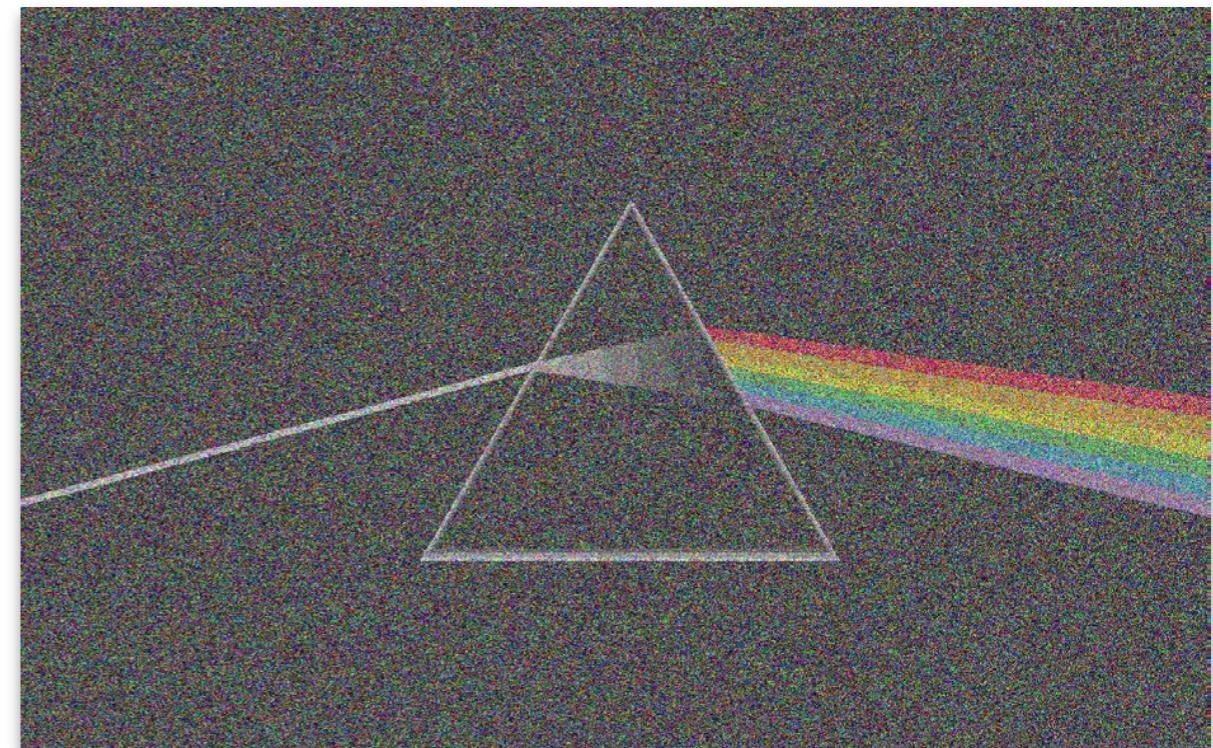
Receiver (Bob)



Secure link



Cloning attack



QKD over the city of Ottawa

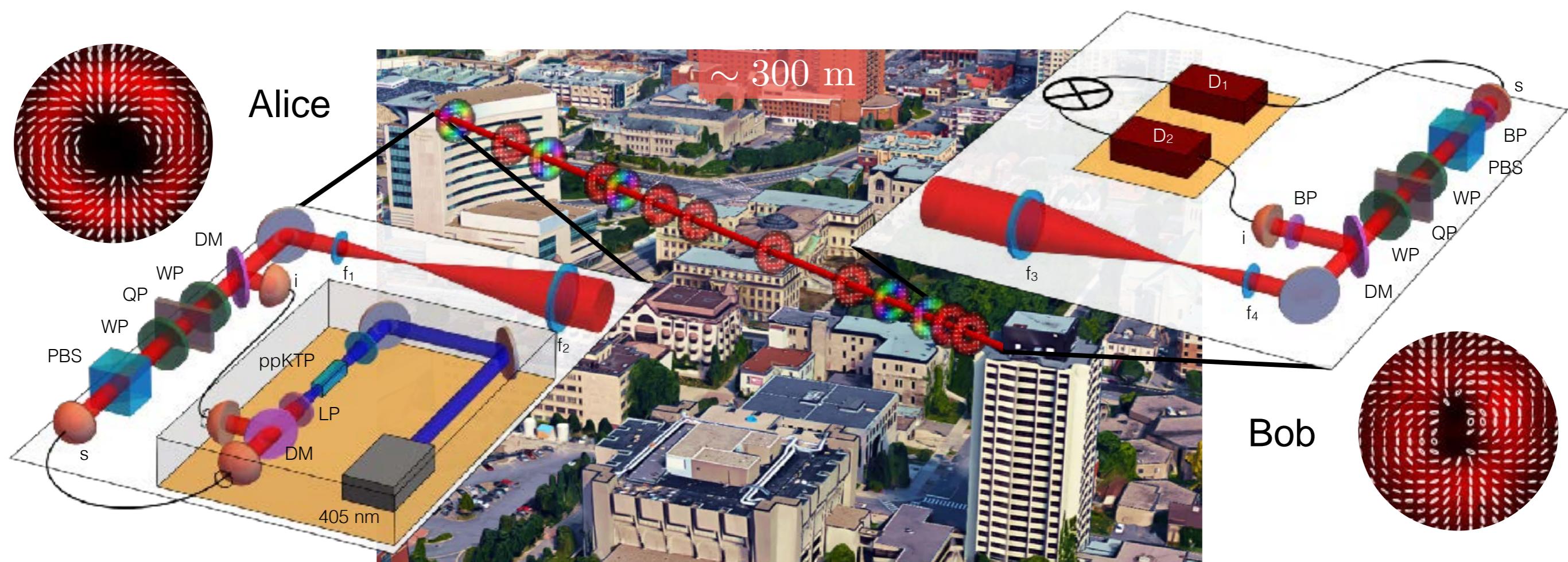


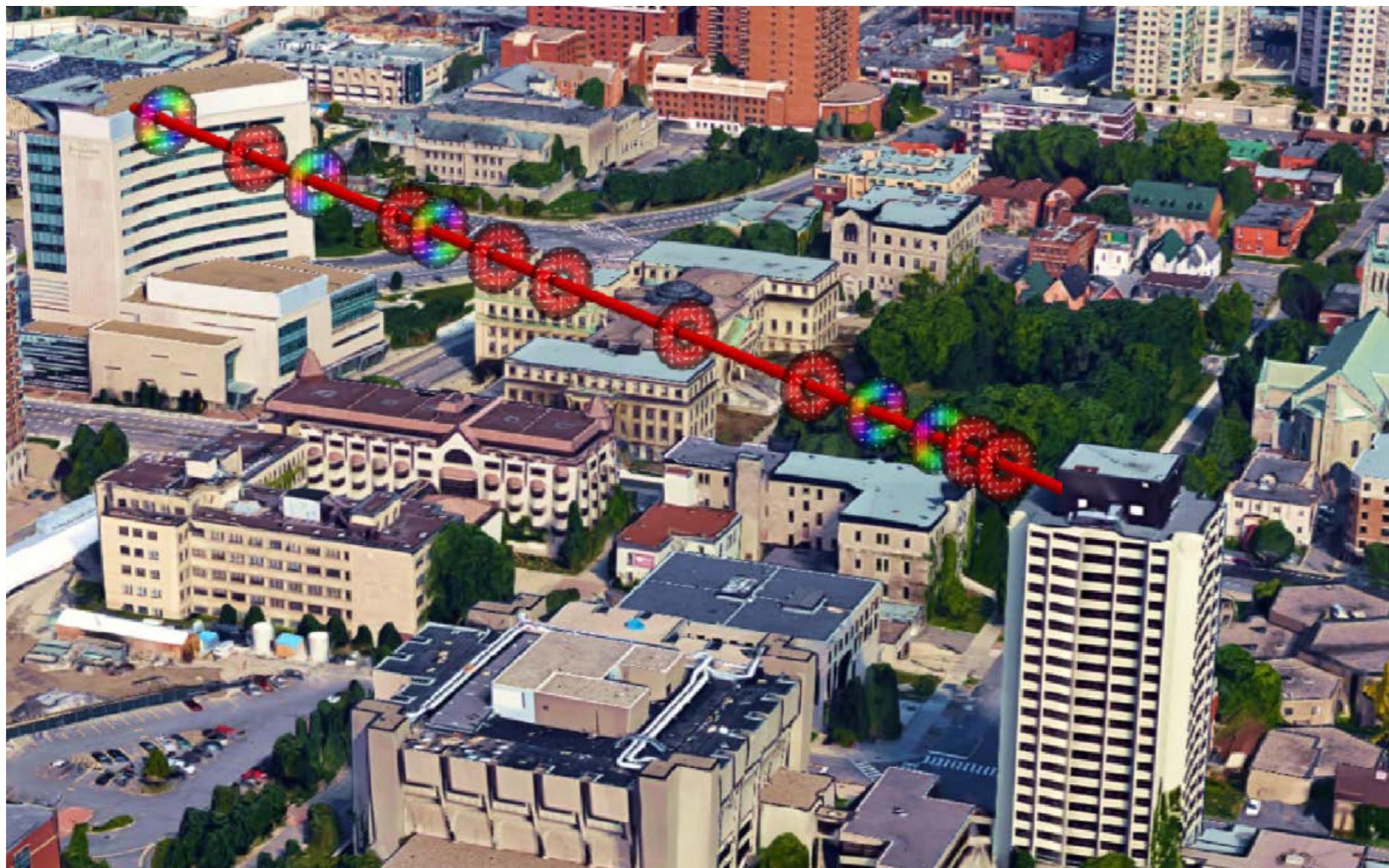


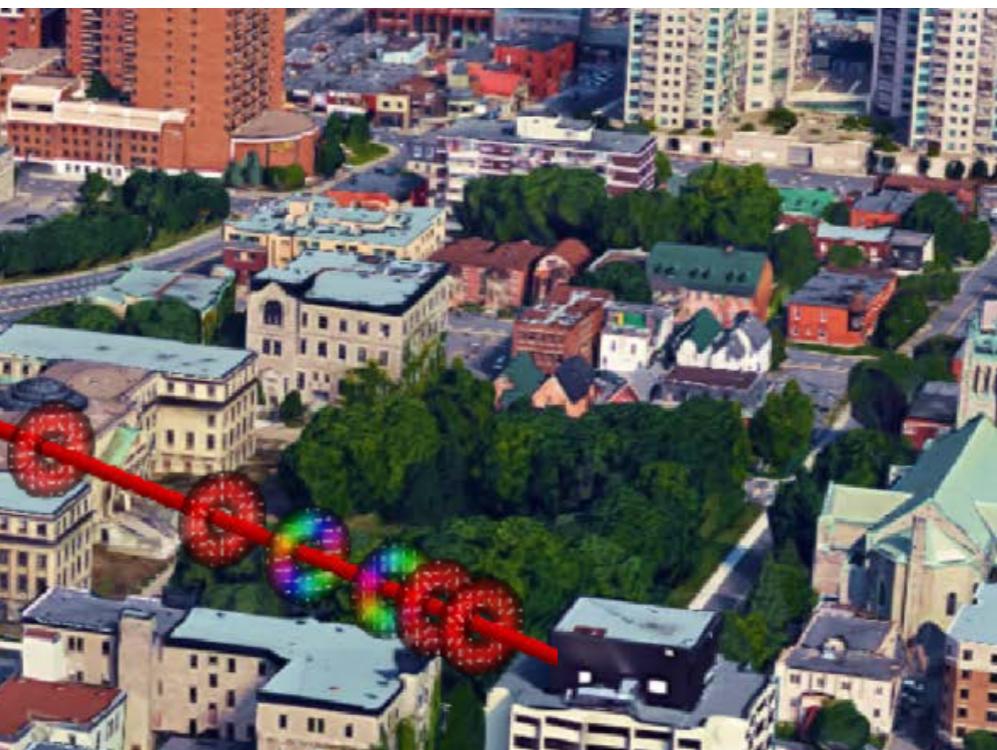
Q X D



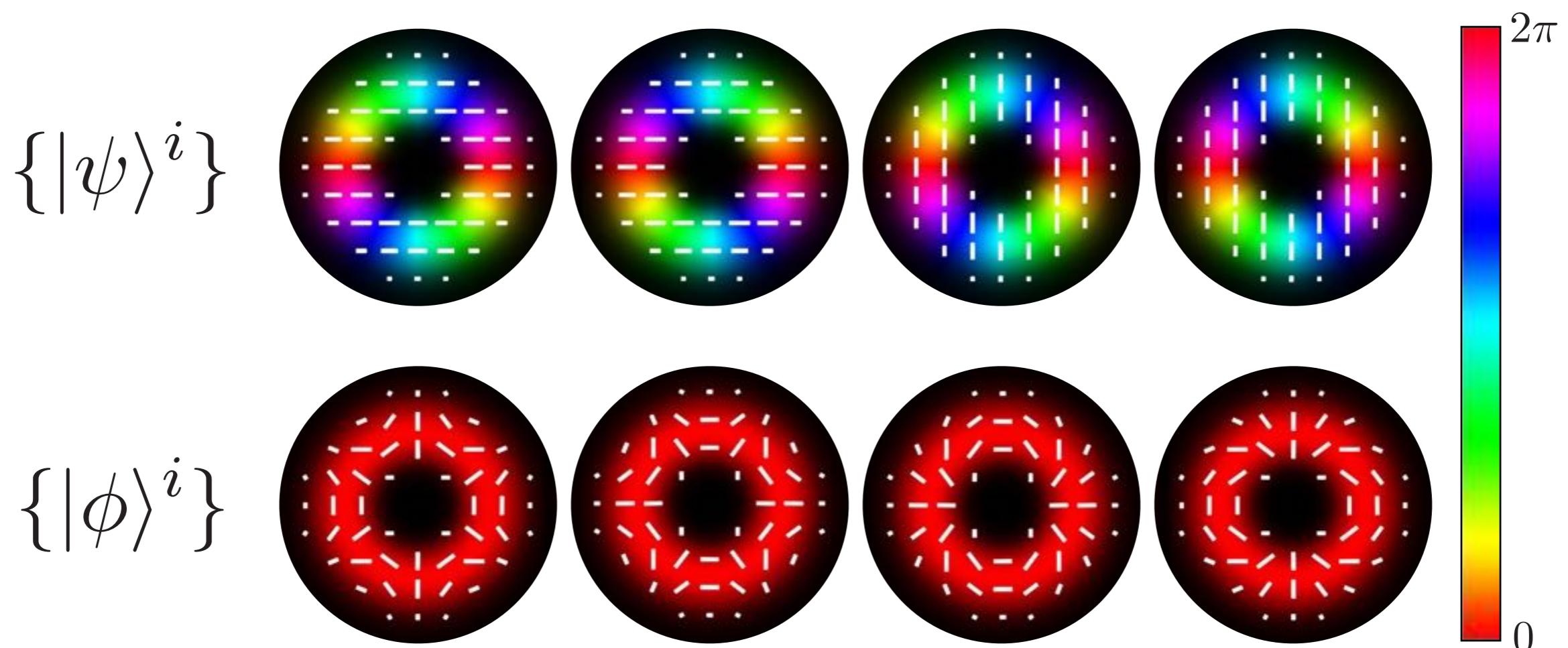
Intra-city QKD experiment





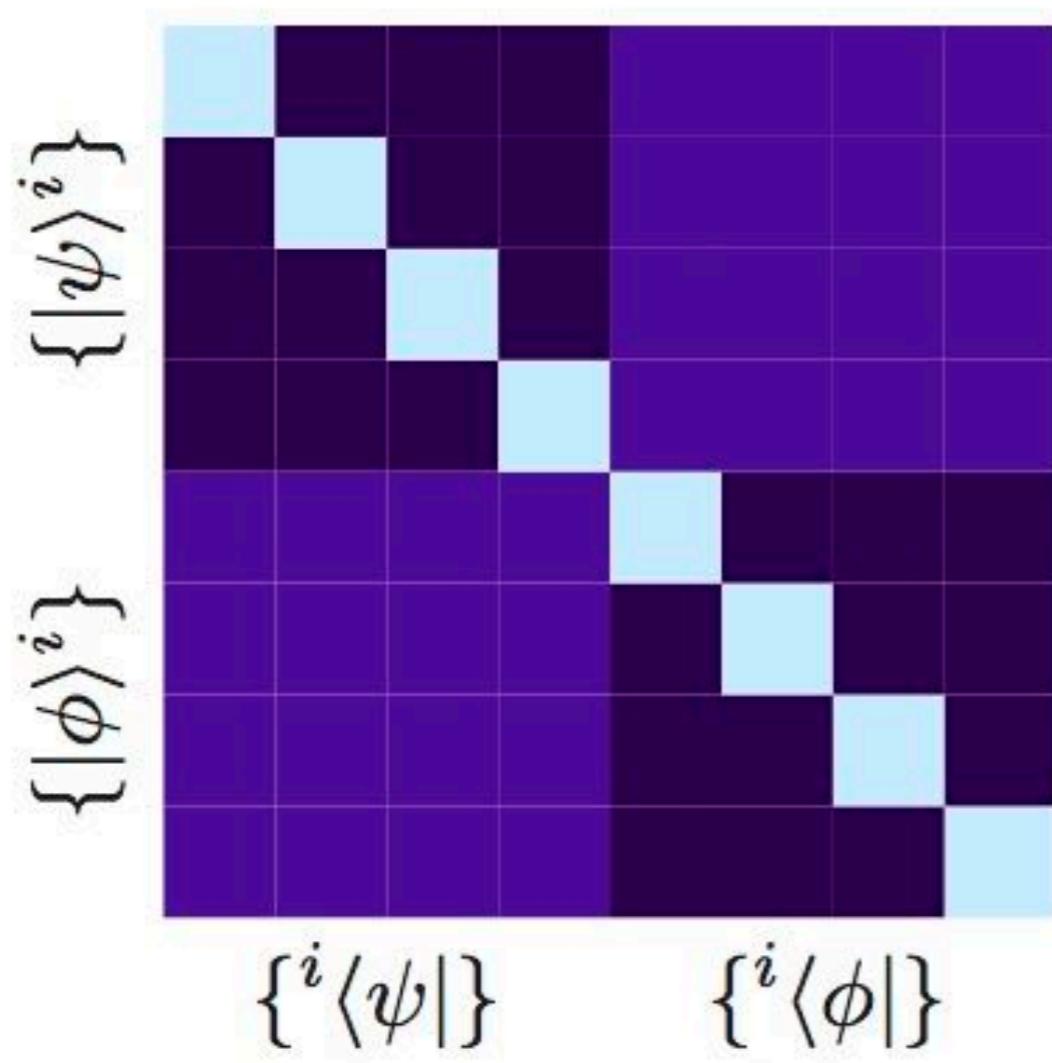


Degrees of freedom of light

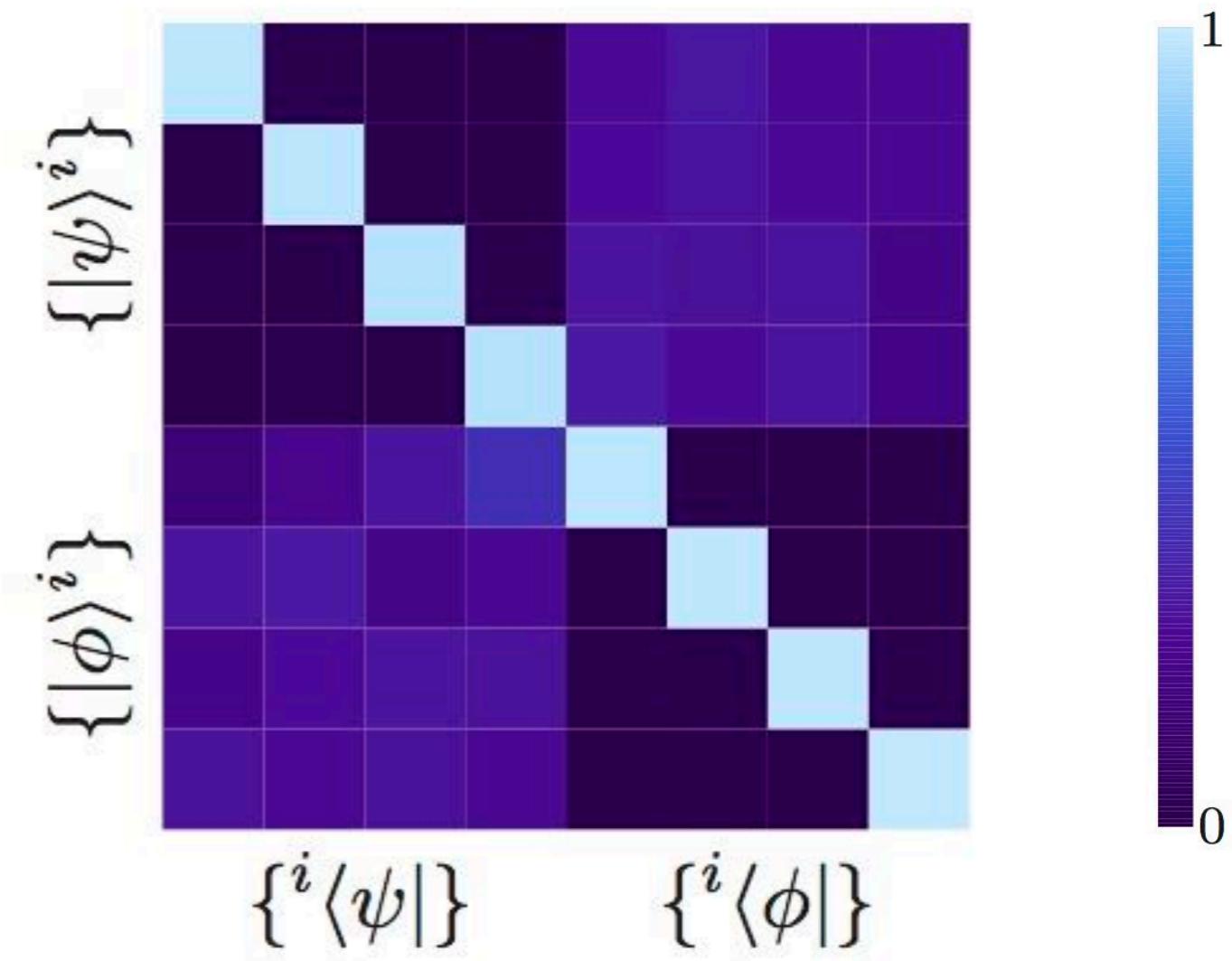


Laboratory results

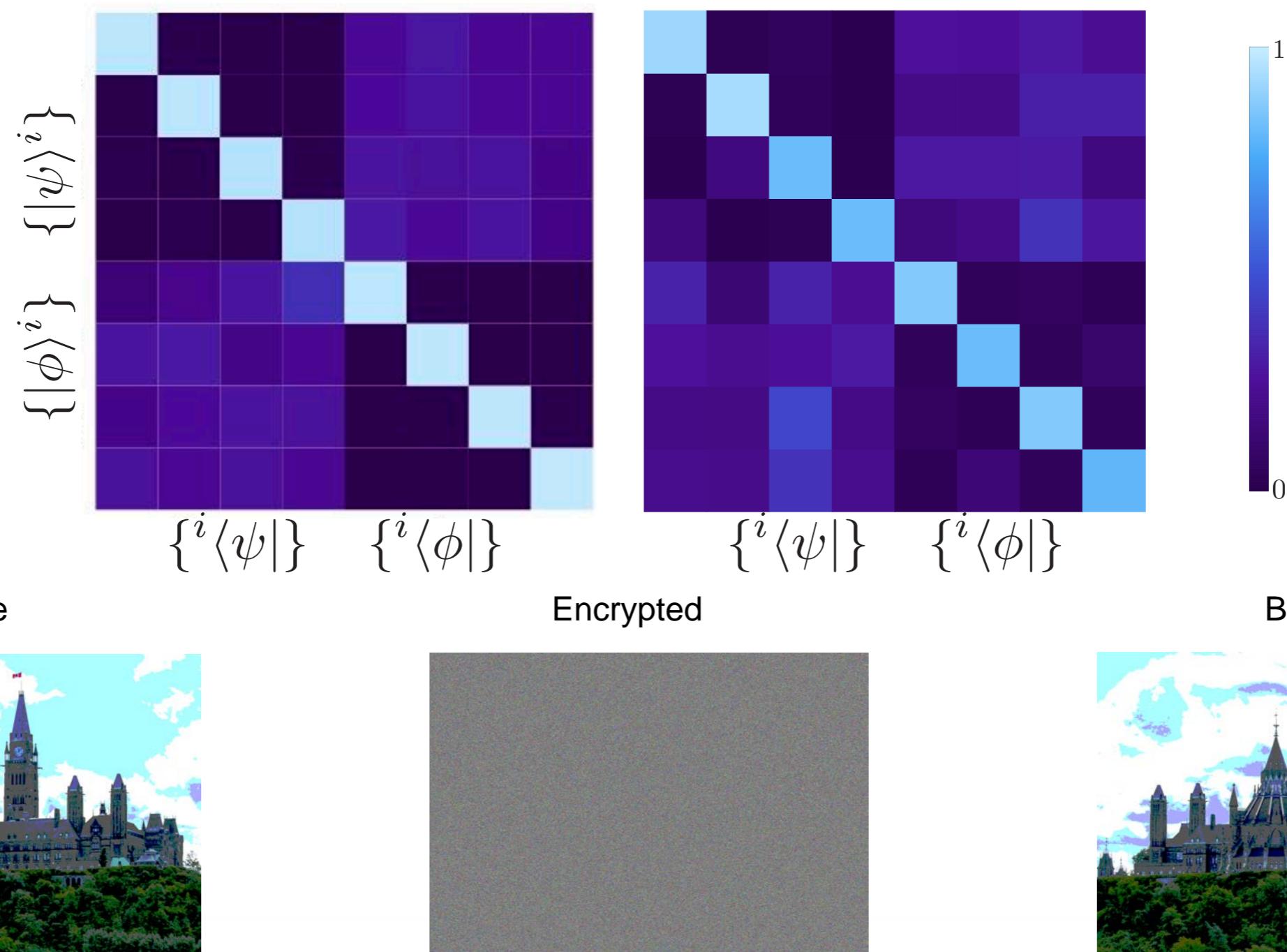
Theory



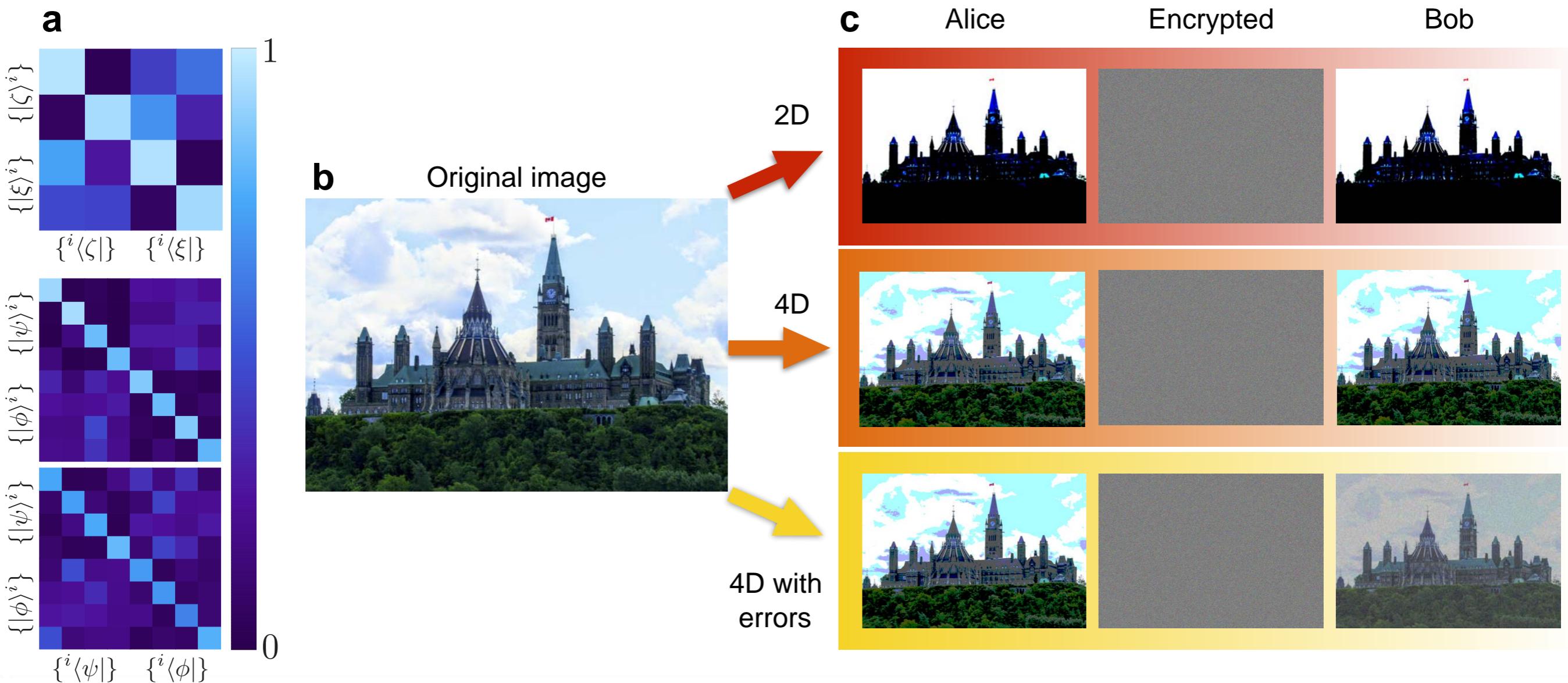
Experiment



Experimental results of intra-city QKD



Experimental results of intra-city QKD

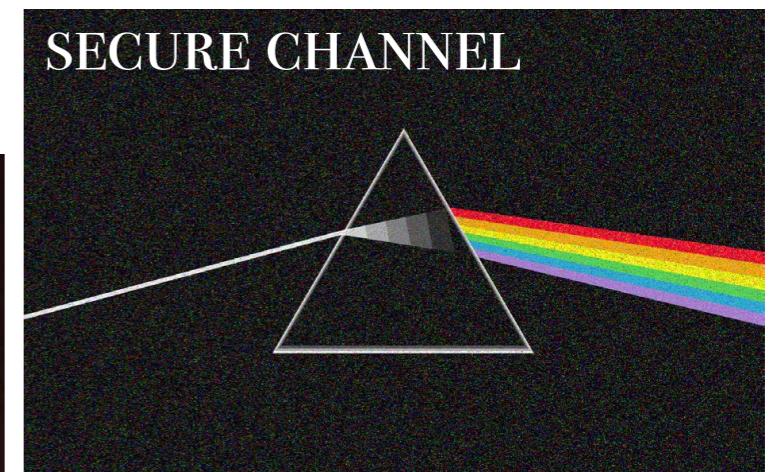


QKD

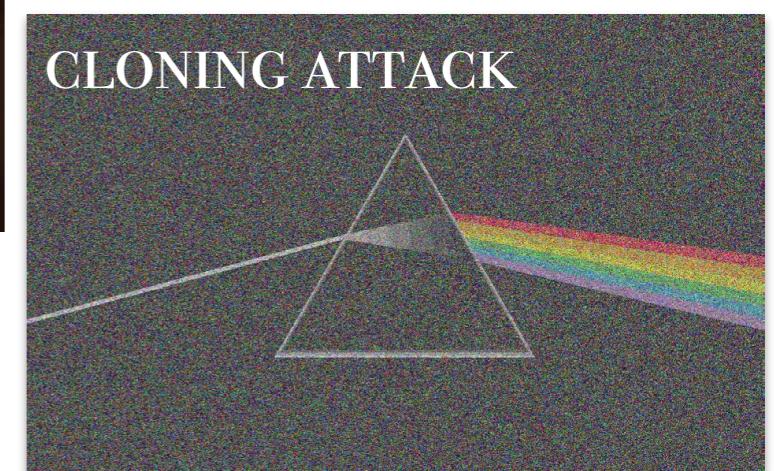
MORE INFORMATION PER CARRIER



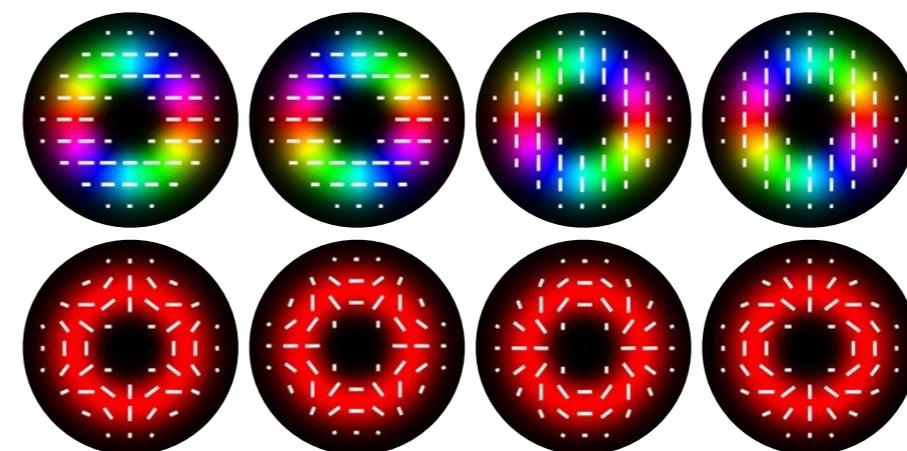
SECURE CHANNEL

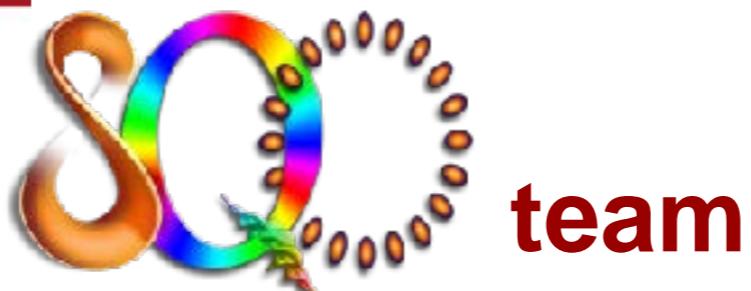


CLONING ATTACK



THEY ARE ROBUST IN A NOISIER CHANNEL





LE SYMPOSIUM SCHAWLOW-TOWNES SUR LA PHOTONIQUE - 2016

THE SCHAWLOW-TOWNES SYMPOSIUM ON PHOTONICS - 2016



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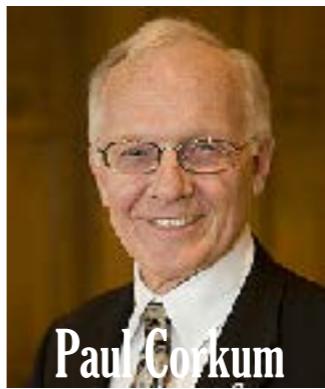
Collaborators



Gerd Luechs



Miles Padgett

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Paul Corkum



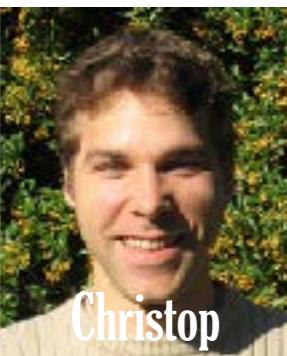
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uOttawa



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Fabio Sciarrino



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Daniele Sanvitto

..., and many junior members of these research teams