Verification of Quantum Computing

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National Investments

Europe 1bn€ UK 270M £ Netherlands 80M \$ US, Singapore,Canada

Quantum Machines

Private Investments

Google, IBM, Intel Big VC founds Startups Companies: D-Wave













Do we need another quantum computer to testing a quantum computer or can we bootstrap a mini quantum device to test a bigger one

Quantum Machines Era



Google Martinis Lab



These devices become relevant at the moment they are no longer classically simulatable

Google Martinis Lab



These devices become relevant at the moment they are no longer classically simulatable

Existing methods of Testing/Validation/Simulation/Monitoring/Tomography ... all become IRRELEVANT

Google Martinis Lab



Efficient verification methods for realistic pseudo quantum computers



Target



Target



Target



Goal

Over the next decades, as quantum technology matures globally we will provide **testing criteria** for industry adaptation

Should we pay \$10000000 for a quantum computer



Should we pay \$10000000 for a quantum computer



Simple test: We ask the box to factor a big number

Should we pay \$10000000 for a quantum computer



Simple test: We ask the box to factor a big number









Quantum Turing Test



Quantum Turing Test


Quantum Turing test



Quantum Turing test

• Can we classically communicate or do we need a bit of **quantumness** to test quantum computer

¥

• Can we do the test **efficiently** or do we need super powerful computer to test quantum computer

Quantum Verification



- Computationally limited verifier
- Powerful quantum server(s)
- Certify the correctness of the computation



- State authentication-Based protocols
- Trapification based
- Measuring entanglement correlation protocols

➡ Interactive Proof System

- Cryptographic Toolkit
- Classically controlled QC







Complexity as Proof System



Yes X satisfies some property

Complexity as Proof System



Complexity as Proof System



with short verifiable certificate proof

Generalisation: Interactive Proofs



Yes X satisfies some property

Generalisation: Interactive Proofs



Yes X satisfies some property

Generalisation: Interactive Proofs



Yes X satisfies some property



A **PROTOCOL** between a

computationally unrestricted prover P and a probabilistic polynomial-time verifier V

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$$(\forall x \in L) \operatorname{Pr}[(V \leftrightarrow P)(x) \ accepts] = 1$$

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Completeness:
$$(\forall x \in L) \operatorname{Pr}[(V \leftrightarrow P)(x) \ accepts] = 1$$

Soundness:
$$(\forall x \notin L)(\forall P') \operatorname{Pr}\left[(V \leftrightarrow P')(x) \ accepts\right] \leq \frac{1}{2}$$









The common input



The common input

Verifier chooses one of the two graphs randomly



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Verifier constructs a graph isomorphic to her choice and send it to the Prover



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Verifier chooses one of the two graphs randomly

Verifier constructs a graph isomorphic to her choice and send it to the Prover

If $G \not\cong H$ prover can find which of the two graphs was send by the verifier

The verifier can check the answer easily



Yes X satisfies some property







Yes X satisfies some property











Rahul Jain, Zhengfeng Ji, Sarvagya Upadhyay, and John Watrous






IP for Quantum Computing



Hiding Complexity

Hiding Complexity



Enables Secure Communication

Access to data is all or nothing

Hiding Complexity



Enables Secure Communication

Access to data is all or nothing

Modern Encryption

Enables arbitrary computation on encrypted data without decrypting

Rivest, Adleman and Dertouzos Can we process encrypted data without decrypting it

Rivest, Adleman and Dertouzos



Rivest, Adleman and Dertouzos



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Gentry 09: A Lattice-based cryptosystem that is fully homomorphic

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32787648736923843984794783947394872349979387983709470059830958309580948503498504984879ut9875937493

590094867-3498674-096759067458976459765-9067459685489765498765468978745943580487568760876508457095



Gentry 09: A Lattice-based cryptosystem that is fully homomorphic



+ %^&&£££\$%

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Gentry 09: A Lattice-based cryptosystem that is fully homomorphic

32787648736923843984794783947394872349979387983709470059830958309580948503498504984879ut9875937493

<u>+ %^&&fff\$%</u>

590094867-3498674-0967590674

Long Key Complicated Server Operations Computational Security 45943580487568760876508457095



Applications

Secure Cloud

Remote File Storage

Secure Multi Party Computation

Verification of Outsourced Computation

Short Proof of Knowledge

. . . .

Universal Blind Quantum Computing (UBQC)

Broadbent, Fitzsimons and Kashefi 09: Quantum Key Distribution + Quantum Teleportation



Unconditional Perfect Privacy

Server learns nothing about client's input/output/computation

Measurement-based QC

- New qubits, to prepare the auxiliary qubits: N
- Entanglements, to build the quantum channel: E
- Measurements, to propagate (manipulate) qubits: M
- Corrections, to make the computation deterministic: C









Circuit Picture



Formal Calculus

Kashefi et.al. Measurement Calculus JACM 2007



Formal Calculus

Kashefi et.al. Measurement Calculus JACM 2007



Program is encoded in the classical control computer Computation Power is encoded in the entanglement



Raussendorf and Briegel, PRL, 2001

Danos, Panangaden, Kashefi, JACM, 2009

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$$J(\alpha) := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\alpha} \\ 1 & -e^{i\alpha} \end{pmatrix}$$

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gate teleportation



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Thinking inside the box

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Gates Composition



Client-Server interactions

Gates Composition



Client-Server interactions

Universal Blind Quantum Computing

$$X = (\tilde{U}, \{\phi_{x,y}\})$$

Universal Blind Quantum Computing



random single qubit generator

 $\left[1/\sqrt{2}\left(\left|0\right\rangle+e^{i\theta}\left|1\right\rangle\right)\right]$ $\theta=0,\pi/4,2\pi/4,\ldots,7\pi/4$














Experimental Implementation S. Barz, E. Kashefi, A. Broadbent, J. Fitzsimons, A. Zeilinger, P. Walther, Science 2012 a 3 $|\theta_2\rangle$ $|\theta_4\rangle$ $|\theta_3\rangle$ b

 $|\theta|$



Experimental Implementation

Client:

limited computational power





Quantum server: full power of Quantum Computation



Entangles qubits

Experimental Implementation

Client:



Random bit flip

Quantum server:

Computes measurement angles





• Correctness: in the absence of any interference, client accepts and the output is correct

• Soundness: Verifier rejects an incorrect output, except with probability at most exponentially small in the security parameter

Fitzsimons and Kashefi, arXiv:1203.5217, 2012



Barnum, Crepeau, Gottesman, Smith and Tapp, FOCS02



Barnum, Crepeau, Gottesman, Smith and Tapp, FOCS02

















Bob

Alice ν

random parameters



 ${\cal V}$

random parameters









$$P_{incorrect}^{\nu} = \left(\mathbb{I} - |\Psi_{ideal}^{\nu}\rangle \left\langle \Psi_{ideal}^{\nu}|\right) \otimes |r_{t}^{\nu}\rangle \left\langle r_{t}^{\nu}\right|$$
Accept Key



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Accept Key

$$\sum_{\nu} p(\nu) Tr(P_{incorrect}^{\nu} B(\nu)) \le \epsilon$$

















Verification with single trap

Theorem. Protocol is (1 - 1/2N)-verifiable in general, and in the case of purely classical output it is (1 - 1/N)-verifiable, where *N* is the total number of qubits in the protocol.






Verification







ε-Verification



$$B_{j}(\nu) = \operatorname{Tr}_{B}\left(\sum_{b} \left|b + c_{r}\right\rangle \left\langle b\right| C_{\nu_{C},b} \Omega \mathcal{P}(\left(\otimes^{B} \left|0\right\rangle \left\langle 0\right|\right) \otimes \left|\Psi^{\nu,b}\right\rangle \left\langle\Psi^{\nu,b}\right|\right) \mathcal{P}^{\dagger} \Omega^{\dagger} C_{\nu_{C},b}^{\dagger} \left|b\right\rangle \left\langle b + c_{r}\right|\right)$$











To increase the probability of any local error being detected

O(N) many traps in random locations

To increase the probability of any local error being detected

O(N) many traps in random locations

To increase the minimum weight of any operator which leads to an incorrect outcome Fault-Tolerance

1.



1.







1.

Challenge: Traps break the graph



3.







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Efficient Verifiable UBQC

[Kashefi-Wallden 16] Dotted triple-graph construction

Dotted triple-graph construction — O(N) overheads for VUBQC

generic resource state, that one can insert multiple independent traps without revealing anything or disrupting the computation



Efficient Verifiable UBQC

[Kashefi-Wallden 16] Dotted triple-graph construction

Dotted triple-graph construction — O(N) overheads for VUBQC

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(a) Trap-colouring



What can we do with 4-qubits



Restricting to Classical Input and Output



Restricting to Classical Input and Output



A Complete new proof of verification was required

Pauli (σ_i)	Trap Stabilizer Measurement			Overall
	$X\otimes \mathbb{I}\otimes Y\otimes Y$	$ Y \otimes X \otimes X \otimes Y $	$Y \otimes Y \otimes \mathbb{I} \otimes X$	
$\boxed{C\otimes C\otimes C\otimes C}$	\checkmark	\checkmark	\checkmark	 ✓
$C \otimes C \otimes C \otimes A$	×	×	×	X
$C \otimes C \otimes A \otimes C$	×	×	\checkmark	X
$C \otimes C \otimes A \otimes A$	\checkmark	\checkmark	×	X
$C \otimes A \otimes C \otimes C$	\checkmark	×	×	X
$C \otimes A \otimes C \otimes A$	×	\checkmark	\checkmark	X
$C \otimes A \otimes A \otimes C$	×	\checkmark	×	X
$C \otimes A \otimes A \otimes A$	\checkmark	×	\checkmark	X
$A \otimes C \otimes C \otimes C$	×	×	×	X
$A \otimes C \otimes C \otimes A$	\checkmark	\checkmark	\checkmark	
$A \otimes C \otimes A \otimes C$	\checkmark	\checkmark	×	X
$A \otimes C \otimes A \otimes A$	×	×	\checkmark	X
$A \otimes A \otimes C \otimes C$	×	\checkmark	\checkmark	X
$A \otimes A \otimes C \otimes A$	\checkmark	×	×	X
$A \otimes A \otimes A \otimes C$	\checkmark	×	\checkmark	X
$A \otimes A \otimes A \otimes A$	×	\checkmark	×	X

A Complete new proof of verification was required

Pauli (σ_i)	Trap Stabilizer Measurement			Overall
	$X \otimes \mathbb{I} \otimes Y \otimes Y$	$ Y \otimes X \otimes X \otimes Y $	$Y \otimes Y \otimes \mathbb{I} \otimes X$	
$C \otimes C \otimes C \otimes C$	✓	\checkmark	\checkmark	\checkmark
$C \otimes C \otimes C \otimes A$	×	×	×	X
$C \otimes C \otimes A \otimes C$	×	×	\checkmark	X
$C \otimes C \otimes A \otimes A$	✓ <i>✓</i>	\checkmark	×	X
$C \otimes A \otimes C \otimes C$	\checkmark	×	×	X
$C \otimes A \otimes C \otimes A$	×	\checkmark	\checkmark	X
$C \otimes A \otimes A \otimes C$	×	\checkmark	×	X
$C \otimes A \otimes A \otimes A$	✓ <i>✓</i>	×	\checkmark	X
$A \otimes C \otimes C \otimes C$	×	×	×	X
$A \otimes C \otimes C \otimes A$	\checkmark	\checkmark	\checkmark	\checkmark
$A \otimes C \otimes A \otimes C$	\checkmark	\checkmark	×	X
$A \otimes C \otimes A \otimes A$	×	×	\checkmark	X
$A \otimes A \otimes C \otimes C$	×	\checkmark	\checkmark	X
$A \otimes A \otimes C \otimes A$	✓ <i>✓</i>	×	×	X
$A \otimes A \otimes A \otimes C$	\checkmark	×	\checkmark	X
$A \otimes A \otimes A \otimes A$	×	\checkmark	×	X







If server knows he is running Bell test, he can create fake outcomes to violate the inequality, the trapificaiton procedure in between prevents this to happen







Blind state generation

Blind Bell test





Non-classical Correlation

Contextuality

Dimentionality

Superposition









VERIFIED PREPARATION VERIFIED COMPUTATION SOC





Adaptation to Computational Capacity of Intermediate Models of QC

D-Wave Boson Sampling Instantaneous QC Quantum Simulator





Adaptation to Computational Capacity of Intermediate Models of QC

D-Wave Boson Sampling Instantaneous QC Quantum Simulator

Simplified Noise Model

Quantum Turing Test for One Pure Qubit


A runnable sequence of commands where for every elementary command the purity doesn't exceed

A runnable sequence of commands where for every elementary command the purity doesn't exceed

 $\pi(\rho_i) < \pi(\rho_{in}) + c$ Input state with constant many pure qubtis

A runnable sequence of commands where for every elementary command the purity doesn't exceed



Constructive Def. An MBQC with surjective flow

$$P_{\boldsymbol{a}} = \prod_{i \in O} X_i^{S_i^x} Z_i^{S_i^z} \prod_{i \in O^c}^{\preceq} \left(\sum_{i \in O^c}^{S_i^z} \left[M_i^{a_i} \right]^{S_i^x} \left(\prod_{\{k:k \sim i,k \succeq i\}} E_{i,k} \right) N_{f(i)}(|+\rangle) \right)$$

A runnable sequence of commands where for every elementary command the purity doesn't exceed

 $\pi(\rho_i) < \pi(\rho_{in}) + c$



Constructive Def. An MBQC with surjective flow

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Verified Delegated QC with One Pure Qubit



Verified Delegated QC with One Pure Qubit



From QKD to verifiable quantum internet



Clients only requires (trusted) preparation of BB84 single qubits states (prepare-and-send).

From QKD to verifiable quantum internet



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From QKD to verifiable quantum internet



Clients only requires (trusted) preparation of BB84 single qubits states (prepare-and-send).

Can we get ride of qubit ?



Alagic, Fefferman, 2016







Classical Verification



Quantum Verification





Breakable Security

Server's Time



The Edinburgh-Paris Quantum Team



The Edinburgh-Paris Quantum Team

Luka Music

