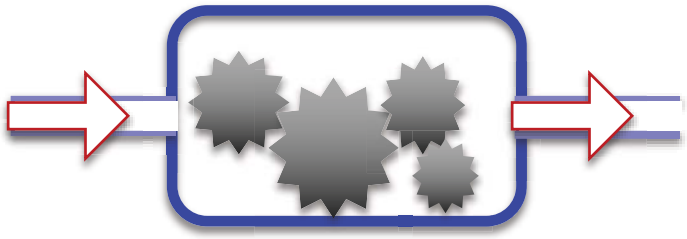
Universal Quantum Emulator

IMAN MARVIAN

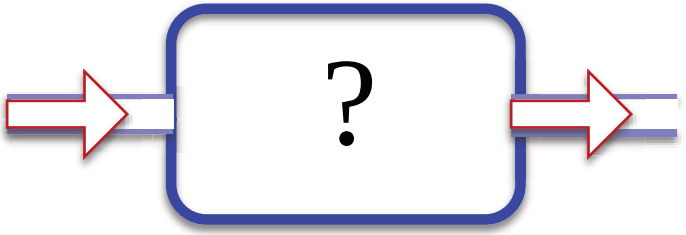
Joint work with Seth Lloyd
arXiv:1606.02734

IPM, Tehran
December 2016

Simulation versus Emulation



Simulator is a machine that can be programmed to mimic the dynamics of other quantum systems. The time evolution of the simulator obeys the same equations of motion as the evolution of the simulated system.

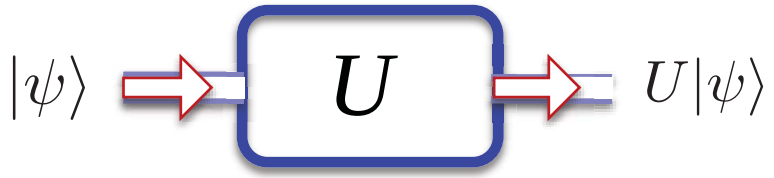


Emulator is a machine that mimics the input-output relation of another system, by looking to the output of that system on some sample input states. Unlike a simulator, an emulator does not need to obey the same dynamical equations as of the emulated system

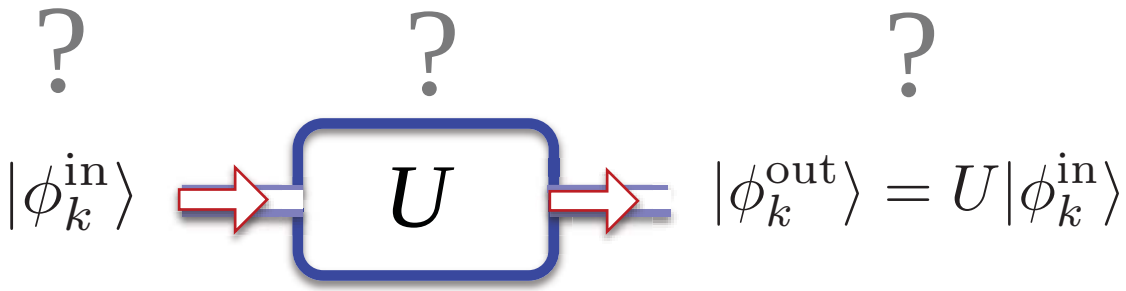
Assumption: We are given multiple copies of some unknown sample input states, and their corresponding output states of an unknown unitary *U*.

Goal: To implement *U* on a system in an unknown state in the span of the sample input states.

Single copy



Multiple copies of samples

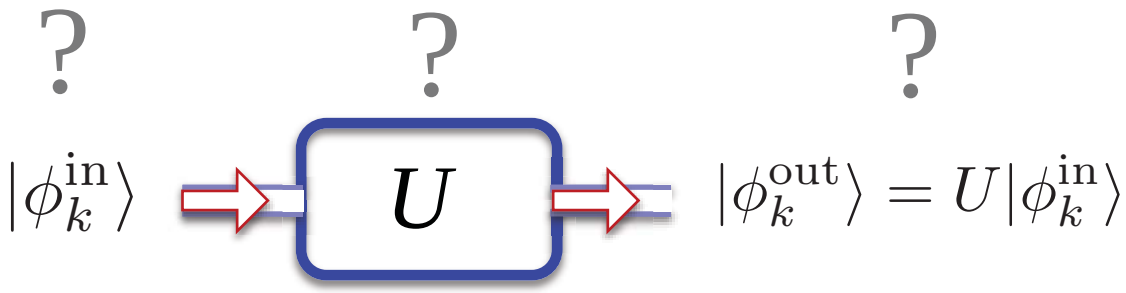


Sample input states $|\phi_k^{\text{in}}\rangle : k = 1, \cdots, K$

Corresponding output states $|\phi_k^{\text{out}}\rangle = U|\phi_k^{\text{in}}\rangle : k = 1, \cdots, K$

Incoherent methods

(based on tomography)



i. Perform state tomography on the given copies of input-output samples.
ii. Find a unitary that relates the inputs to the outputs.
iii. Implement the unitary on new given states.

NOTE: Once we find the description of this unitary via tomography we can use it arbitrary number of times. However, note that the unitary we find in this way is, in general, a complicated unitary. Therefore, every time we want to emulate the action of *U* we need to implement a complicated unitary.

	Runtime	**Sample Complexity**
Incoherent Methods	$D \times d$	$D \times d$
Universal Quantum Emulator	$\mathrm{Log}(D) \times \mathrm{poly}(d)$	$\mathrm{poly}(d)$

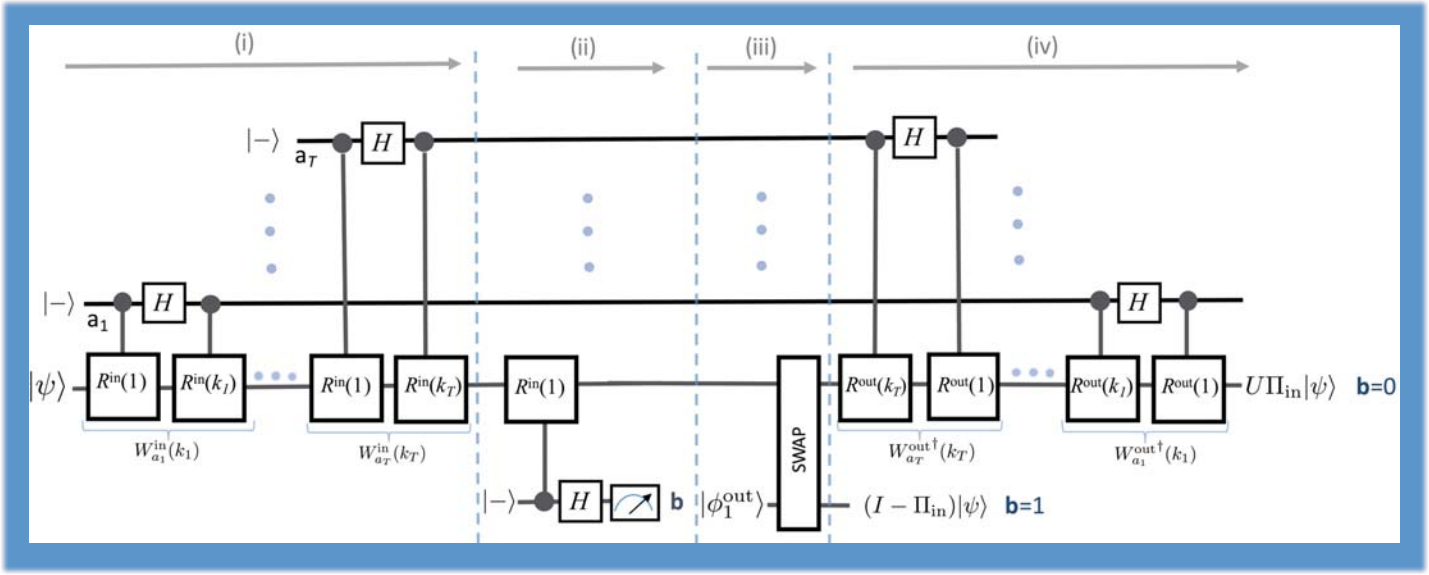
D: Dimension of the Hilbert space

d: Dimension of the subspace spanned by the sample input states

Sample complexity*:* The total number of copies of sample inputs and outputs needed to run the algorithm.

Overview

1. Preliminaries
2. The Algorithm
3. How it works



Necessary Condition

Example: Emulating unitary $U = e^{i\theta\sigma_z}$

$$|0\rangle \longrightarrow e^{i\theta}|0\rangle$$

$$|1\rangle \longrightarrow e^{-i\theta}|1\rangle$$

Even if we have infinite copies of the sample states we may not be able to uniquely determine the action of *U*.

General Condition (Also necessary for tomography)

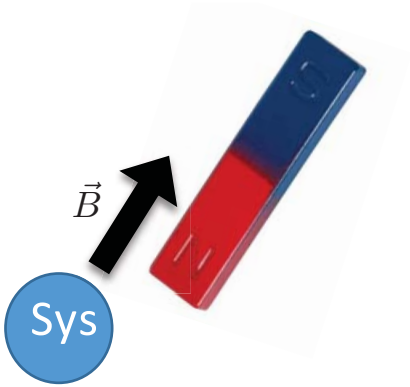
$\mathcal{H}_{\text{in}}$: The subspace spanned by the sample input states $|\phi_k^{\text{in}}\rangle : k = 1, \cdots, K$

Having copies of the sample input states and their corresponding output states, we can uniquely determine the action of *U* on states in this subspace, if and only if

$$\text{Alg}\{|\phi_k^{\text{in}}\rangle\langle\phi_k^{\text{in}}| : k = 1, \cdots, K\} = \text{Full matrix algebra on } \mathcal{H}_{\text{in}}$$

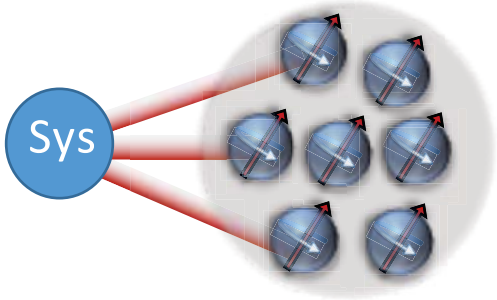
From states to Hamiltonians and unitaries

$$H_{\text{sys}} = \vec{B} \cdot \vec{\sigma}_{\text{sys}}$$



Heisenberg interaction

$$H_{\text{int}} = J\vec{\sigma}_{\text{sys}} \cdot \vec{\sigma}_{\text{anc}}$$
$$= J(2\text{Swap} - I)$$



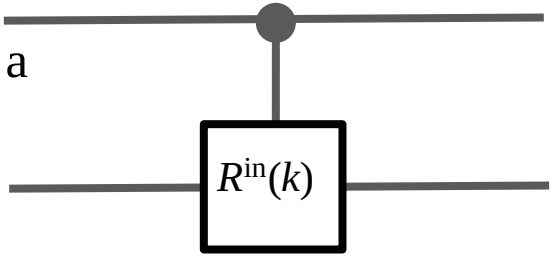
$$\text{Swap}|\psi\rangle|\phi\rangle = |\phi\rangle|\psi\rangle$$

Theorem Using n copies of state $|\psi\rangle$ we can simulate the unitary $e^{-it|\psi\rangle\langle\psi|}$ in time $n \times \log D$, with error $\mathcal{O}(1/n)$.

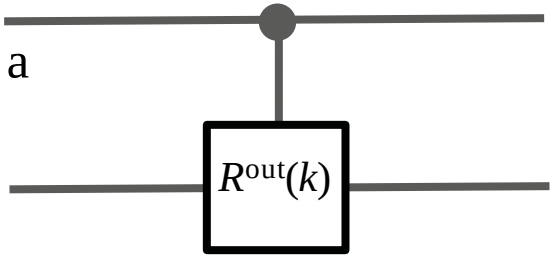
S. Lloyd, M. Mohseni, P. Rebentrost, Nature Physics 10, 631 (2014).

Samples states remain almost unaffected in this process.

Controlled-Reflection



Controlled-Reflection about the input sample $|\phi_k^{\text{in}}\rangle$



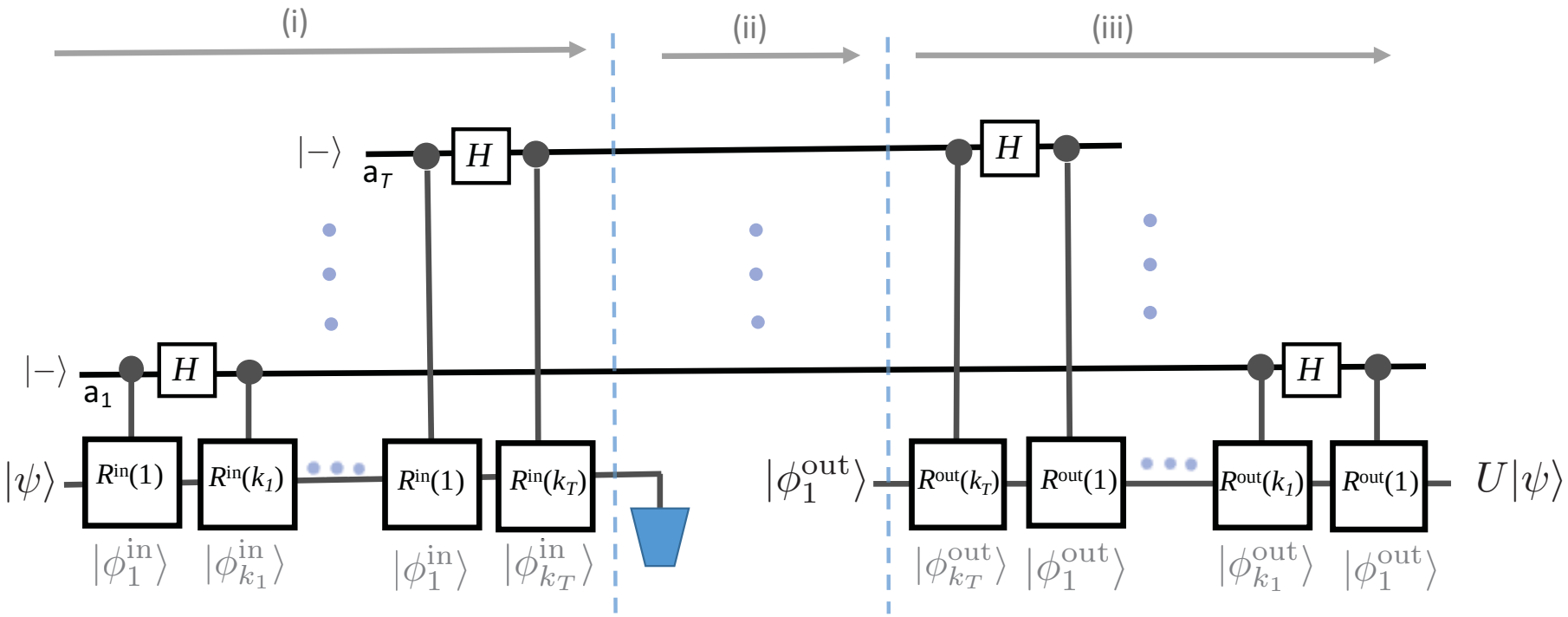
Controlled-Reflection about the input sample $|\phi_k^{\text{out}}\rangle$

$$R_a(k) = |0\rangle\langle 0|_a \otimes I + |1\rangle\langle 1|_a \otimes e^{i\pi|\phi_k\rangle\langle\phi_k|}$$

Using n copies of state $|\phi_k\rangle$ we can simulate the controlled-reflection $R_a(k)$ in time $n \times \log D$, with error $\mathcal{O}(1/n)$.

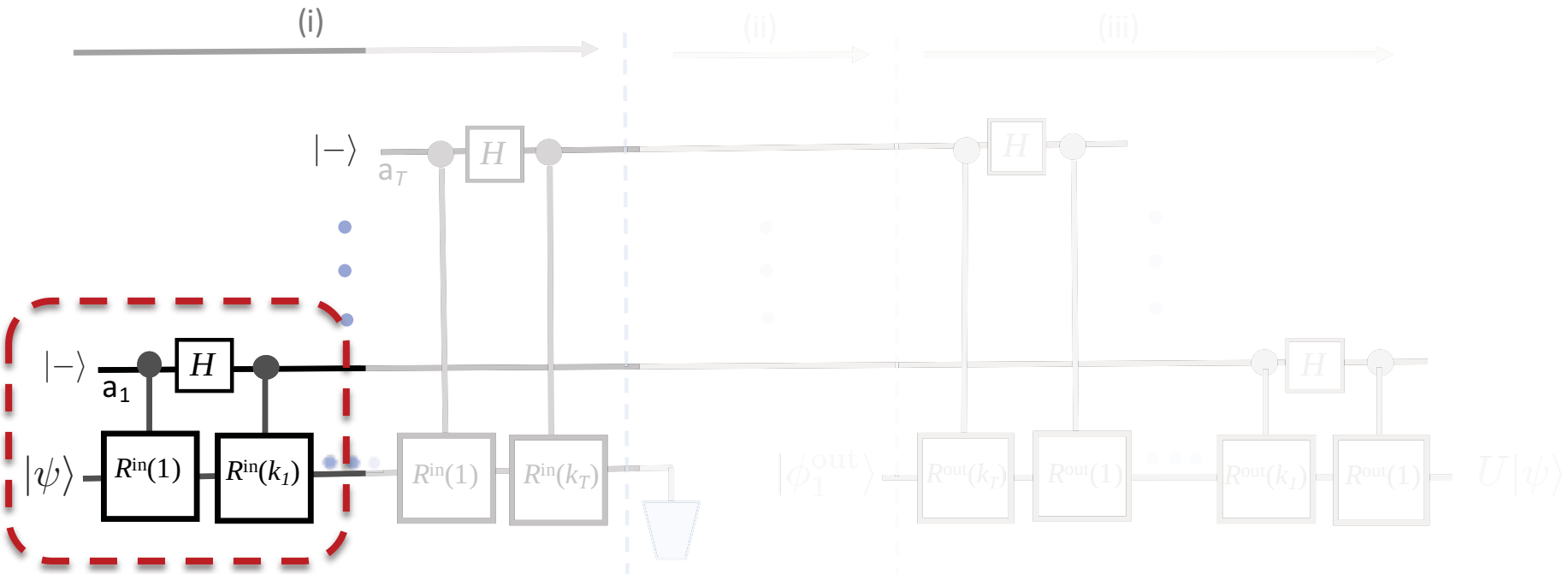
Samples states remain almost unaffected in this process.

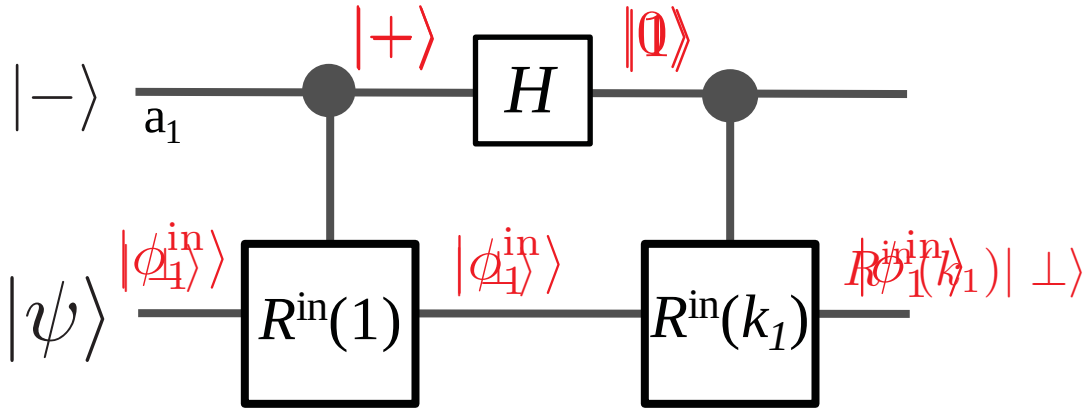
Simplified version of the Universal Quantum Emulator



- Integer T is a parameter that determines the accuracy of emulation.
- T ancillary qubits are all initialized in state $|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$.
- $k_1 \ldots k_T$ are T intgers between *1 to K*, chosen uniformly at random, and independent of each other.
- State $|\phi_1^{\text{in}}\rangle$ (and $|\phi_1^{\text{out}}\rangle$) is one of the sample input states (and its corresponding output), which is chosen randomly at the begining of the algorithm.

Note: This is a simplified version, which assumes the given state is in the span of the sample input states. Also, it is not efficient in terms of the number of ancillas.

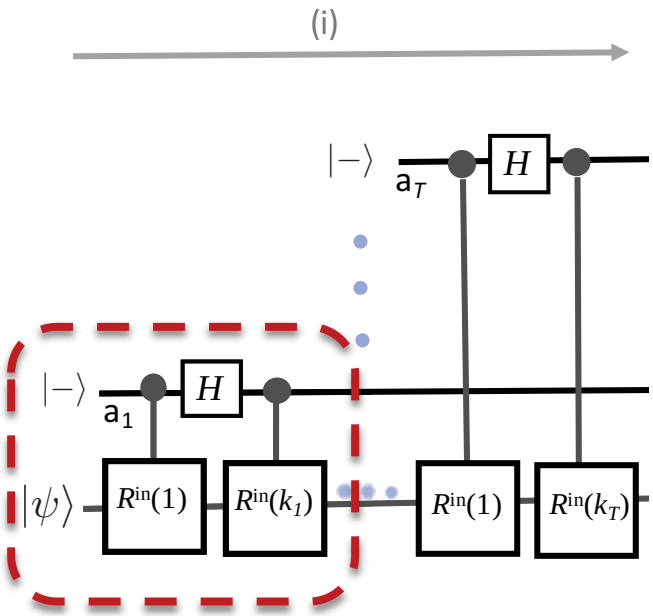




$$R^{in}(1) = e^{i\pi|\phi_1^{in}\rangle\langle\phi_1^{in}|}$$

$$R^{in}(k_1) = e^{i\pi|\phi_{k_1}^{in}\rangle\langle\phi_{k_1}^{in}|}$$

- If the system is in state $|\phi_1^{in}\rangle$, then it remains unaffected.
- If the system is orthogonal to $|\phi_1^{in}\rangle$, then it is reflected about the randomly chosen state $|\phi_{k_1}^{in}\rangle$.



Average reduced state of system

$$\mathcal{W}(|\psi\rangle\langle\psi|) = P|\psi\rangle\langle\psi|P + \mathcal{D}(P^{\perp}|\psi\rangle\langle\psi|P^{\perp})$$

where $\mathcal{D}(\rho) = \frac{1}{K}\sum_{k=1}^{K} R^{\mathrm{in}}(k)\ \rho\ R^{\mathrm{in}}(k)$

$\{P = |\phi_1^{\mathrm{in}}\rangle\langle\phi_1^{\mathrm{in}}|, P^{\perp} = I - P\}$ $\quad R^{\mathrm{in}}(k) = e^{i\pi|\phi_k^{\mathrm{in}}\rangle\langle\phi_k^{\mathrm{in}}|}$

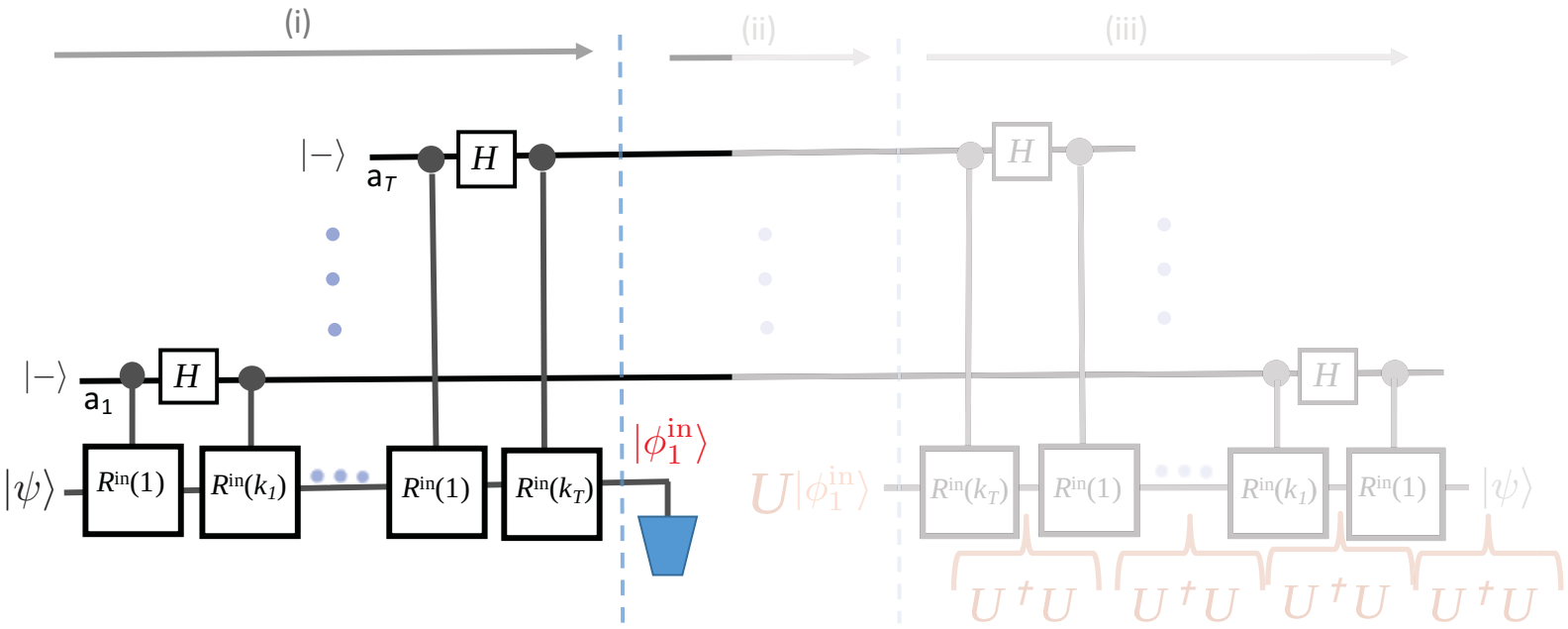
(1) Ancillary qubits are initially uncorrelated with each other, and
(2) Different random integers k_1.. k_T are statistically independent of each other.

Therefore, at the end of step (i) the average reduced state of system is $\mathcal{W}^T(|\psi\rangle\langle\psi|)$.

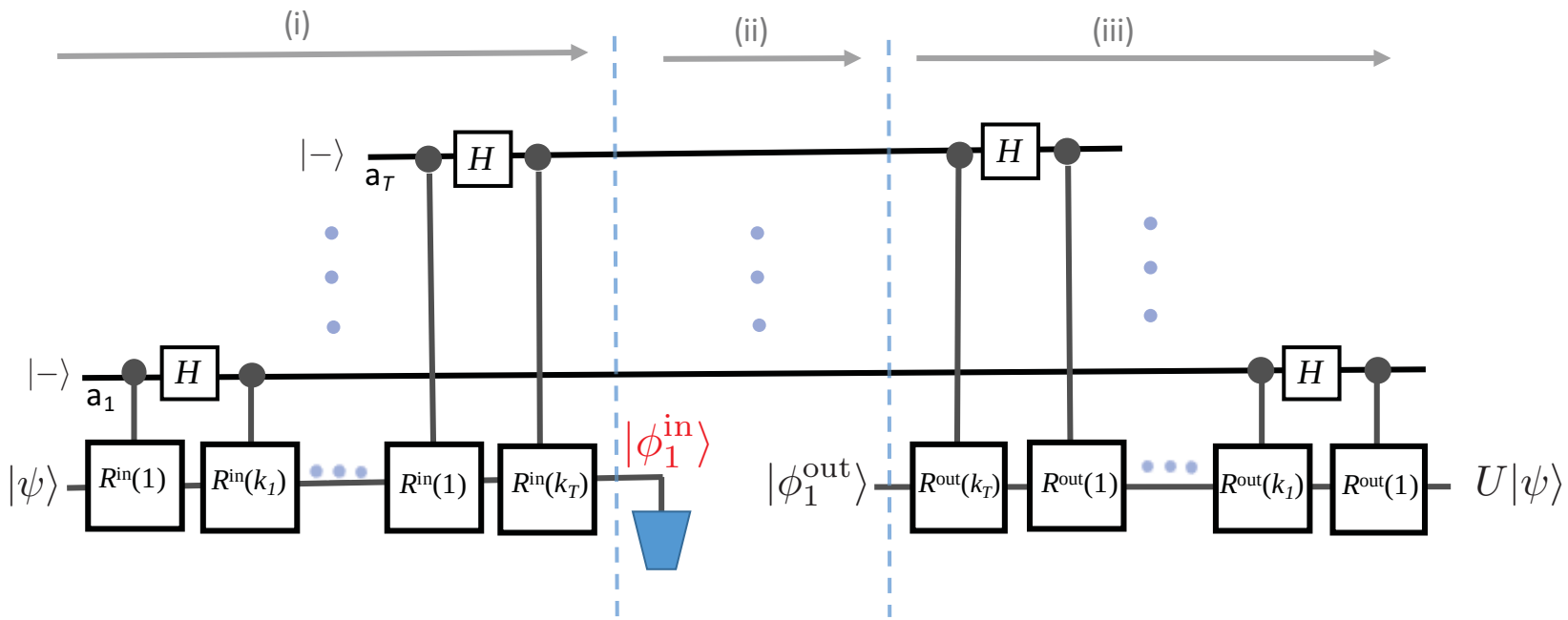
Recall the assumption $\mathrm{Alg}\{|\phi_k^{\mathrm{in}}\rangle\langle\phi_k^{\mathrm{in}}| : k = 1, \cdots, K\} =$ Full matrix algebra on $\mathcal{H}_{\mathrm{in}}$

It follows that channels $\mathcal{D}$ and $\mathcal{W}$ have unique fixed points inside $\mathcal{H}_{\mathrm{in}}$.

The probability of finding the system in a state orthogonal to $|\phi_1^{\mathrm{in}}\rangle$ is exponentially small in T.



- But quantum information is conserved during a unitary evolution!
- Therefore, all the information about the input state should now be encoded in the ancillary qubits.
- Step (iii) is the complex conjugate of step (i) up to the unknown unitary U.



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Uhlmann Fidelity

$$\mathrm{F}(\mathcal{E}_U(\rho), U\rho U^\dagger) \geq p_{\text{erase}}(\rho) = \langle \phi_1^{\text{in}} | \mathcal{W}^T(\rho) | \phi_1^{\text{in}} \rangle$$

The overall effect of the circuit on the system

The probability that we have successfully erased the state of system in step (i)

Emulating via coherent erasing.

Runtime and error analysis

- Implementing this circuit with **ideal** controlled reflections, we can emulate the desired unitary with error less than or equal ϵ provided that we choose

$$T \geq \frac{d \times \log(8d\epsilon^{-2})}{1 - |\lambda_{\mathcal{D}}|}$$

$\lambda_{\mathcal{D}}$ is the eigenvalue of channel $\mathcal{D}$ with the second largest magnitude $\quad \mathcal{D}(\rho) = \frac{1}{K}\sum_{k=1}^{K} R^{\text{in}}(k)\ \rho\ R^{\text{in}}(k)$

- In the actual algorithm we use the given copies of the samples to simulate the reflections.

Sample Complexity (Total copies of sample states)	**Runtime**
$N_{\text{tot}} = \tilde{\mathcal{O}}\left(\frac{d^2 \times \epsilon^{-1}}{(1 - \lvert\lambda_{\mathcal{D}}\rvert)^2}\right)$	$t_{\text{tot}} = \tilde{\mathcal{O}}(N_{\text{tot}} \times \log D)$

- The runtime and sample complexity are mainly determined by the mixing time, or equivalently the spectral gap, of channel $\mathcal{D}$.
- This gap has an information theoretic interpretation.

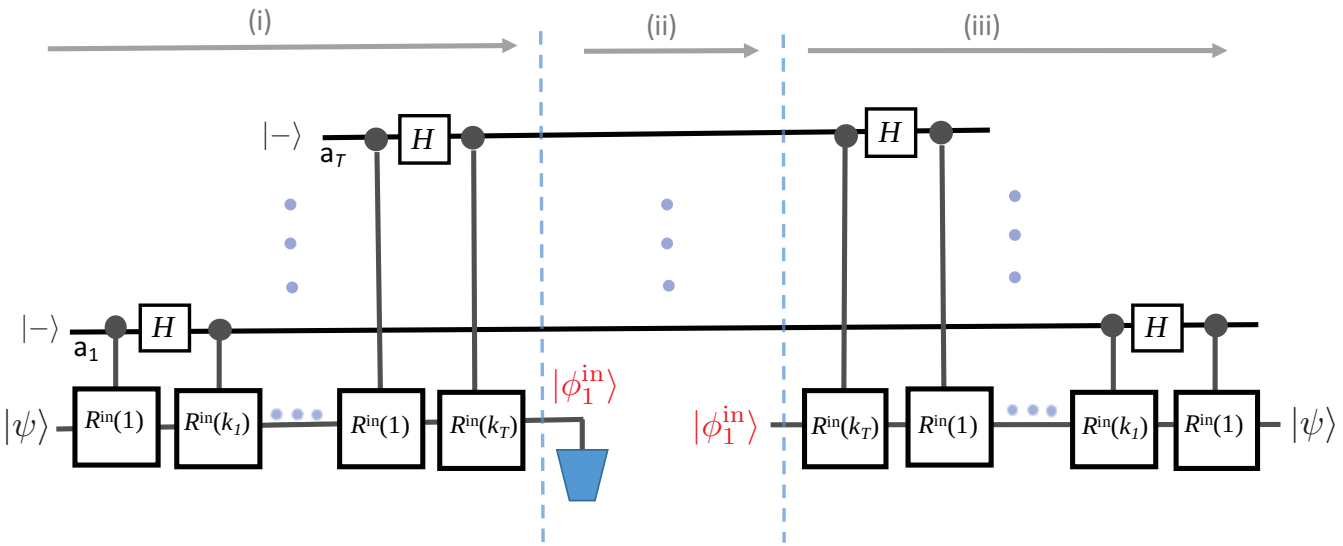
An interpretation of this algorithm

At the end of step (i), the state encoded in the ancillary qubits can be interpreted as the **coordinates** of the input state relative to a frame defined by the sample input states.

- In step (i) of the algorithm we find the coordinates of the input relative to the basis defined by the sample input states. $|\phi_k^{\text{in}}\rangle : k = 1, \cdots, K$

- Then, in step (iii), we reconstruct the state which has exactly the same coordinates relative to the frame defined by the sample output states.

$$|\phi_k^{\text{out}}\rangle = U|\phi_k^{\text{in}}\rangle : k = 1, \cdots, K$$

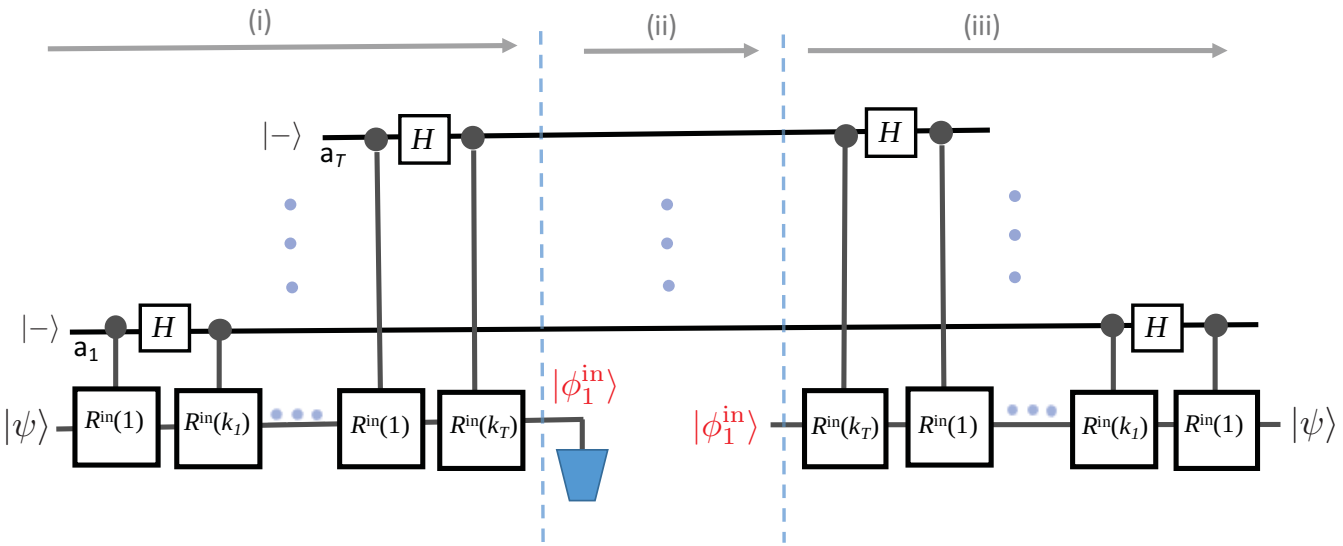


State Compression

- In the circuit presented here, the information about the new input state is encoded in the ancillas using a **unary** encoding. Using a **binary** encoding instead, we can implement the same transformation with exponentially less ancillas.

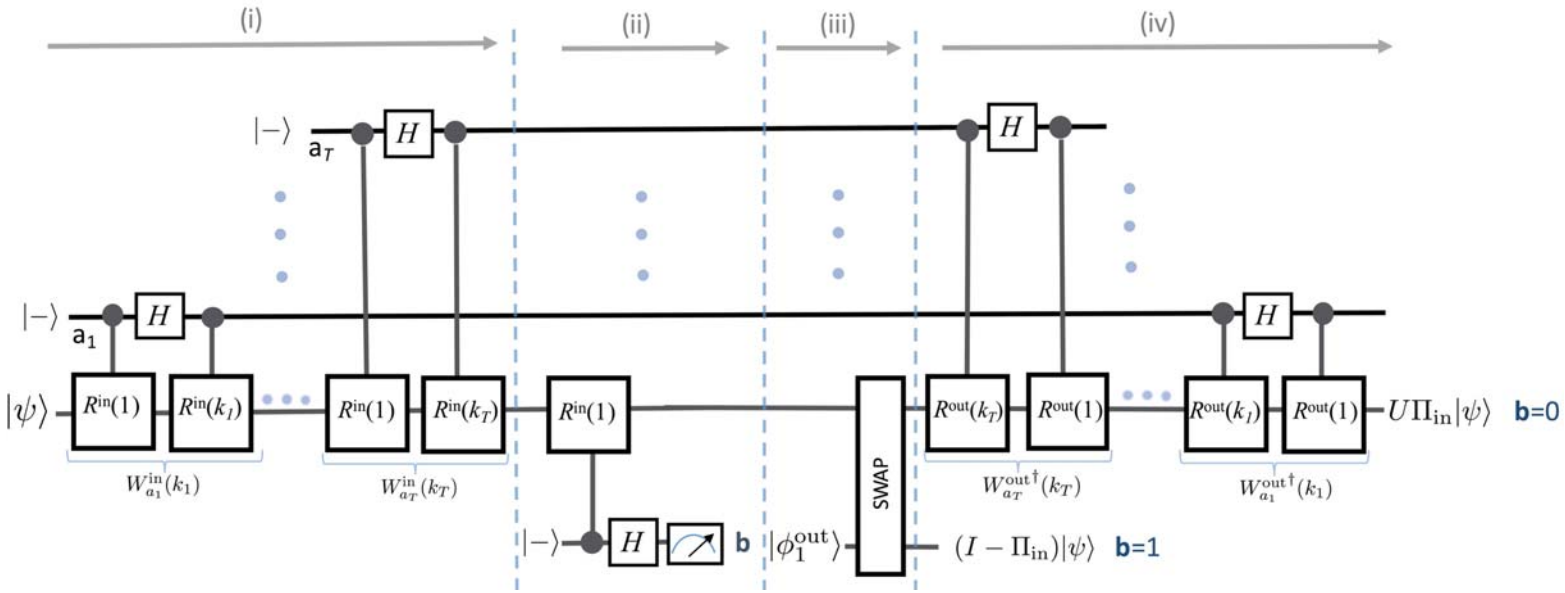
- It follows that we can use this algorithm to **compress** an unknown state of Log D qubits in a d-dimensional subspace, in Log(T)=O(Log d) qubits.

$$T \geq \frac{d \times \log(8d\epsilon^{-2})}{1 - |\lambda_{\mathcal{D}}|}$$



Thanks for your attention.

I. Marvian, S. Lloyd, **Universal Quantum Emulator**, arXiv:1606.02734



Coordinates of the input state with respect to samples

- At the end of step (i) the joint state of system and ancillary qubits is given by

$$|\phi_1^{\rm in}\rangle \sum_{t=0}^{T-1} \langle \phi_1^{\rm in}|\psi(t,\mathbf{k})\rangle \ |t\rangle_{\mathbf{a}} + |\psi(T,\mathbf{k})\rangle |T\rangle_{\mathbf{a}}$$

$$|t\rangle_{\mathbf{a}} \equiv |1\rangle^{\otimes t} \otimes |0\rangle^{\otimes (T-t)}$$

$$|\psi(t+1,\mathbf{k})\rangle = R^{\rm in}(k_{t+1}) P^{\perp} |\psi(t,\mathbf{k})\rangle$$

$$|\psi(0,\mathbf{k})\rangle = |\psi\rangle$$

- The coordinates of the input state with respect to sample input states: $\{\langle \phi_1^{\rm in}|\psi(t,\mathbf{k})\rangle \ : \ t = 0, \cdots, T-1\}$

- Because of the no-cloning theorem, to find the coordinates we need to erase the state of system.