Quantum Optomechanical Cavity

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IPM-Tehran (29.12.2016)

Presentation outline

- Mechanical resonators
- Radiation pressure
- Optomechanical cavity
- Microwave cavity and electromechanics
- Theory of the Optomechanical cavity/cooling

Presentation outline

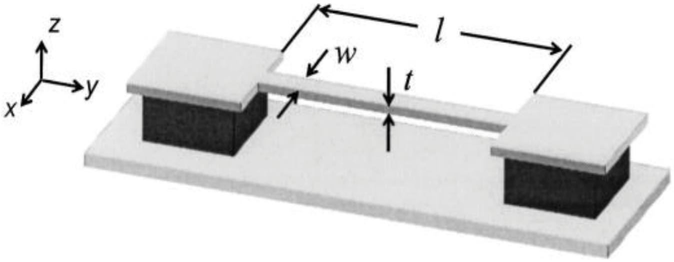
✓Mechanical resonators

☐ Radiation pressure

☐ Optomechanical cavity

☐Microwave cavity and electromechanics

☐Theory of the Optomechanical cavity/cooling



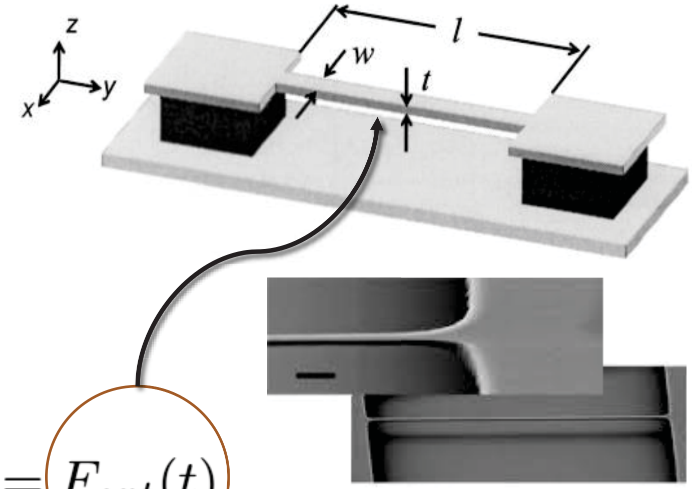
Mechanical resonators

Vibrational modes of any object is given by Euler beam theory

$$m_{\text{eff}}\frac{dx^2(t)}{dt^2}+m_{\text{eff}}\Gamma_m\frac{dx(t)}{dt}+m_{\text{eff}}\Omega_m^2x(t)=F_{ext}(t)$$

Rev. Mod. Phys. **86**, 1391(2014)

Mechanical resonators



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damping

external forces

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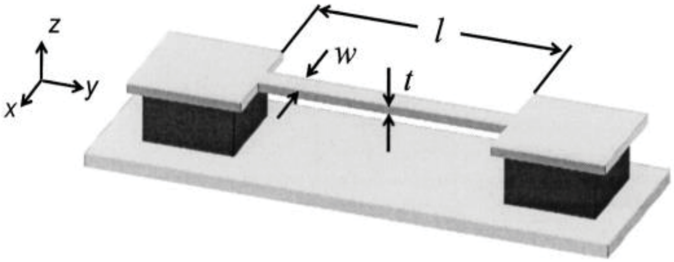

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$$m_{\mathrm{eff}}\frac{dx^2(t)}{dt^2}+\underbrace{m_{\mathrm{eff}}\Gamma_m\frac{dx(t)}{dt}}_{\text{damping}}+m_{\mathrm{eff}}\Omega_m^2x(t)=\underbrace{F_{ext}(t)}_{\text{external forces}}$$

$$\hat{x}=x_{\mathrm{ZPF}}(\hat{b}+\hat{b}^{\dagger}),\ \hat{p}=-im_{\mathrm{eff}}\Omega_{\mathrm{m}}x_{\mathrm{ZPF}}(\hat{b}-\hat{b}^{\dagger}),$$

$$[\hat{x}_{ZPF},\hat{p}_{ZPF}]=i\hbar.$$

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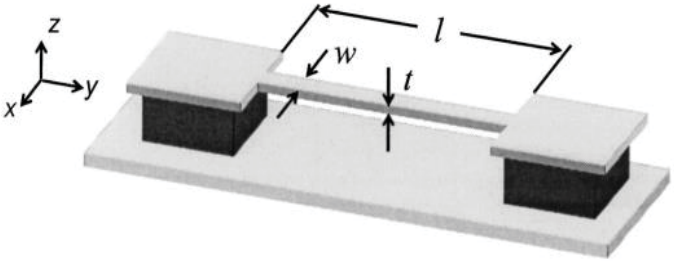

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$$\left[b,b^\dagger\right]=1$$

$$\left\langle 0\left|\hat{x}^2\boxed{\left|0\right\rangle}\right.\right.=x_{\mathrm{ZPF}}^2$$

mechanical vacuum state

$$x_{\mathrm{ZPF}}=\sqrt{\frac{\hbar}{2m_{\mathrm{eff}}\Omega_{\mathrm{m}}}}$$

the spread of the coordinate in the ground-state

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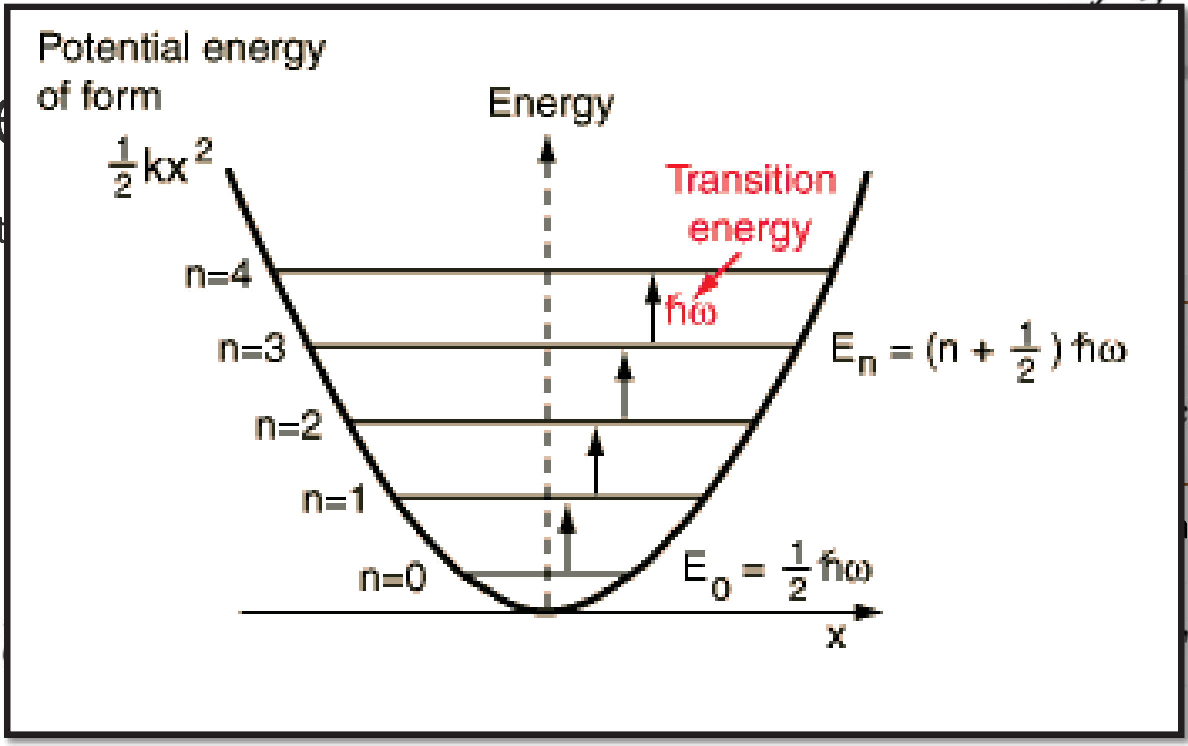

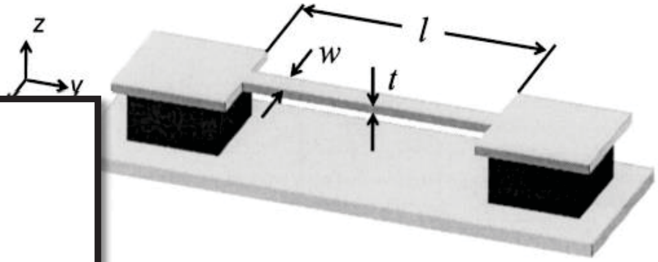

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the spread of the coordinate in the ground-state

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Effect of dissipation

$n = b^\dagger b$

$$\frac{d}{dt}\langle n\rangle = -\Gamma_m(\langle n\rangle - \bar{n}_{th}) \qquad \langle n\rangle(t = 0) = 0$$

$$\langle n\rangle(t) = \bar{n}_{th}(1 - e^{-t\Gamma_m})$$

average phonon number of the environment

$$\bar{n}_{\mathrm{th}} = (e^{\hbar\Omega_{\mathrm{m}}/k_B T} - 1)^{-1}$$

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$$\bar{n}_{\rm th} = (e^{\hbar\Omega_{\rm m}/k_B T} - 1)^{-1} \approx k_B T_{bath}/\hbar\Omega_m$$

Rate of heating out from ground state

$$\Gamma_m = \Omega_{\rm m}/Q_{\rm m}$$

$$\boxed{\frac{d}{dt}\langle n(t=0)\rangle = \bar{n}_{th}\cdot\Gamma_m \approx \frac{k_B T_{bath}}{\hbar Q_m}}$$

Cold particle in the box

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Effect of dissipation

- Viscous damping
- Clamping losses
- Thermoelastic damping and phonon-phonon interactions
- Materials-induced losses

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Effect of dissipation

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Cryogenic temperature ~7mk

$\langle n$

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$_{th} = (e^{\hbar\Omega_m/k_B T} - 1)^{-1}$

Rate of heating out from ground state

$$\Gamma_m = \Omega_m/Q_m$$

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Noise spectrum of a damped mechanical resonator

- One measures the motion of a single harmonic oscillator in thermal equilibrium, one will observe a trajectory $x(t)$ oscillating at the eigenfrequency Ωm.

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- ✓ A randomly time-varying amplitude and phase
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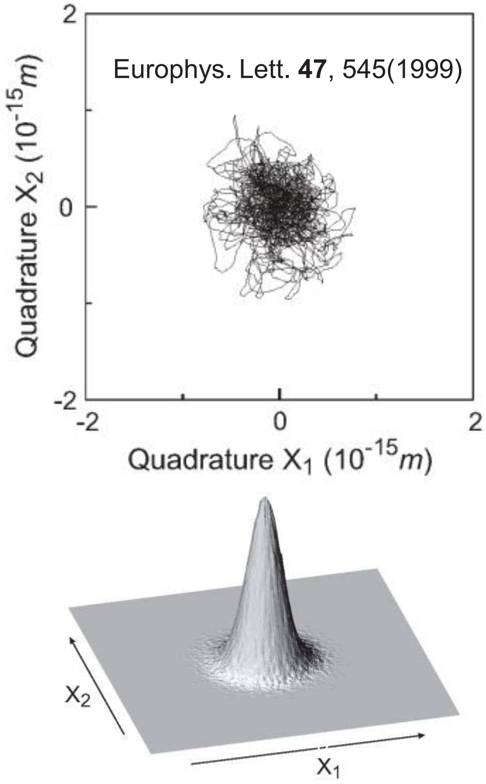
Europhys. Lett. **47**, 545(1999)

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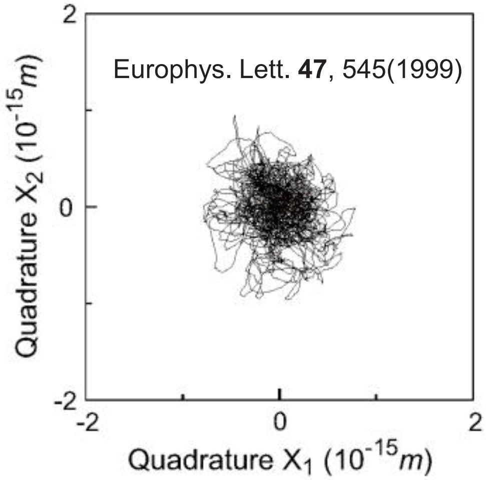
✓ Amplitude and phase change on the time scale Γ_m^{-1}

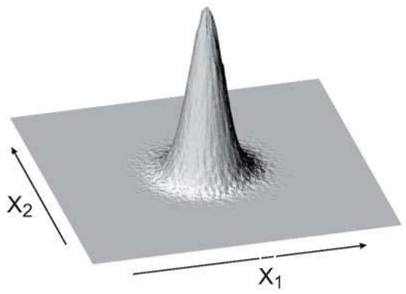
$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^{\tau} x(t) e^{i\omega t} dt \,. \xrightarrow{\tau \to \infty} \left\langle |\tilde{x}(\omega)|^2 \right\rangle = S_{xx}(\omega) \,.$$

noise power spectral density

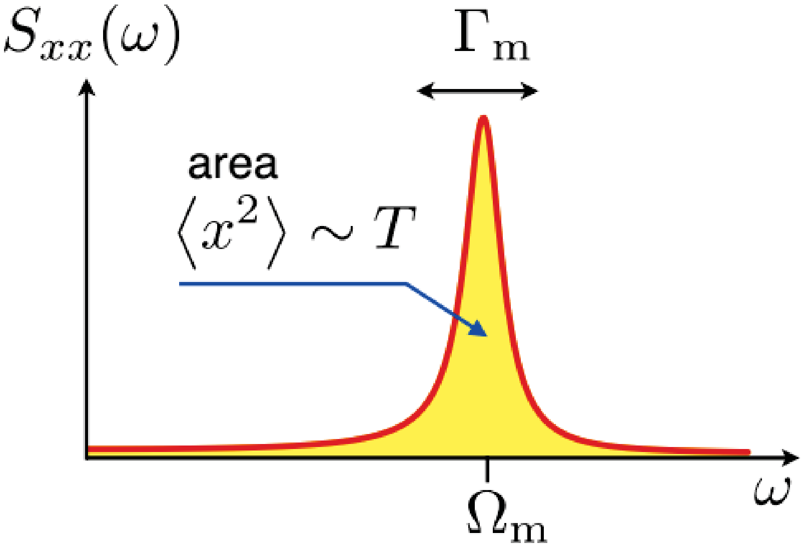
$$S_{xx}(\omega) \equiv \int_{-\infty}^{+\infty} \langle x(t) x(0) \rangle \, e^{i\omega t} \, dt = 2 \frac{k_B T}{\omega} \mathrm{Im} \chi_{xx}(\omega) \,,$$

$$\chi_{xx}(\omega) = \left[m_{\mathrm{eff}} (\Omega_m^2 - \omega^2) - i m_{\mathrm{eff}} \Gamma_m \omega \right]^{-1} .$$





weak damping $(\Gamma_m \ll \Omega_m)$



Quantum regime $\langle \hat{x}(t)\hat{x}(0) \rangle$

$$\boxed{S_{xx}(\omega) = \frac{2\hbar}{1 - e^{-\hbar\omega/k_B T}} \mathrm{Im}\chi_{xx}(\omega)}$$

Why mechanical resonators? applications

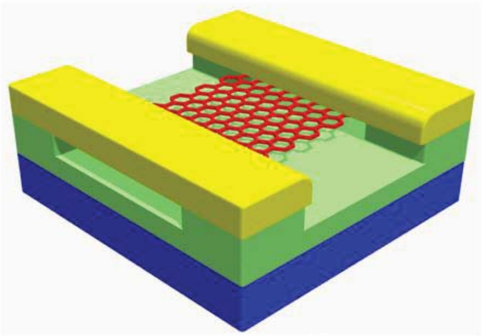
Bridges different systems with different charactristics [Phys. Scr. T 137 014001(2009)]:

a. Coupling with several qubits: atom, ion, and molecule
b. Coupling with BECs
c. Superconducting qubits(transmon)

Quantum information processing [Phys. Rev. Lett. **110**, 120503(2013)]

Optical-microwave conversion [*ShB*, Phys. Rev. Lett. **109**, 130503 (2012)]

Strong Nonlinearity [Phys. Rev. B **67** 134302 (2003)]



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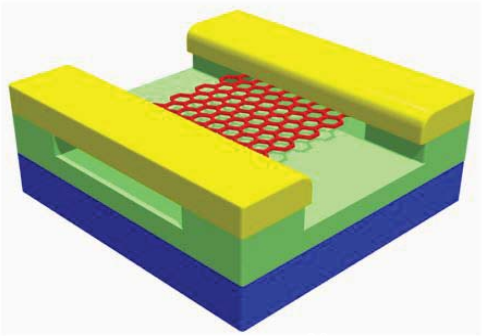

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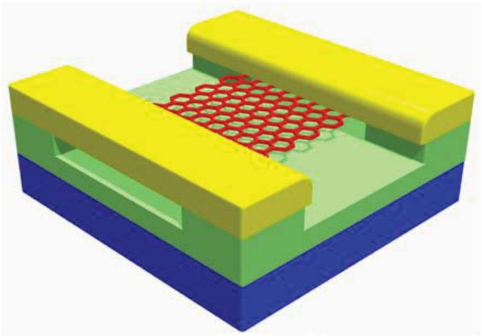

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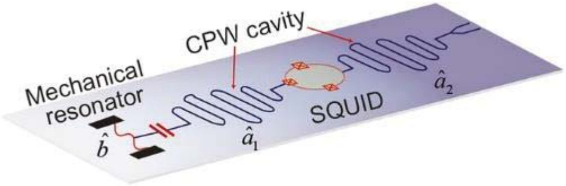

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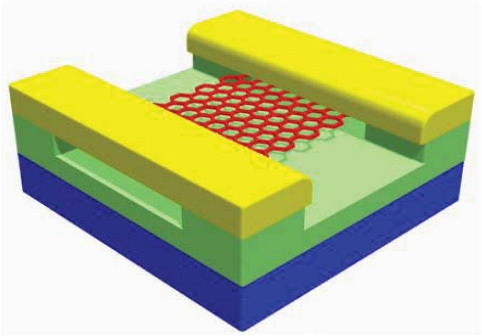

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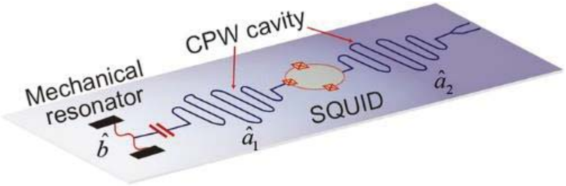

Quantum information processing [Phys. Rev. Lett. **110**, 120503(2013)](Memory, repeater)

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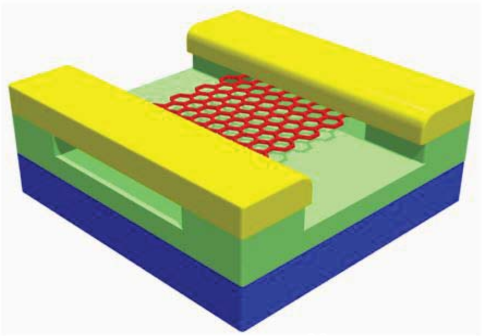

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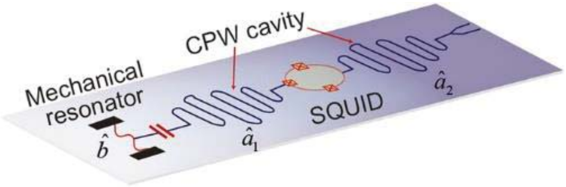

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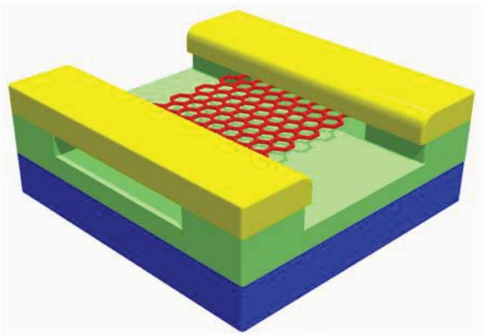

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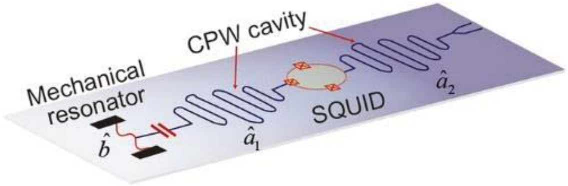

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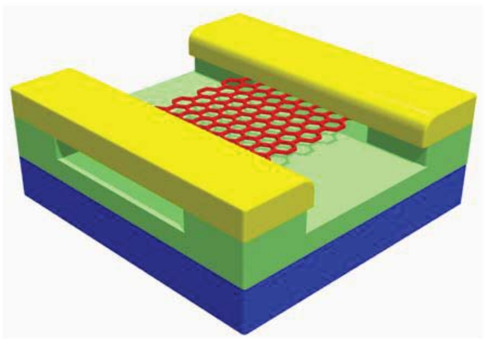


Why mechanical resonators? applications

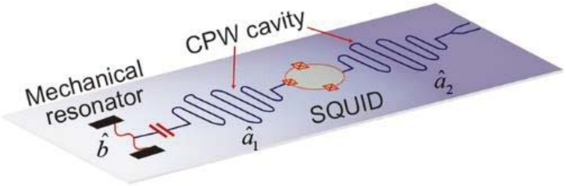
Bridges s. Scr. T 137 014001(2009)]:

a. Cou

b. Cou

We need mechanical cooling

c. Superconducting qubits(transmon)



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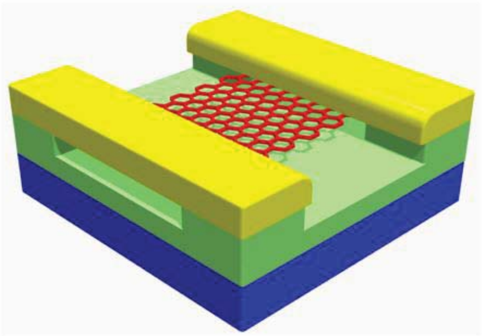

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- ☐ Optomechanical cavity
- ☐ Microwave cavity and electromechanics
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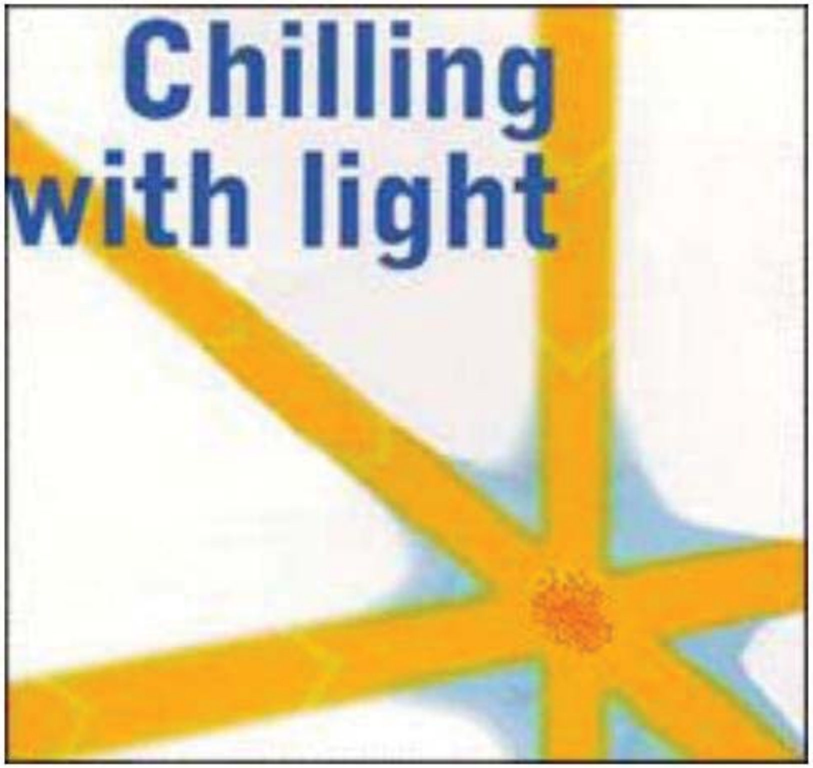
Radiation Pressure

- Effects of radiation pressure on **massive matter**?
- **Radiation pressure** on astronomic scales:

J. Keppler
De Cometis, 1619

- **Radiation pressure** on microscopic scales:



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The Nobel Prize in Physics 1997
"for development of methods to cool and trap atoms with laser light”


S. Chu


C. Cohen-Tannoudji


W. Phillips

Radiation pressure

Simplest form of radiation pressure coupling is the momentum transfer due to reflection.

Single photon transfers the momentum

$$|\Delta p| = 2h/\lambda$$

$$\langle \hat{F} \rangle = 2\hbar k \frac{\langle \hat{a}^\dagger \hat{a} \rangle}{\tau_c} = \hbar \frac{\omega}{L} \langle \hat{a}^\dagger \hat{a} \rangle$$

$\tau_c = 2L/c$ the cavity round trip time

the radiation pressure force caused by one intracavity photon

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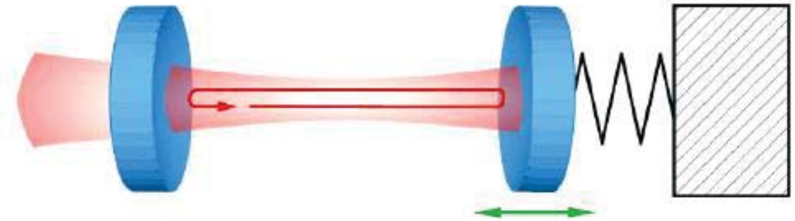
Optomechanical cavity



$$\hat{H}_0 = \hbar\omega_{\rm cav}\hat{a}^\dagger\hat{a} + \hbar\Omega_m\hat{b}^\dagger\hat{b}$$

$$\omega_{\rm cav}(x) \approx \omega_{\rm cav} + x\partial\omega_{\rm cav}/\partial x + \ldots$$

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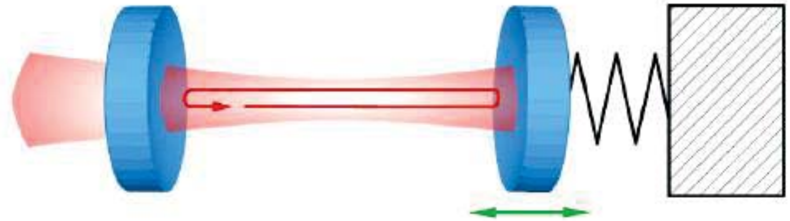
Optomechanical cavity



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$$\omega_{\text{cav}}(x) \approx \omega_{\text{cav}} + x\partial\omega_{\text{cav}}/\partial x + \ldots \longrightarrow \hbar\omega_{\text{cav}}(x)\hat{a}^\dagger\hat{a} \approx \hbar(\omega_{\text{cav}} - G\hat{x})\hat{a}^\dagger\hat{a}\,.$$

$$G = \omega_{\text{cav}}/L$$

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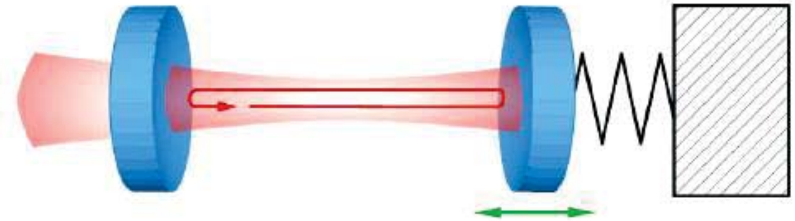

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$$\downarrow$$

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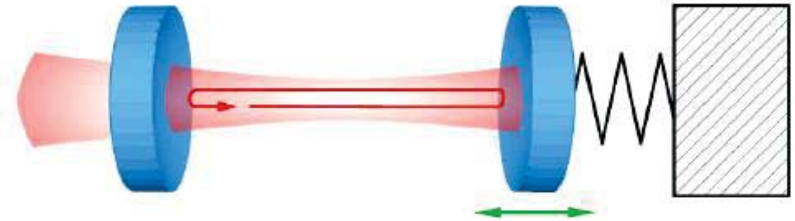
$$G = \omega_{\mathrm{cav}}/L$$

$$\hat{x} \;=\; x_{\mathrm{ZPF}}(\hat{b} + \hat{b}^\dagger)$$

$$g_0 = \underbrace{G}_{\to\, 10^{15} - 10^{19} Hz/m} x_{\mathrm{ZPF}} \longrightarrow \sim 10^{-16} m$$

1ng mass and 1MHz frequency

Optomechanical cavity



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$$G = \omega_{\text{cav}}/L$$

$$\hat{x} \;=\; x_{\text{ZPF}}(\hat{b} + \hat{b}^\dagger)$$

$$\boxed{\hat{H}_{\text{int}} = -\hbar g_0\hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger)}$$

$$g_0 = Gx_{\text{ZPF}}$$

equency

$10^{15} - 10^{19} Hz/m$

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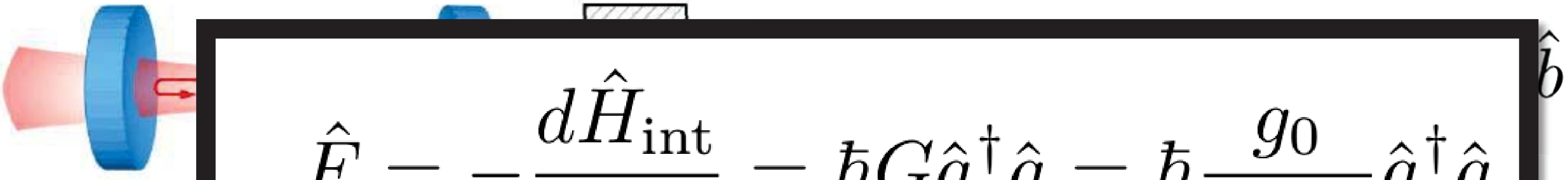
Optomechanical cavity



$$\hat{F} = -\frac{d\hat{H}_{\mathrm{int}}}{d\hat{x}} = \hbar G\hat{a}^\dagger\hat{a} = \hbar\frac{g_0}{x_{\mathrm{ZPF}}}\hat{a}^\dagger\hat{a}$$

$$\omega_{\mathrm{cav}}(x) \approx \omega_{\mathrm{cav}} + x\partial\omega_{\mathrm{cav}}/\partial x + \ldots$$

$$G\hat{x})\hat{a}^\dagger\hat{a}\,.$$

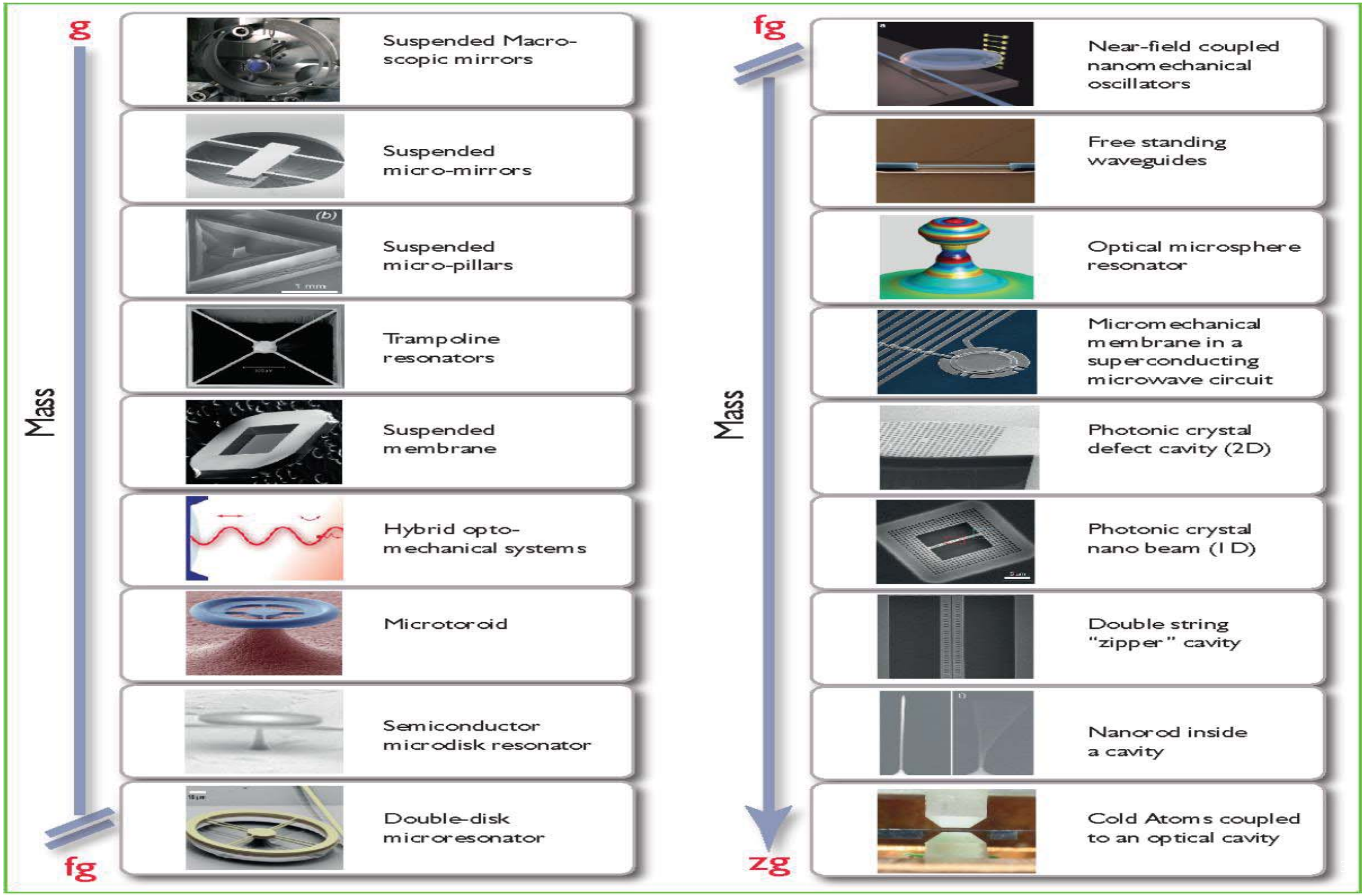
$$G = \omega_{\mathrm{cav}}/L$$

$$\hat{x} \;=\; x_{\mathrm{ZPF}}(\hat{b} + \hat{b}^\dagger)$$

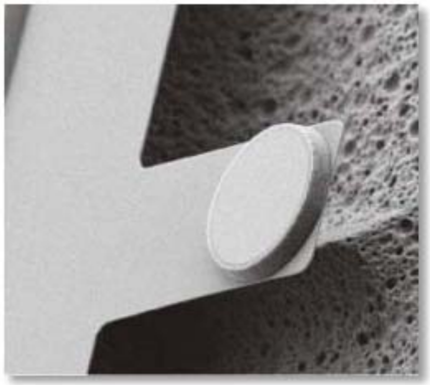
$$g_0 = Gx_{\mathrm{ZPF}}$$

$$\boxed{\hat{H}_{\mathrm{int}} = -\hbar g_0\hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger)}$$

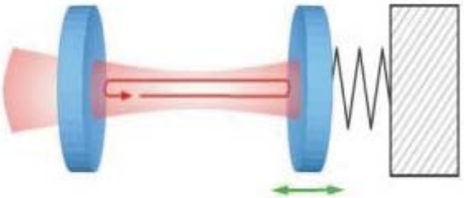
Rev. Mod. Phys. **86**, 1391(2014)


g
Mass
fg
Suspended Macro-scopic mirrors
Suspended micro-mirrors
(b)
1 mm
Suspended micro-pillars
Trampoline resonators
Suspended membrane
Hybrid opto-mechanical systems
Microtoroid
Semiconductor microdisk resonator
Double-disk microresonator
fg
Mass
zg
Near-field coupled nanomechanical oscillators
Free standing waveguides
Optical microsphere resonator
Micromechanical membrane in a superconducting microwave circuit
Photonic crystal defect cavity (2D)
Photonic crystal nano beam (1 D)
Double string "zipper" cavity
Nanorod inside a cavity
Cold Atoms coupled to an optical cavity

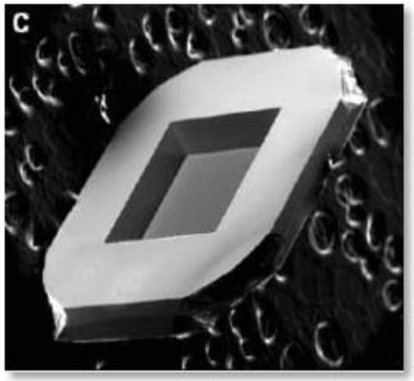
All optical cavities



Micromirrors

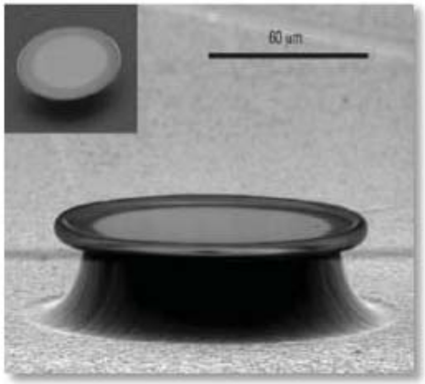


Aspelmeyer (Vienna)
Heidmann (Paris)
Bouwmeester (St Barbara, Leiden)

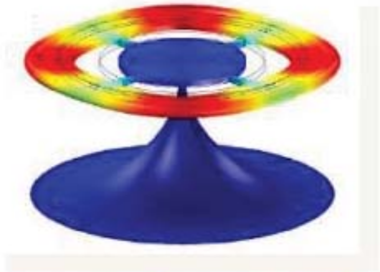


Micromembranes

Harris (Yale)
Kimble (Caltech)
Treutlein (Basel)



Microtoroids



Kippenberg (MPQ)
Weig (LMU)
Vahala (Caltech)
Bowen (UQ)

Presentation outline

☐ Mechanical resonators

☐ Radiation pressure

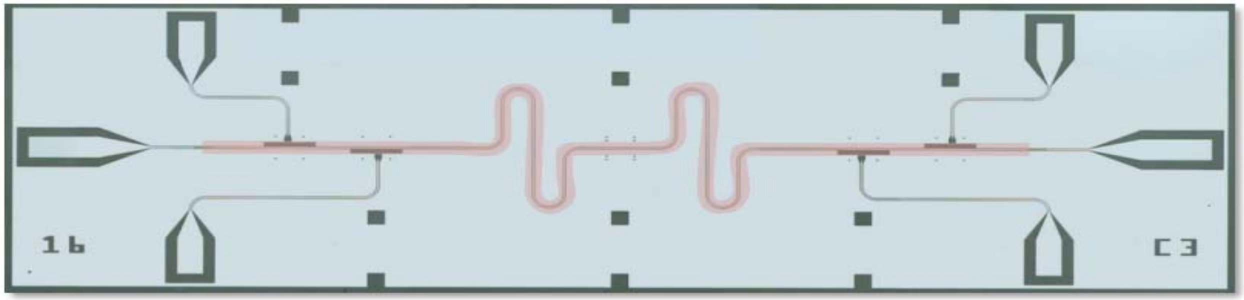
☐ Optomechanical cavity

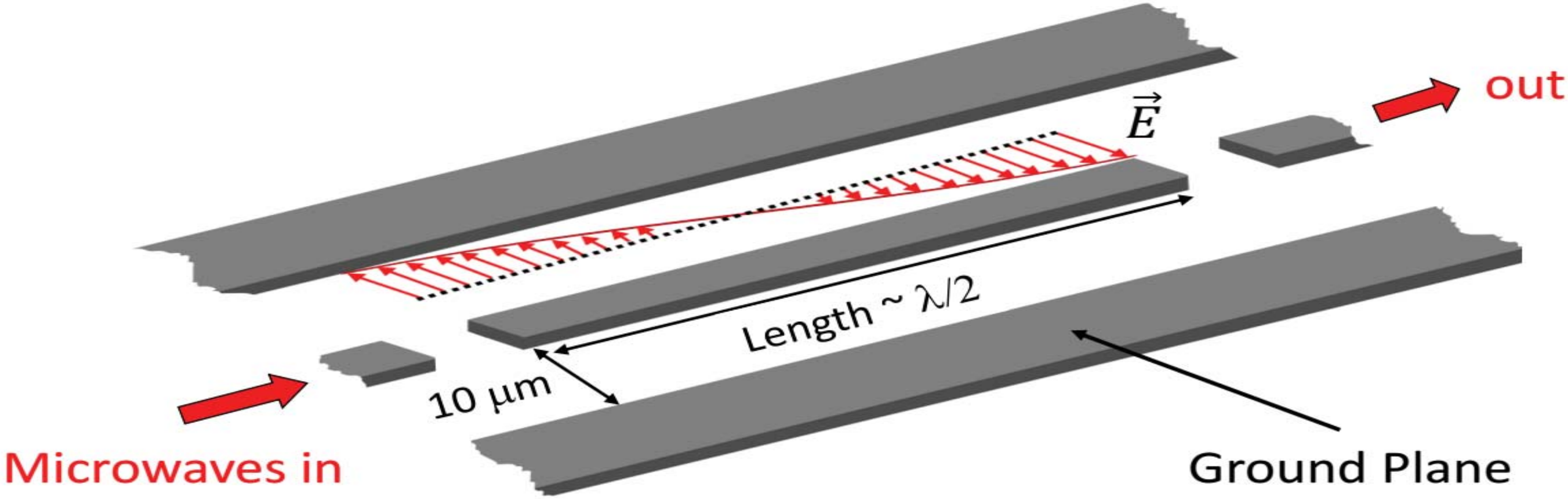
✓ Microwave cavity and electromechanics

☐ Theory of the Optomechanical cavity/cooling

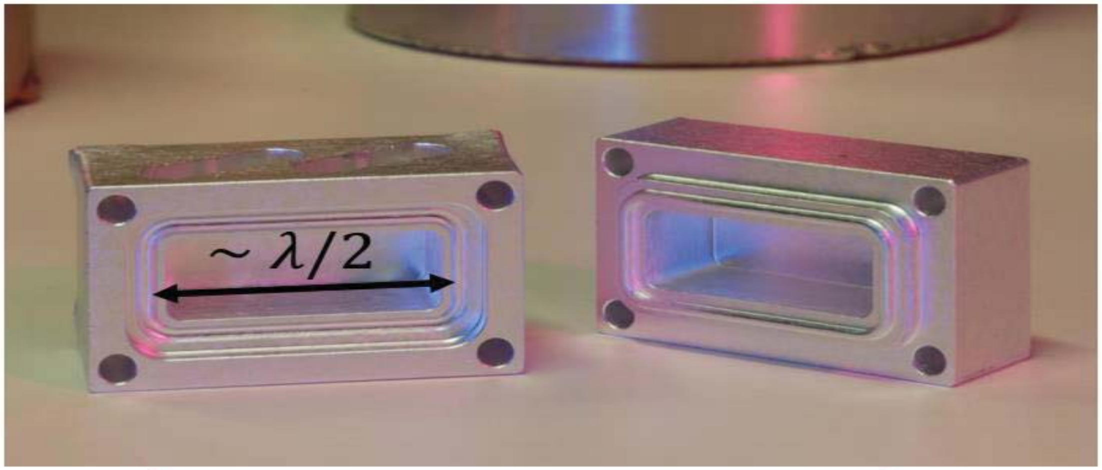
Resonators and Cavities

Coplanar Waveguide Resonators

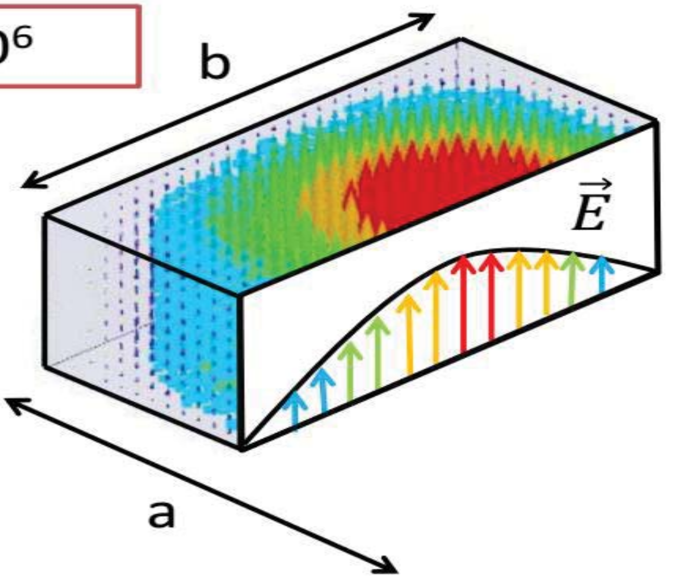




Waveguide microwave resonator



Observed Q's > 10^6

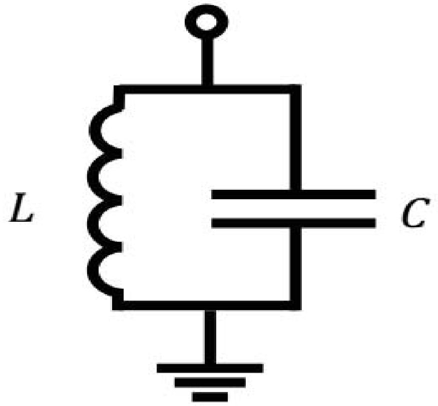


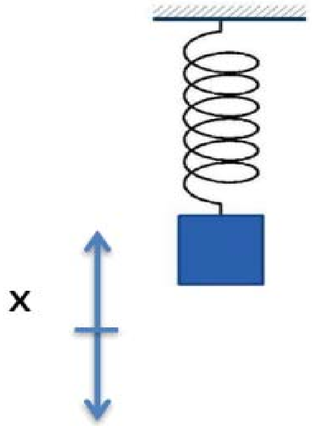
$$\upsilon_{m,n} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Reagor et.al. Appl. Phys. Lett. 102, 192604 (2013)

Quantum Circuits

Around a resonance:





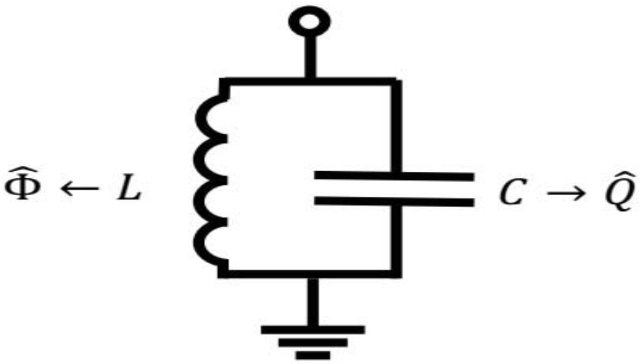
Lagrangian ⟶ $H = \frac{\hat{Q}^{\,2}}{2\,C} + \frac{\hat{\Phi}^{\,2}}{2\,L}$ ⇔ $H = \frac{\hat{p}^{\,2}}{2\,m} + \frac{m\omega^2 \hat{x}^{\,2}}{2}$

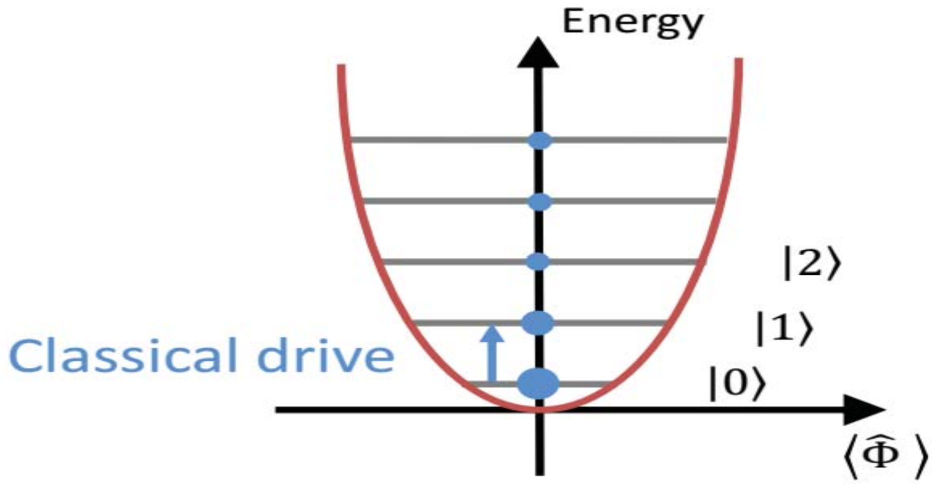
energy in magnetic field ⇔ potential energy

energy in electric field ⇔ kinetic energy

Quantum Circuits

Around a resonance:





$$H = \frac{\hat{Q}^{\,2}}{2\,C} + \frac{\hat{\Phi}^{\,2}}{2\,L}$$

$$\hat{Q} = \sqrt{\frac{\hbar}{2\,Z_0}}\left(a + a^\dagger\right) \quad \hat{\Phi} = i\sqrt{\frac{\hbar\,Z_0}{2}}\left(a - a^\dagger\right)$$

$$\boxed{H = \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right)}$$

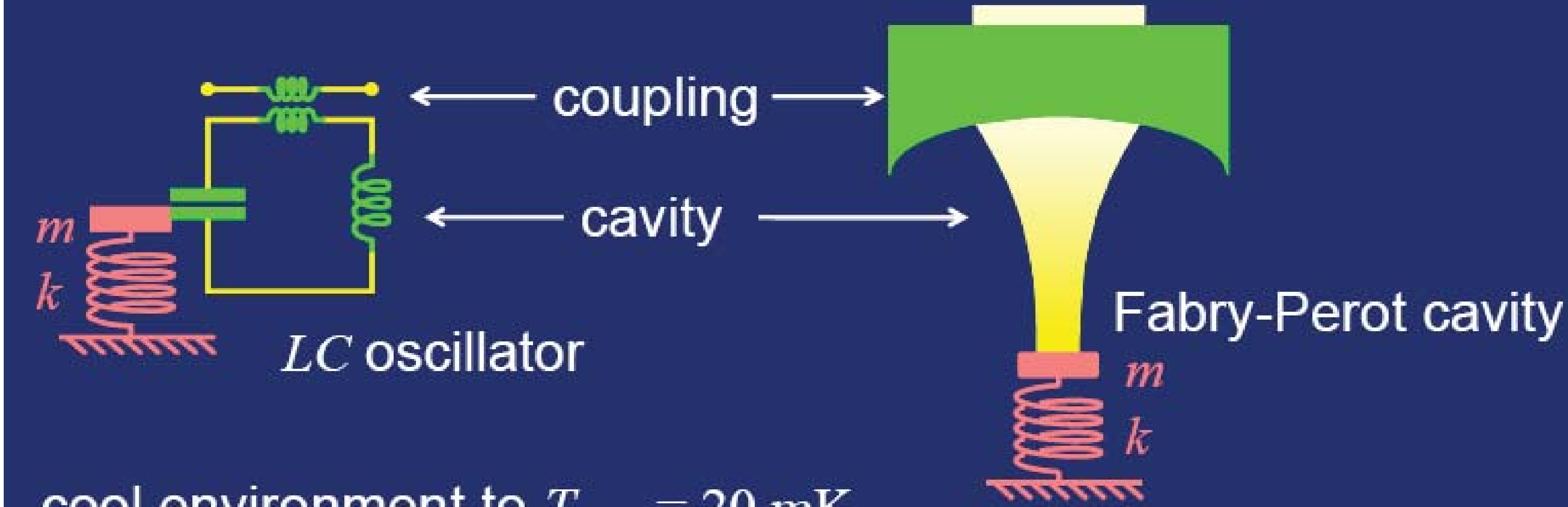
Quantum Harmonic Oscillator

$$\boxed{Z_0 = \sqrt{\frac{L}{C}} \longrightarrow 1 \ldots 100\ \Omega}$$

$$\boxed{\omega_0 = \sqrt{\frac{1}{LC}} \longrightarrow 4 \ldots 10\ GHz}$$

Reduce coupling to the environment by lowering temperature: microwave optomechanics

Microwave "light" in ultralow temperature cryostat

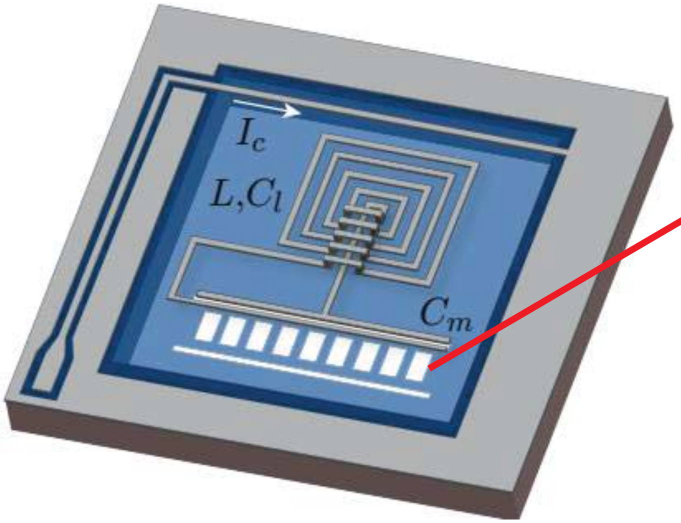


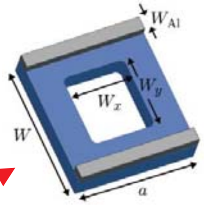
cool environment to $T_{\text{env}} = 20\ \text{mK}$

for 10 MHz oscillator $n_{\text{env}} = 40$

goal: $\Gamma > n_{\text{env}}\gamma$

Si3N4 electromechanics

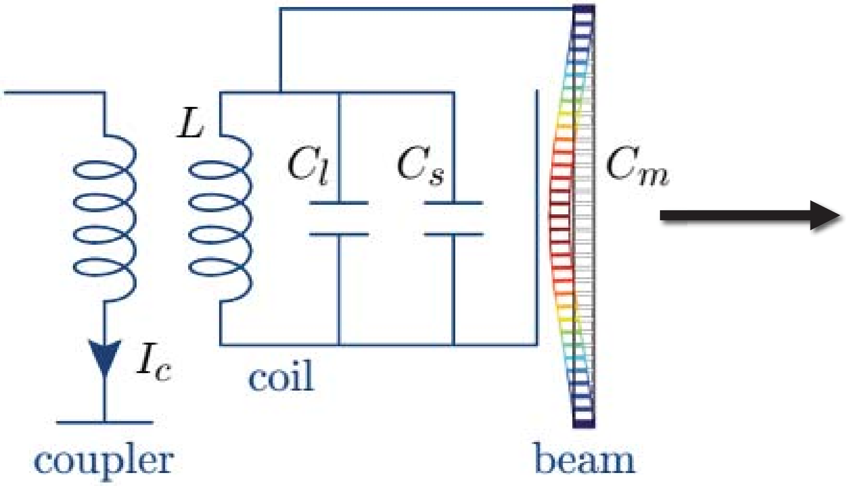


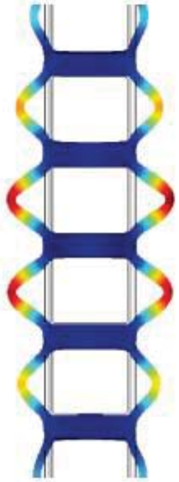


$$a = W = 2.23\ \mu\mathrm{m},\ W_x = W_y = 1.52\ \mu\mathrm{m}$$

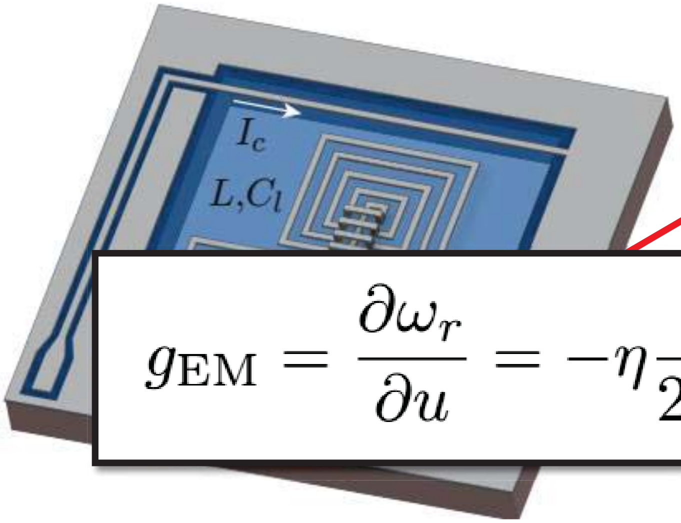
$$W_{\mathrm{Al}} = 170\ \mathrm{nm}$$

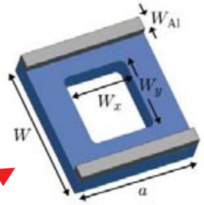
$$t_{\mathrm{mem}} = 300\ \mathrm{nm}\ \mathrm{and}\ t_{\mathrm{Al}} = 65\ \mathrm{nm}$$





Si3N4 electromechanics





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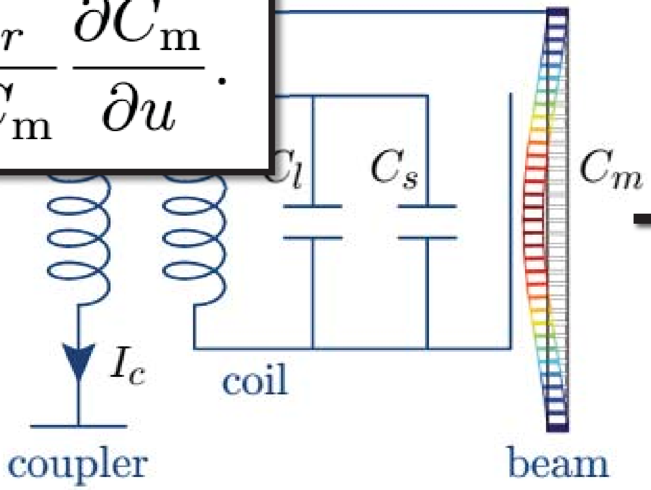
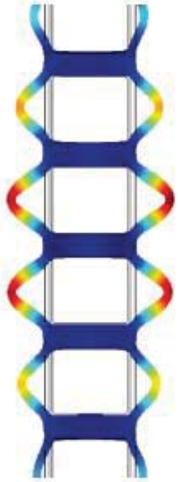
$t_{\text{mem}} = 300\ \text{nm and } t_{\text{Al}} = 65\ \text{nm}$

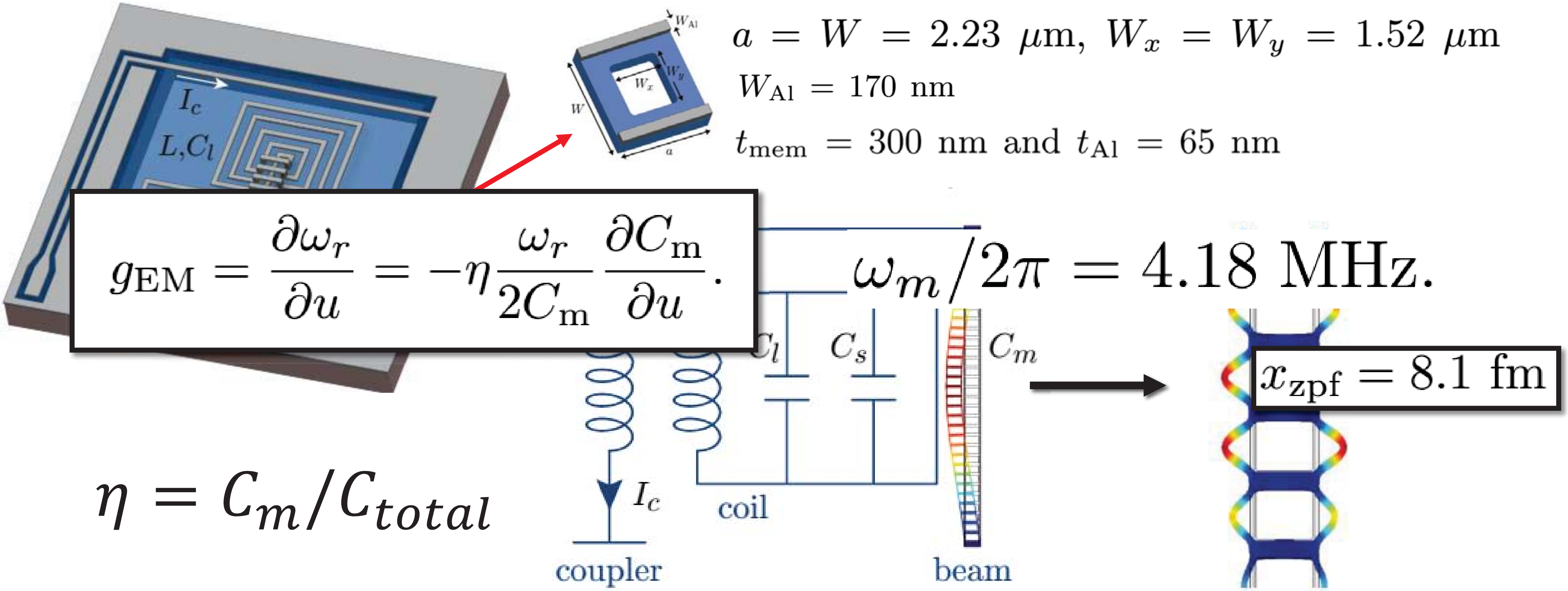
$$g_{\text{EM}} = \frac{\partial \omega_r}{\partial u} = -\eta \frac{\omega_r}{2C_{\text{m}}} \frac{\partial C_{\text{m}}}{\partial u}.$$

$$\eta = C_m / C_{total}$$





Si3N4 electromechanics



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$t_{\text{mem}} = 300\ \text{nm}$ and $t_{\text{Al}} = 65\ \text{nm}$

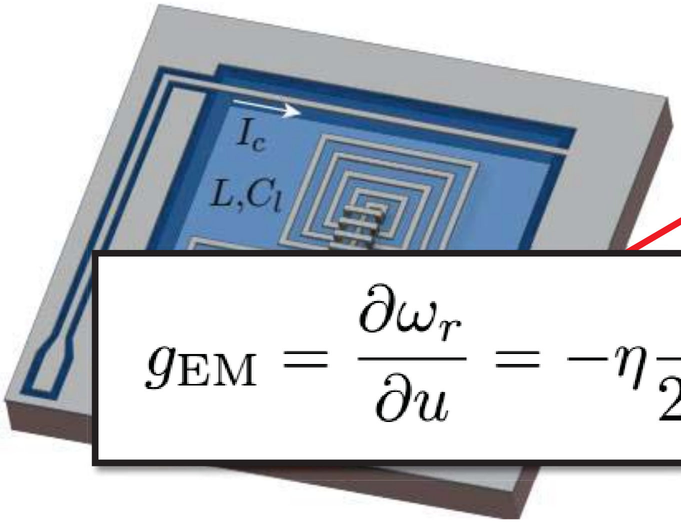
$$g_{\text{EM}} = \frac{\partial \omega_r}{\partial u} = -\eta \frac{\omega_r}{2C_{\text{m}}} \frac{\partial C_{\text{m}}}{\partial u}.$$

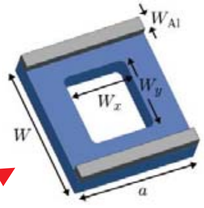
$$\omega_m/2\pi = 4.18\ \text{MHz}.$$

$$x_{\text{zpf}} = 8.1\ \text{fm}$$

$$\eta = C_m/C_{total}$$

Si3N4 electromechanics





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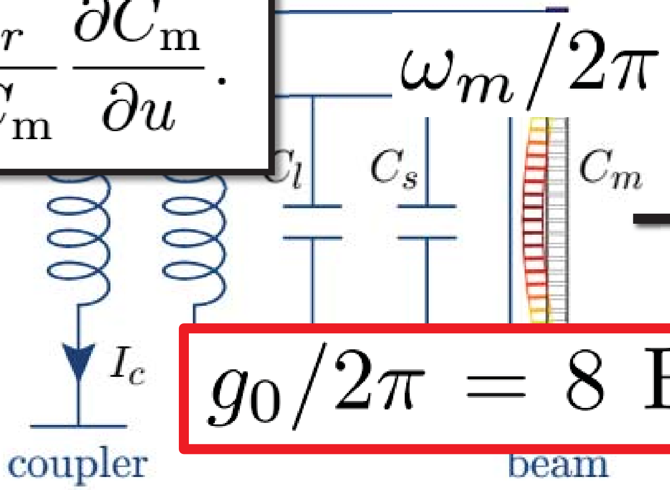
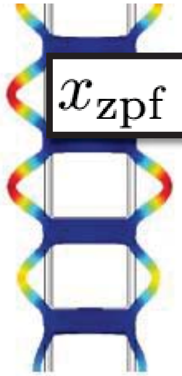
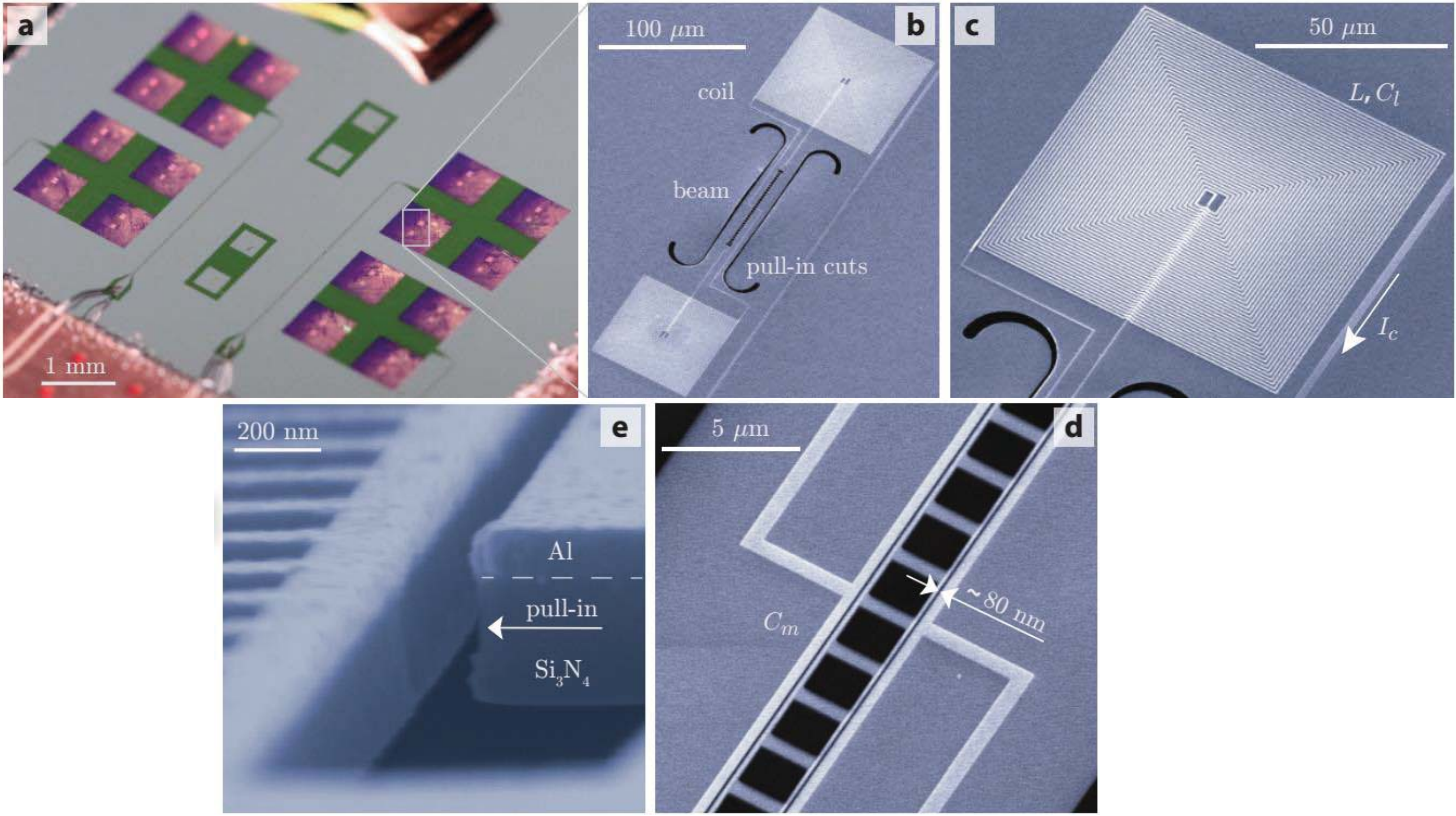


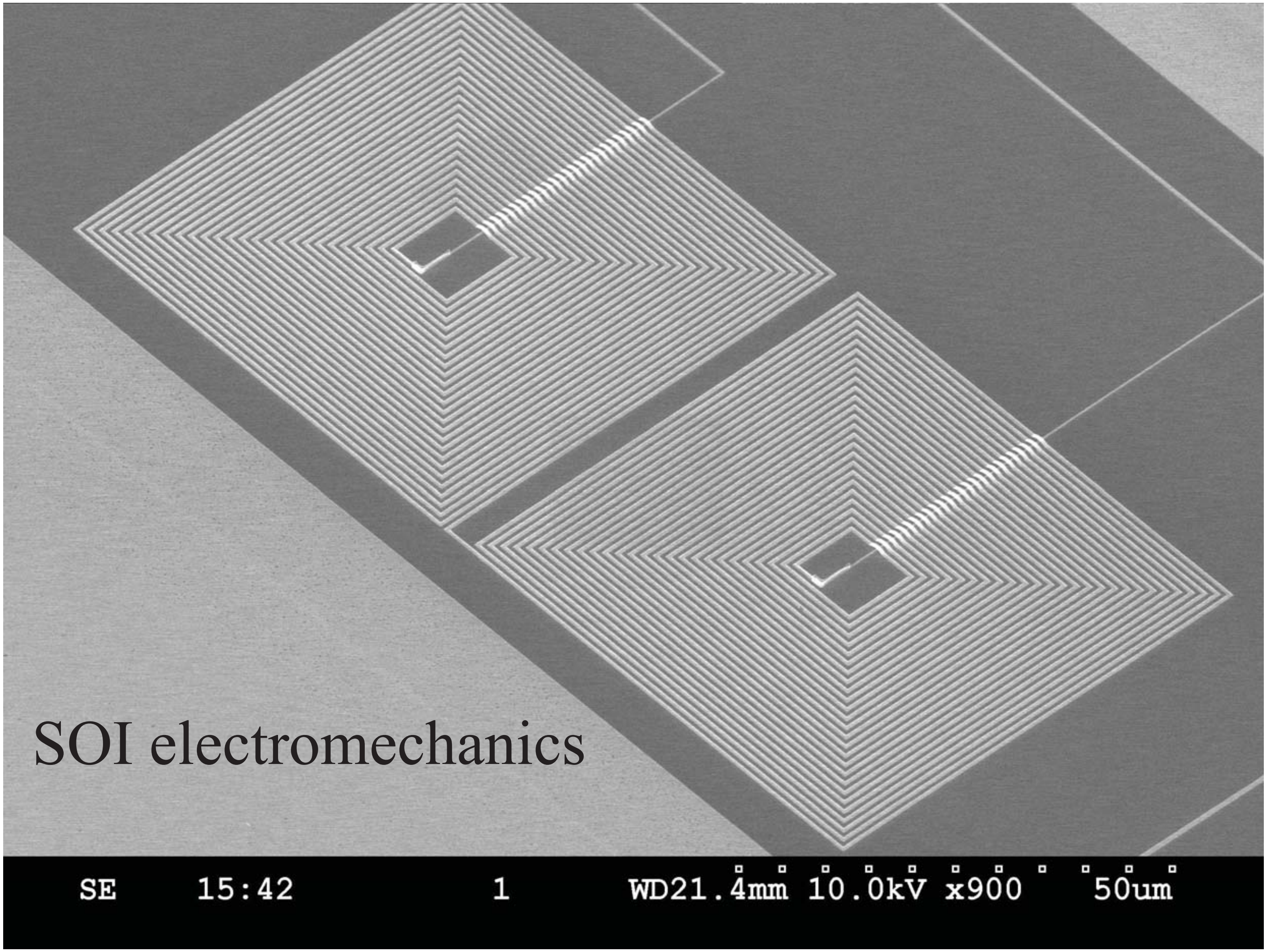


$x_{\mathrm{zpf}} = 8.1\ \mathrm{fm}$

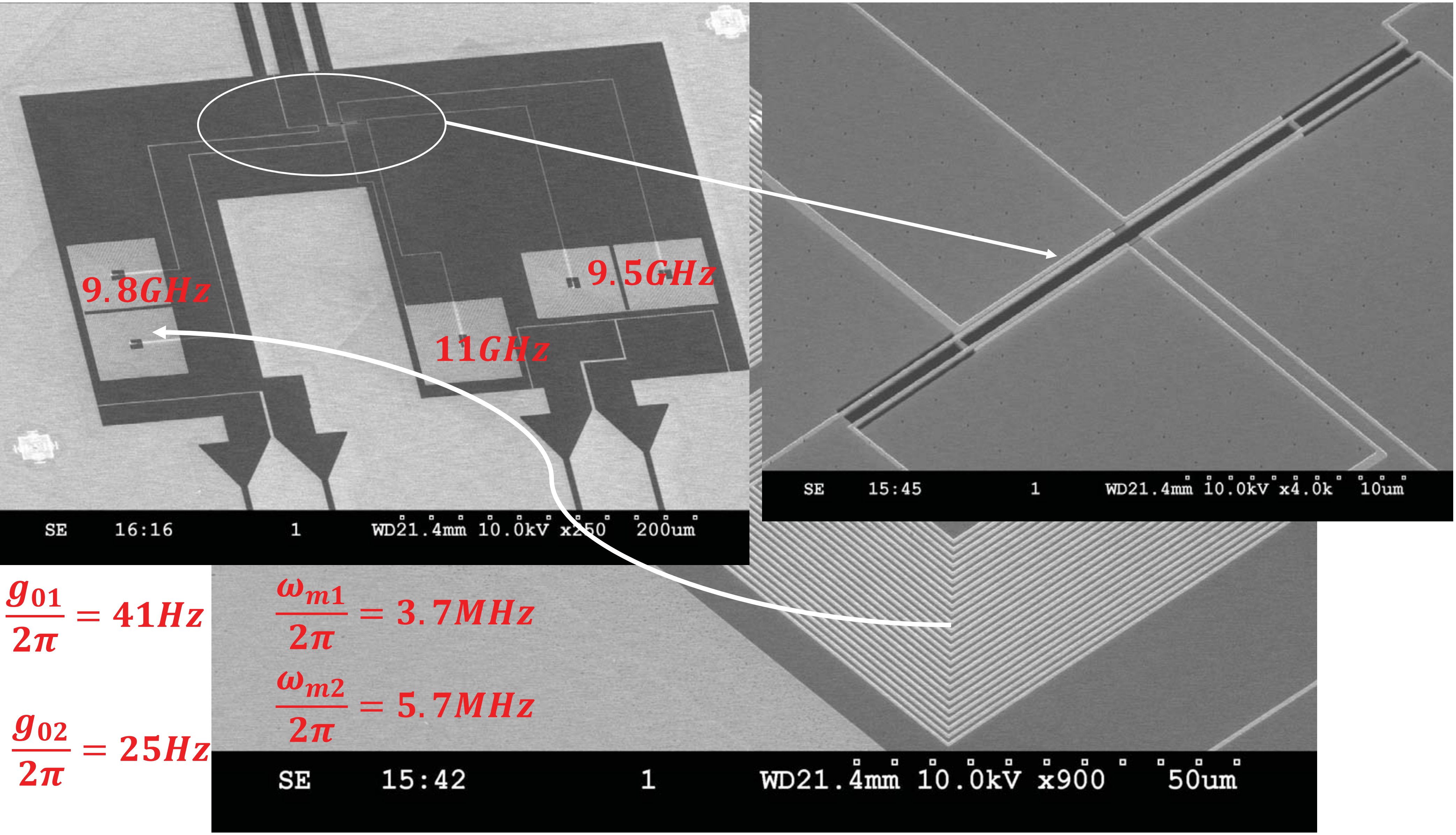
$$\eta = C_m/C_{total}$$

$$g_0/2\pi = 8\ \mathrm{Hz}$$


a
1 mm
b
100 μm
coil
beam
pull-in cuts
c
50 μm
L, C_l
I_c
e
200 nm
Al
pull-in
Si_3N_4
d
5 μm
C_m
~80 nm



SOI electromechanics



$$\frac{g_{01}}{2\pi} = 41Hz$$

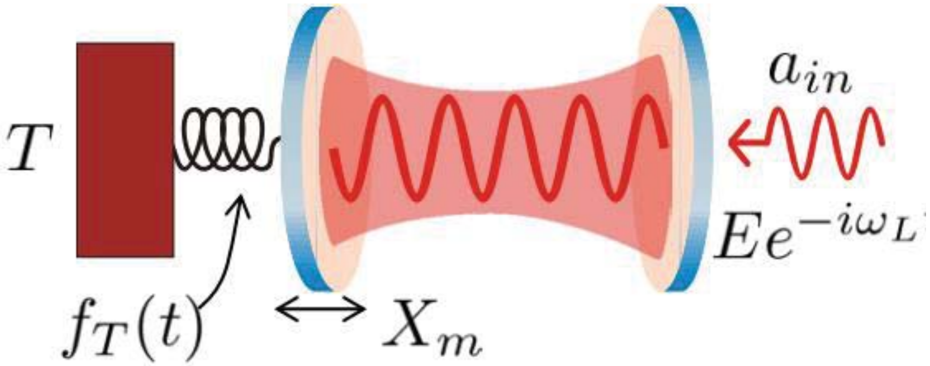
$$\frac{\omega_{m1}}{2\pi} = 3.7MHz$$

$$\frac{\omega_{m2}}{2\pi} = 5.7MHz$$

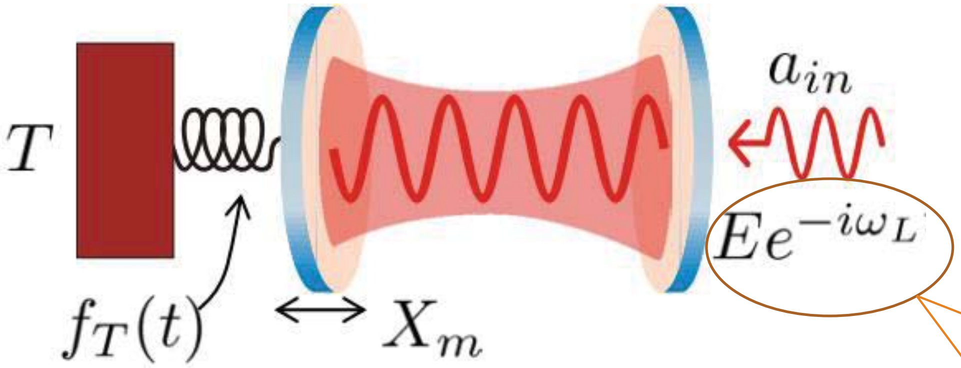
$$\frac{g_{02}}{2\pi} = 25Hz$$

Presentation outline

- ☐ Mechanical resonators
- ☐ Radiation pressure
- ☐ Optomechanical cavity
- ☐ Microwave cavity
- ✓ Theory of the Optomechanical cavity/cooling



$$H = \omega_m b^\dagger b + \omega_c a^\dagger a - g_0 a^\dagger a x_m$$

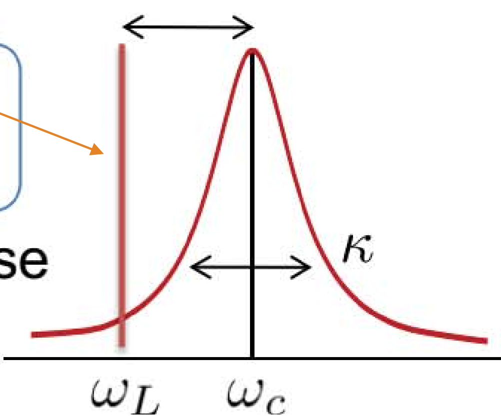


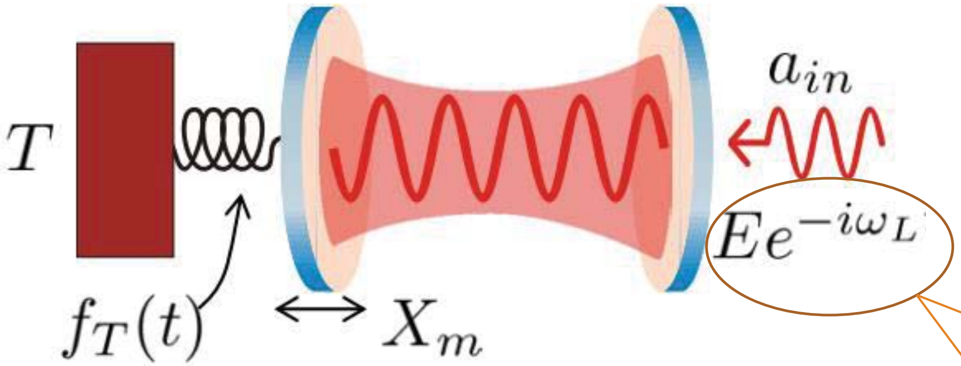
cavity

$$H = \omega_m b^\dagger b + \omega_c a^\dagger a - g_0 a^\dagger a x_m$$

$$\Delta = \omega_c - \omega_L$$

$$\boxed{\dot{a} = -(\underbrace{\kappa}_{\text{decay}} + i\omega_c)a - ig_0 a x_m + \underbrace{Ee^{-i\omega_L t}}_{\text{drive}} - \underbrace{\sqrt{2\kappa}\, a_{in}}_{\text{vacuum noise}}}$$

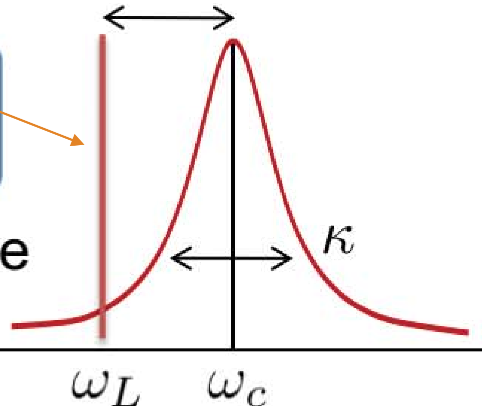




cavity

$$H = \omega_m b^\dagger b + \omega_c a^\dagger a - g_0 a^\dagger a x_m$$

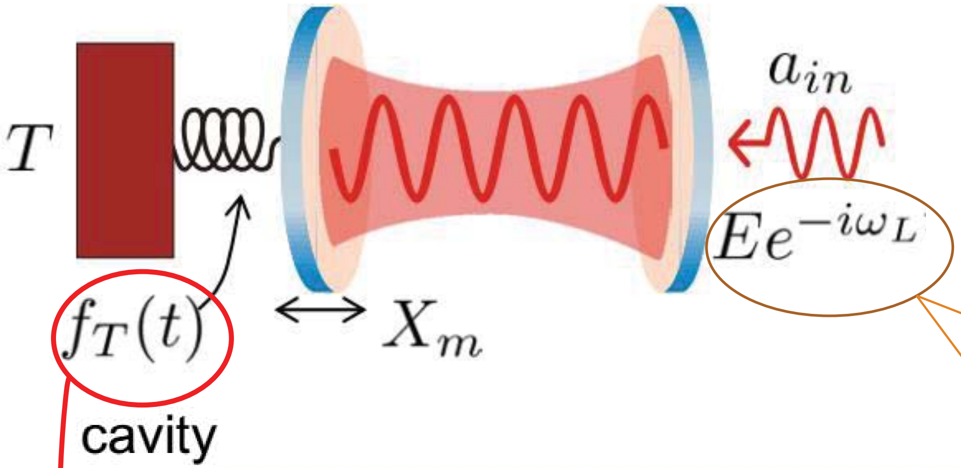
$$\Delta = \omega_c - \omega_L$$



$$\dot{a} = -(\underbrace{\kappa}_{\text{decay}} + i\omega_c)a - ig_0 a x_m + \underbrace{Ee^{-i\omega_L t}}_{\text{drive}} - \underbrace{\sqrt{2\kappa}\, a_{in}}_{\text{vacuum noise}}$$

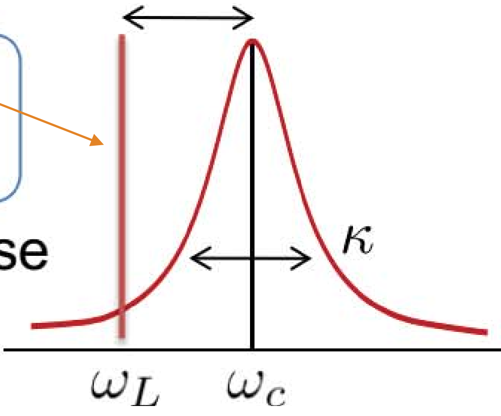
$$\dot{x}_m = \omega_m p_m$$

$$\dot{p}_m = -\omega_m x_m - \underbrace{\gamma p_m}_{\text{decay}} + g_0 a^\dagger a - \underbrace{\sqrt{2\gamma}\, f(t)}_{\text{thermal force}}$$



$$H = \omega_m b^\dagger b + \omega_c a^\dagger a - g_0 a^\dagger a x_m$$

$$\Delta = \omega_c - \omega_L$$



$$\dot{a} = -(\underbrace{\kappa}_{\text{decay}} + i\omega_c)a - ig_0 a x_m + \underbrace{Ee^{-i\omega_L t}}_{\text{drive}} - \underbrace{\sqrt{2\kappa}\, a_{in}}_{\text{vacuum noise}}$$

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Optomechanical **Quantum** equations of motion

$$\boxed{\begin{aligned} \dot{a} &= -(\kappa - i\Delta)a - ig_0 a x_m + E - \sqrt{2\kappa}\, a_{in} \\ \dot{x}_m &= \omega_m p_m \\ \dot{p}_m &= -\omega_m x_m - \gamma p_m + g_0 a^\dagger a - \sqrt{2\gamma}\, f_T(t) \end{aligned}}$$

Optomechanical **Quantum** equations of motion

$$\dot{a} = -(\kappa - i\Delta)a - ig_0 a x_m + E - \sqrt{2\kappa}\, a_{in}$$

$$\dot{x}_m = \omega_m p_m$$

$$\dot{p}_m = -\omega_m x_m - \gamma p_m + g_0 a^\dagger a - \sqrt{2\gamma}\, f_T(t)$$

Optomechanical **Classical** equations of motion

$$\dot{\alpha} = -(\kappa - i\Delta)\alpha - ig_0 \alpha x_m + E - \sqrt{2\kappa} a_{in}$$

$$\dot{x}_m = \omega_m p_m$$

$$\dot{p}_m = -\omega_m x_m - \gamma p_m + g_0 |\alpha|^2 - \sqrt{2\gamma}\, f_T(t)$$

Optomechanical **Quantum** equations of motion

$$\begin{aligned}
\dot{a} &= -(\kappa - i\Delta)a - ig_0 a x_m + E - \sqrt{2\kappa}\, a_{in} \\
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\end{aligned}$$

Optomechanical **Classical** equations of motion

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\dot{\alpha} &= -(\kappa - i\Delta)\alpha - ig_0 \alpha x_m + E - \sqrt{2\kappa} a_{in} \\
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\end{aligned}$$

no vacuum fluctuations

Classical random force

Optomechanical **Quantum** equations of motion

$$
\begin{aligned}
\dot{a} &= -(\kappa - i\Delta)a - ig_0 a x_m + E - \sqrt{2\kappa}\, a_{in} \\
\dot{x}_m &= \omega_m p_m \\
\dot{p}_m &= -\omega_m x_m - \gamma p_m + g_0 a^\dagger a - \sqrt{2\gamma}\, f_T(t)
\end{aligned}
$$

Optomechanical **Classical** equations of motion

$$
\begin{aligned}
\dot{\alpha} &= -(\kappa - i\Delta)\alpha - ig_0 \alpha x_m + E - \sqrt{2\kappa a_{in}} \\
\dot{x}_m &= \omega_m p_m \\
\dot{p}_m &= -\omega_m x_m - \gamma p_m + g_0 |\alpha|^2 - \sqrt{2\gamma}\, f_T(t)
\end{aligned}
$$

no vacuum fluctuations

Classical random force

multistability, self induced oscillations,
chaos,…

Linearization

Analyze the **quantum** dynamics around classical mean values due to classical random forces and quantum noise on mirror and cavity

$$\delta a = a - \bar{\alpha} \qquad \bar{\alpha} \gg 1 \qquad [\delta a, \delta a^\dagger] = 1$$

$$\delta x_m = x_m - \bar{x}_m \qquad [\delta x_m, \delta p_m] = i$$

$$\delta p_m = p_m - \bar{p}_m$$

$$\delta \dot{x}_m = \omega_m \delta p_m$$

$$\delta \dot{a} = -(\kappa - i\Delta)\delta a - i g_0 \bar{\alpha}\, \delta x_m - i g_0\, \delta a \delta x_m - \sqrt{2\kappa}\, a_{in}$$

$$\delta \dot{p}_m = -\omega_m \delta x_m - \gamma \delta p_m + g_0 \bar{\alpha}(\delta a + \delta a^\dagger) + g_0 \delta a^\dagger \delta a - \sqrt{2\gamma}\, f_T(t)$$

Linearization

Analyze the **quantum** dynamics around classical mean values due to classical random forces and quantum noise on mirror and cavity

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$$\delta x_m = x_m - \bar{x}_m \qquad [\delta x_m, \delta p_m] = i$$

$$\delta p_m = p_m - \bar{p}_m$$

$$\delta \dot{x}_m = \omega_m \delta p_m$$

$g = g_0\bar{\alpha}$

$$\delta \dot{a} = -(\kappa - i\Delta)\delta a - i\boxed{g_0\bar{\alpha}}\,\delta x_m - i g_0\, \delta a \delta x_m - \sqrt{2\kappa}\, a_{in}$$

$$\delta \dot{p}_m = -\omega_m \delta x_m - \gamma \delta p_m + \boxed{g_0\bar{\alpha}}(\delta a + \delta a^\dagger) + g_0 \delta a^\dagger \delta a - \sqrt{2\gamma}\, f_T(t)$$

Linearization

Analyze the **quantum** dynamics around classical mean values due to classical random forces and quantum noise on mirror and cavity

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$$\delta p_m = p_m - \bar{p}_m$$

$$\delta \dot{x}_m = \omega_m \delta p_m$$

$g = g_0\bar{\alpha}$

Small nonlinear term $\bar{\alpha} \gg 1$

$$\delta \dot{a} = -(\kappa - i\Delta)\delta a - i\boxed{g_0\bar{\alpha}}\,\delta x_m - i g_0\, \delta a \delta x_m - \sqrt{2\kappa}\, a_{in}$$

$$\delta \dot{p}_m = -\omega_m \delta x_m - \gamma \delta p_m + \boxed{g_0\bar{\alpha}}(\delta a + \delta a^\dagger) + g_0\, \delta a^\dagger \delta a - \sqrt{2\gamma}\, f_T(t)$$

Linearization

An random forces

and

$$\delta\dot{a} = -(\kappa - i\Delta)\delta a - ig\,\delta x_m - \sqrt{2\kappa}\,a_{in}$$

$$\delta\dot{x}_m = \omega_m \delta p_m$$

$$\delta\dot{p}_m = -\omega_m \delta x_m - \gamma\delta p_m + g(\delta a + \delta a^\dagger) - \sqrt{2\gamma}\,f_T(t)$$

$$\delta\dot{x}_m = \omega_m \delta p_m$$

$g = g_0\bar{\alpha}$

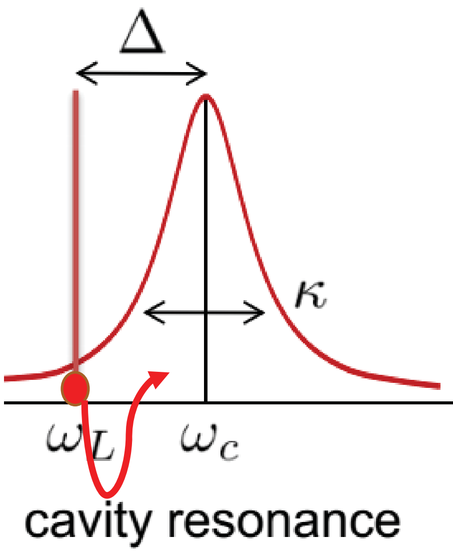
Small nonlinear term $\bar{\alpha} \gg 1$

$$\delta\dot{a} = -(\kappa - i\Delta)\delta a - i\boxed{g_0\bar{\alpha}}\,\delta x_m - ig_0\boxed{\delta a\delta x_m} - \sqrt{2\kappa}\,a_{in}$$

$$\delta\dot{p}_m = -\omega_m \delta x_m - \gamma\delta p_m + \boxed{g_0\bar{\alpha}}(\delta a + \delta a^\dagger) + g_0\boxed{\delta a^\dagger\delta a} - \sqrt{2\gamma}\,f_T(t)$$

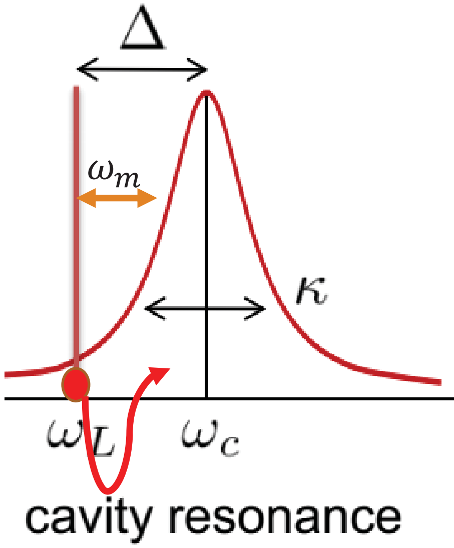
Optomechanical cooling

$$\Delta = \omega_m$$



Optomechanical cooling

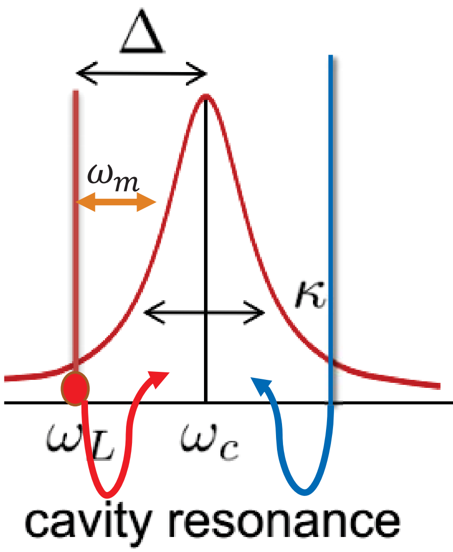
$$\boxed{\Delta = \omega_m}$$



Anti-Stokes scattering

$\omega_L \rightarrow \omega_L + \omega_m$ + annihilation of one phonon

Optomechanical cooling

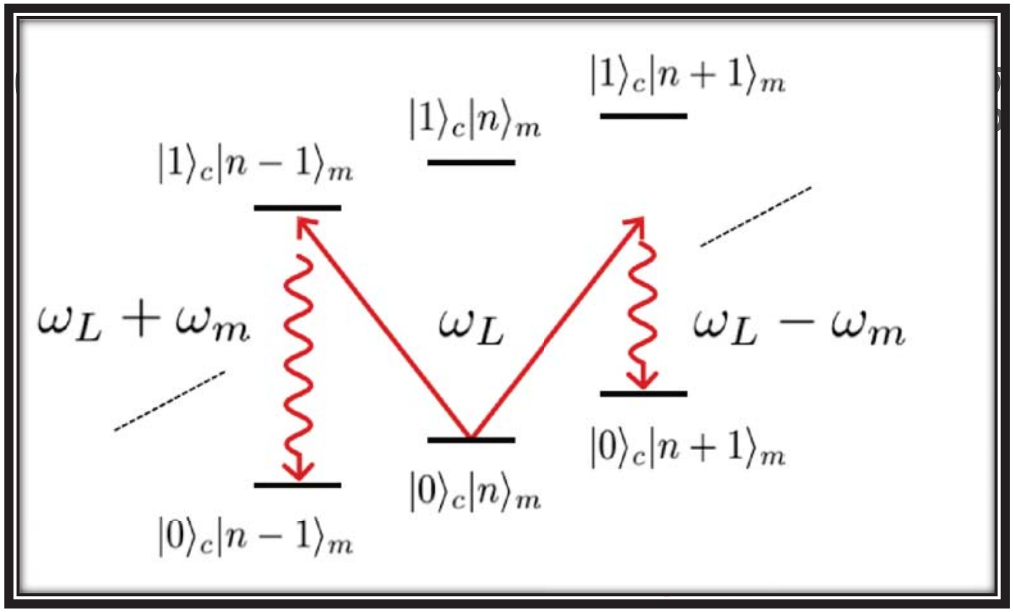


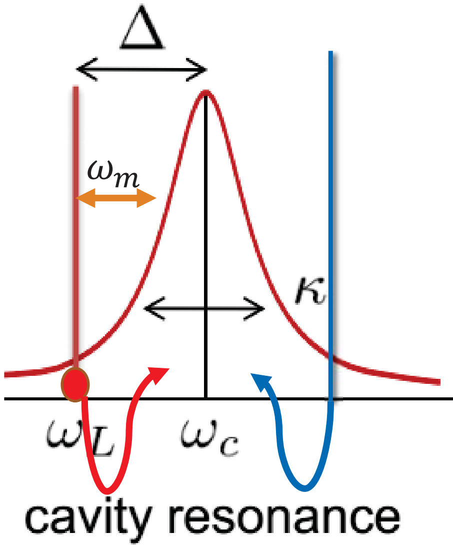
$$\Delta = \omega_m$$

Anti-Stokes scattering

$\omega_L \rightarrow \omega_L + \omega_m$ + annihilation of one phonon

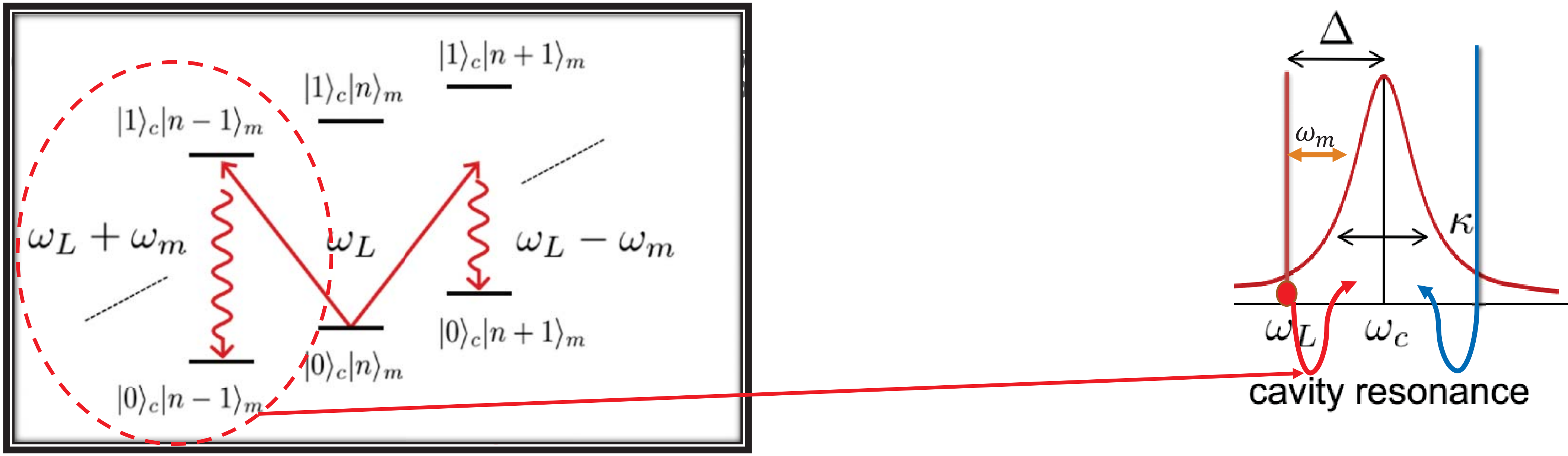
Stokes scattering $\omega_L \rightarrow \omega_L - \omega_m$ + creation of one phonon





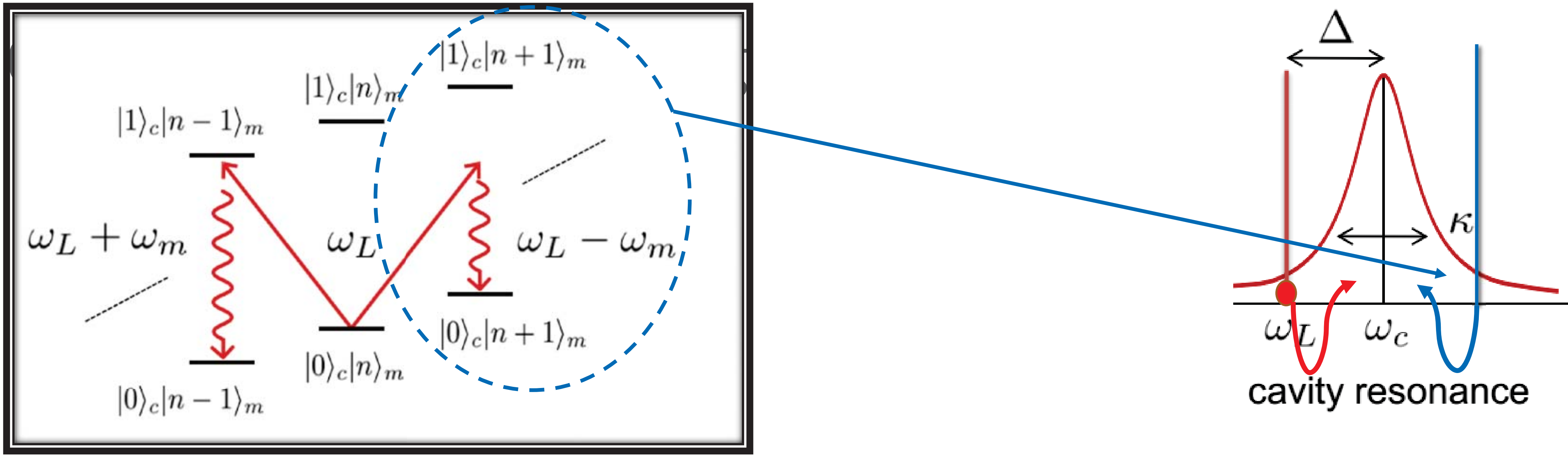
$\omega_L \to \omega_L + \omega_m$ + annihilation of one phonon

Stokes scattering $\omega_L \to \omega_L - \omega_m$ + creation of one phonon



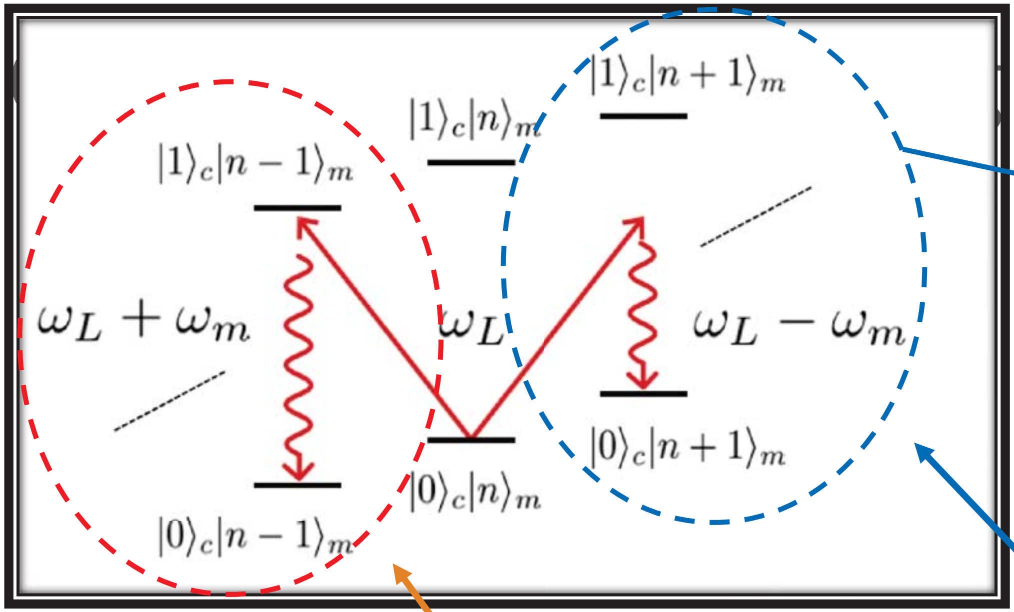
$\omega_L \rightarrow \omega_L + \omega_m$ + annihilation of one phonon

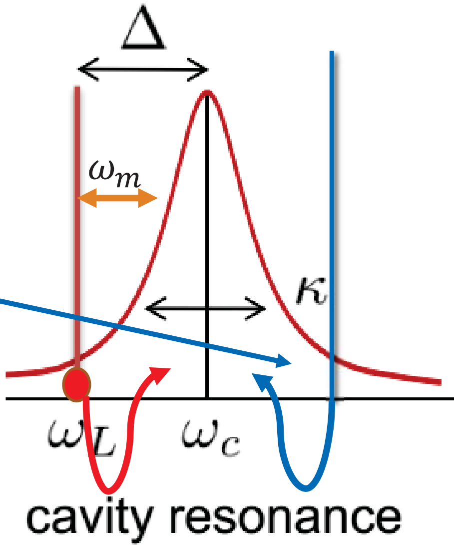
Stokes scattering $\omega_L \rightarrow \omega_L - \omega_m$ + creation of one phonon



$\omega_L \rightarrow \omega_L + \omega_m$ + annihilation of one phonon

Stokes scattering $\omega_L \rightarrow \omega_L - \omega_m$ + creation of one phonon





$$H_{int} = -\frac{g}{\sqrt{2}}(\delta a+\delta a^\dagger)(\delta b+\delta b^\dagger)$$

n

$$\propto \boxed{(\delta a\delta b^\dagger+\delta a^\dagger\delta b)} + \boxed{(\delta a\delta b+\delta a^\dagger\delta b^\dagger)}$$

$\Delta = \omega_c - \omega_L = \omega_m$

$\Delta = \omega_c - \omega_L = -\omega_m$

- Cavity field mediates mechanical cooling, weak coupling regime $\boxed{g \ll \kappa \ll \omega_m}$

$$\delta a(t) \simeq -\frac{ig}{\sqrt{2}}\left(\frac{1}{\kappa}\,\delta b(t) + \frac{1}{2i\omega_m}\,\delta b^\dagger(t)\right) - \frac{\sqrt{2\kappa}}{i\omega_m}\,a_{in}(t)$$

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backaction

$$\delta\dot{b} = -(\gamma + i\omega_m)\delta b - \frac{ig}{\sqrt{2}}(\delta a + \delta a^\dagger) - \sqrt{2\gamma}\,f_T(t)$$

- Cavity field mediates mechanical cooling, weak coupling regime $\boxed{g \ll \kappa \ll \omega_m}$

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backaction

$$\delta\dot{b} = -(\gamma + i\omega_m)\delta b - \frac{ig}{\sqrt{2}}(\delta a + \delta a^\dagger) - \sqrt{2\gamma}\,f_T(t)$$

$$\delta\dot{b} = -\left[\left(\gamma + \frac{g^2}{2\kappa}\right) + i\left(\omega_m + \frac{g^2}{4\omega_m}\right)\right]\delta b + \frac{g\sqrt{\kappa}}{\omega_m}(a_{in} - a_{in}^\dagger) - \sqrt{2\gamma}\,f_T(t)$$

optomechanical cooling rate: $\Gamma = \dfrac{g^2}{2\kappa}$

Optical spring effect(softening)

- **Average number of phonons** in steady state

using $\langle a_{in}(t)a^{\dagger}_{in}(t')\rangle = \delta(t-t')$ vacuum noise

$\langle f_T(t) f_T(t')\rangle \simeq \bar{n}_T \delta(t-t')$ thermal noise $\bar{n} \simeq \frac{k_B T}{\hbar\omega_m}$

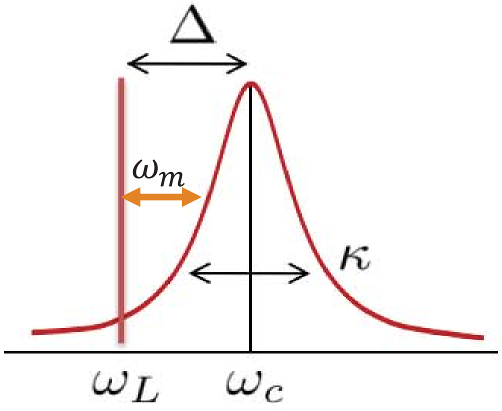
$$\boxed{\langle \delta b^{\dagger} \delta b\rangle = \frac{\Gamma}{\gamma+\Gamma}\left(\frac{\kappa}{\omega_m}\right)^2 + \frac{\gamma}{\gamma+\Gamma}\bar{n}_T \simeq \left(\frac{\kappa}{\omega}\right)^2}$$

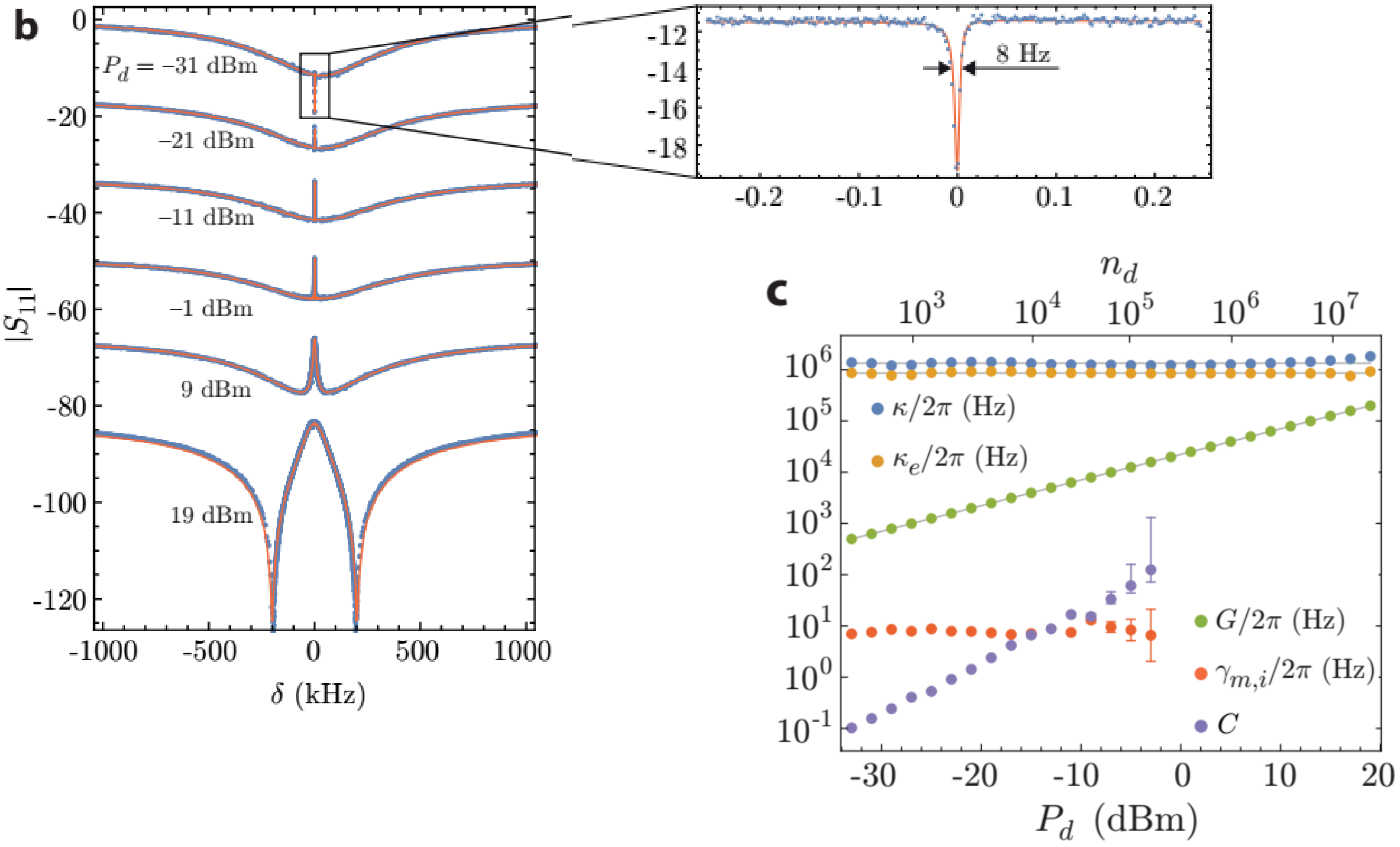
for efficient cooling $\gamma \bar{n}_T \ll \Gamma$

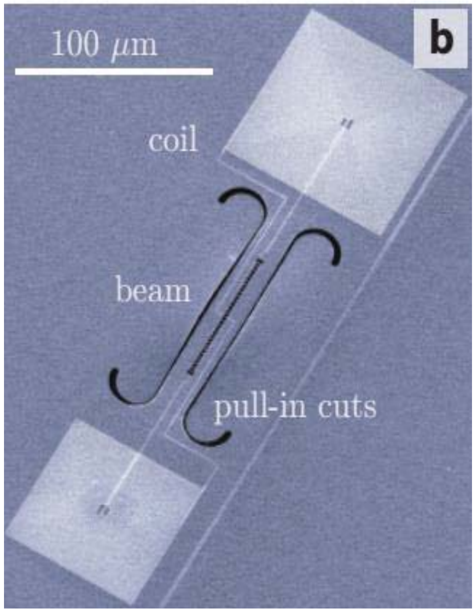
summary of ground state cooling conditions:

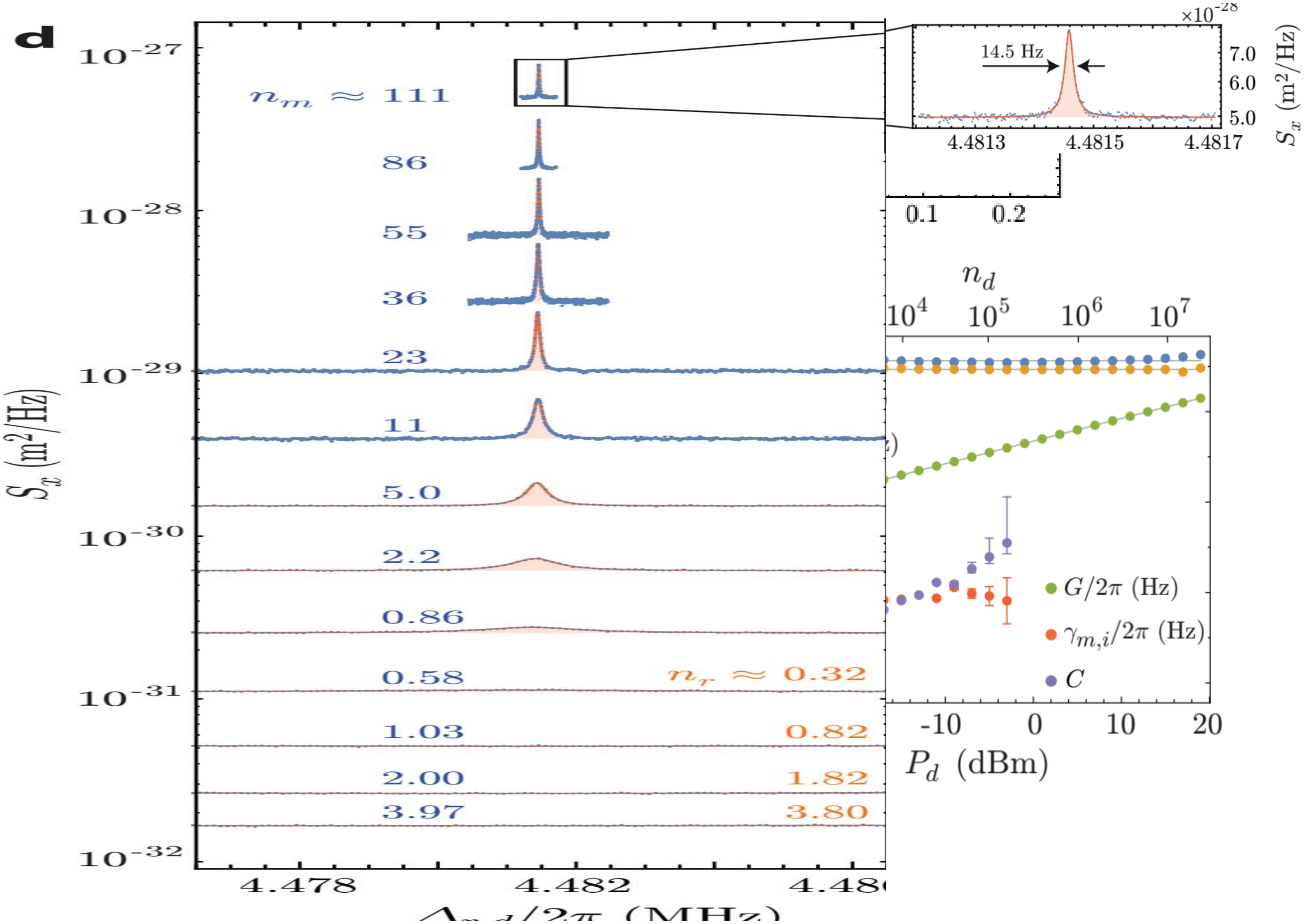
- red detuned drive $\boxed{\Delta = \omega_m}$
- sideband resolution $\omega_m \gg \kappa$
- strong coupling

$\gamma \bar{n}_T \ll \Gamma$ $g^2 \gg (\gamma \bar{n}_T)\kappa$




b
c
$P_d = -31$ dBm
-21 dBm
-11 dBm
-1 dBm
9 dBm
19 dBm
$|S_{11}|$
0
-20
-40
-60
-80
-100
-120
-1000
-500
0
500
1000
δ (kHz)
-12
-14
-16
-18
-0.2
-0.1
0
0.1
0.2
8 Hz
n_d
10^3
10^4
10^5
10^6
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
● $\kappa/2\pi$ (Hz)
● $\kappa_e/2\pi$ (Hz)
● $G/2\pi$ (Hz)
● $\gamma_{m,i}/2\pi$ (Hz)
● C
-30
-20
-10
0
10
20
P_d (dBm)

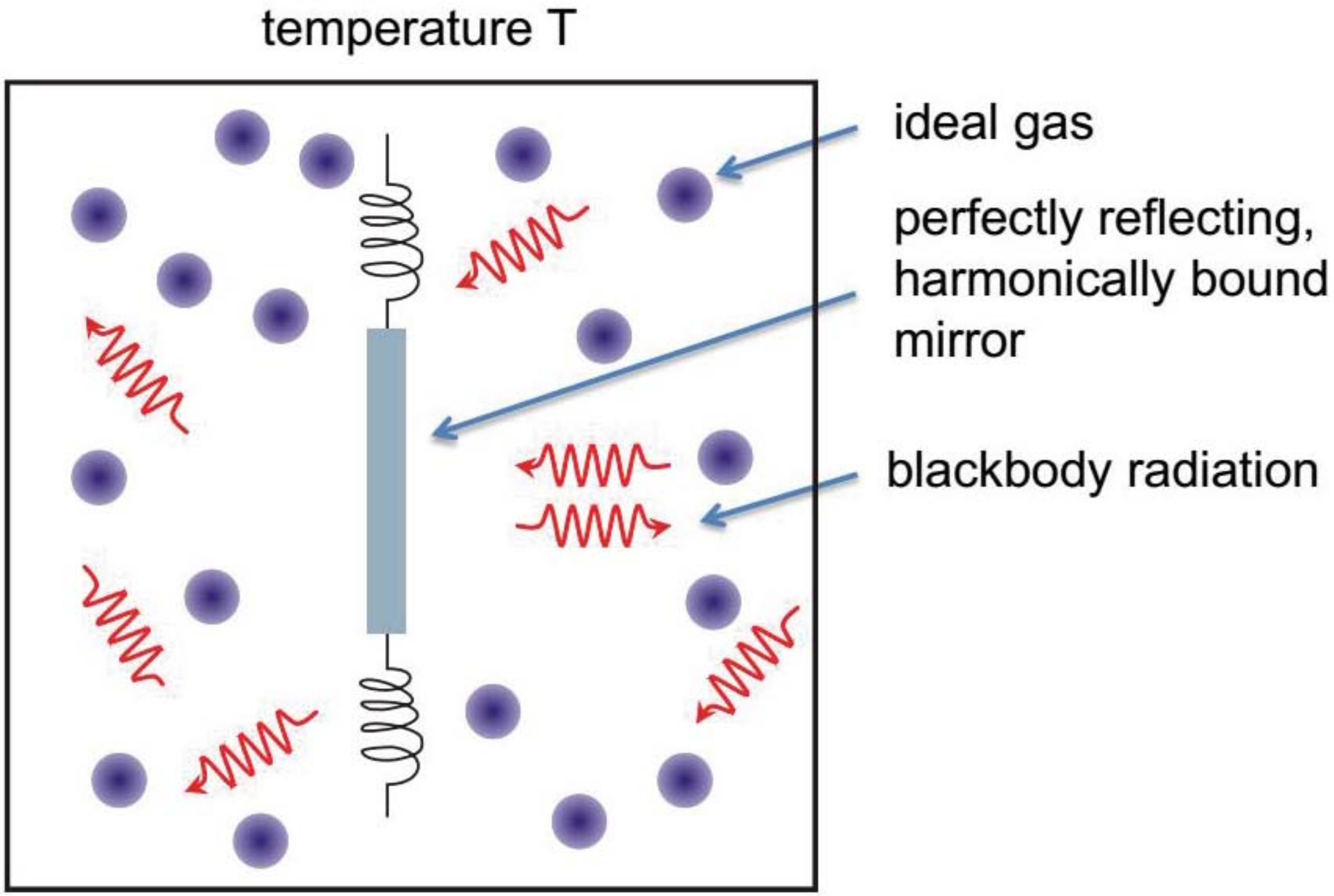

100 μm
b
coil
beam
pull-in cuts

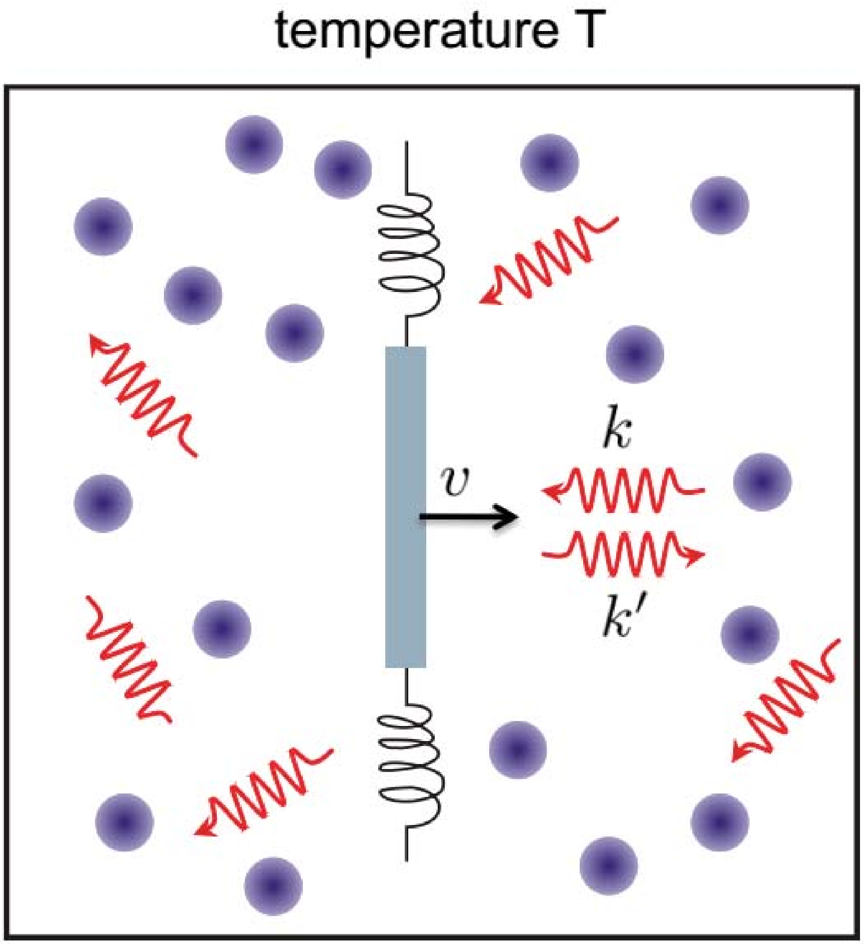


d
10^{-27}
10^{-28}
10^{-29}
10^{-30}
10^{-31}
10^{-32}
S_x (m^2/Hz)
$n_m \approx$ 111
86
55
36
23
11
5.0
2.2
0.86
0.58
1.03
2.00
3.97
$n_r \approx$ 0.32
0.82
1.82
3.80
4.478
4.482
$\Delta_{r,d}/2\pi$ (MHz)
$\times 10^{-28}$
14.5 Hz
7.0
6.0
5.0
S_x (m^2/Hz)
4.4813
4.4815
4.4817
0.1
0.2
n_d
10^4
10^5
10^6
10^7
$G/2\pi$ (Hz)
$\gamma_{m,i}/2\pi$ (Hz)
C
-10
0
10
20
P_d (dBm)

Conclusion

- Applications of mechanical resonators
- Physic of mechanical resonators
- Optomechanical cavities
- Microwave eletromechanics
- Optomechanical cooling


temperature T
ideal gas
perfectly reflecting, harmonically bound mirror
blackbody radiation



Doppler shift of reflected wave:

$$k' = -k\left(1 - \frac{2v}{c}\right)$$

Radiation pressure force due to momentum $\sim k - k'$ transfer on mirror:

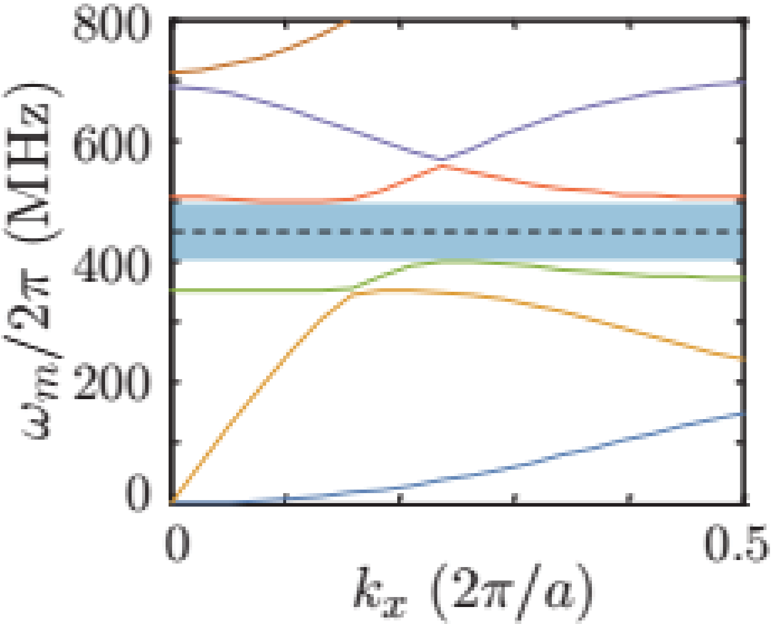
$$F_{\mathrm{rp}} = -\frac{2P}{c}\left(1 - \frac{v}{c}\right) \qquad \text{power } P$$

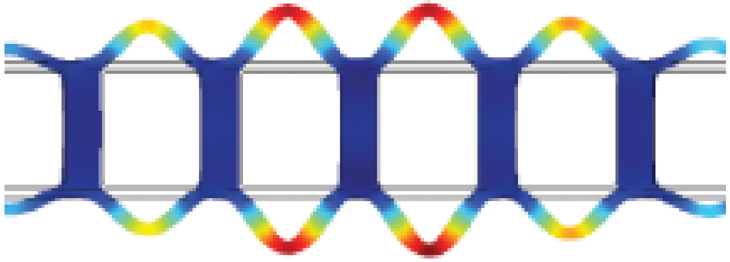
Radiation provides friction:

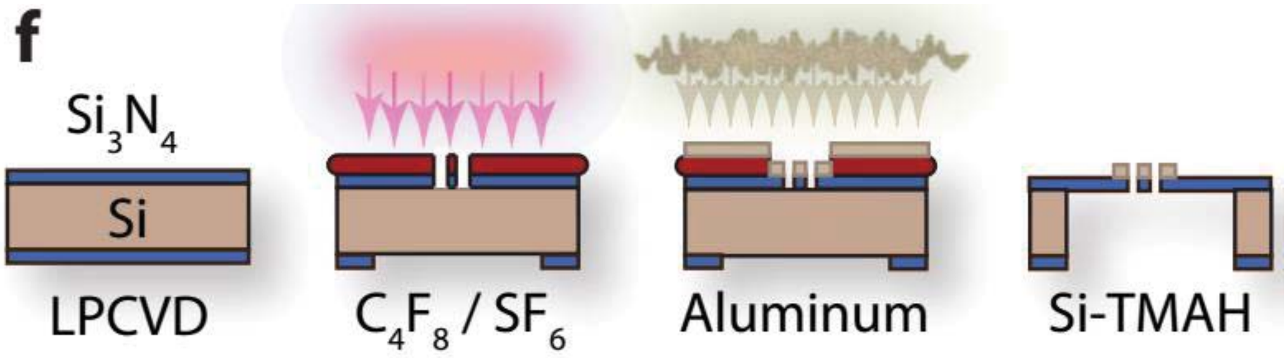
$$\dot{p} = -\gamma p \qquad \gamma = \frac{2P}{mc^2}$$

“Doppler cooling” of mirror

→ Thermal equlibrium?


800
600
400
200
0
$\omega_m/2\pi$ (MHz)
0
0.5
k_x $(2\pi/a)$





low-pressure chemical vapor deposition
(LPCVD)

(i) LPCVD of stoichiometric Si3N4 on both sides of a 200 μm thick silicon substrate,
(ii) C4F8:SF6 plasma etch through the nitride membrane defining the mechanical beam resonator and pull-in cuts on the top side, and membrane windows on the bottom side,
(iii) electron beam lithography, aluminum deposition, and lift-off steps to pattern the microwave circuit
(iv) final release of the nitride membrane using a silicon-enriched tetramethylammonium hydroxide (TMAH) solution

1 cm × 1 cm chips diced from a high-resistivity siliconon-insulator (SOI) wafer manufactured by SOITEC using the Smart Cut process [22]. The SOI wafer consists of a 300 nm thick silicon device layer with (100) surface orientation and *p*-type (Boron) doping with a specified resistivity of 500 Ω-cm. Underneath the device layer is a 3 *μ*m buried silicon dioxide (SiO_2) BOX layer. The device and BOX layers sit atop a silicon (Si) handle wafer of thickness 675 *μ*m and a specified resistivity of 750 Ω-cm. Both the Si device layer and handle wafer are grown using the Czochralski crystal growth method.

Fabrication of the coupled coil resonator and H-slot resonator can be broken down into the following six steps.

In step (1), we pattern the H-slot resonator using electron beam (e-beam) lithography in ZEP-520A resist, and etch this pattern into the Si device layer using an inductively coupled plasma reactive ion etch (ICP-RIE). After the ICP-RIE etch, we clean the chips with a 4 min piranha bath and a 12 sec buffered hydrofluoric acid (BHF) dip. In step (2), we pattern the capacitor electrodes and ground plane region using ZEP-520A resist and use electron beam evaporation to deposit 60 nm of Al on the chip. In step (3), we define a protective scaffold formed out of LOR 5B e-beam resist to create the crossover re-