Microwave Quantum Illumination(Quantum Sensor)

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Presentation outline

- ❑ Optical Quantum Illumination
- ❑ Microwave-Optical Convertor
- ❑ Quantum Illumination at the Microwave Wavelengths

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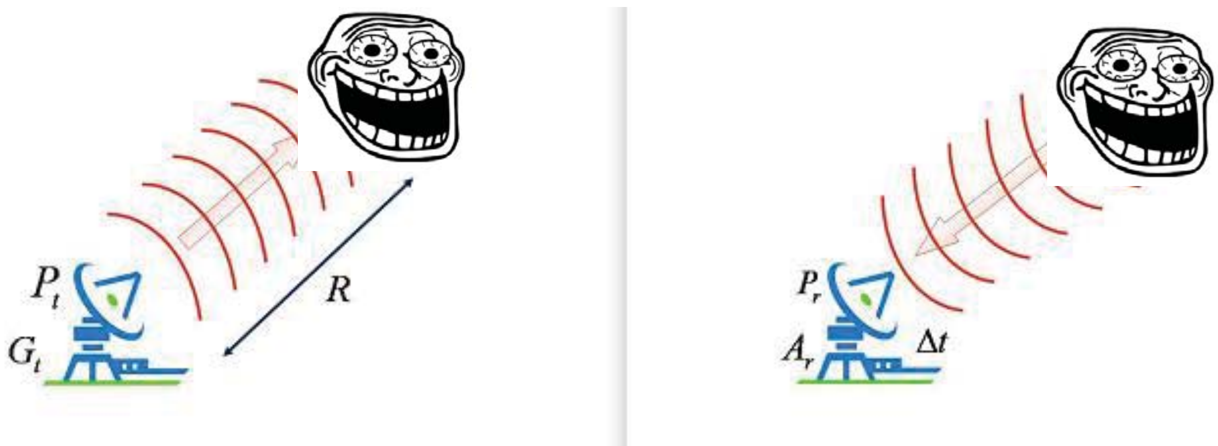
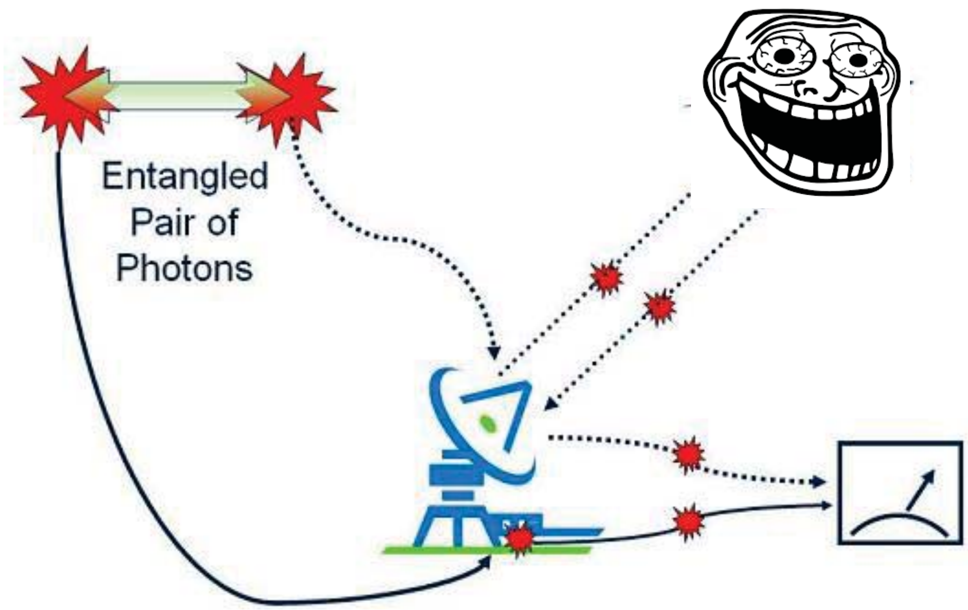
Standard Illumination(Sensor)



Quantum Illumination

Quantum illumination is a quantum-optical sensing technique in which an entangled source is exploited to improve the detection of a low-reflectivity object that is immersed in a bright thermal background.



S. Lloyd, Science, **321**, 1463(2008).

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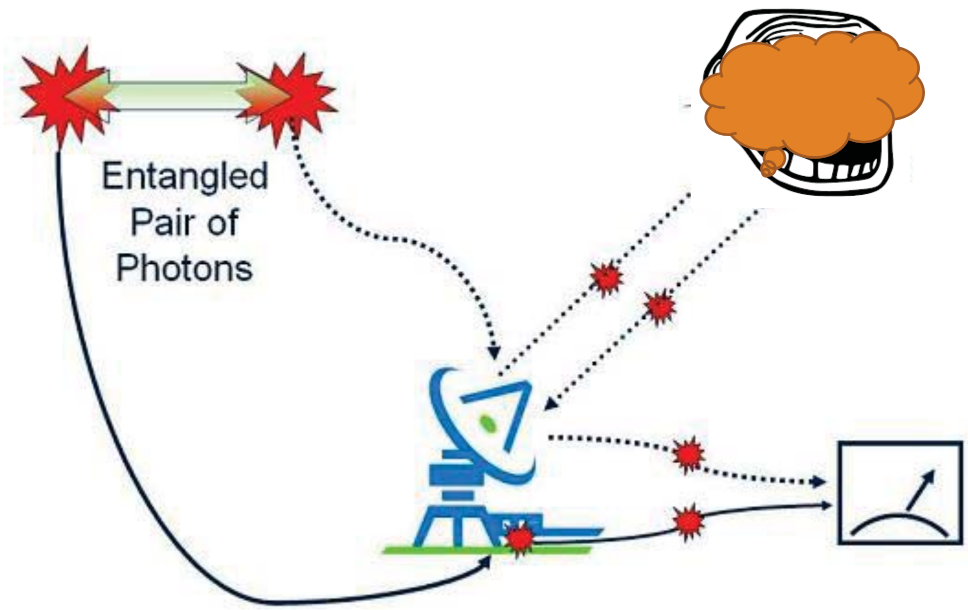


Only works for

- Thermal noises(Environment)
- High temperature
- Low reflection

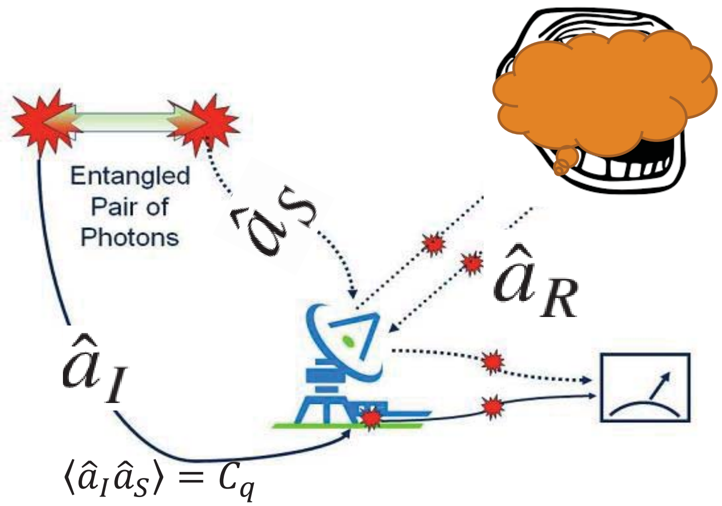
S. Lloyd, Science, **321**, 1463(2008).

Quantum Illumination

$$|\psi\rangle_{SI} = \sum_{n=0}^{\infty} \sqrt{\frac{N_S^n}{(N_S+1)^{n+1}}} |n\rangle_S |n\rangle_I,$$

$$\mathbf{\Lambda}_{SI} = \frac{1}{4} \begin{bmatrix} S & 0 & C_q & 0 \\ 0 & S & 0 & -C_q \\ C_q & 0 & S & 0 \\ 0 & -C_q & 0 & S \end{bmatrix},$$

where $S \equiv 2N_S + 1$ and $C_q \equiv 2\sqrt{N_S(N_S+1)}$.



Quantum Illumination

$\langle \hat{a}_s{}^\dagger \hat{a}_s \rangle + \langle \hat{a}_s \hat{a}_s{}^\dagger \rangle$

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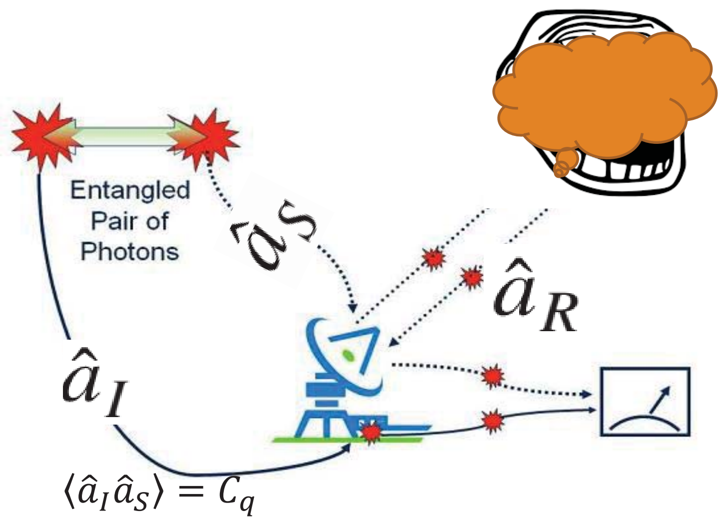

Quantum Illumination

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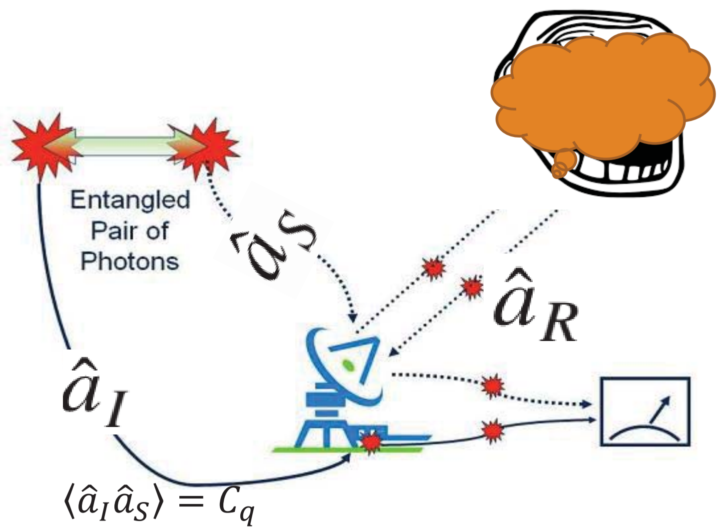
$\langle \hat{a}_s{}^{\dagger} \hat{a}_s \rangle + \langle \hat{a}_s \hat{a}_s{}^{\dagger} \rangle$

$\langle \hat{a}_I{}^{\dagger} \hat{a}_I \rangle + \langle \hat{a}_I \hat{a}_I{}^{\dagger} \rangle$



Quantum Illumination

$\langle \hat{a}_I \hat{a}_S \rangle \propto C_q$

$$|\psi\rangle_{SI} = \sum_{n=0}^{\infty} \sqrt{\frac{N_S^n}{(N_S+1)^{n+1}}} |n\rangle_S |n\rangle_I,$$

$$\mathbf{\Lambda}_{SI} = \frac{1}{4}\begin{bmatrix} S & 0 & C_q & 0 \\ 0 & S & 0 & -C_q \\ C_q & 0 & S & 0 \\ 0 & -C_q & 0 & S \end{bmatrix},$$

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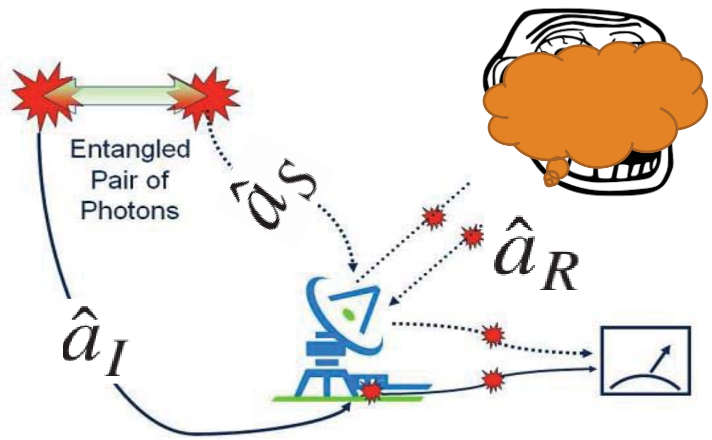
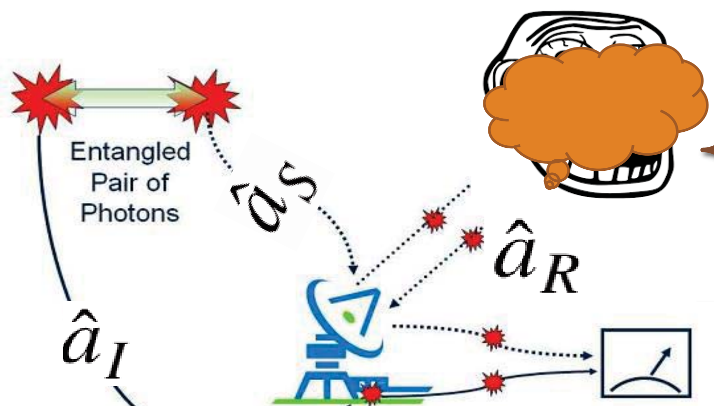


Quantum Illumination

$\langle \hat{a}_I \hat{a}_S \rangle \leq \sqrt{N_S N_I}$

$$|\psi\rangle_{\mathrm{SI}} = \sum_{n=0}^{\infty} \sqrt{\frac{N_S^n}{(N_S+1)^{n+1}}} |n\rangle_S |n\rangle_I, \qquad \boldsymbol{\Lambda}_{\mathrm{SI}} = \frac{1}{4}\begin{bmatrix} S & 0 & C_q & 0 \\ 0 & S & 0 & -C_q \\ C_q & 0 & S & 0 \\ 0 & -C_q & 0 & S \end{bmatrix}, \quad \text{where } S \equiv 2N_S + 1 \text{ and } C_q \equiv 2\sqrt{N_S(N_S+1)}.$$



$\langle \hat{a}_I \hat{a}_S \rangle = C_q$

hypothesis H_0 (target absent) $\quad \hat{a}_R = \hat{a}_B$

hypothesis H_1 (target present) $\quad \hat{a}_R = \sqrt{\kappa}\hat{a}_S + \sqrt{1-\kappa}\hat{a}_B \qquad \kappa \ll 1$

Thermal-photon number $\langle \hat{a}_B{}^{\dagger} \hat{a}_B \rangle = N_B \gg N_S$

Covariance matrix after reflection

under H_0

$$\mathbf{\Lambda}_{\mathrm{RI}}^{(0)} = \frac{1}{4}\begin{bmatrix} B & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & S \end{bmatrix}$$

under H_1

$$\mathbf{\Lambda}_{\mathrm{RI}}^{(1)} = \frac{1}{4}\begin{bmatrix} A & 0 & \sqrt{\kappa}C_q & 0 \\ 0 & A & 0 & -\sqrt{\kappa}C_q \\ \sqrt{\kappa}C_q & 0 & S & 0 \\ 0 & -\sqrt{\kappa}C_q & 0 & S \end{bmatrix}$$

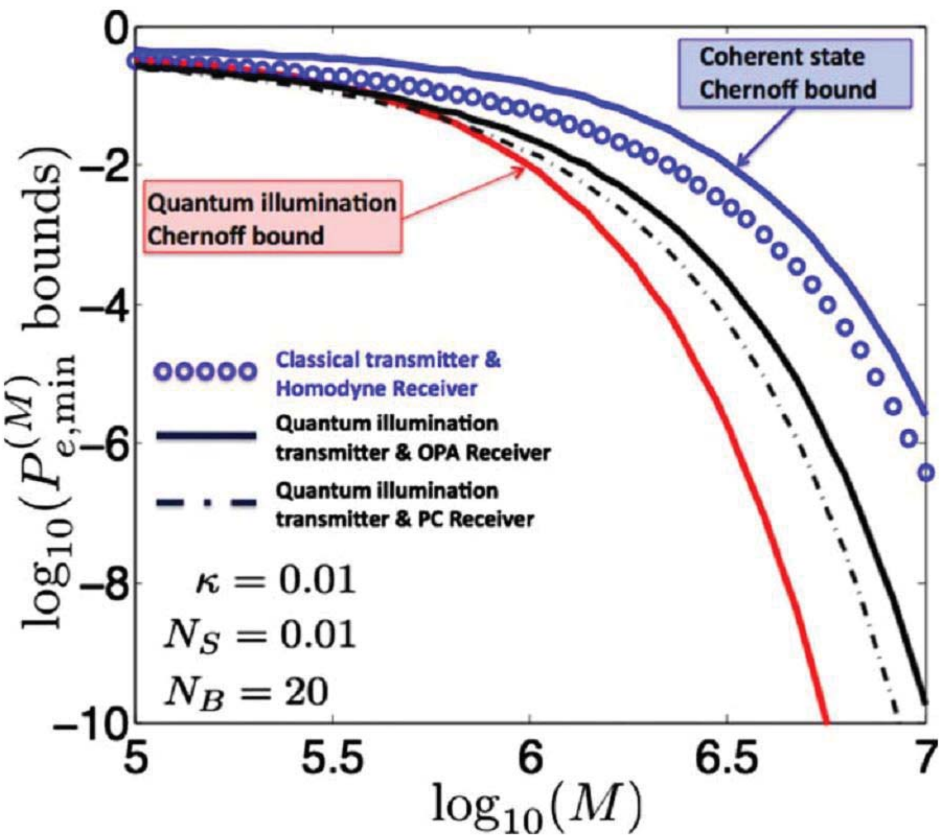
$$B \equiv 2N_B + 1 \quad \text{and} \quad A \equiv 2\kappa N_S + B$$

$$\frac{1}{2}\{\min_{0\le s\le 1} \mathrm{tr}[(\hat{\rho}^{(0)})^s(\hat{\rho}^{(1)})^{(1-s)}]\}^M.$$

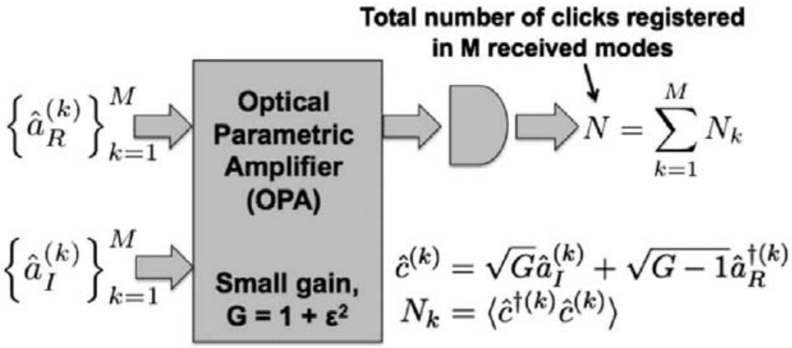
$$\Pr(e)_{\mathrm{QI}} \le e^{-M\kappa N_S/N_B}/2.$$

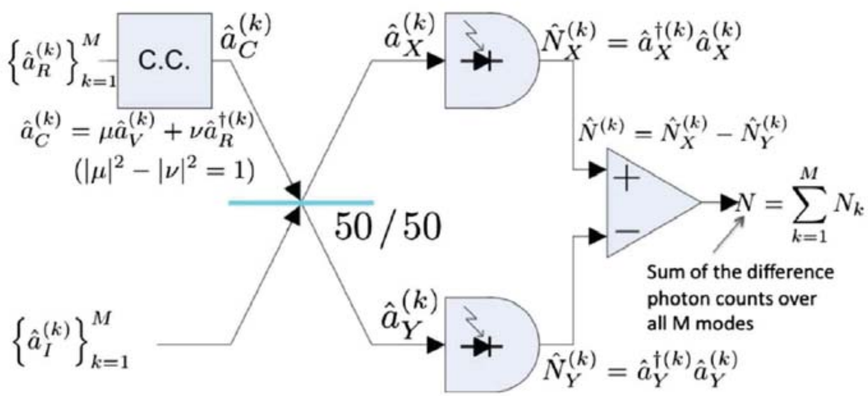
$$\Pr(e)_{\mathrm{CS}} \le e^{-M\kappa N_S(\sqrt{N_B+1}-\sqrt{N_B})^2}/2$$

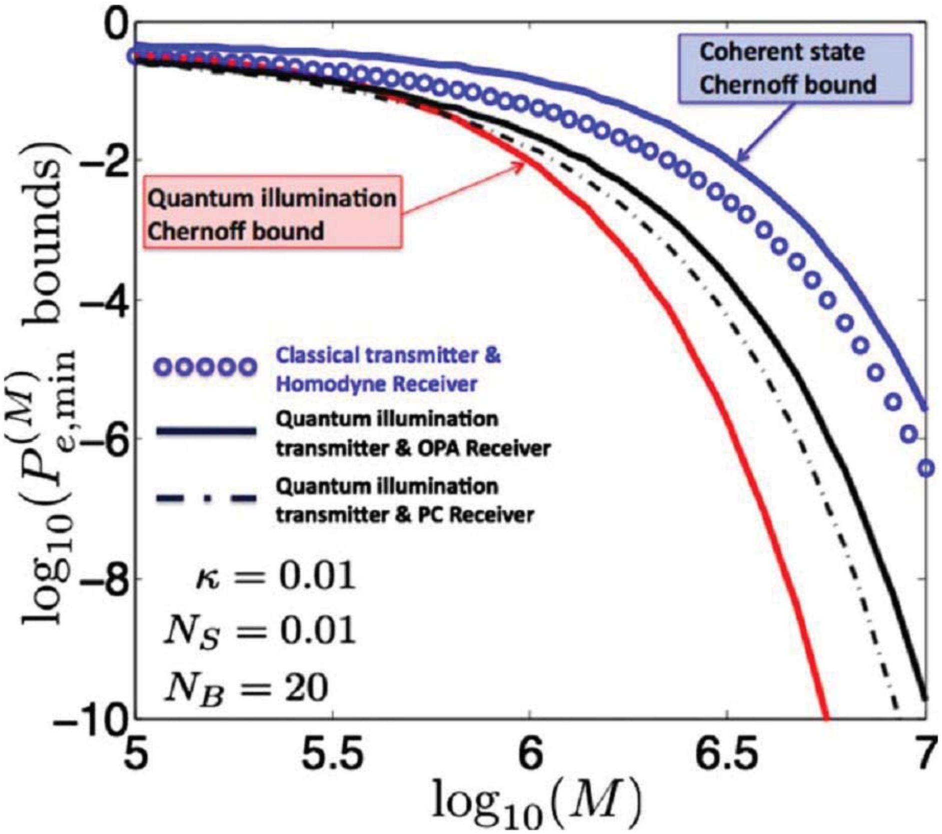
$$\approx e^{-M\kappa N_S/4N_B}/2, \quad \text{when} \quad N_B \gg 1.$$



Phys. Rev. Lett. **101**, 253601 (2008).







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Benefits

I. non-invasive,
II. high-resolution detection of low reflective objects in noisy environment.
III. Improving detection efficiency even if the original entanglement is completely destroyed by noisy environment

Benefits

NO OPTIMUM RECEIVER

I. non-i…
II. H… …ction of low reflective objects in noisy en…
III. Imp…ving detection efficiency even if the original entanglement is completely destroyed by noisy environment

Benefits

I. non-i

II. I ction of low r noisy en

III. Imp ving detection efficiency original entanglement is completely dest ed by noisy environment

NO OPTIMUM RECEIVER

NO MICROWAVE ANALOGOUS

In Optical domain $10^{15} Hz$ $N_B \sim 0$

What about Microwave $frequencies$ $\sim 10^9 Hz$?

- Very noisy environment $N_B \gg 0$

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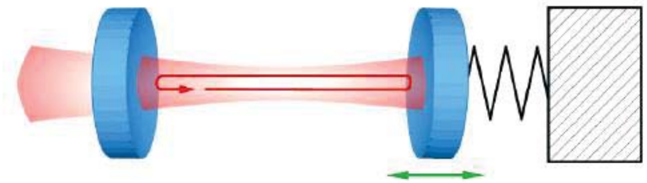
Optomechanics



$$\hat{F}_{Rad} \propto \hat{a}^{+}\hat{a}$$

$$\hat{H} \rightarrow \hat{x}\hat{F}_{Rad} \Rightarrow \hat{x}\hat{a}^{+}\hat{a}$$

$$H = \hbar\omega_c(x)a^{+}a + \frac{1}{2}\hbar\omega_m(x^2 + p^2) + i\hbar E(a^{+}e^{-i\omega_L t} - a\ e^{i\omega_L t})$$

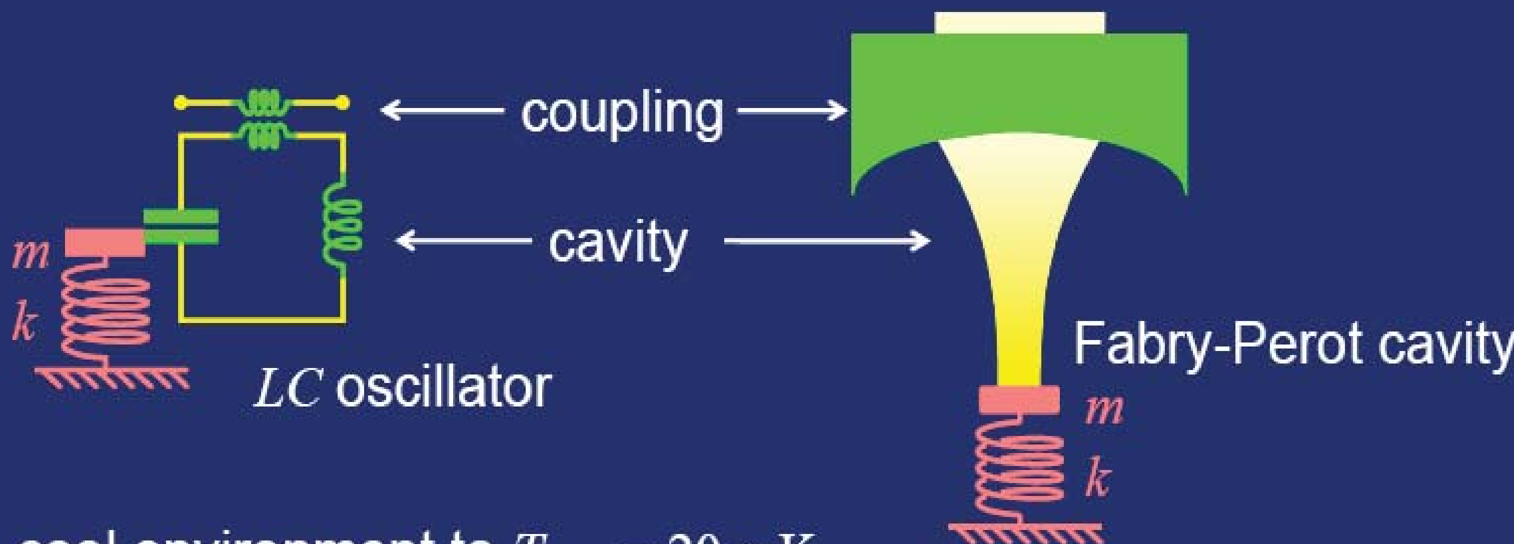
$$= \hbar\omega_c a^{+}a + \frac{1}{2}\hbar\omega_m(x^2 + p^2) + i\hbar E(a^{+}e^{-i\omega_L t} - a\ e^{i\omega_L t}) - \hbar G_0 a^{+}a\,x$$

$$G_0 = (\omega_c / L)\sqrt{\hbar / m\omega_m} \qquad |E| = \sqrt{2P\kappa / \hbar\omega_L}$$

J. D. Mc. Cullen, P. Meystre and E. M. Wright, Opt. Lett. **9**, 193 (1984).

Reduce coupling to the environment by lowering temperature: microwave optomechanics

Microwave "light" in ultralow temperature cryostat



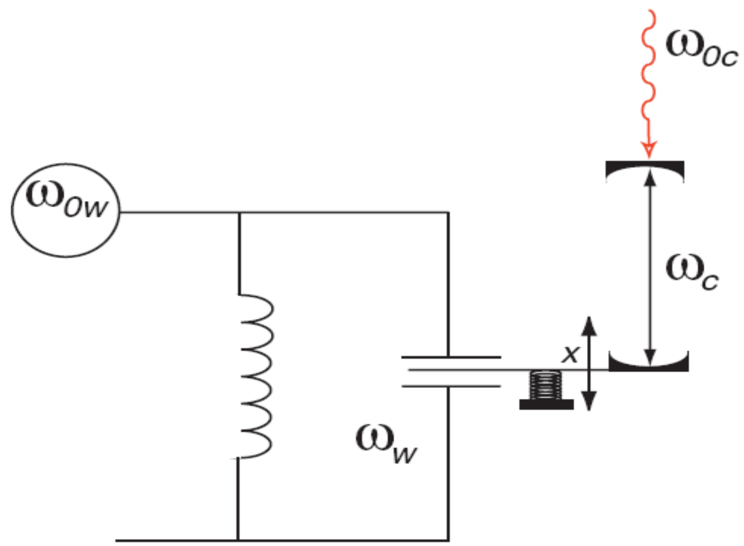
cool environment to $T_{\mathrm{env}} = 20\ \mathrm{mK}$

for 10 MHz oscillator $n_{\mathrm{env}} = 40$

goal: $\Gamma > n_{\mathrm{env}}\gamma$

Optical to Microwave interface

$$H = \frac{p_x^2}{2m} + \frac{m\omega_m^2 x^2}{2} + \frac{\Phi^2}{2L} + \frac{Q^2}{2[C + C_0(x)]} - e(t)Q$$
$$+\hbar\omega_c a^\dagger a - \hbar G_{0c} a^\dagger a x + i\hbar E_c (a^\dagger e^{-i\omega_{0c}t} - a e^{i\omega_{0c}t}).$$

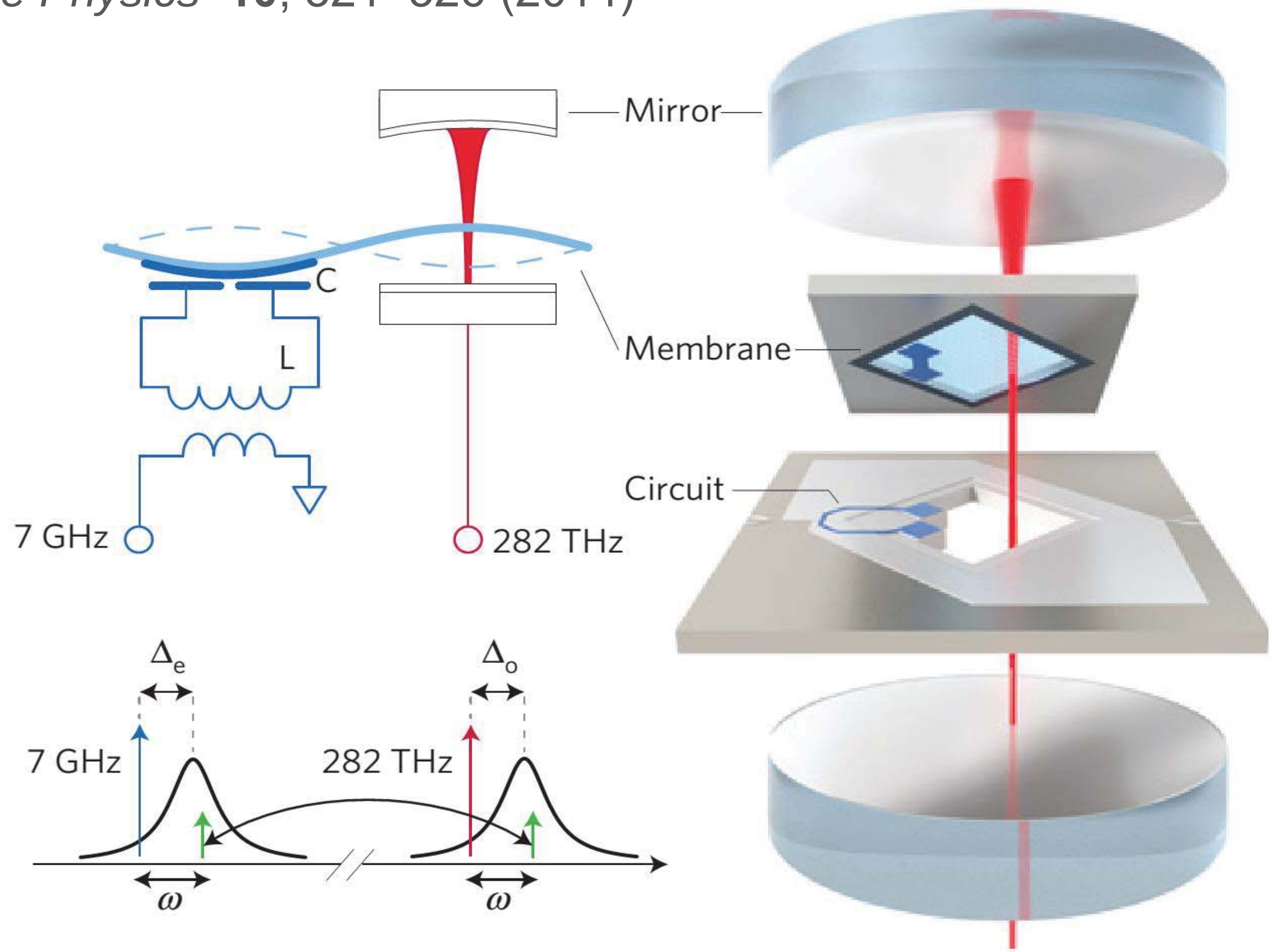


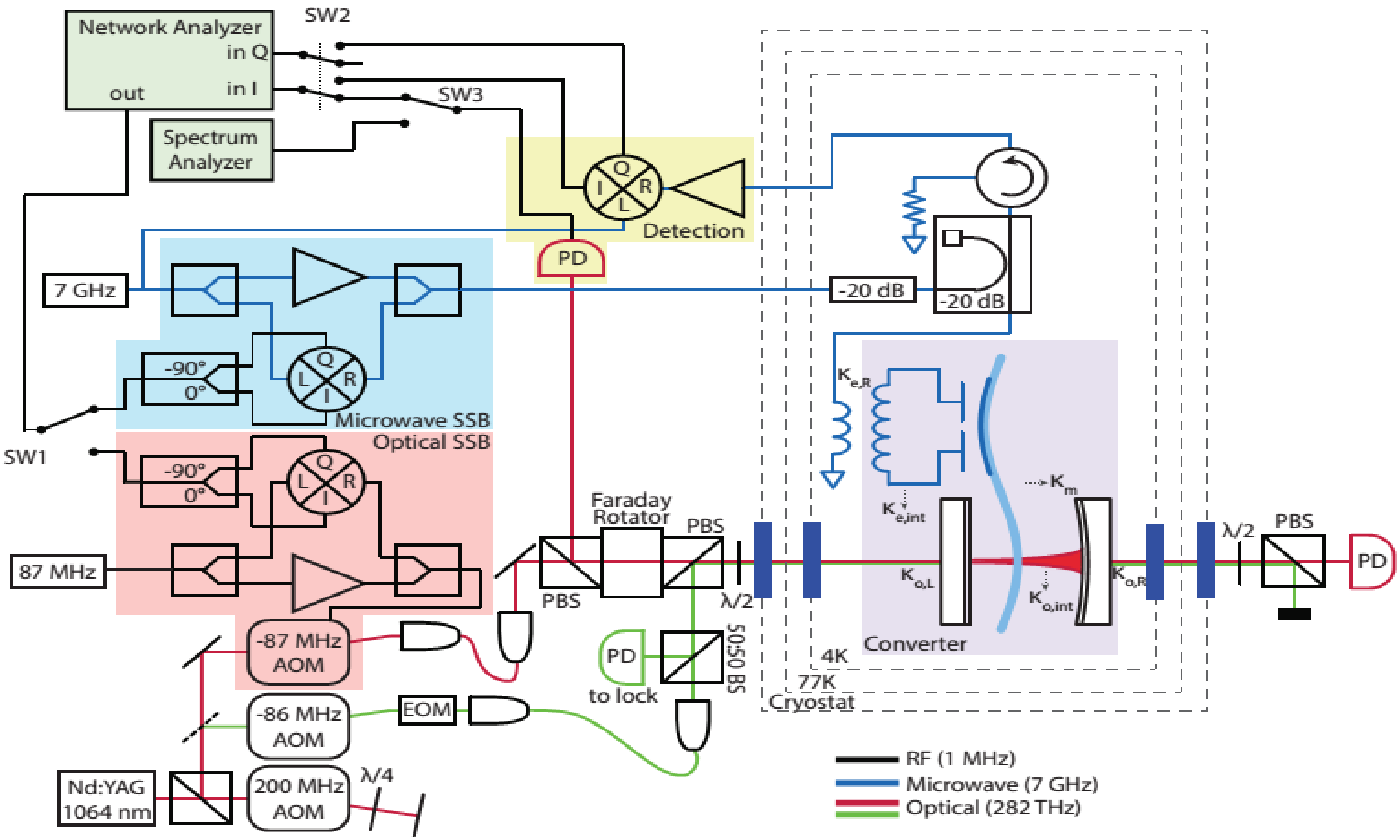
$$\frac{Q^2}{2[C + C_0(x)]} = \frac{Q^2}{2C_\Sigma} - \frac{\mu}{2dC_\Sigma} x(t) Q^2,$$

$$C_\Sigma = C + C_0 \text{ and } \mu = C_0/C_\Sigma$$

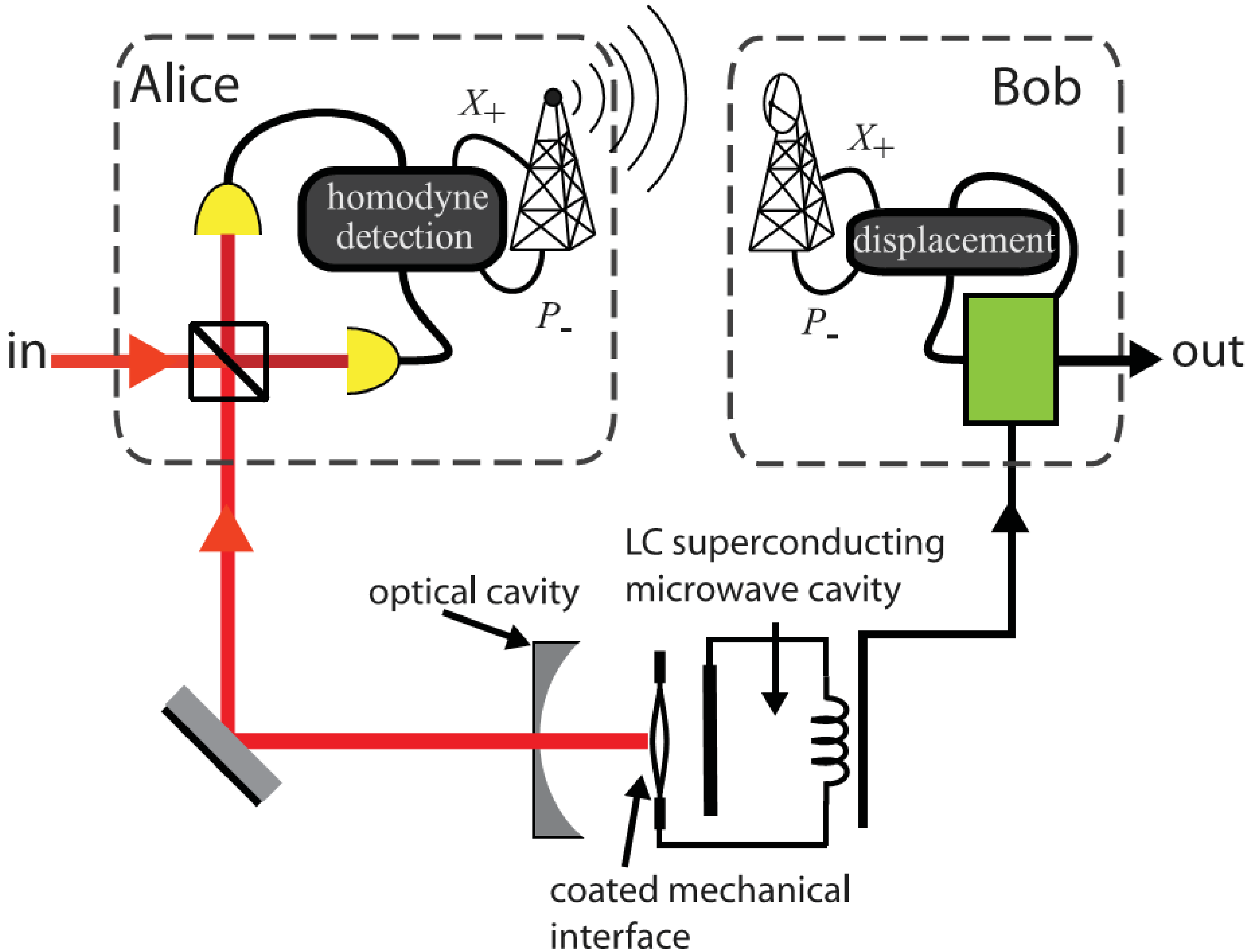
ShB. *et. al*., Rev. A **84**, 042342 (2011).

Nature Physics **10**, 321–326 (2014)





Network Analyzer
in Q
in I
out
SW2
SW3
Spectrum Analyzer
Q
I
R
L
Detection
PD
7 GHz
-20 dB
-20 dB
-90°
0°
Microwave SSB
Optical SSB
SW1
87 MHz
Faraday Rotator
PBS
PBS
λ/2
PD
to lock
50:50 BS
-87 MHz AOM
-86 MHz AOM
EOM
Nd:YAG 1064 nm
200 MHz AOM
λ/4
4K
77K
Cryostat
Converter
λ/2
PBS
PD
RF (1 MHz)
Microwave (7 GHz)
Optical (282 THz)



ShB, et.al, PRL, **109**, 130503 (2012).

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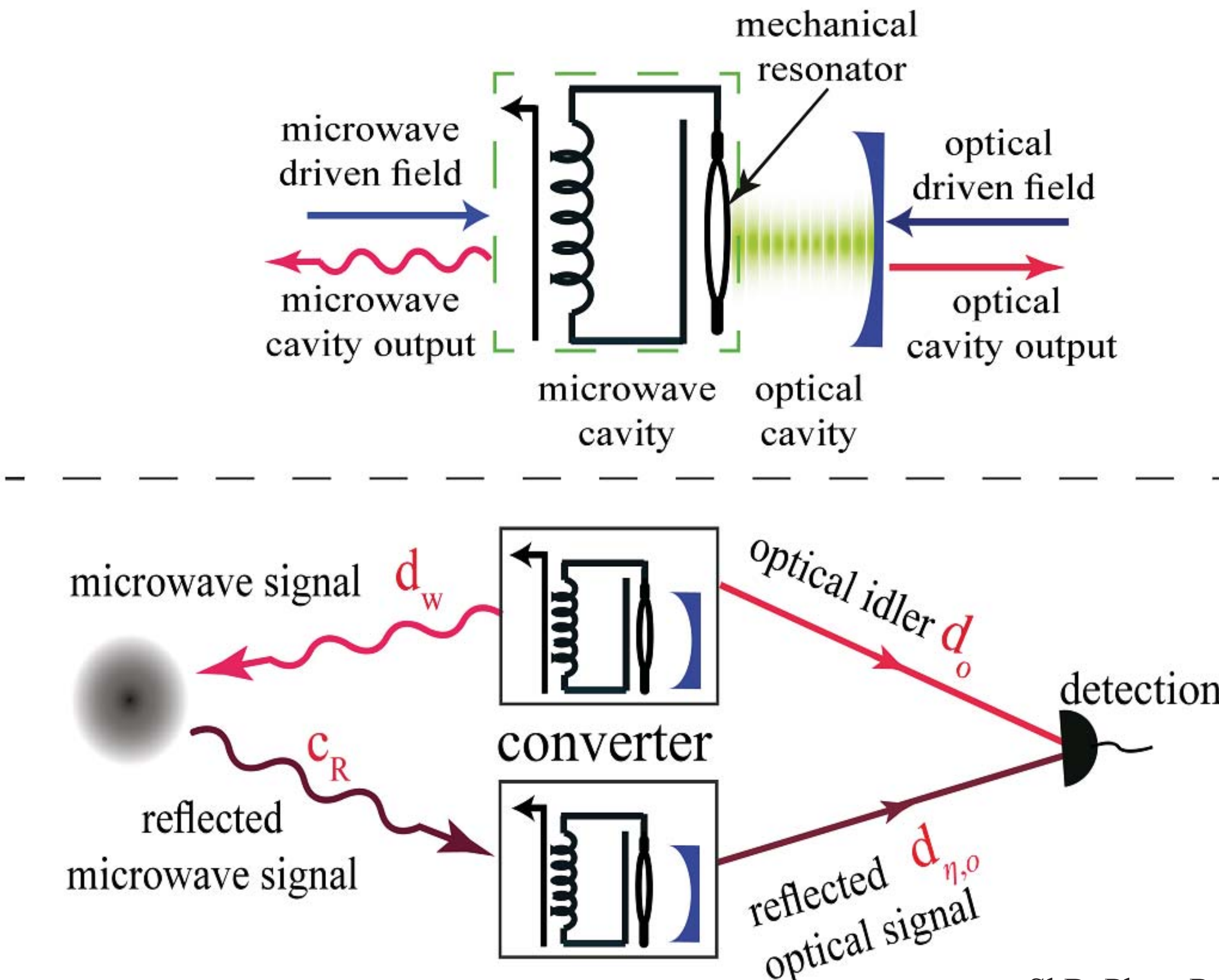
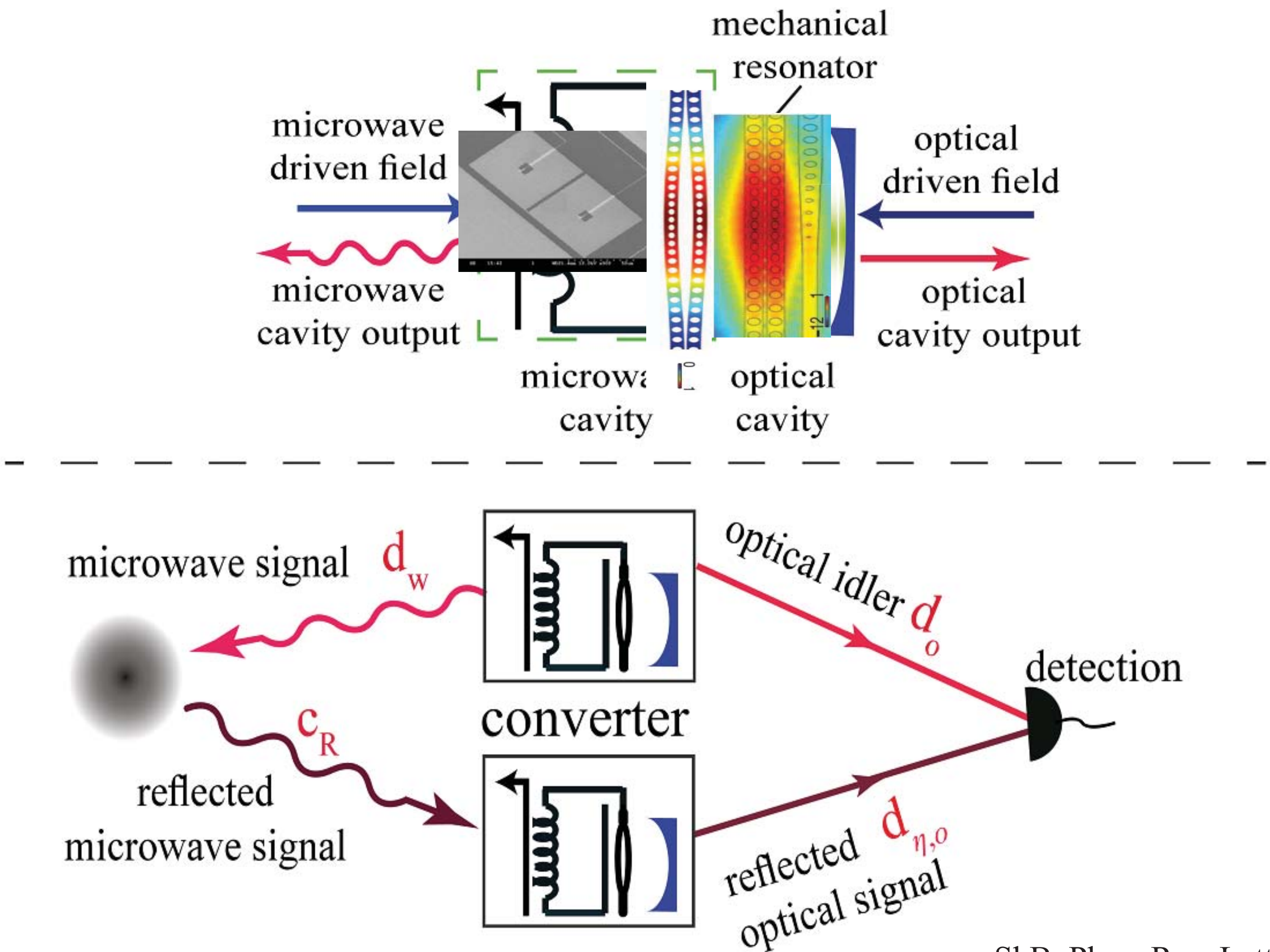


ShB. Phys. Rev. Lett. **114**, 080503(2015)



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$$\hat{H} = \hbar\omega_M \hat{b}^\dagger \hat{b} + \hbar \sum_{j=\mathrm{w},o} \omega_j \hat{a}_j^\dagger \hat{a}_j + \frac{\hbar g_\mathrm{w}}{2}(\hat{b}^\dagger + \hat{b})(\hat{a}_\mathrm{w} + \hat{a}_\mathrm{w}^\dagger)^2 + \hbar g_o (\hat{b}^\dagger + \hat{b})\hat{a}_o^\dagger \hat{a}_o$$
$$+ i\hbar E_\mathrm{w}(e^{i\omega_{\mathrm{d,w}}t} - e^{-i\omega_{\mathrm{d,w}}t})(\hat{a}_\mathrm{w} + \hat{a}_\mathrm{w}^\dagger) + i\hbar E_o(\hat{a}_o^\dagger e^{-i\omega_{\mathrm{d},o}t} - \hat{a}_o e^{i\omega_{\mathrm{d},o}t}),$$

In a rotating frame with respect to $\hbar\omega_{\mathrm{d,w}} a_\mathrm{w}^\dagger a_\mathrm{w} + \hbar\omega_{\mathrm{d},o} a_o^\dagger a_o$

$$H = \hbar\omega_M \hat{b}^\dagger \hat{b} + \hbar \sum_{j=\mathrm{w},o} \left[\Delta_{0,j} + g_j(\hat{b}^\dagger + \hat{b})\right] \hat{a}_j^\dagger \hat{a}_j + H_\mathrm{dri},$$

the interaction picture with respect to the free Hamiltonian

$$H = \hbar \sum_{j=\mathrm{w},o} G_j(\hat{b}\mathrm{e}^{-\mathrm{i}\omega_M t} + \hat{b}^\dagger \mathrm{e}^{\mathrm{i}\omega_M t})(\hat{c}_j^\dagger \mathrm{e}^{\mathrm{i}\Delta_j t} + \hat{c}_j \mathrm{e}^{-\mathrm{i}\Delta_j t}),$$

By setting the cavity detunings $\Delta_\mathrm{w} = -\Delta_\mathrm{o} = \omega_M$

$$\hat{H} = \hbar G_o(\hat{c}_o \hat{b} + \hat{b}^\dagger \hat{c}_o^\dagger) + \hbar G_\mathrm{w}(\hat{c}_\mathrm{w} \hat{b}^\dagger + \hat{b}\hat{c}_\mathrm{w}^\dagger).$$

$$\mathbf{V}(\omega) = \begin{pmatrix} V_{11} & 0 & V_{13} & 0 \\ 0 & V_{11} & 0 & -V_{13} \\ V_{13} & 0 & V_{33} & 0 \\ 0 & -V_{13} & 0 & V_{33} \end{pmatrix},$$

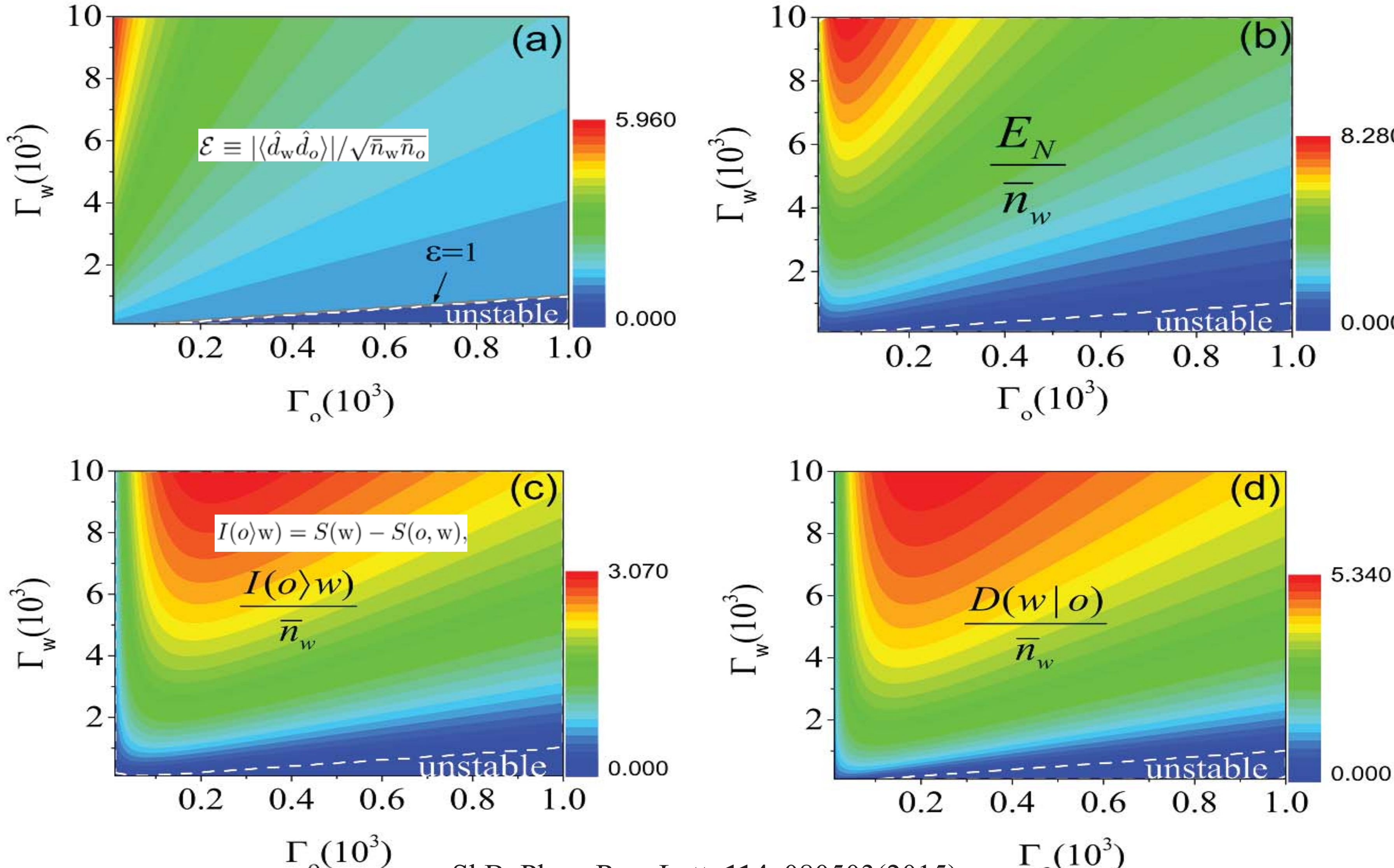
$$V_{11} = \frac{\langle X_{\mathrm{w}}(\omega) X_{\mathrm{w}}(\omega')\rangle}{\delta(\omega+\omega')} = \bar{n}_{\mathrm{w}} + 1/2,$$

$$V_{33} = \frac{\langle X_o(\omega) X_o(\omega')\rangle}{\delta(\omega+\omega')} = \bar{n}_o + 1/2,$$

$$V_{13} = \frac{\langle X_{\mathrm{w}}(\omega) X_o(\omega') + X_o(\omega') X_{\mathrm{w}}(\omega)\rangle}{2\delta(\omega+\omega')} = \langle \hat{d}_{\mathrm{w}} \hat{d}_o \rangle,$$

$$E_N = \max[0, -\log(2\zeta^-)], \qquad \zeta^- = 2^{-1/2}\left(V_{11}^2 + V_{33}^2 + 2V_{13}^2 - \sqrt{(V_{11}^2 - V_{33}^2)^2 + 4V_{13}^2(V_{11}+V_{33})^2}\right)^{1/2}.$$

smallest partially-transposed symplectic eigenvalue of $\mathbf{V}(\omega)$



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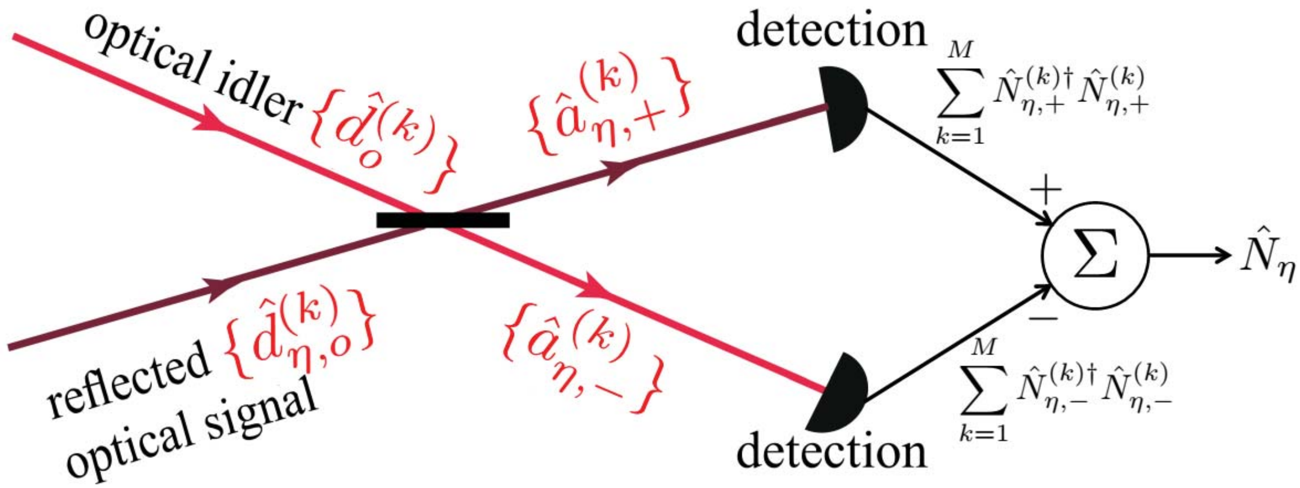
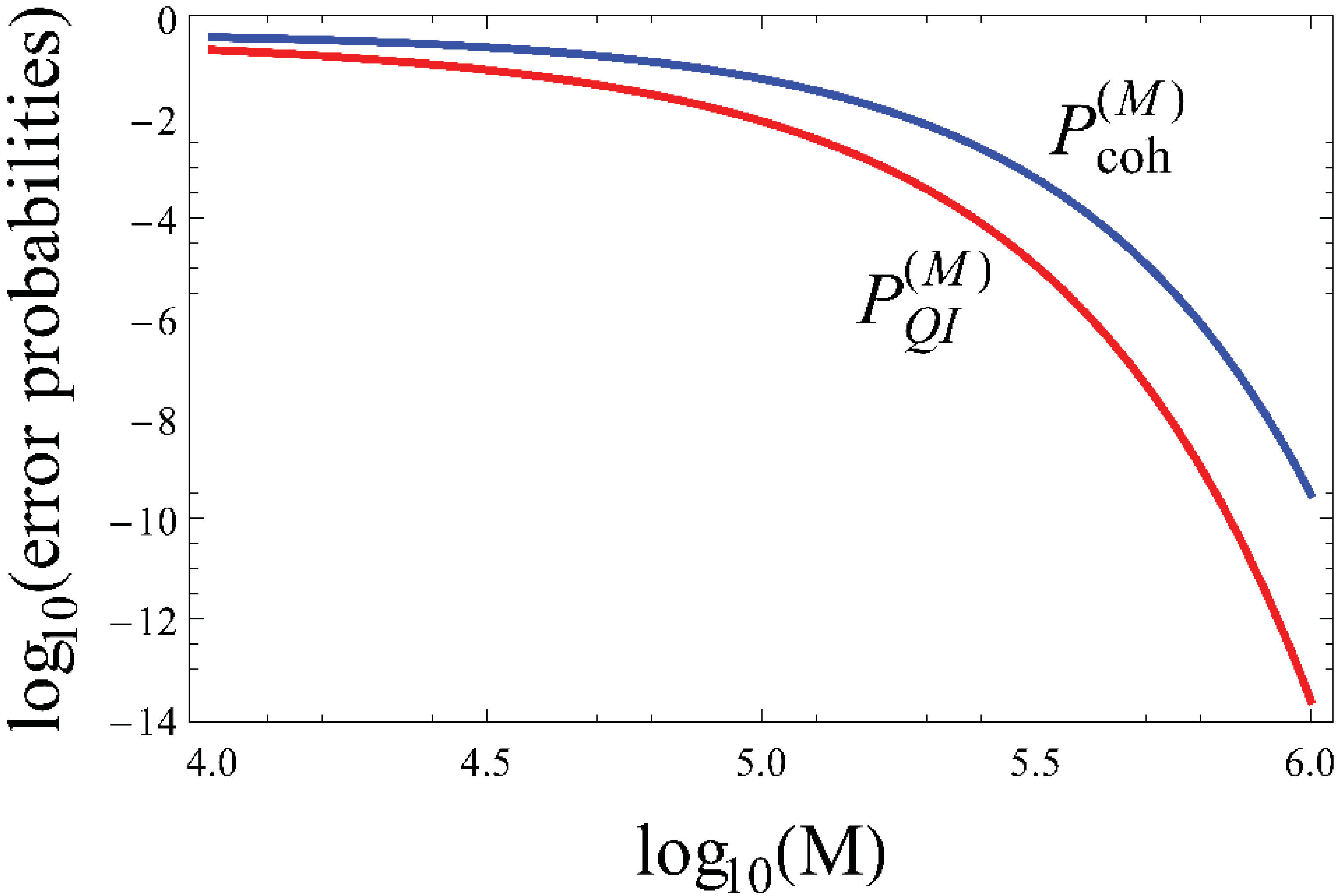


$$\hat{N}^{(k)} = \hat{a}_{\eta,+}^{\dagger(k)}\hat{a}_{\eta,+}^{(k)} - \hat{a}_{\eta,-}^{\dagger(k)}\hat{a}_{\eta,-}^{(k)}.$$

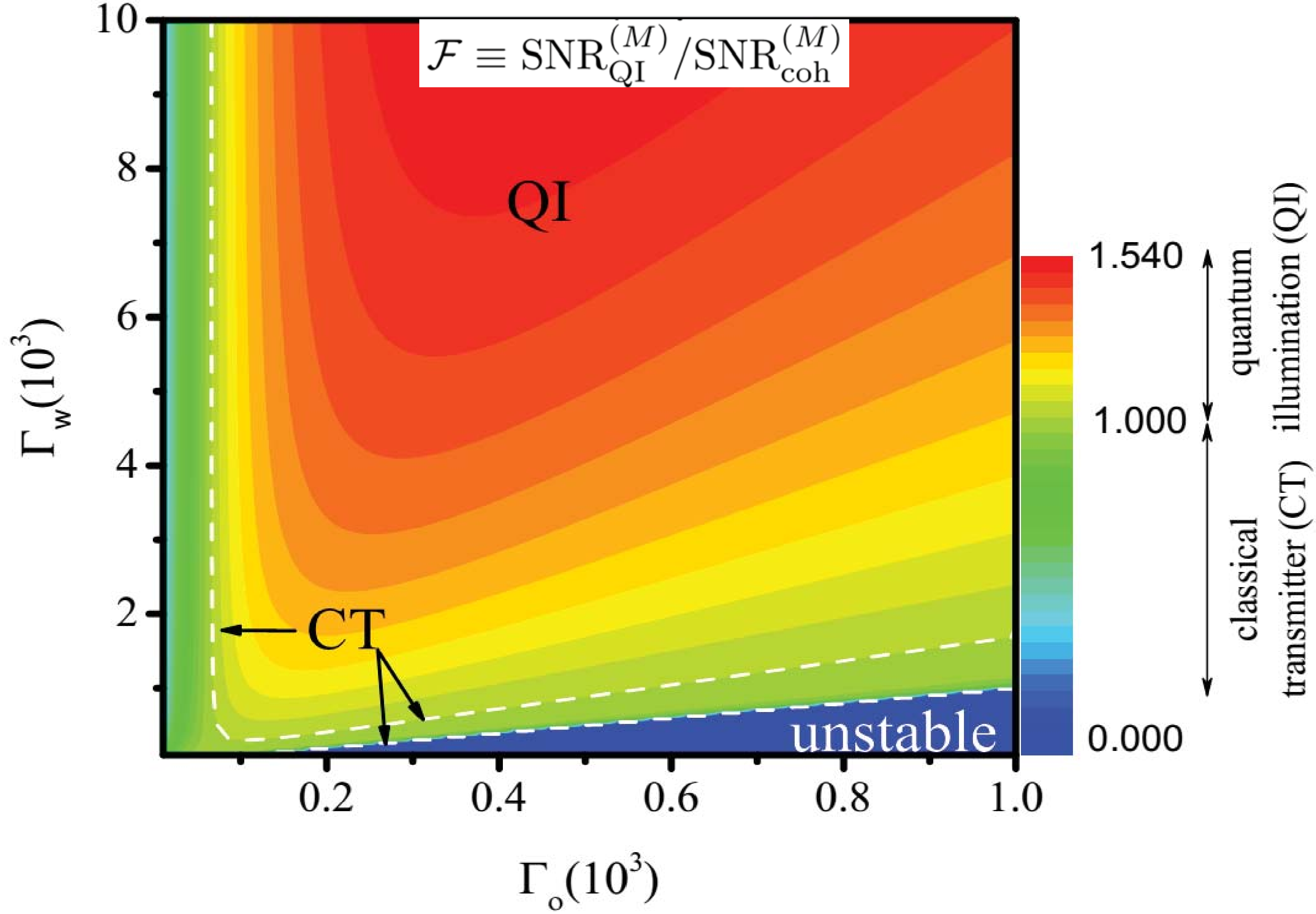
$$P_{\mathrm{EOM}}^{(M)} = \frac{1}{2}\mathrm{erfc}\Big(\sqrt{\frac{M}{2}}\gamma\Big),$$

where the signal-to-noise ratio γ is defined as

$$\gamma = \frac{(\bar{N}_{\eta\neq 0} - \bar{N}_{\eta=0})}{(\Delta\bar{N}_{\eta\neq 0} + \Delta\bar{N}_{\eta=0})}.$$



log10(error probabilities)
0
−2
−4
−6
−8
−10
−12
−14
4.0
4.5
5.0
5.5
6.0
log10(M)
$P_{\mathrm{coh}}^{(M)}$
$P_{QI}^{(M)}$



$\mathcal{F} \equiv \mathrm{SNR}_{\mathrm{QI}}^{(M)}/\mathrm{SNR}_{\mathrm{coh}}^{(M)}$
QI
CT
unstable
1.540
1.000
0.000
quantum illumination (QI)
classical transmitter (CT)
$\Gamma_w(10^3)$
$\Gamma_o(10^3)$
10
8
6
4
2
0.2
0.4
0.6
0.8
1.0

Conclusion

I. Quantum Illumination is superior to classical illumination

II. Microwave-optical converter is an interesting device for implementing microwave quantum illumination

III. Microwave quantum illumination shows 3dB improvement compare to standard microwave illumination

Thank you

S. Pirandola,
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Camerino

J. Shapiro,
MIT

D. DiVincenzo
Aachen

ShB. Phys. Rev. Lett. **114**, 080503(2015)