Holographic Renormalization Group

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Outline

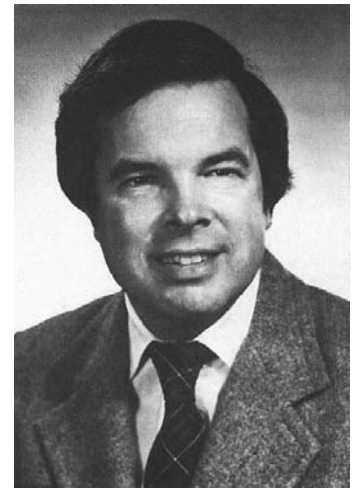
- Wilsonian Renormalization Group (WRG)
- AdS/CFT Correspondence
- Construction of Holographic WRG
- Open Issues

Wilsonian Renormalization Group

Integrating out high energy degrees freedom
of a microscopic Theory

⇓

Effective field theory



K. Wilson (1936)

$$Z = \int \mathcal{D}\phi\Big|_{k>\Lambda} \mathcal{D}\phi\Big|_{k<\Lambda} e^{-S[\phi]} = \int \mathcal{D}\phi\Big|_{k<\Lambda} e^{-S_\Lambda[\phi]}$$

Wilsonian Renormalization Group

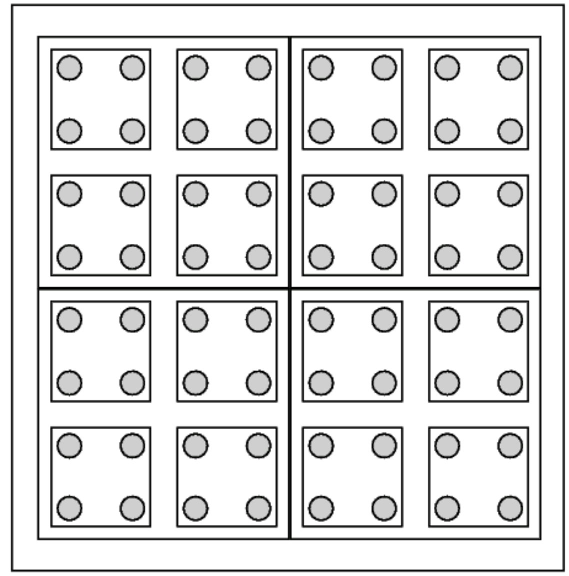
$$Z = \int \mathcal{D}\phi\Big|_{k>\Lambda} \mathcal{D}\phi\Big|_{k<\Lambda} e^{-S[\phi]} = \int \mathcal{D}\phi\Big|_{k<\Lambda} e^{-S_\Lambda[\phi]}$$

- Separate the path integral variables into high and low energy.
- Integrate out the high energy variables. Result: action with an infinite number of couplings.
- Cutoffs do not play nicely with symmetries (gauge, coordinate, extended supersymmetry, dualities!).

Wilsonian Renormalization Group

Considering all sites, the physics is described by *H(T,J).*

Now consider the blocks, if the physics is described by *H(T',J'),* the theory describing the system is renormalizable.



H(T,J)→H(T',J')→H(T'',J'')→…→flowing toward a fixed point

RG Equations

Physics does not depend on the energy scale leads to Callan-Symanzik equation:

$$\frac{d}{d\Lambda} G^{(n)}(x_1, ..., x_n) = 0$$

- Fix the value of the parameters at an arbitrary energy scale.
- Solve CS equation using the renormalization conditions.
- Result: running of the parameters.

AdS/CFT Correspondence

A classical weakly coupled theory of gravity living in *d+1* dimension is dual to a strongly coupled conformal field theory in large *N* limit living in *d* dimension.

$$\left\langle \exp\left\{ \int d^d x\, \phi_0(x)\mathcal{O}(x) \right\} \right\rangle_{\mathrm{CFT}} = Z_{\mathrm{gravity}}[\phi_0]$$

Holographic WRG

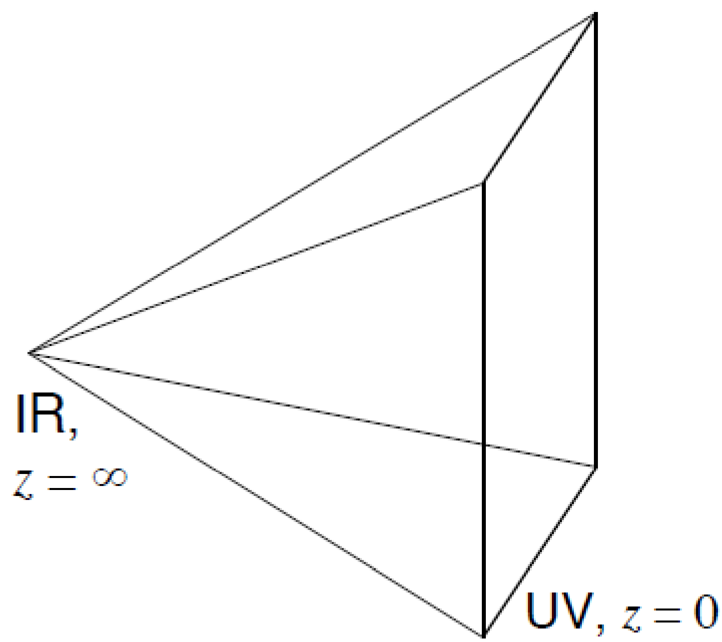
Gravity	⟷	Gauge Theory
↓		↓
?		Wilsonian RG

Idea: pull the Wilson RG back to the gravity theory, where it becomes the holographic RG.

Holographic WRG

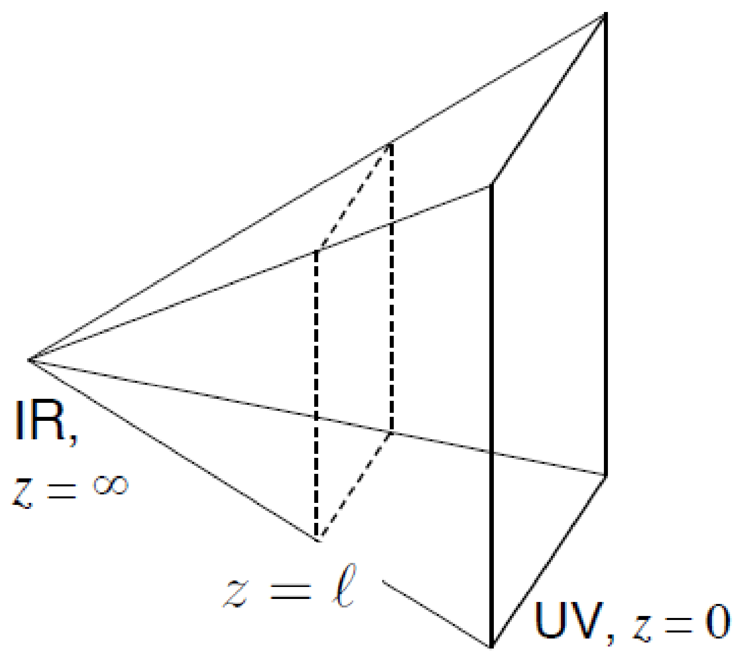
AdS/CFT maps, CFT energy to 1/z, where z is the emergent coordinate:

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + dx^\mu dx_\mu\right)$$



Holographic WRG

This suggests that we should integrate fields at small z first, and progressively move the cutoff to the IR.



Outline for construction of HWRG

1- Splitting the path integral in the bulk.

2- Splitting the path integral in the field theory, and looking for parallels.

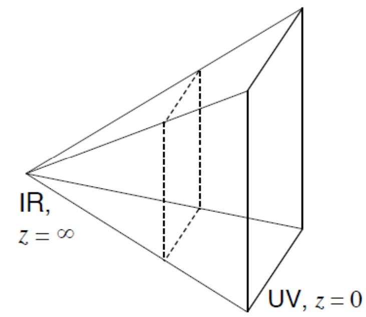
Splitting the bulk path integral

- Consider some scalar fields in a *fixed* AdS background:

$$\begin{aligned} Z &= \int \mathcal{D}\varphi \, e^{-\int_0^\infty dz\, \mathcal{L}(z)} \\ &= \int \mathcal{D}\varphi\big|_{z>\ell} \mathcal{D}\tilde{\varphi} \, \mathcal{D}\varphi\big|_{z<\ell} \, e^{-\int_\ell^\infty dz\, \mathcal{L}(z) - \int_0^\ell dz\, \mathcal{L}(z)} \\ &= \int \mathcal{D}\tilde{\varphi} \, \Psi_{\mathrm{IR}}(\ell, \tilde{\varphi}) \Psi_{\mathrm{UV}}(\ell, \tilde{\varphi}) \, . \end{aligned}$$

where $\tilde{\varphi}(x) = \varphi(z = \ell, x)$

$$Z = \int \mathcal{D}\tilde{\phi}\, \Psi_{\mathrm{IR}}(\ell, \tilde{\phi}) \Psi_{\mathrm{UV}}(\ell, \tilde{\phi})$$



- It is plausible to interpret the large-z part of the path integral in terms of a CFT with a UV cutoff (Susskind & Witten, 1998)

$$\Psi_{\mathrm{IR}}(\ell, \tilde{\varphi}) = \int \mathcal{D}M|_{k\ell<1} \exp\left\{ - \int d^d x\, \tilde{\varphi}^I(x) \mathcal{O}_I(x) \right\}$$

Where *M* stands for matrix fields of the CFT. This is the usual dictionary between bulk fields and single-trace boundary interactions but this time with a UV cutoff.

$$Z = \int \mathcal{D}\tilde{\phi}\, \Psi_{\rm IR}(\ell, \tilde{\phi}) \Psi_{\rm UV}(\ell, \tilde{\phi})$$

- In this stage assume that $\Psi_{\rm UV}$ has a local representation in scale ℓ so it can be expanded in an infinite number of higher derivative local terms (like the Wilsonian action).

$$Z = \int \mathcal{D}\tilde{\phi}\, \Psi_{\rm IR}(\ell, \tilde{\phi}) \Psi_{\rm UV}(\ell, \tilde{\phi})$$

The role of the functional integral over $\tilde{\phi}$ on the interface looks like some weighted average over couplings.

$$Z = \int \mathcal{D}\tilde{\phi}\, \Psi_{\rm IR}(\ell, \tilde{\phi}) \Psi_{\rm UV}(\ell, \tilde{\phi})$$

- For example considering a single scalar with a Gaussian UV factor

$$\Psi_{\rm UV}(\ell, \tilde{\varphi}) = \exp\left\{ -\frac{1}{2h} \int d^d x \, (\tilde{\varphi}(x) - g(x))^2 \right\}$$

Using our postulate for $\Psi_{\rm IR}$ and carrying out the integral over $\tilde{\phi}$ gives

$$Z \propto \int \mathcal{D}A|_{k\ell<1} \exp\left\{ - \int d^d x \left(g(x)\mathcal{O}(x) - \frac{h}{2}\mathcal{O}(x)^2 \right) \right\}$$

Double-trace terms are generated!

$$Z = \int \mathcal{D}\tilde{\phi}\, \Psi_{\rm IR}(\ell, \tilde{\phi}) \Psi_{\rm UV}(\ell, \tilde{\phi})$$

- In general if we read off the field theory effective action using our postulate on $\Psi_{\rm IR}$ it leads to

$$e^{-S_\ell} = \int \mathcal{D}\tilde{\varphi}\, \exp\left\{ - \int d^d x\, \tilde{\varphi}^I(x) \mathcal{O}_I(x) \right\} \Psi_{\rm UV}(\ell, \tilde{\varphi})$$

Thus the Wilsonian action contains multi-trace terms, localized on scale ℓ .

HRG Equations

Varying ℓ gives radial Schrodinger equations

$$\begin{aligned} \partial_\ell \Psi_{\rm IR}(\ell,\tilde\varphi) &= H(\tilde\varphi, \delta/\delta\tilde\varphi)\Psi_{\rm IR}(\ell,\tilde\varphi)\,, \\ \partial_\ell \Psi_{\rm UV}(\ell,\tilde\varphi) &= -H(\tilde\varphi, \delta/\delta\tilde\varphi)\Psi_{\rm UV}(\ell,\tilde\varphi) \end{aligned}$$

The 'Wilsonian action' is the integral transform of UV and satisfies.

$$\partial_\ell e^{-S_\ell} = -H(\delta/\delta\mathcal{O}, \mathcal{O})e^{-S_\ell}$$

This should be compared with the RG on the path integral is independent of where the splitting is done, e field theory side.

$$0 = \frac{d}{d\ell}Z = \frac{d}{d\ell}\left\langle e^{-S_\ell}\right\rangle_\ell$$

Outlook

- WRG leads to the running of the parameters.
- AdS/CFT is correspondence between a gravity theory with a QFT (without gravity).
- Applying the AdS/CFT recipe to a part of AdS space leads to a recipe for HWRG.
- The relation between the cutoffs (AdS space and QFT) is not determined yet.
- We don’t have a recipe of HWRG for the general gauge/gravity duality.