بسم الله الرحمن الرحيم

Non-Gaussianity

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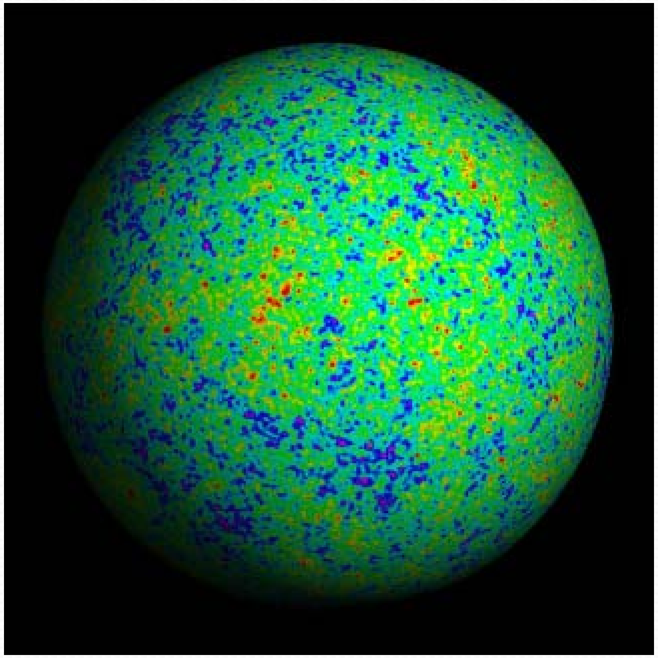
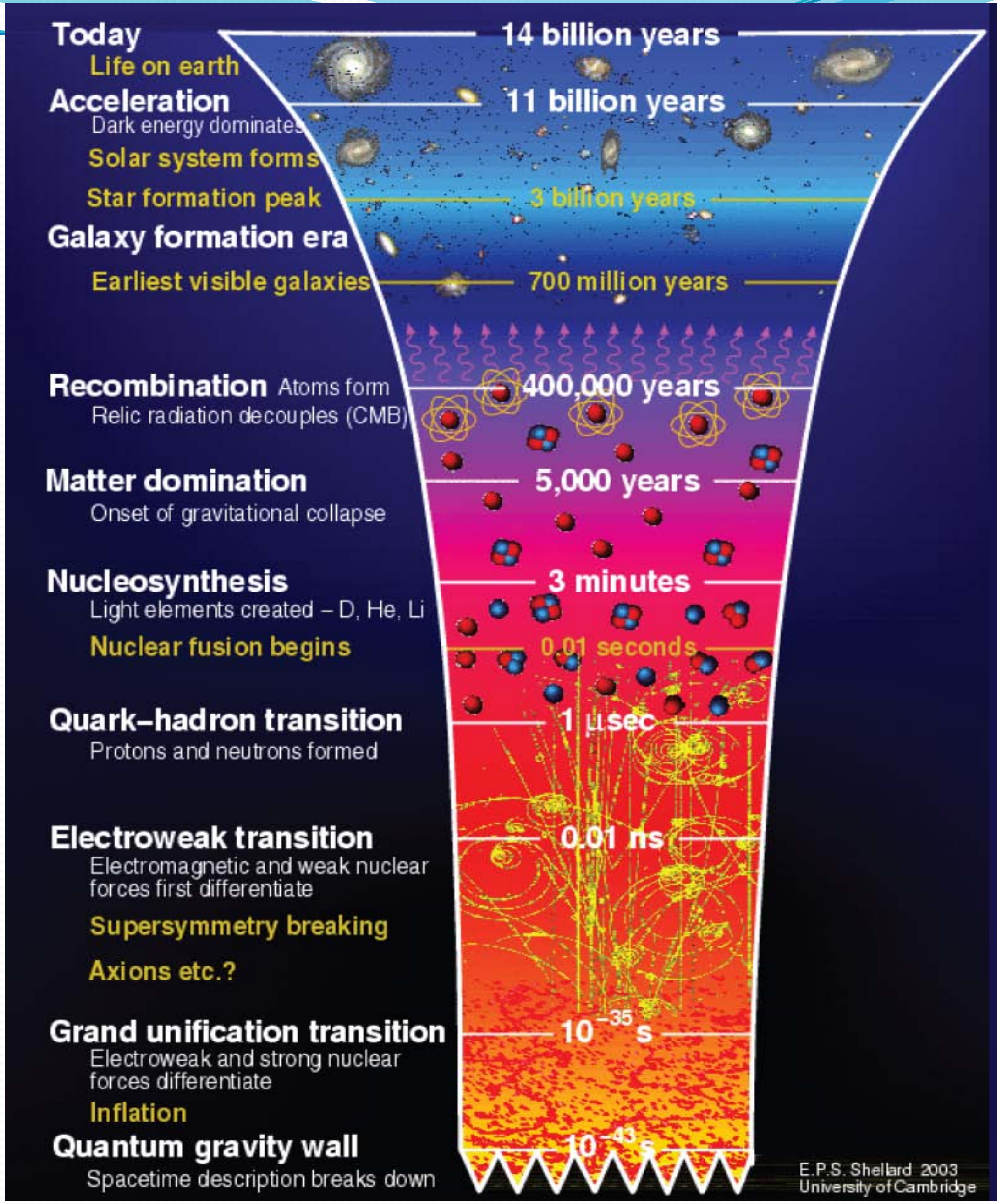
Outline

- Inflation
- Observational cosmology
- Theoretical cosmology
- Non-Gaussianity and its charcteristics



Standard Model of Cosmology



Problems of standard model

- Flatness problem
- Horizon problem
- Monopole problem

Solving the problems: Inflation

- Accelerating universe can solve the problems.

A.Guth 1980

- Solution to:
  - Flatness problem
  - Horizon problem
  - Monopoles problem

Dynamics of Inflation

Friedmann equations

$$\begin{cases} H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{m_{P\ell}^2}\rho \\ \frac{\ddot{a}}{a} = -\frac{4\pi}{3m_{P\ell}^2}(\rho + 3P) \end{cases}$$

$\longrightarrow$ Accelerating the universe needs extraordinary content

- Matter content of universe at inflationary phase is inflaton, a scalar field

- Accelerating is possible:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad , \quad P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

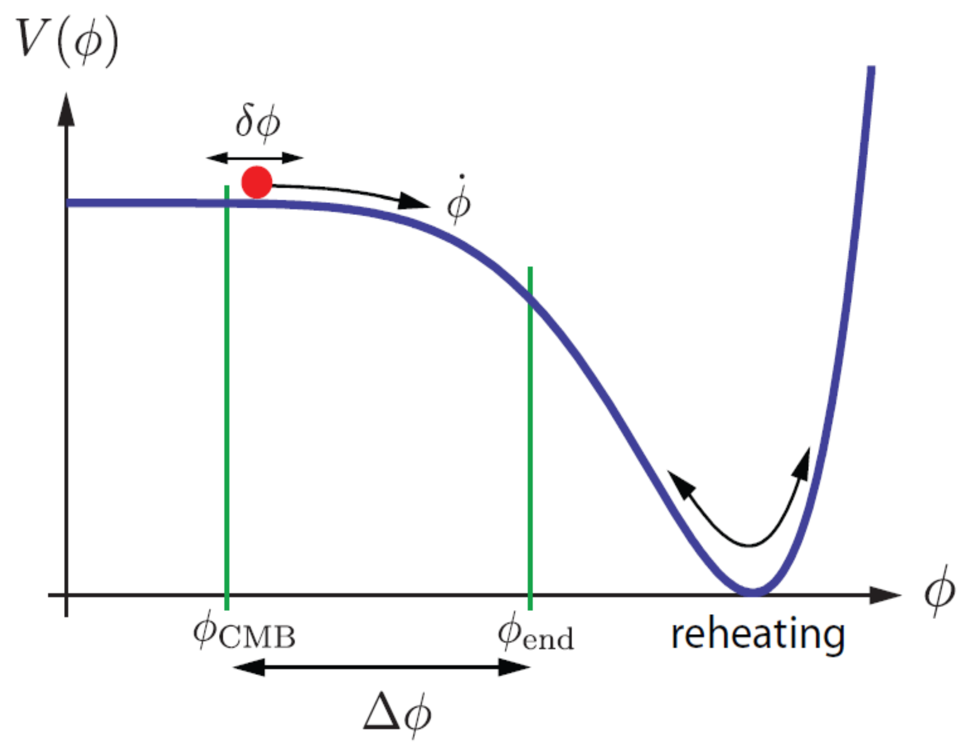
conditions for inflation:

- Slow-Roll conditions:

$$\epsilon = \frac{m_{\rm pl}^2}{16\pi}\left(\frac{V_\phi}{V}\right)^2 , \quad \eta = \frac{m_{\rm pl}^2 V_{\phi\phi}}{8\pi V}$$

$$\epsilon \ll 1 \text{ and } |\eta| \ll 1$$

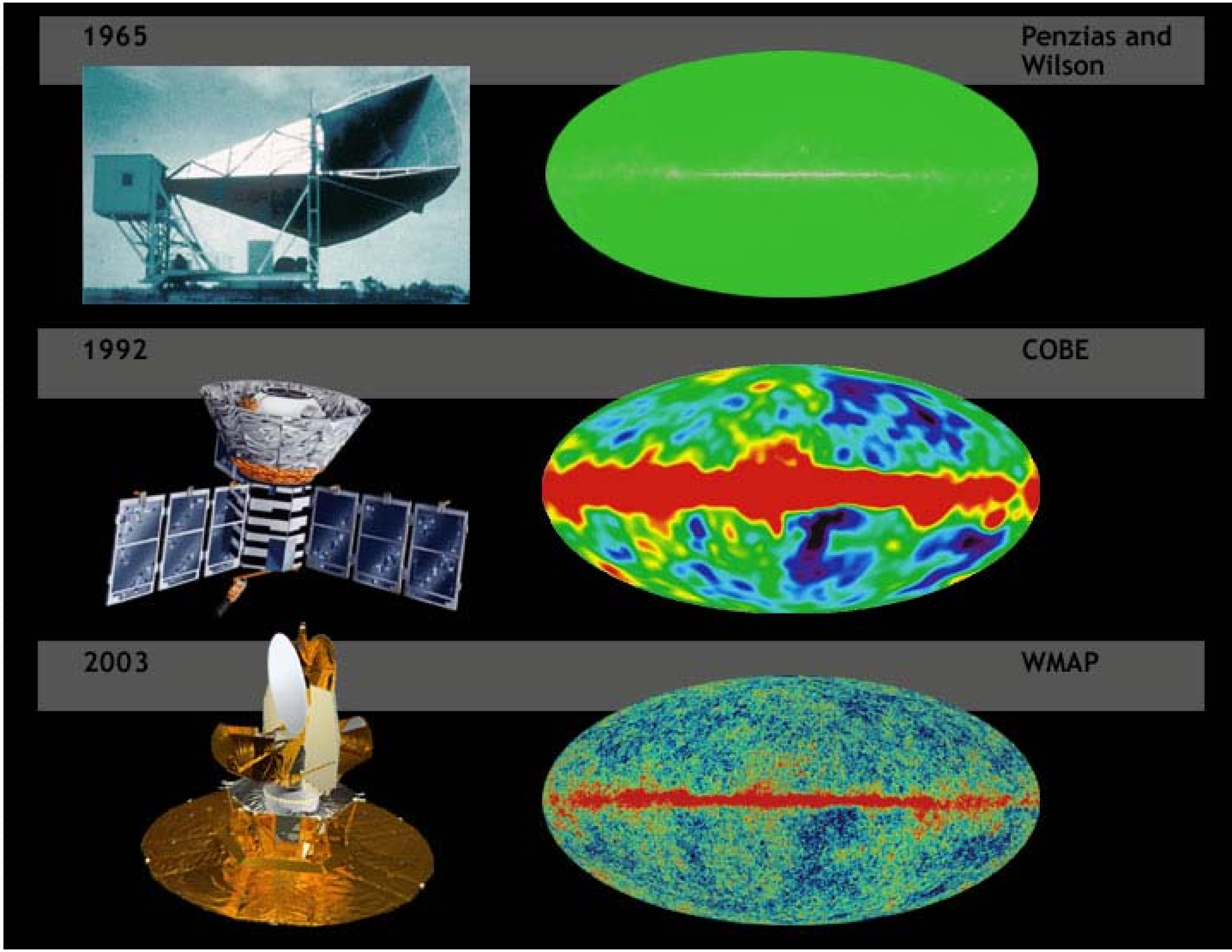
- In order to inflation occurs and lasts sufficiently long time.



B.~A.~Bassett, S.~Tsujikawa and D.~Wands, %``Inflation dynamics and reheating,'' Rev.\ Mod.\ Phys.\ \ {\bf 78}, 537 (2006) [astro-ph/0507632].

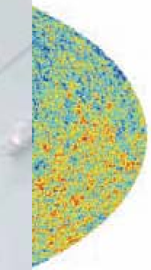
CMB: The observations

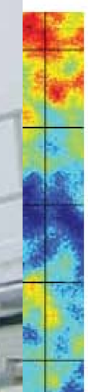
CMB: Radio waves remained from early universe.
We observe them when they reach to our horizon.



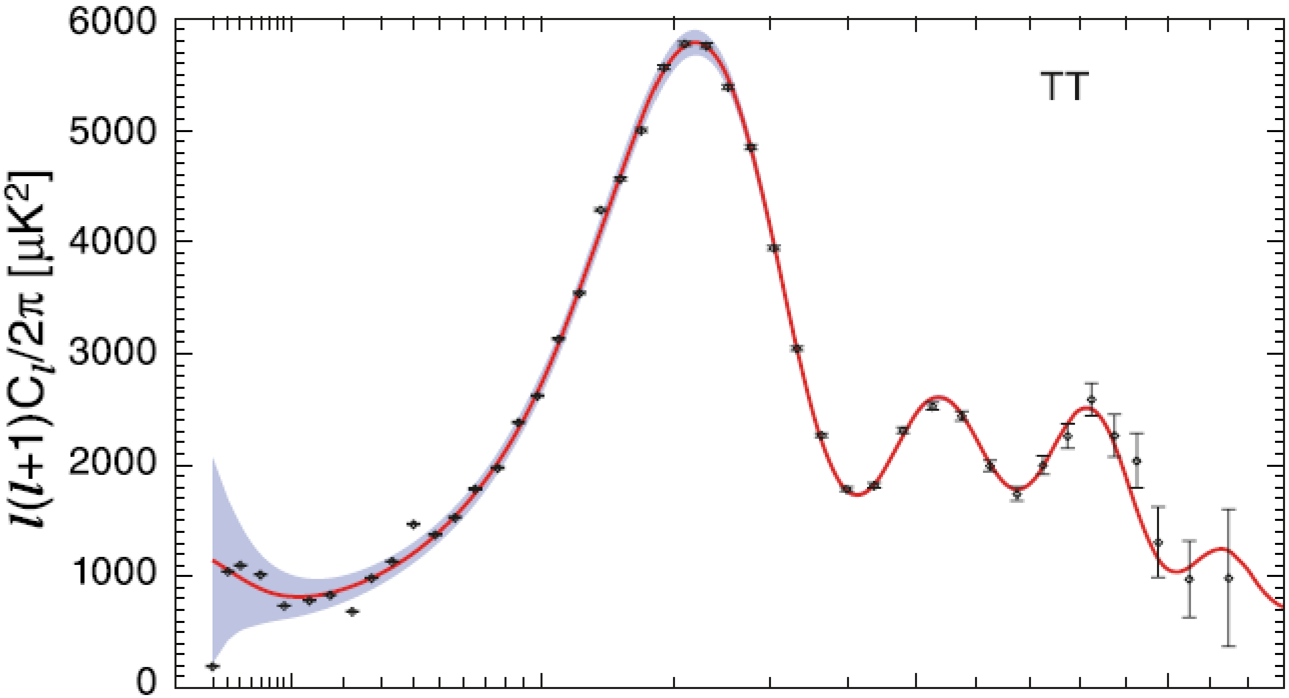
- W

7$^{\text{th}}$ y





What we learn from CMB data?



- Power spectrum:

The relation between CMB anisotropy and quantum fluctuations of early universe

What we learn from CMB data?

Non-Gaussianity:

Measures the correlation between different scales or different parts of the universe.

- Gaussian perturbations have zero non-Gaussianity

How can we build fluctuations on CMB?

- Quantum fluctuations exist naturally
- Can they be the initial seeds of CMB fluctuations?

Inflation can do that

How can we model the CMB?

Perturbation theory in cosmology

- Perturbation of metric around FRW background:

$$\begin{aligned} \mathrm{d}s^2 &= -(1+2A)\mathrm{d}t^2 + 2a(\partial_i B - S_i)\mathrm{d}x^i \mathrm{d}t \\ &\quad + a^2\left[(1-2\psi)\delta_{ij} + 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}\right]\mathrm{d}x^i \mathrm{d}x^j \end{aligned}$$

+ Perturbation of matter:

$$\phi = \phi_0(t) + \delta\phi(t,x)$$

- A good quantity!

Curvature perturbation: $\mathcal{R} \equiv \psi + \frac{H}{\dot{\phi}}\delta\phi$

Power spectrum/non-Gaussianity and CMB

Power spectrum of curvature perturbation:

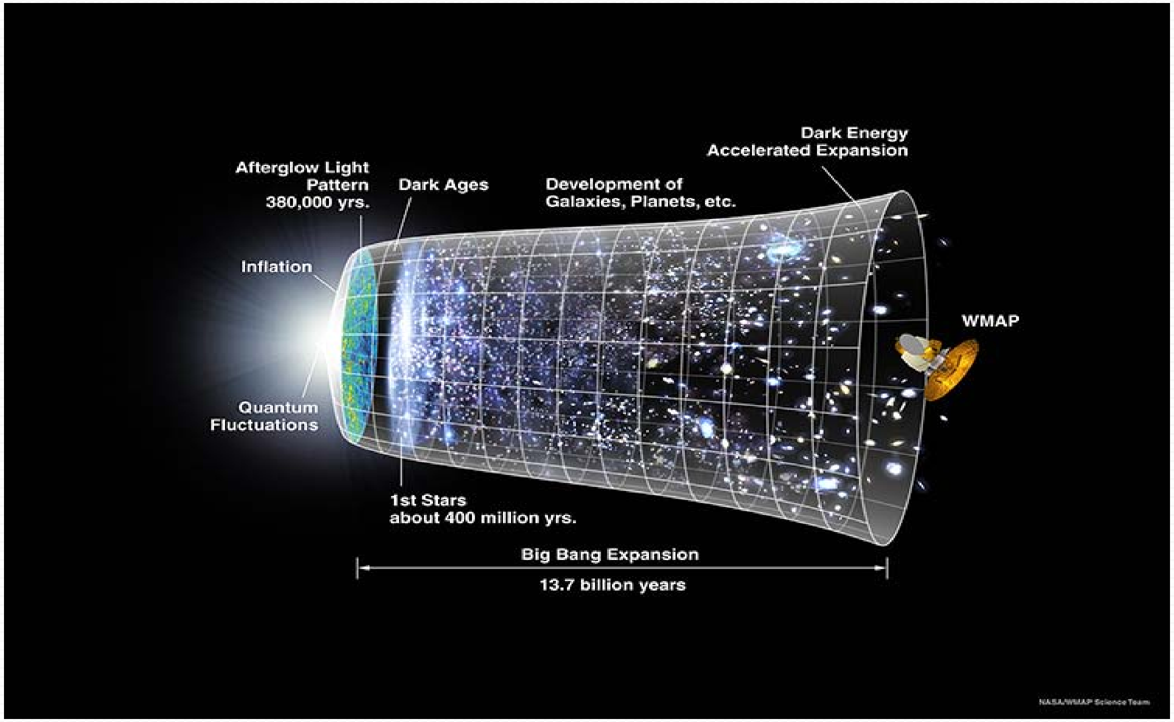
$$< \mathcal{R}_{k_1}\mathcal{R}_{k_2} > \longrightarrow \mathcal{P}_{\mathcal{R}} \equiv \frac{4\pi k^3}{(2\pi)^3}|\mathcal{R}^2|$$

Non-Gaussianity:

$$< \mathcal{R}_{k_1}\mathcal{R}_{k_2}\mathcal{R}_{k_3} >= -6/5 f_{NL}(< \mathcal{R}_{k_1}\mathcal{R}_{k_2} > +2perm.)$$

To gather…

- Inflation
- Observation of temperature fluctuations
- Cosmological perturbation theory and inflation
- Power spectrum and non-Gaussianity



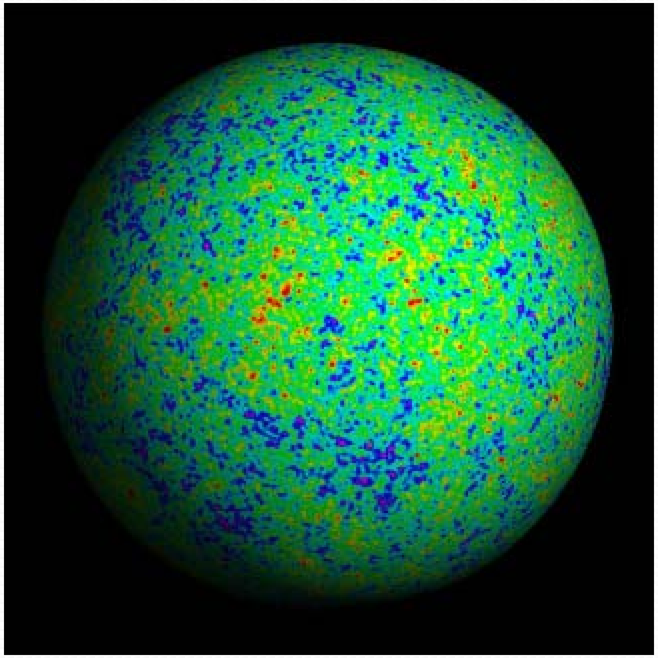
http://en.citizendium.org/wiki/File:060915_CMB_Timeline75.jpg

Why non-Gaussianity is important?

- Includes information about statistics of fluctuations
- Can constrain or rule out models of inflation
- Is both detectable and calculable (in principle)

Characteristics of Non-Gaussianity

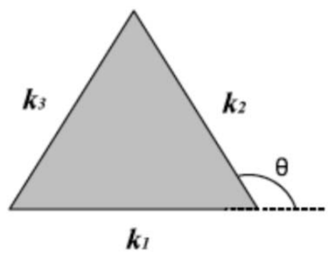
- Size
- Shape
- Sign



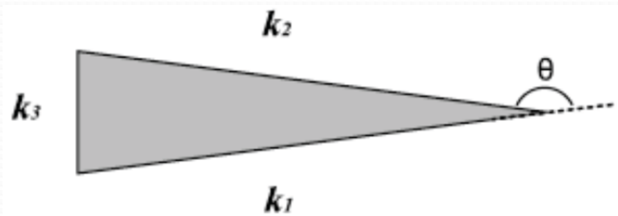
- These characteristics are detectable distinguishable

Shape of Non-Gaussianity

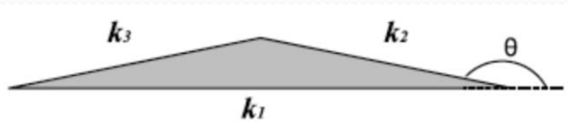
Shape is the scale dependence of non-Gaussianity



(2) Equilateral



(1) Squeezed



(3) Folded

http://www.umich.edu/~mctp/SciPrgPgs/events/2011/CosmoNongaussianity/index.html (by Matarrese)

Size of Non-Gaussianity

WMAP constraints

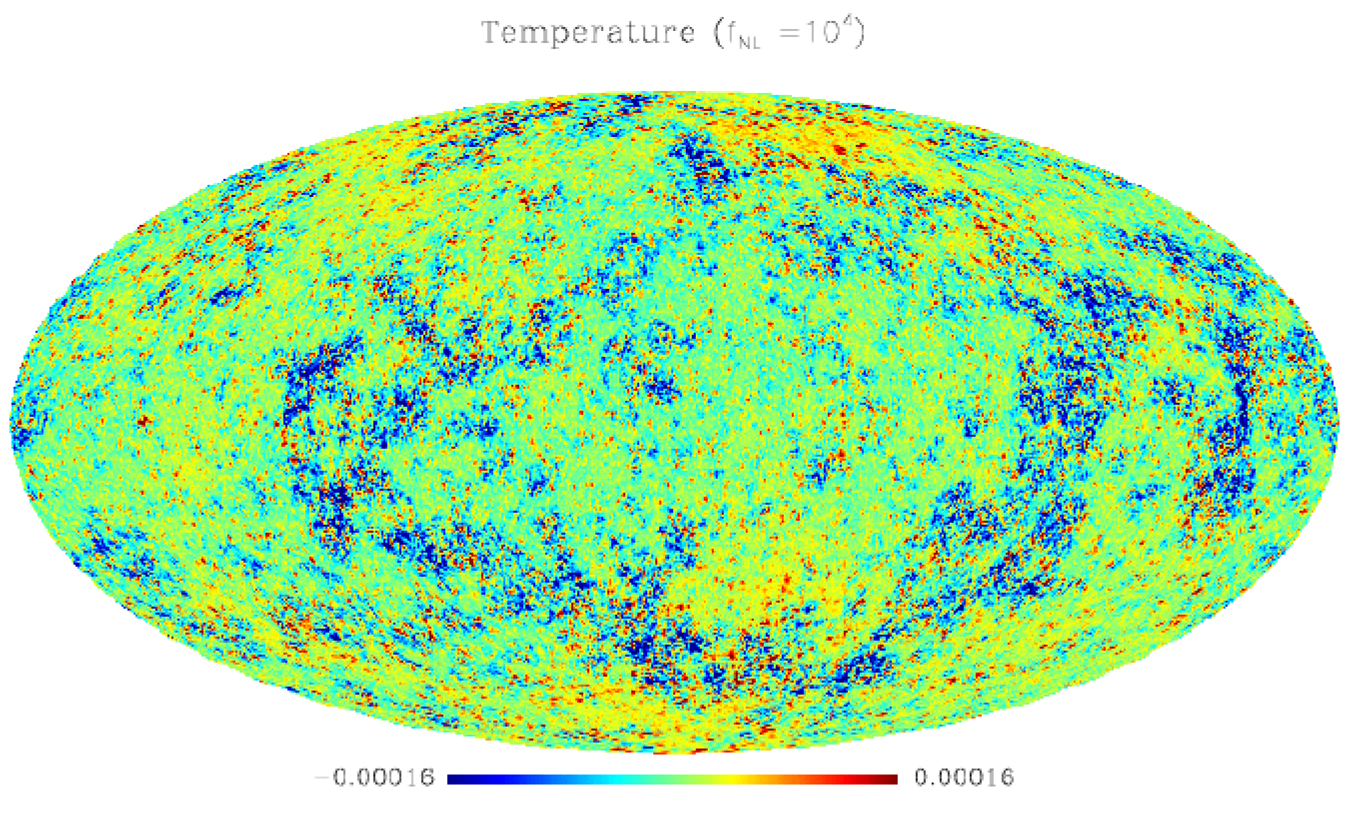
http://www.umich.edu/~mctp/SciPrgPgs/events/2011/CosmoNongaussianity/index.html

	WMAP 7-yrs	(Komatsu et al. 2010)
Local	$-10 < f_{NL} < 74$	
Equilateral	$-214 < f_{NL} < 266$	(95% c.l)
Orthogonal	$-410 < f_{NL} < 6$	

$$< \mathcal{R}_{k_1} \mathcal{R}_{k_2} \mathcal{R}_{k_3} > = -6/5 f_{NL} (< \mathcal{R}_{k_1} \mathcal{R}_{k_2} > + 2 perm.)$$

Planck will have more to say about non-Gaussianity

Is the non-Gaussianity large?



http://ipht.cea.fr/Pisp/pontavignon2008/programme.htm (by Wandelt & Yadav)

How to calcualte the non-Gaussianity?

- Maldacena’s Method
- δN formalism

Maldacena's method

- Quadratic action(The propagator):

$$S_2 = \int \mathrm{d}\tau\, \mathrm{d}^3x\; a^2 \left[\frac{\Sigma}{H^2}(\mathcal{R}')^2 - \varepsilon(\partial\mathcal{R})^2\right]$$

%``Primordial non-Gaussianities in single field inflation,''
JCAP\ {\bf 0506}, 003 (2005)
[astro-ph/0503692].

Maldacena's method

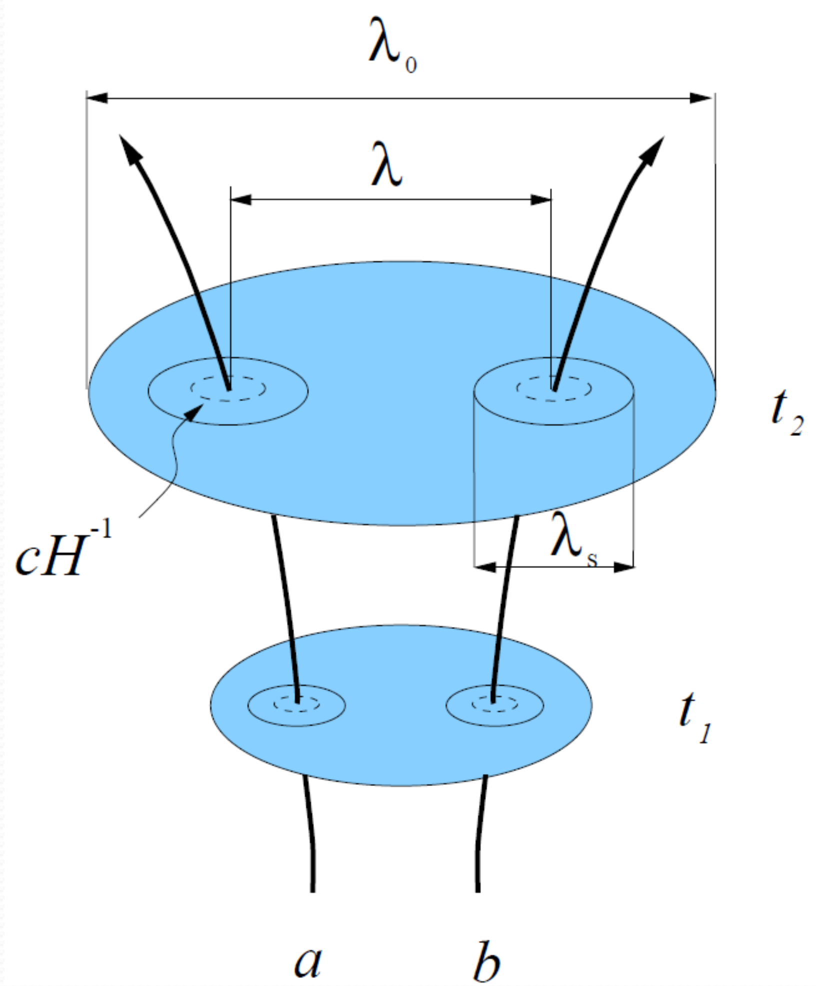
- Cubic action(The interaction):

$$S_3 = \int \mathrm{d}t\, \mathrm{d}^3x \left[-\frac{2}{3}a^3\varepsilon \left(u + \frac{\varepsilon}{\varepsilon_X}\frac{s}{3} \right) \frac{\dot{\mathcal{R}}^3}{H} + 4a^5\varepsilon(2u+\varepsilon)H\dot{\mathcal{R}}^2\partial^{-2}\dot{\mathcal{R}} \right.$$
$$\left. - 4a^3\varepsilon\dot{\mathcal{R}}\partial^2\mathcal{R}\partial^{-2}\dot{\mathcal{R}} \right],$$

- coupling constants!

%``Primordial non-Gaussianities in single field inflation,''
JCAP\ {\bf 0506}, 003 (2005)
[astro-ph/0503692].

δN Formalism



D.~Wands, K.~A.~Malik, D.~H.~Lyth and A.~R.~Liddle, %``A New approach to the evolution of cosmological perturbations on large scales,''
Phys.\ Rev.\ D\ {\bf 62}, 043527 (2000)
[astro-ph/0003278].

Some Examples

- Typical slow-roll single field inflation:

The size of non-Gaussianity is of order of slow-roll parameters and non-detectable.

- Multiple field inflation:

Entropic perturbations can induce large non-Gaussianity

Some Examples

- Dirac-Born-Infield (DBI) inflation:

M.~Alishahiha, E.~Silverstein and D.~Tong, %``DBI in the sky,''
Phys.\ Rev.\ D\ {\bf 70}, 123505 (2004)
[hep-th/0404084].

Non-standard kinetic term

$$\mathcal{L}_{\rm eff} = -\frac{1}{g_s}\sqrt{-g}\left(f(\phi)^{-1}\sqrt{1 + f(\phi) g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} + V(\phi)\right)$$

$$\frac{d^2 v_k}{d\tau^2} + \left(c_s^2 k^2 - \frac{\nu^2 - \frac{1}{4}}{\tau^2}\right) v_k = 0$$

N sponsible for large non-Gaussianity

$$S_3 = \frac{1}{2}\int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \Bigg[-\frac{4}{3}\varepsilon\left(\frac{1}{3c_{\rm s}^2}\frac{\varepsilon}{\varepsilon_X}s + u\right)\frac{\dot{\mathcal{R}}^3}{Hc_{\rm s}^2} + \frac{2\varepsilon}{c_{\rm s}^2}\left(3u + \frac{\varepsilon}{c_{\rm s}^2}\right)\mathcal{R}\dot{\mathcal{R}}^2$$
$$+\frac{2\varepsilon}{a^2c_{\rm s}^2}(\varepsilon - 2s - uc_{\rm s}^2)\mathcal{R}(\partial\mathcal{R})^2$$
$$-\frac{4}{a^2}\frac{\varepsilon}{c_{\rm s}^2}\dot{\mathcal{R}}\mathcal{R}_{,i}\chi_{,i} + \mathcal{R}^2\dot{\mathcal{R}}\frac{\varepsilon}{c_{\rm s}^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\eta}{c_{\rm s}^2}\right) - \frac{\varepsilon^3}{c_{\rm s}^4}\mathcal{R}\dot{\mathcal{R}}^2 + \frac{1}{a^2}\varepsilon\mathcal{R}\chi_{,ij}\chi_{,ij}\Bigg]$$

%``Primordial non-Gaussianities in single field inflation,''
JCAP\ {\bf 0506}, 003 (2005)
[astro-ph/0503692].

Some Examples

- Ghost inflation:

N.~Arkani-Hamed, P.~Creminelli, S.~Mukohyama and M.~Zaldarriaga, %``Ghost inflation,''
JCAP\ {\bf 0404}, 001 (2004)
[hep-th/0312100].

The non-standard dispersion relation results large non-Gaussinaity

$$S = \int d^4x \, \frac{1}{2}\dot{\pi}^2 - \frac{\alpha^2}{2M^2}(\nabla^2\pi)^2$$

Conclusion

- Non-Gaussianity is the three point correlation function of temperature fluctuations on CMB.
- Different models predict different non-Gaussianity
- The detection of size, shape and sign of Non-Gaussianity in forthcoming observations constrains models of inflation.

متشکرم

Thank you