

# BILAYER GRAPHENE FIELD EFFECT TRANSISTORS

Fariborz Parhizgar  
Institute for research in Fundamental Science



# OUTLINE

Intro.

- Transistors (From Ge to C)
- Importance of Graphene

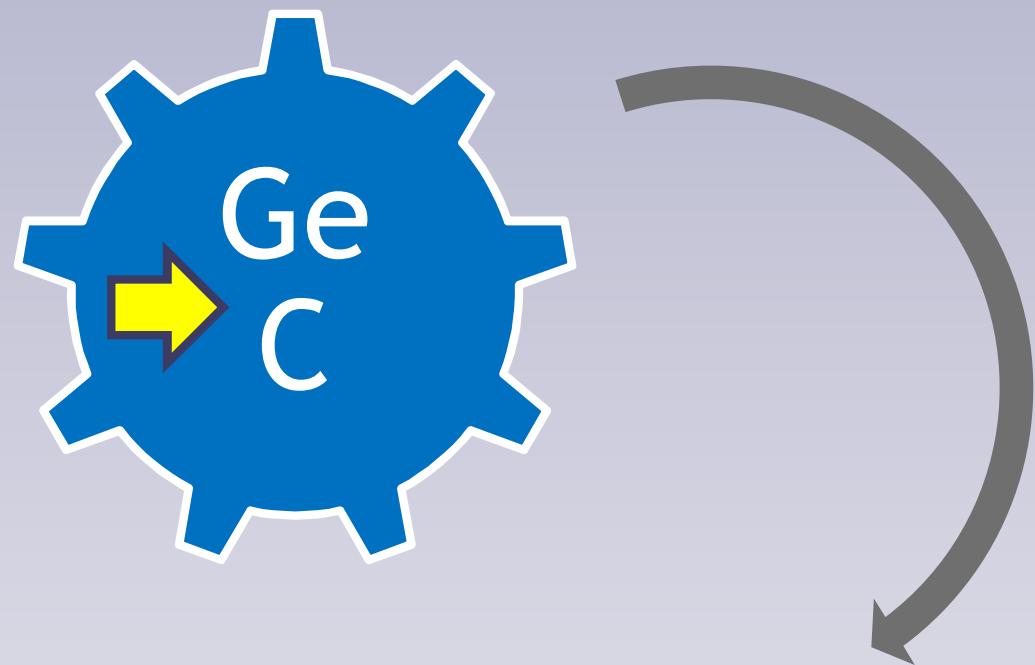
Model

- monolayer Graphene
- Bilayer Graphene

conclusion

- Bilayer: a good candidate for FET

# INTORDUCTION



# SI & GE TRANSISTORS

Small  
size



High  
mobility

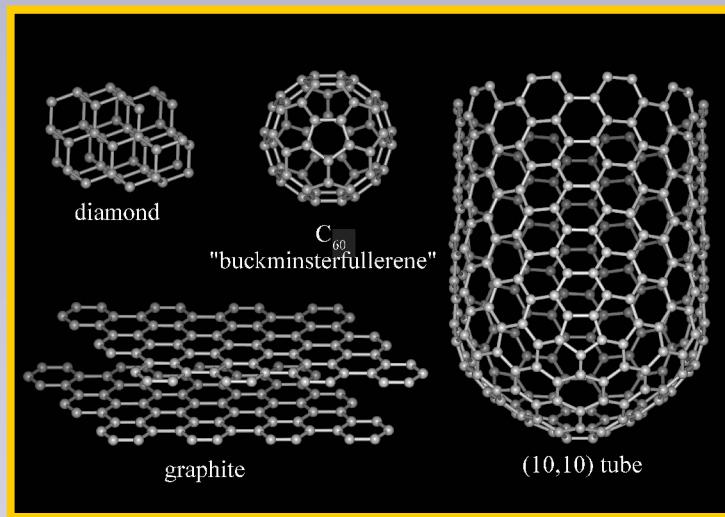
2 important  
Properties  
that a  
transistors  
must have

# Si & Ge

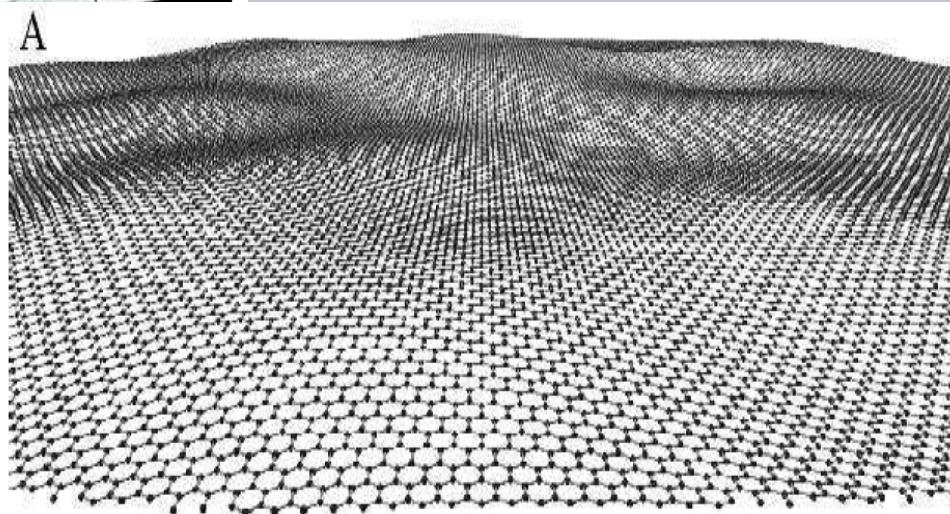
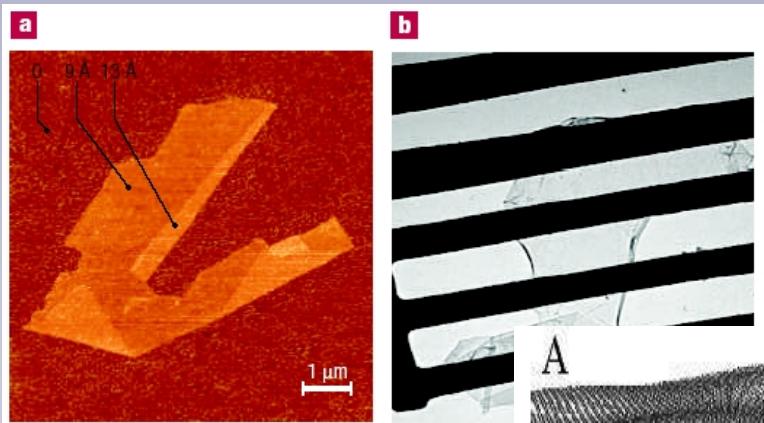
Properties	Si	Ge	GaAs
Atoms/cm <sup>3</sup>	5.02 x 10 <sup>22</sup>	4.42 x 10 <sup>22</sup>	4.42 x 10 <sup>22</sup>
Effective mass electrons ( $m/m_0$ )	0.26	0.082	0.067
Effective mass holes ( $m/m_0$ )	0.69	0.28	0.57
Electron affinity (V)	4.05	4.0	4.07
Energy gap (eV)	1.12	0.67	1.42
Mobility electrons (cm <sup>2</sup> /V s)	1500	3900	8500
Mobility holes (cm <sup>2</sup> /V s)	450	1900	450

C  
Si  
Ge  
Sn  
Pb

# CARBON ALLOTROPES



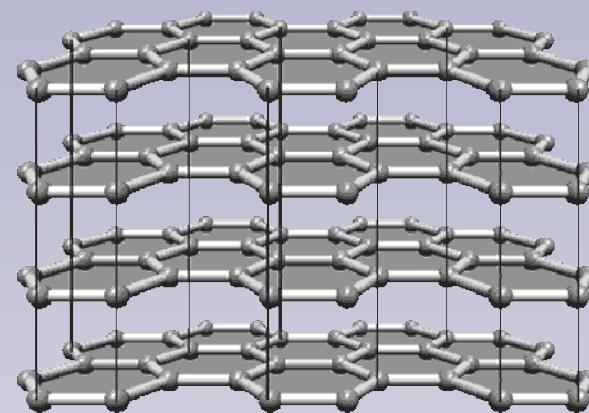
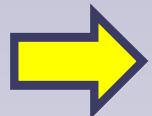
# SINGLE LAYER OF CARBON ATOMS



Wallace Phys. Rev. 104, 666  
Geim et. al. Nat. Mat. 6, 183

# GRAPHENE

- ◎ Carbon: smallest atom in group 4



sp<sub>2</sub> orbitals which makes σ-bond

p<sub>z</sub> Orbital which makes π-bond

# GRAPHENE

Crystal  
upto nm  
sizes

High  
mobility  
 $\sim 10^5$   
 $\text{cm}^2/\text{Vs}$

Good  
candidate to  
make  
transistors

# MODEL I

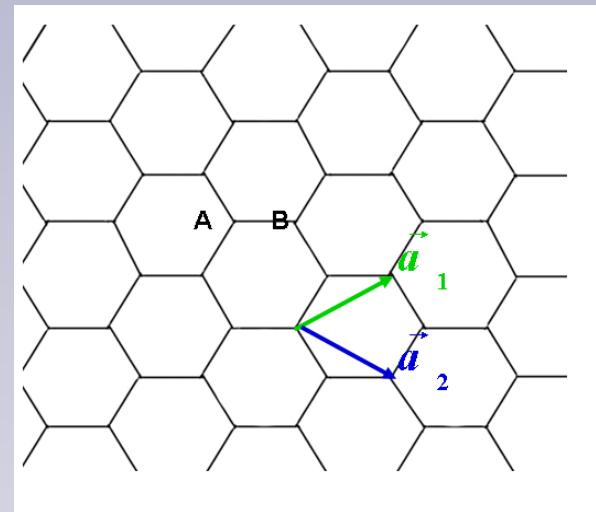


# GRAPHENE: HONEYCOMB LATTICE

- Honeycomb lattice is not a Bravious Lattice
- Hoping of  $P_z$  Orbitals



$$H = \sum_{\langle ij \rangle} t_{ij} C_i^\dagger C_j + H.C.$$



# GRAPHENE: TB MODEL

- ◎ Taking Fourier Transform:

$$c_\alpha(k) = \sum_i c_{\alpha,i} e^{ik \cdot r_i}$$



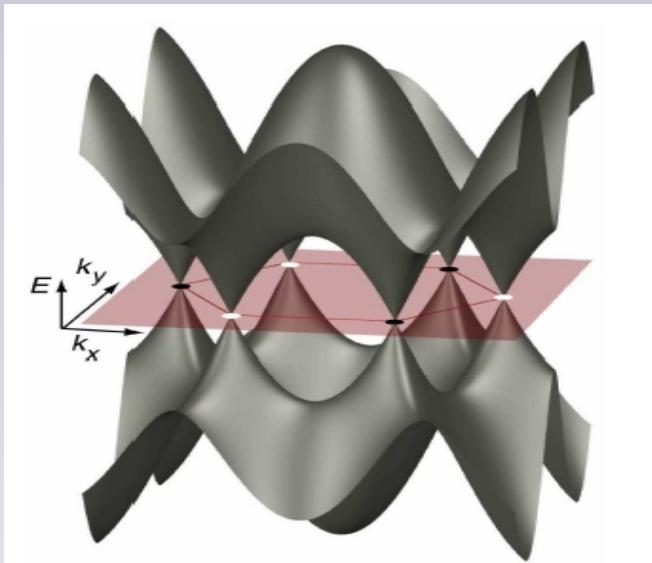
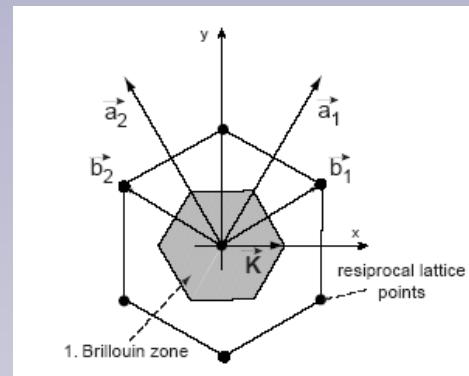
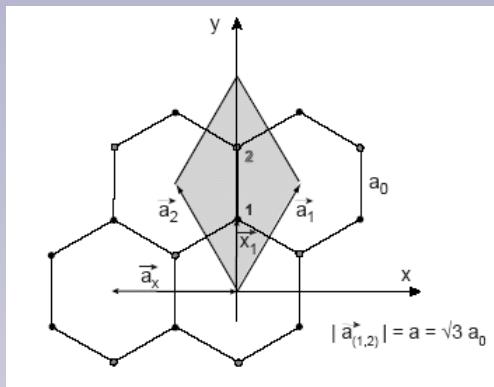
$$H \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \begin{pmatrix} E_p & f(\vec{k}) \\ f^*(\vec{k}), & E_p \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

$$f(\vec{k}) = \gamma [1 + e^{i\vec{k} \cdot \vec{a}_1} + e^{i\vec{k} \cdot \vec{a}_2}]$$



$$E(k) = E_p \pm \sqrt{3 + 2 \cos \vec{k} \cdot \vec{a}_1 + 2 \cos \vec{k} \cdot \vec{a}_2 + 2 \cos \vec{k} (\vec{a}_2 - \vec{a}_1)}$$

# GRAPHENE: B.Z.



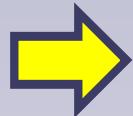
Two independent K-Point

# GRAPHENE: LOW ENERGY LIMIT

$H =$

- Expanding the function  $f(k)$ :

$$f(k) = v_F (k_x \pm i k_y) \begin{cases} K \\ K' \end{cases}$$



$$H = \sum_{\vec{k}} \hat{\sigma} \cdot \vec{k}$$

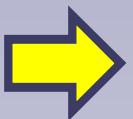
- massless Dirac Fermions  $E(k) = v_F k$

$$v_F \sim 10^6 \text{ m/s} \sim c / 300$$

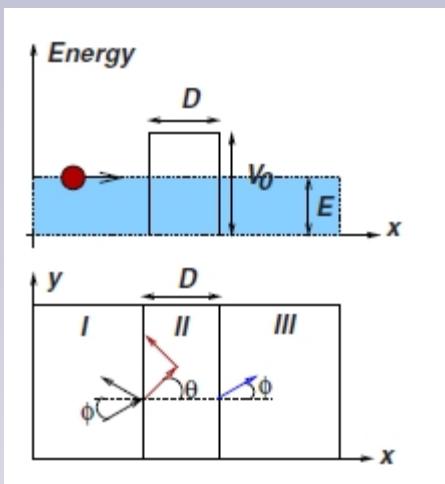
- Where  $\hat{\sigma}$  is the Pauli matrix for pseudospins

# GRAPHENE: CHIRALITY

$$H = \sum_{\vec{k}} \hat{\sigma} \cdot \vec{k}$$



- Absence of e-backscattering:



$$\psi_I(r) = \frac{1}{\sqrt{2}} \left( \begin{matrix} 1 \\ se^{i\varphi} \end{matrix} \right) e^{i(k_x x + k_y y)} + \frac{r}{\sqrt{2}} \left( \begin{matrix} 1 \\ se^{i(\pi - \varphi)} \end{matrix} \right) e^{i(-k_x x + k_y y)}$$

$$\psi_{II}(r) = \frac{a}{\sqrt{2}} \left( \begin{matrix} 1 \\ s' e^{i\theta} \end{matrix} \right) e^{i(q_x x + k_y y)} + \frac{b}{\sqrt{2}} \left( \begin{matrix} 1 \\ s' e^{i(\pi - \theta)} \end{matrix} \right) e^{i(-q_x x + k_y y)}$$

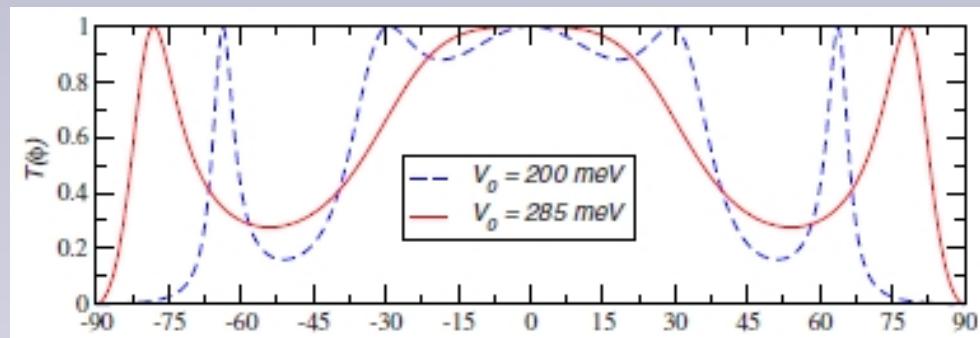
$$\psi_{III}(r) = \frac{t}{\sqrt{2}} \left( \begin{matrix} 1 \\ se^{i\varphi} \end{matrix} \right) e^{i(k_x x + k_y y)}$$

$$s = \text{sgn}(E), s' = \text{sgn}(E - V_0), \theta = \tan^{-1}(k_y / q_x)$$

Castro Neto et. Al. Rev. Mod. Phys. 81,109

# GRAPHENE: KLEIN PARADOX

$$T(\phi) = \frac{\cos^2 \theta \cos^2 \phi}{[\cos(Dq_x)\cos \phi \cos \theta]^2 + \sin^2(Dq_x)(1 - ss' \sin \phi \sin \theta)^2}.$$



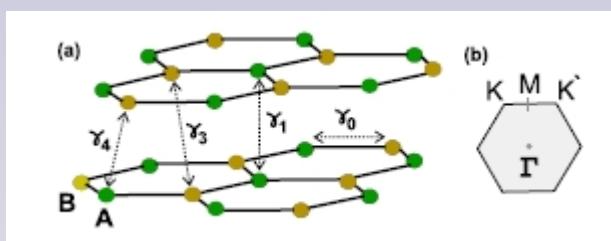
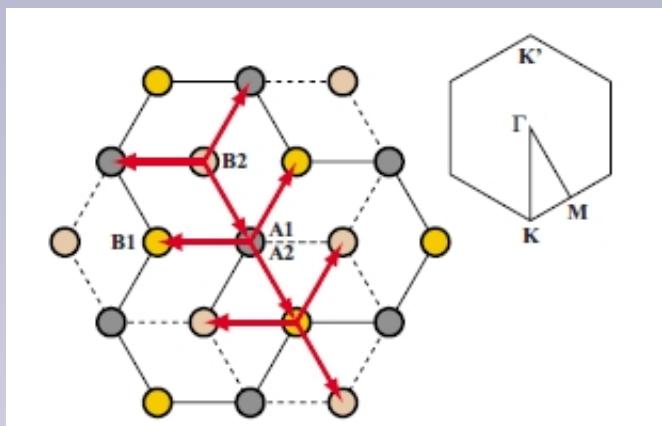
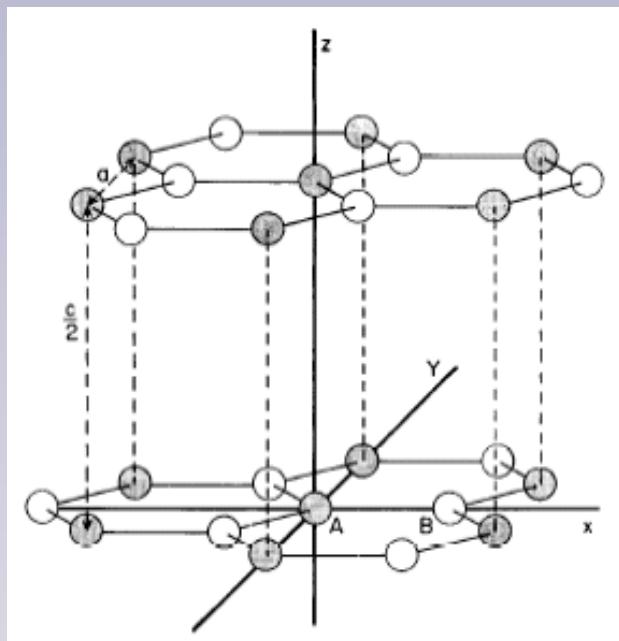
- Quasi particles can not be trapped →  
your PC will not turn off

## MODEL II



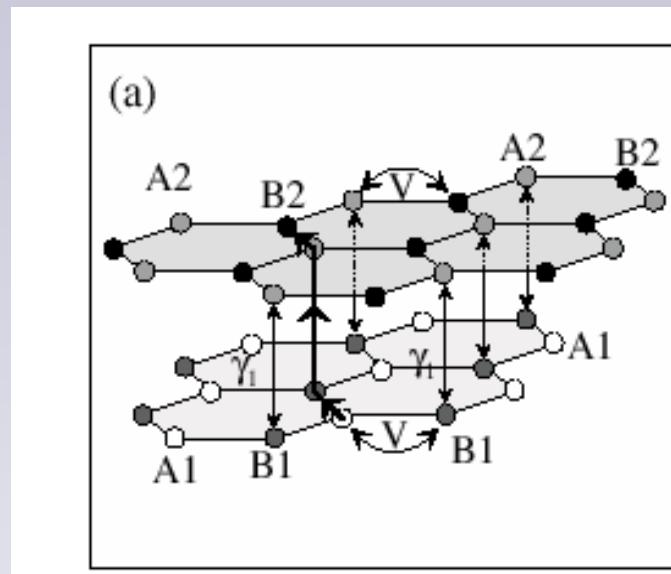
# BLG: BERNAL STACKING

- Most possible config.



# TIGHT BINDING MODEL

$$H = -t \sum_{\langle ij \rangle m\sigma} (a_{m,i,\sigma}^\dagger b_{m,j,\sigma} + H.C.) - t_\perp \sum_{j\sigma} (a_{1,j,\sigma}^\dagger a_{2,j,\sigma} + H.C.) \\ - tv_4 \sum_{\langle ij \rangle m\sigma} (a_{1,i,\sigma}^\dagger b_{2,j,\sigma} + H.C.) - tv_4 \sum_{\langle ij \rangle m\sigma} (a_{2,i,\sigma}^\dagger b_{1,j,\sigma} + H.C.) \\ - tv_3 \sum_{\langle ij \rangle m\sigma} (b_{1,i,\sigma}^\dagger b_{2,j,\sigma} + H.C.)$$



# HOPING COEFFICIENT

	$t \equiv \gamma$	$\gamma_1$	$\gamma_3$	$\gamma_4$	$\Delta$
Kuzmenko [147] (IR spec.)		0.378 (0.005)			0.015 (0.005)
Kuzmenko [164] (IR spec.)	3.16 (0.03)	0.381 (0.003)	0.38 (0.06)	0.14 (0.03)	0.022 (0.003)
Zhang [165] (IR spec.)	3.0	0.40	0.3	0.15	0.018
Malard [166] (Raman)	2.9	0.30	0.10	0.12	
Malard [167] (Raman)	3.0	0.35	0.13	0.13	
Min [168] (ab init.)	2.6	$\approx 0.34$	0.3		
Gava [169] (ab init.)	-3.4013	0.3963	0.3301	0.1671	

Table 1. Tight-binding parameters for bilayer graphene, given in eV. Methods of determination of the parameters include infra-red spectroscopy (IR spec.), Raman spectroscopy (Raman), and ab initio density functional theory calculations (ab init.). Note that Min et al. claim that  $\gamma_1$  varies slightly with the interlayer potential  $U$ . Bracketed values are stated uncertainties.

T.Chakraborty et. Al. advance in phys. April 2010

# BLG: MODEL

Fourier transform:

$$H = \sum_k \Psi^\dagger(k) \begin{pmatrix} \Delta & f(k) & t_\perp & v_4 f^*(k) \\ f^*(k) & \Delta & v_4 f^*(k) & v_3 f(k) \\ t_\perp & v_4 f(k) & -\Delta & f^*(k) \\ v_4 f(k) & v_3 f^*(k) & f(k) & -\Delta \end{pmatrix} \Psi(k)$$

$$\Psi^\dagger(k) = (a_{1,k,\sigma}^\dagger, b_{1,k,\sigma}^\dagger, a_{2,k,\sigma}^\dagger, b_{2,k,\sigma}^\dagger)$$

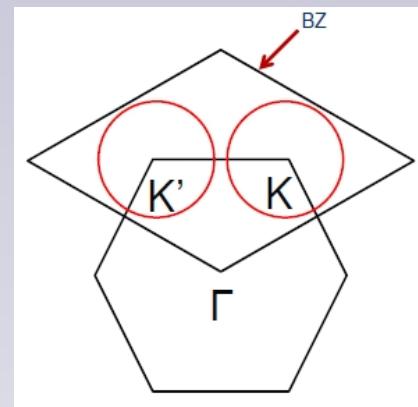
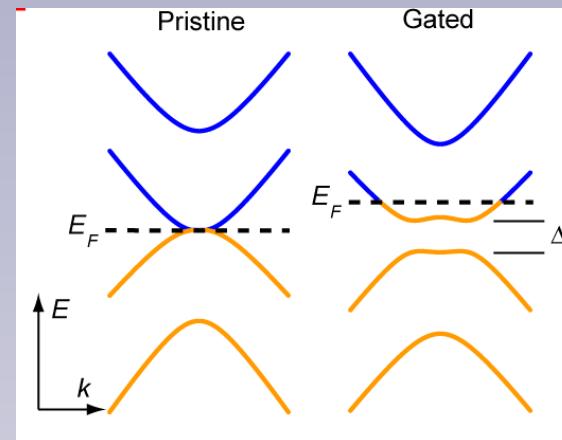
$$f(k) = t \sum_{i=1}^3 e^{ik \cdot d_i} \simeq v_F q \Phi(q)$$

# BLG: MODEL

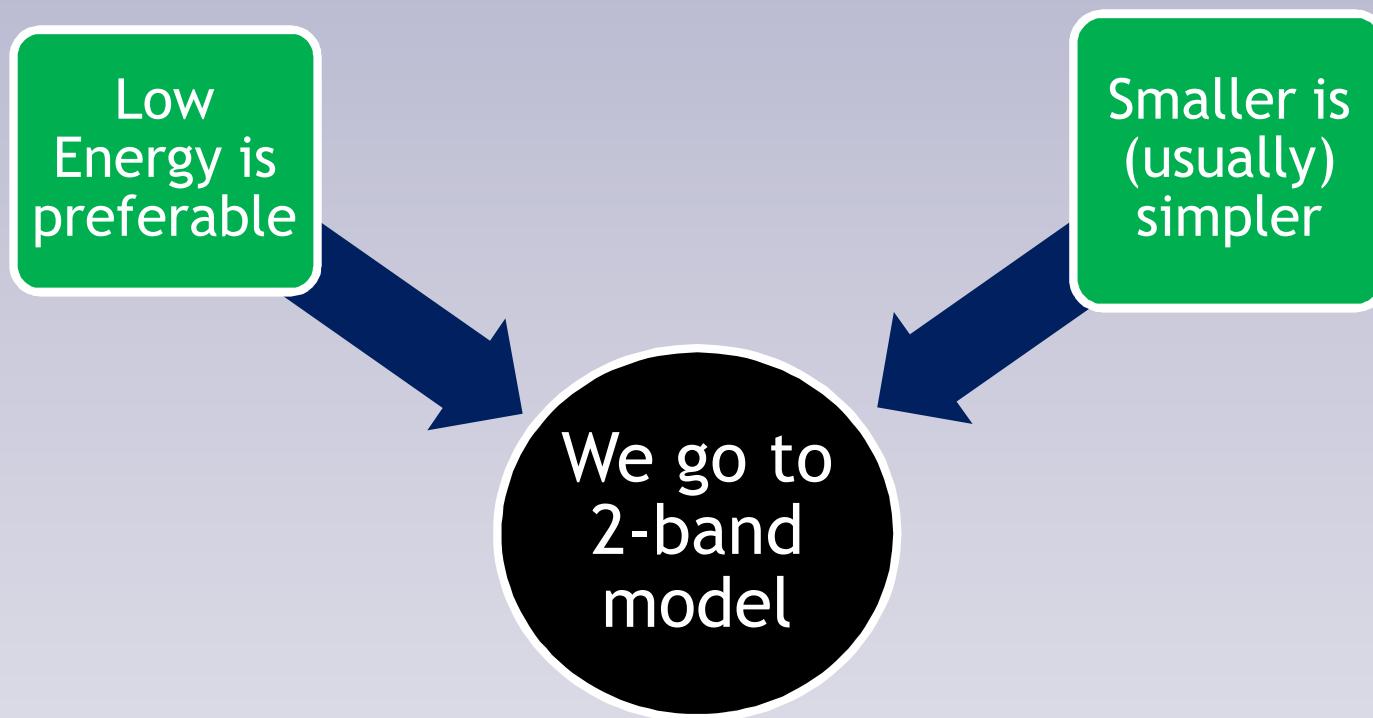
- Dispersion Relation:

$$\varepsilon^2 = v^2 p^2 + \frac{t_{\perp}^2}{2} + \frac{V^2}{4} - \sqrt{v^2 p^2 (V^2 + t_{\perp}^2) + t_{\perp}^4 / 4}$$

- Independent of  $\Phi(k)$



# BLG LOW ENERGY MODEL



# 2-BAND LOW ENERGY MODEL

In low energy limit ( $\varepsilon \ll t_{\perp}$ ):

$$H = K_0 + K_1 + K_2$$

$$K_0 = \begin{pmatrix} V/2 & 0 & t_{\perp} & 0 \\ 0 & V/2 & 0 & 0 \\ t_{\perp} & 0 & -V/2 & 0 \\ 0 & 0 & 0 & -V/2 \end{pmatrix} \quad K_1 = \begin{pmatrix} 0 & pe^{i\phi} & 0 & v_4 pe^{-i\phi} \\ f^*(k) & 0 & v_4 pe^{-i\phi} & 0 \\ 0 & v_4 pe^{i\phi} & 0 & pe^{-i\phi} \\ v_4 pe^{i\phi} & 0 & pe^{i\phi} & 0 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_3 pe^{i\phi} \\ 0 & 0 & 0 & 0 \\ 0 & v_3 pe^{-i\phi} & 0 & 0 \end{pmatrix}$$

# LOW ENERGY 2-BAND MODEL

using degenerate 2<sup>nd</sup> order perturbation:

$$\langle l | P_0 H P_0 | l' \rangle = \langle l | P_0 [K_0 + \lambda K_2] P_0 | l' \rangle + \lambda^2 \left\langle l \left| P_0 K_1 P_1 \frac{1}{E - K_0} P_1 K_1 P_0 \right| l' \right\rangle$$

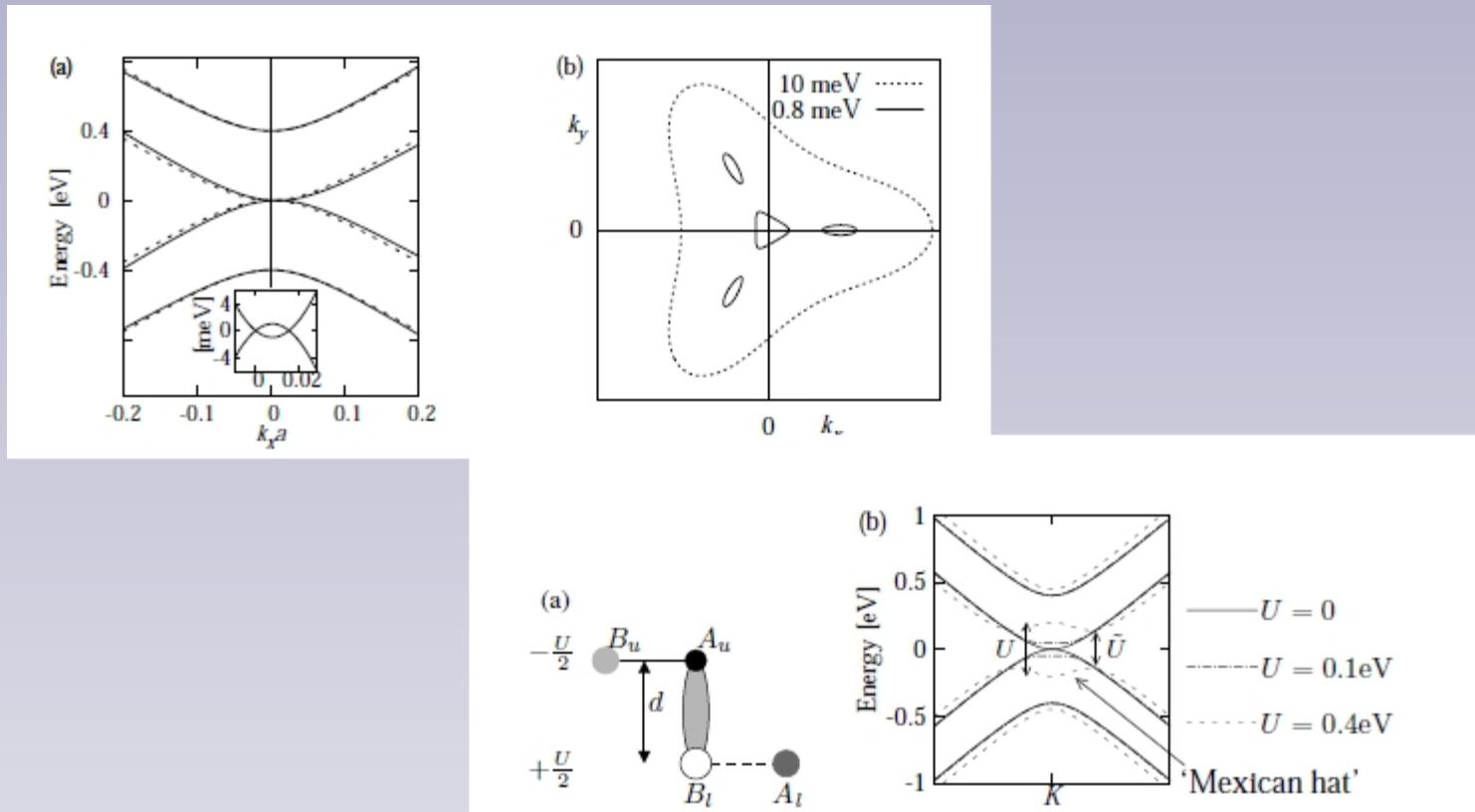


$$H_{low} = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^\dagger)^2 \\ (\pi)^2 & 0 \end{pmatrix} + v_3 \begin{pmatrix} 0 & \pi^\dagger \\ \pi & 0 \end{pmatrix} + \frac{V}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

McCann et. al. PRL 96, 086805

Nilsson et. al. PRB 78, 045405

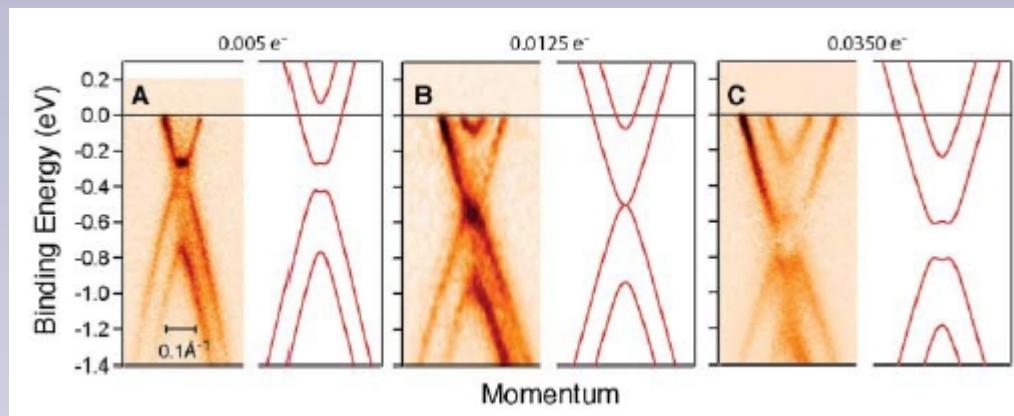
# BAND DISPERSION: TRIGONAL WARPING



T.Chakraborty et. Al. advance in phys. April 2010

# BLG: BAND GAP

Experimental evidence of the gap:



Ohta et. al. Science 313, 951

# MAGNITUDE OF THE GAP

- Low energy band with the gap:

$$\varepsilon^2 = v^2 p^2 + \frac{t_{\perp}^2}{2} + \frac{V^2}{4} - \sqrt{v^2 p^2 (V^2 + t_{\perp}^2) + t_{\perp}^4 / 4}$$

→  $\tilde{U} = \frac{t_{\perp} V}{\sqrt{V^2 + t_{\perp}^2}}$  at momentum:

$$p^2 = \frac{V^2}{4v^2} \frac{V^2 + 2t_{\perp}^2}{V^2 + t_{\perp}^2}$$



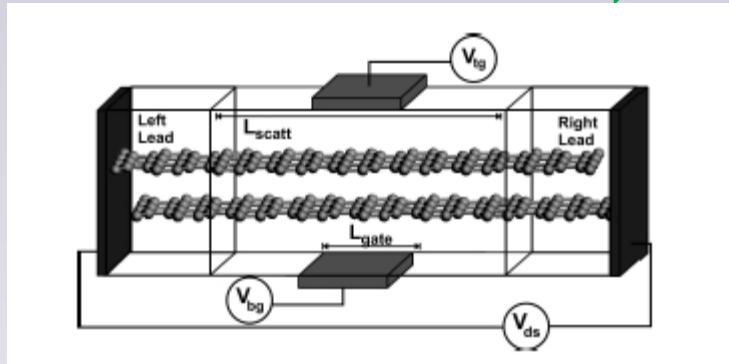
$$V \sim .2eV$$

# BIASED BLG

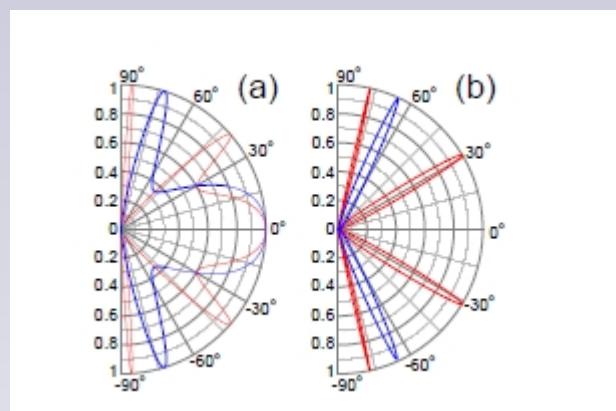
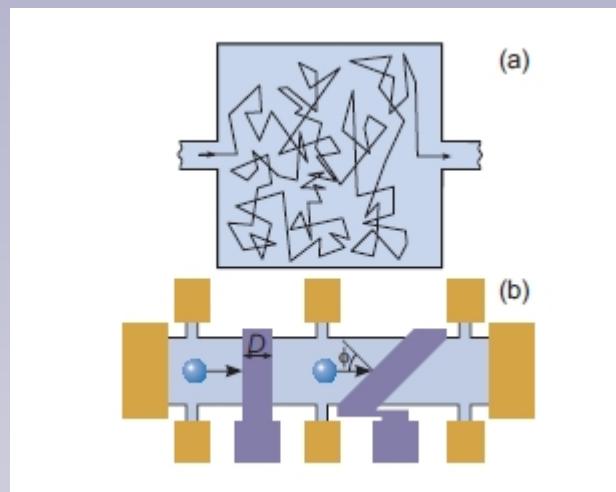
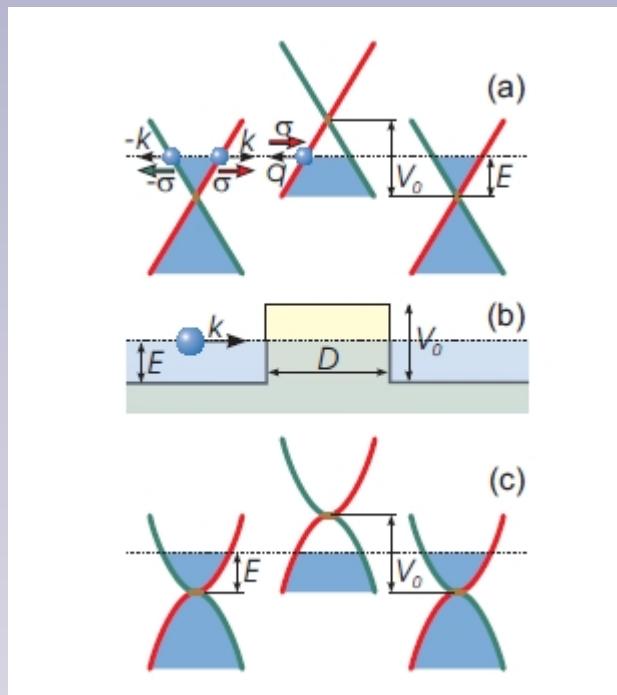
Gate Voltage  
Change the  
Fermi  
energy.

Using Dual  
gated BLG:

2 tuning  
parameter



# KLEIN TUNELLING IN BLG



# CONCLUSION



# BILAYER APPLICATIONS

Digital Electronics

Pseudospintronics

Terahertz Technology

Infrared nanophotonics

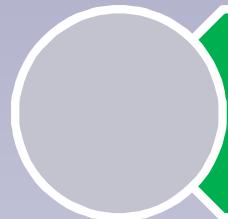
# GAPLESS MONOLAYER

SLG is  
Gapless

On/off  
ratio ~ 5

We need  
Gap

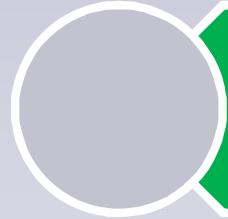
# SUGGESTION TO PRODUCE GAP



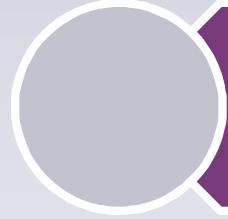
Uniaxial Strain



Substrate Interaction



Lateral Confinement



Biased BLG

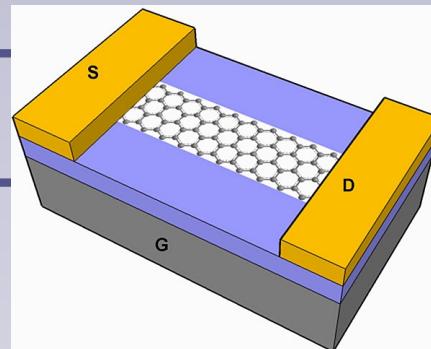
# GRAPHENE NANORIBBON

Advantages

- Gap~Hundereds of meV
- High On/Off ratio

Disadvantages

- Low mobility
- Undesirable shape

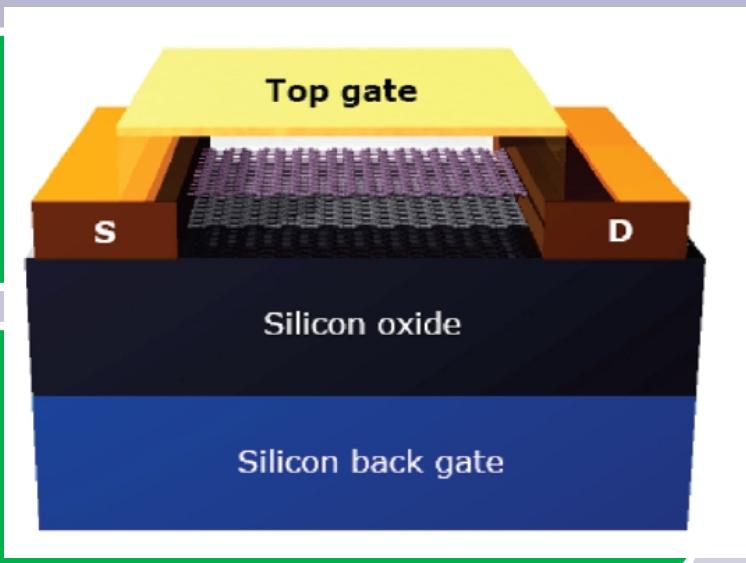


# BLG FET

Gap upto .3eV

On/off ratio: (100-2000)

Nano lett. 2010,10,715



A photograph of a forest floor. The ground is covered in a thick layer of fallen leaves in shades of brown, orange, and yellow. A single, small green sprout with two leaves is growing through the center of the leaf pile. In the background, many tall, thin trees stand in a dense forest, their trunks and branches creating a vertical pattern.

Thanks for your  
attention