Gravity as an emergent phenomenon

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IPM, Tehran
Fall 1391

Contents

- Newtonian approach to gravity
- Einstein's approach to gravity
- Thermodynamic approach to gravity

Newtonian gravity

- **Newton's novel paradigm**
  - **Equation of motion (Kinematics)**

$$F = ma$$

  - **Equation of fields (Dynamics)**

$$F = GMm / r^2$$

- ***Inertial frames***

Accident in Newtonian Gravity

$$\begin{Bmatrix} F = m_I a \\ F = GMm_g / r^2 \end{Bmatrix}$$

$$\boxed{m_I = m_g}$$

Equivalence *phenomenon : The freely falling observer doesn't see gravity*

Reversing the Logic

$$m_I = m_g$$

Equivalence phenomenon

Einstein →

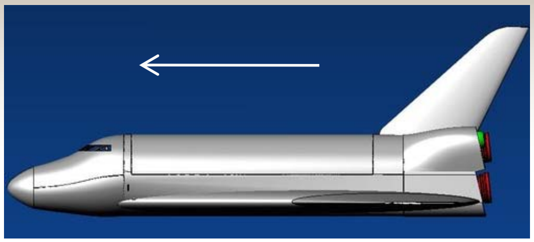
Equivalence *Principle*

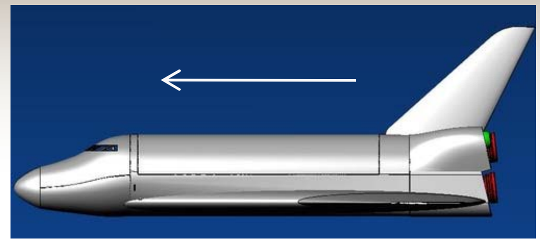
$$m_I = m_g$$

First principles of GR

1. There is always a locally inertial frame
2. Physics is special relativity in the Locally Inertial Frame (without Gravity)
3. General Covariance

Geometry of space-time



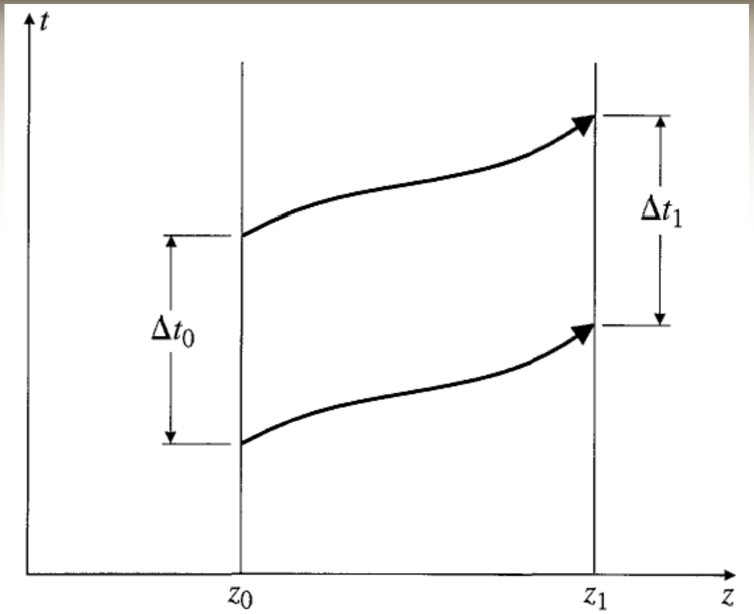


$$\frac{\Delta\lambda}{\lambda_0} = \frac{a_g z}{c^2}$$

Equivalence principle

$$\begin{aligned}\frac{\Delta\lambda}{\lambda_0} &= \frac{1}{c}\int \nabla\Phi \, dt \\ &= \frac{1}{c^2}\int \partial_z\Phi \, dz \\ &= \Delta\Phi,\end{aligned}$$

Geometry of space-time



S. Carroll, Geometry and space-time, Addison Wesley, 2004

Space-time is curved!

General Relativity

- Space-time exists!
- No gravitational Force
- Interaction of matter and space-time
- General Covariance (minimal coupling)

$$\partial_\mu \to \nabla_\mu$$

$$\eta_{\mu\nu} \to g_{\mu\nu}$$

$$d^n x \to \sqrt{-g}\, d^n x$$

General Relativity

- **Kinematics:** space-time says how particles move **(Geodesic equation)**

$$\frac{du^{\mu}}{d\tau} = 0 \Rightarrow u^{\nu}\partial_{\nu}u^{\mu} = 0 \rightarrow u^{\mu}\nabla_{\mu}u^{\nu} = 0$$

- **Dynamics:** matter says how metric changes **(No Guiding principle)**

Action Principle

- **Mathematical requirements on dynamical equations**
  1. **2^{nd} order in metric**
  2. **General covariance(general relativity)**
  3. **Symmetric and divergenceless "e.o.m"**

  $\Longrightarrow$ **Lanczos- Lovelock Action**

- **Hilbert Action (1^{st} order of Lovelock action)**

$$S_g = \frac{1}{16\pi G}\int d^n x\sqrt{-g}\,R \xrightarrow{\delta S=0} R_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Solutions to GR
General Features

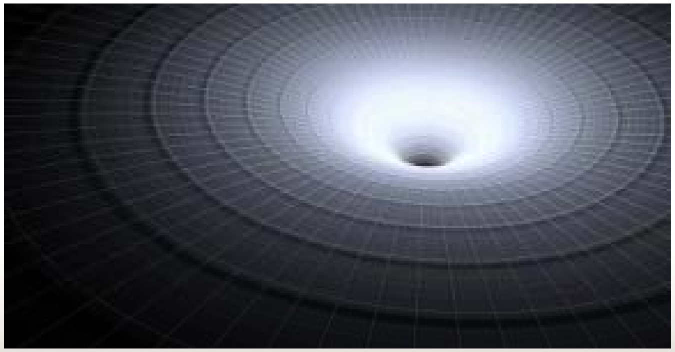
- Black holes
  - Curvature invariants diverge
  - Unknown physics
- Horizons
  - One-way membranes
- Cosmic Censorship conjecture (Penrose)

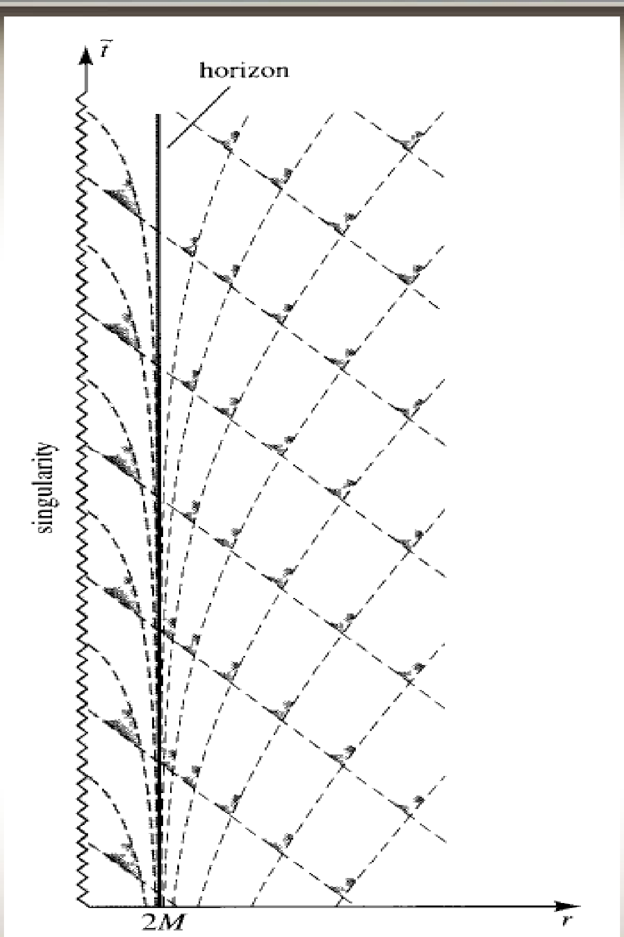
Solutions to GR
Vacuum

- **Schwarzshild**

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

$$f(r) = 1 - \frac{2GM}{r}$$





J. Hartle, Gravity, Addison Wesley, 2003

Solutions to GR
Flat space

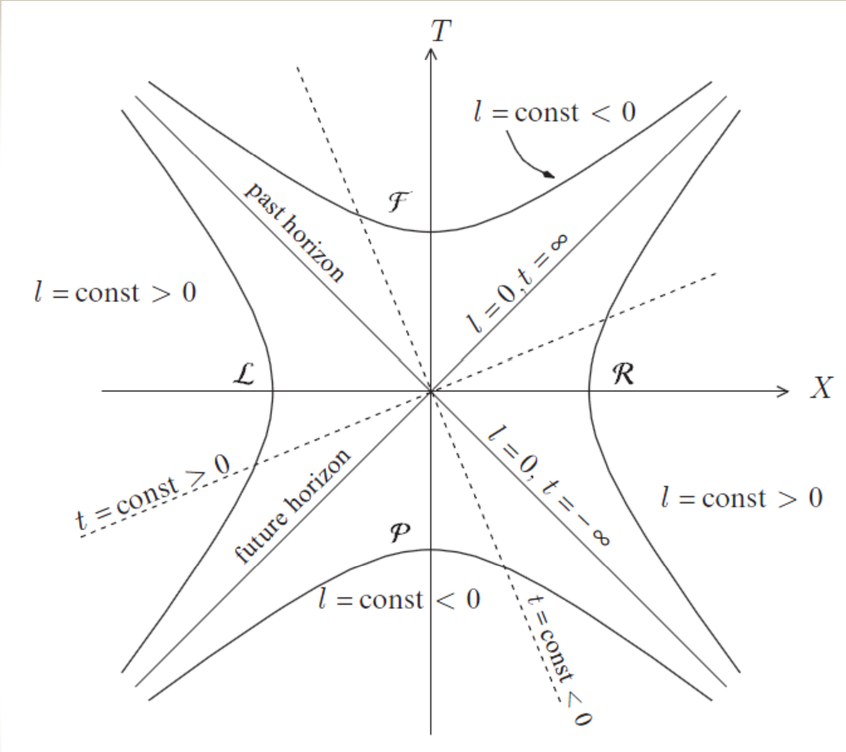
- **Flat space**

$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2$$

- **Accelerated observer**

$$\begin{cases} X = \dfrac{e^{gx}}{g}\cosh(gt) \\ T = \dfrac{e^{gx}}{g}\sinh(gt) \end{cases}$$

$$ds^2 = e^{2gx}(-dt^2 + dx^2) + dY^2 + dZ^2$$



T. Padmanabhan, Gravity; Foundations and Frontiers, Cambridge 2010

Black hole vs. Thermodynamics

- Entropy decreasing process(Wheeler)
- Horizon Entropy (Bekenstein)

- 2nd law of Black holes: Horizon area is a non-decreasing function of time

Unruh effect

- **Davis, Unruh,1975:**

  Rindler horizon in flat space-time has a temperature

$$\nu\,|\,f(\nu)\,|^2 = \frac{\beta}{e^{\beta\nu}-1}, \qquad \beta^{-1} = k_B T = \frac{h}{c}\frac{g}{2\pi}$$

Hawking radiation

- **Distant observers receive a thermal radiation from the black hole at late times with**

$$k_B T = \frac{h}{c}\frac{g}{2\pi}, \quad g = GM / r_{hor}^2 \ \text{surface gravity}$$

- **0th law of Black holes: surface gravity is constant on Horizon**

Black holes and horizons

- **Horizons are One-way membranes**
- **Horizons have a temperature**
- **Horizons have entropy**

Accidents in GR

Field equations can be interpreted as Thermodynamic Identities

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$schwarzshild \rightarrow (1-f) - rf'(r) = -(8\pi G/c^4) P\, r^2$$

$$\frac{c^4}{G}\left[\frac{1}{2} f'(a) a - \frac{1}{2}\right] = 4\pi P a^2$$

$$\xrightarrow{\times da} \underbrace{\frac{hg}{2\pi c}}_{T} \underbrace{\frac{c^3}{Gh} d\left(\frac{1}{4} 4\pi a^2\right)}_{dS} - \underbrace{\frac{1}{2}\frac{c^4}{G} da}_{dE} = \underbrace{P d\left(\frac{4\pi a^3}{3}\right)}_{dV}$$

$$TdS - dE = PdV$$

Accidents in GR

$$N = \frac{A}{l_{pl}^2} = \frac{4\pi r^2 c^3}{Gh} \quad Horizon\ \ area\ \ bits$$

$$k_B T = \frac{hg}{2\pi c} = \frac{hGM}{2\pi c r^2}$$

$$\frac{1}{2} N.k_B T = \frac{1}{2}\frac{4\pi r^2 c^3}{Gh}\frac{hGM}{2\pi c r^2} = M$$

$$\boxed{\Rightarrow E = \frac{1}{2} N k_B T, \qquad \text{Energy Equipartition}}$$

Boltzmann’s principle

- Thermal behavior $\Longrightarrow$ microscopic physics

Or

“if you can heat something ,

it has sub-structure”

Reversing the Logic Emergence

- Gravity as a Fundamental theory
  - accidents
- Gravity as an Emergent phenomena
  - GR is the statistical mechanics of some unknown degrees of freedom

Universality of Thermodynamics

- Large irrelevance of underlying theory

- Finite parameters
  - Viscosity
  - Specific heat
  - Conductivity

Alternative perspective

- **Kinematics → Equivalence principle**
- **Dynamics → Thermodynamic Principle**

The new principle

- **2^{nd} law of thermodynamics should hold for any observer**

$$\delta(S_{matter} + S_{grav}) \geq 0 \quad \text{for all observers}$$

Gravitational Entropy

$$S_{matter}[n] = \int_V d^D x \sqrt{-g}\, T_{ab} n^a n^b$$

$$S_{grav}[n] = \int_V d^D x \sqrt{-g}\, P_{ab}^{cd} \nabla_c n^a \nabla_d n^b$$

$$P_{ab}^{cd} = \delta_a^c \delta_b^d - \delta_a^d \delta_b^c$$

Field equations

$$S[n] = \int_V d^D x \sqrt{-g}\left[P_{ab}^{cd} \nabla_c n^a \nabla_d n^b - T_{ab} n^a n^b - \lambda(x) g_{ab} n^a n^b\right]$$

$\delta S = 0$ for any vector n

$$\left(2P_b^{ijk} R_{ijk}^a - T_b^a + \lambda \delta_b^a\right) n_a = 0$$
$$\Rightarrow \left(G_{ab} - T_{ab} + \lambda g_{ab}\right) n_a = 0$$

Summary and conclusion

- Gravity can be an Emergent phenomenon
- There is no need to quantize Gravity
- We should search for the underlying theory

References

- T.Padmanabhan, *Gravitation,2010*
- T.Padmanabhan, “*Gravity*; *A different perspective*” Lecture at Perimeter Institute, 2010

Thank you for your attention