The Spectrum Of Modified Theories Of Gravity

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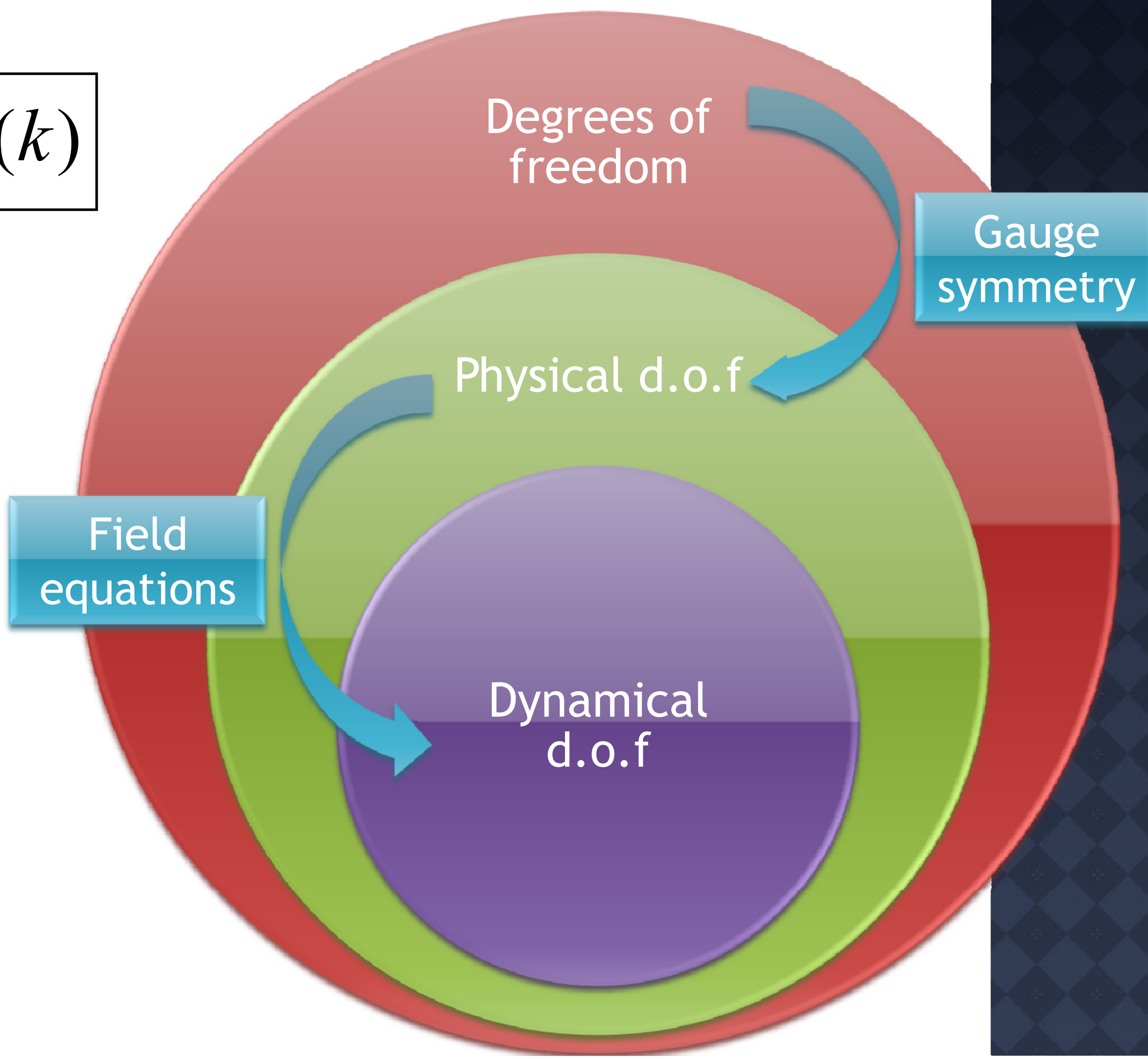
CONTENTS

DYNAMICAL DEGREES OF FREEDOM

$$\boxed{\Phi(x) = \int d^4k.e^{ik.x}\Phi(k)}$$



Green's Function

$$\left(\Box - m^2\right)\Phi = 0$$

$$(\partial^2 - m^2)G(x-x') = \delta(x-x')$$

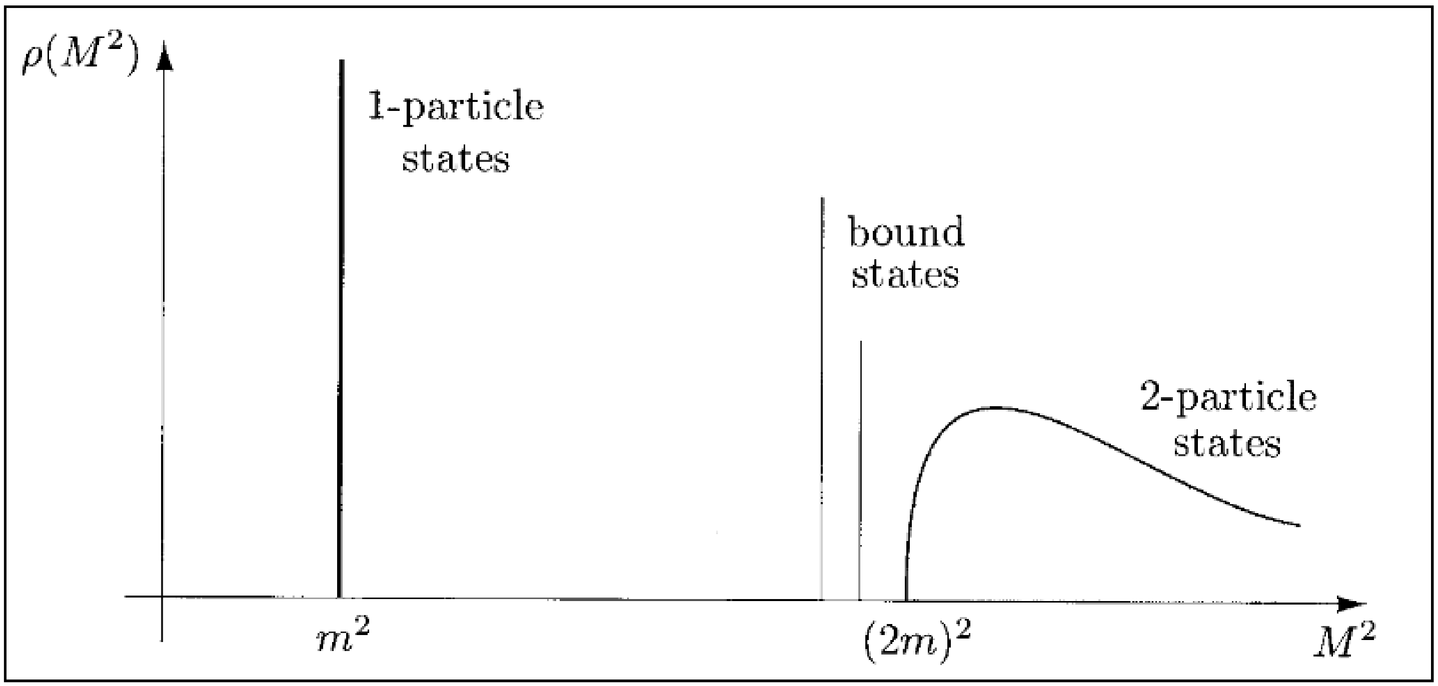
$$\xrightarrow{\text{Fourier Trans.}} -\int d^4k.e^{ik.(x-x')}(k^2+m^2)\widetilde{G}(k) = \int d^4k.e^{ik.(x-x')}$$

$$\Rightarrow \widetilde{G}(k) = -\frac{1}{(k^2+m^2)} = \frac{1}{(\omega^2 - |\vec{k}|^2 - m^2)}$$

Propagator in QFT

- Propagators in QFT

$$\langle\Omega|\,T\phi(x)\phi(y)\,|\Omega\rangle = \int_0^\infty \frac{dM^2}{2\pi}\,\rho(M^2)\,D_F(x-y;M^2)$$



Peskin,Schroeder, An Intro to QFT ,1995

EM

$$\boxed{A_\mu(x) = \int d^4k . e^{ik.x} A_\mu(k)}$$

$$\begin{cases} \Box A_\mu = 0 \\ \partial_\mu A^\mu = 0 \end{cases} \qquad \begin{cases} k^2 A_\mu(k) = 0 \\ k_\mu A^\mu(k) = 0 \end{cases}$$

$$k = (1,1,0,0) \rightarrow A_\mu = (\Phi, \Phi, A_y, A_z)$$

$$gauge \quad freedom \quad \Rightarrow A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \varphi$$

$$A_\mu = (0, 0, A_y, A_z)$$

Massless field(particle) with 2 degrees of freedom

METHOD

Fourier Transformation

fix the gauge freedom

Apply the E.O.M

Extract dynamical D.O.F

Analyze # and spin of D.O.F

Spectrum Of Different Theories

Theory	Spin	Mass	Dynamical Degrees of freedom
EM	Spin 1 Photon	0	2 transverse Left/right polarizations
GR	Spin 2 Graviton	0	2 transverse
Modified gravity	?	?	?

F(R) Theories of Gravity

- Action $$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R)$$

- Field equation

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)f'(R) = 8\pi G T_{\mu\nu}$$

- Linearization $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = \frac{1}{d}\eta_{\mu\nu}h + j_{\mu\nu}$$

$$\partial^\mu j_{\mu\nu} = 0$$

$$\boxed{-\frac{f'(0)}{2}\Box j_{\mu\nu} + [\Box\eta_{\mu\nu} - \partial_\mu\partial_\nu]\Big(\frac{f'(0)}{2}(d-2) - f''(0)(d-1)\Box\Big)\frac{h}{d} = 0}$$

SPECTRUM

Spectrum	Number of dynamical modes	Spin	Mass
scalar	1	0	$m^2_{scalar} = \frac{f_0'}{2f_0''}\frac{d-2}{d-1} - \frac{R_0}{d-1}$
graviton	2	2	0

f(R,Ricci) Theories of Gravity

K.S Stelle (1977)

Renormalizability of f(R,Ricci) Theories of Gravity

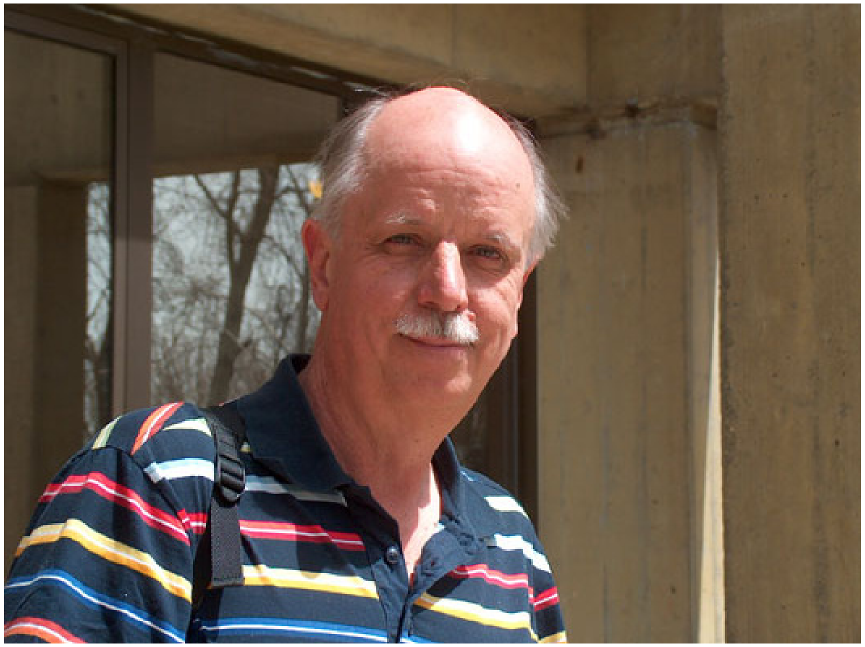


Photo: www.physics.ipm.ac.ir

f(R,Ricci) Theories of Gravity

- *Action*

$$S = \int d^4x\sqrt{-g}\Big(\frac{1}{16\pi G}(R-2\Lambda) + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2\Big)$$

- *Field equtions*

$$\begin{aligned} &R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} + 2\beta R(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}) \\ &+(\alpha+2\beta)(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)R + \alpha\Box(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) + 2\alpha(R_{\mu\rho\nu\sigma} - \frac{1}{4}R_{\rho\sigma}g_{\mu\nu})R^{\rho\sigma} = 0 \end{aligned}$$

- *Linearized fied equations*

$$-\frac{1}{2}\Box(1+\alpha\Box)j_{\mu\nu} + [\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu]\Big(\frac{d-2}{2d} - (\frac{\alpha}{2} + 2\beta(\frac{d-1}{d}))\Box\Big)h = 0$$

4^{th} order field equation

$$(\Box - m_1^2)(\Box - m_2^2)\phi = 0$$

- Green's function

$$\frac{1}{(p^2+m_1^2)(p^2+m_2^2)} = \frac{1}{m_2^2-m_1^2}\left(\frac{1}{(p^2+m_1^2)} - \frac{1}{(p^2+m_2^2)}\right)$$

- Ghosts!
  - Negative Norm in Hilbert space
  - Negative probability
  - Non-Unitarity
  - Negative Energy excitations

SPECTRUM

Spectrum	Number of dynamical modes	Spin	Mass
scalar	1	0	$m^2_{scalar} = \frac{1}{2(\alpha + 3\beta)}$
graviton	2	2	0
tensor	5	2	$M^2_{spin2} = -\frac{1}{\alpha}$

The tensor mode is a ghost
and can be a Tachyon

CONCLUSION

- *a method for extracting dynamical degrees of freedom*
- *Modified gravity theories can be Renormalizable but they have ghost degrees of freedom so make the theory problematic*
- *Maybe the ghost can disappear with a suitable fine tuning of parameters*

References

• *K. S. Stelle. "Renormalization of Higher Derivative Quantum Gravity", Phys. Rev. D16. 953-969 (1977).*

•*K. S. Stelle. "Classical Gravity with Higher Derivatives. Gen. Rel. Grav. 9.353-371(1978).*

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Thank you for your attention