# Radiative Corrections to Deep Inelastic Scattering

ARIF AKHUNDOV
University of Valencia
Spain

# Overview

- Deep Inelastic Scattering
  at SLAC, CERN and HERA
- Kinematics of DIS
- RC to DIS at SPS (CERN)
- RC to DIS at HERA (DESY)

# Deep Inelastic Scattering

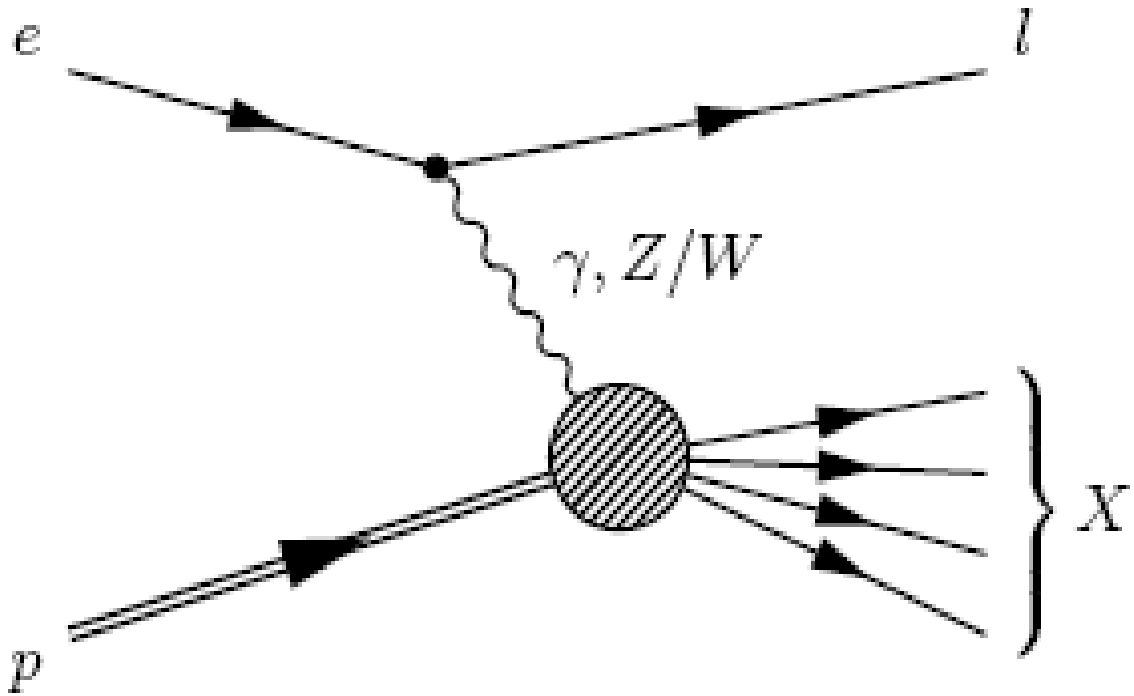



Figure 2.1: Deep inelastic lepton-proton scattering

- Standard variables are:

$$
\begin{aligned}
x &= \frac{-q^2}{2p\cdot q} = \frac{Q^2}{2M(E-E')} \\
y &= \frac{q\cdot p}{k\cdot p} = 1 - \frac{E'}{E}
\end{aligned}
$$

where $Q^2 = -q^2 > 0$, $M^2 = p^2$ and energies refer to target rest frame.

- Elastic scattering has $(p+q)^2 = M^2$, i.e. $x = 1$. Hence deep inelastic scattering (DIS) means $Q^2 \gg M^2$ and $x < 1$.

The energy of the incident electron beam is accurately known
The proton is the target particle; in the SLAC experiments (and many later experiments at CERN) the target is at rest in the laboratory. This defines the LAB frame.

DIS experiments have also been done with muons and with neutrinos.

Since 1992 DIS experiments are also done on the electron-proton collider ***HERA***.

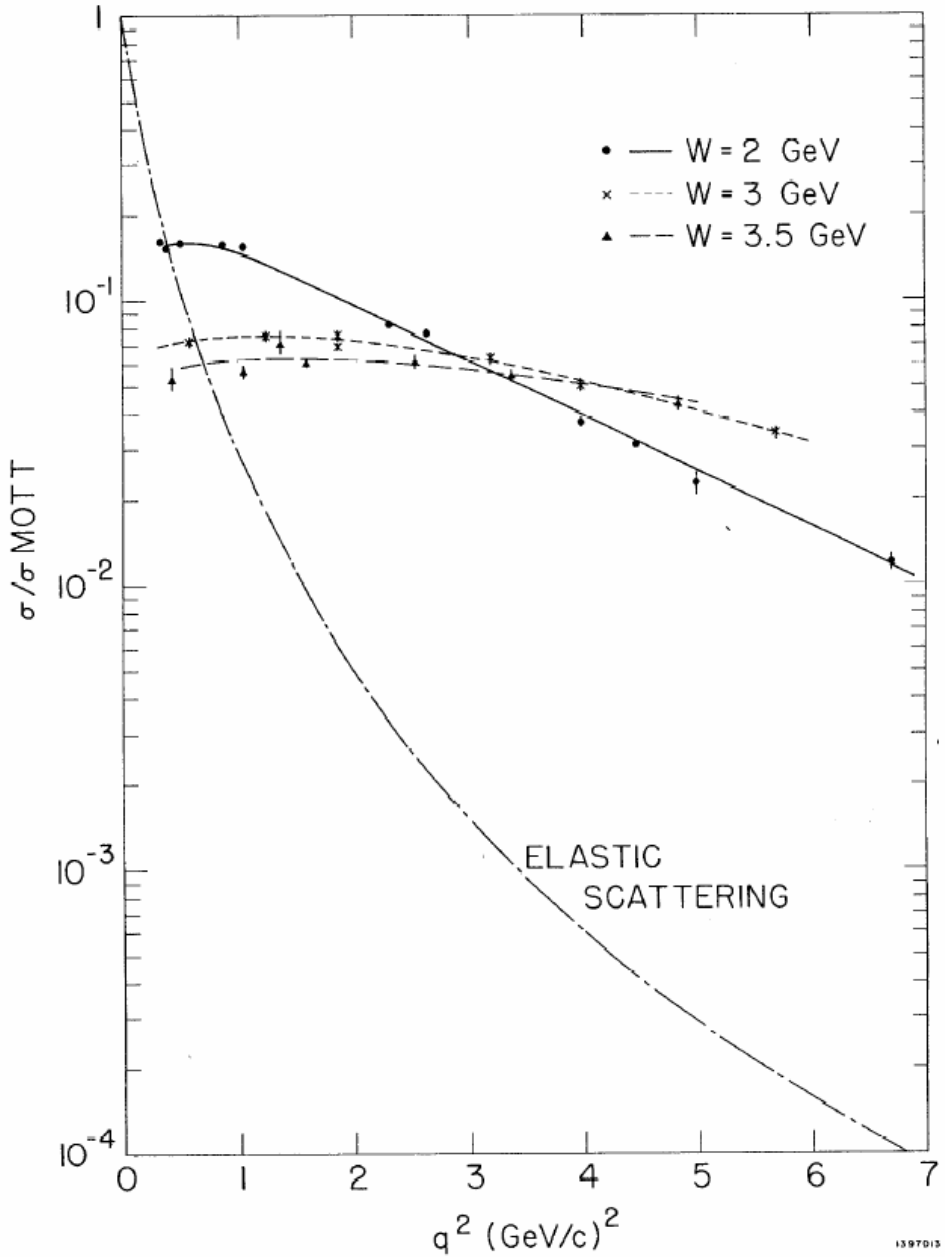
- Structure functions $F_i(x, Q^2)$ parametrise target structure as 'seen' by virtual photon. Defined in terms of cross section

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{Q^2xy}\left[\left(\frac{1+(1-y)^2}{2}\right)2xF_1 + (1-y)(F_2 - 2xF_1) - (M/2E)xyF_2\right].$$

On general grounds the structure functions are functions of two kinematical invariants

They were expected to drop with increasing $Q^2$ as rapidly as the form factors

The surprising experimental result was different!



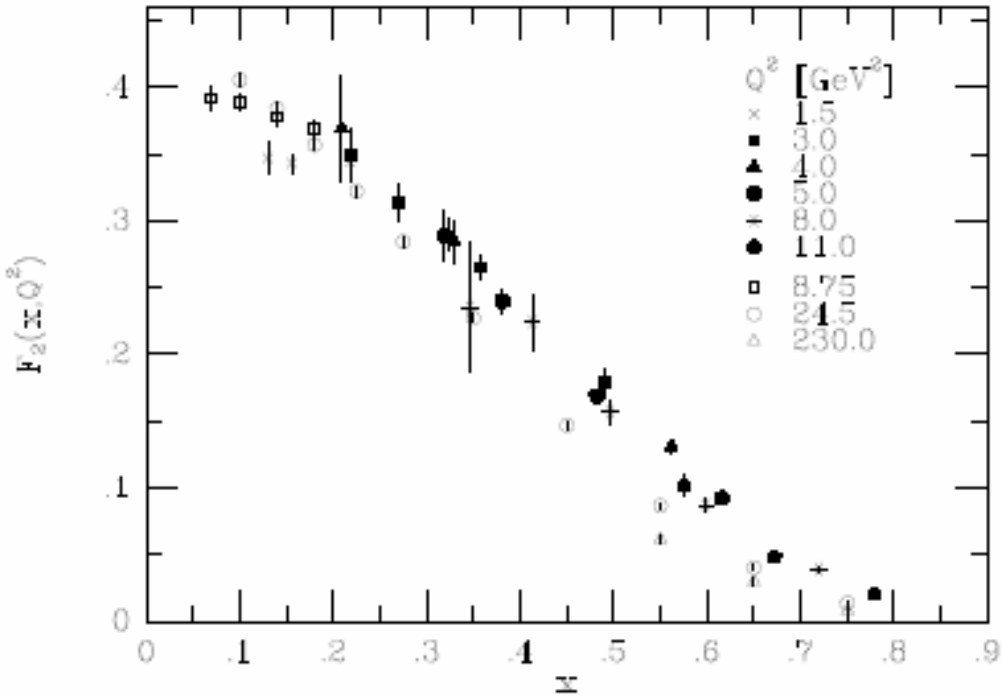

SLAC-PUB-650
August 1969
(EXP) and (TH)

OBSERVED BEHAVIOR OF HIGHLY INELASTIC
ELECTRON-PROTON SCATTERING

M. Breidenbach, J. I. Friedman, H. W. Kendall
Department of Physics and Laboratory for Nuclear Science,*
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

E. D. Bloom, D. H. Coward, H. DeStaebler,
J. Drees, L. W. Mo, R. E. Taylor
Stanford Linear Accelerator Center,† Stanford, California 94305

- Deep inelastic:
  - $W >> M_P$
- $\sigma/\sigma_{Mott} \approx const.$

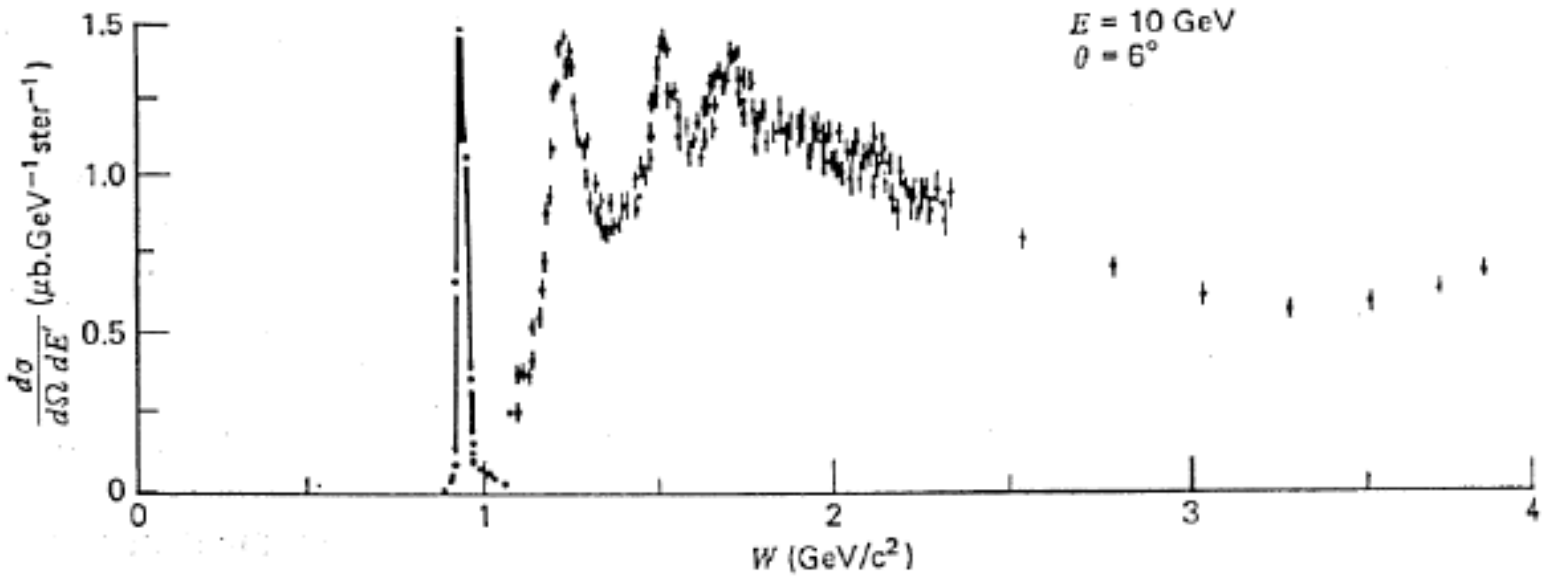



- Figure shows $F_2$ structure function for proton target. Although $Q^2$ varies by two orders of magnitude, in first approximation data lie on universal curve.

- Bjorken limit is $Q^2$, $p \cdot q \to \infty$ with $x$ fixed. In this limit structure functions obey approximate Bjorken scaling law, i.e. depend only on dimensionless variable $x$:

$$
\begin{aligned}
F_i(x, Q^2) &\longrightarrow F_i(x) \\
\sigma_i(x, Q^2) &= \frac{4\pi^2\alpha}{Q^2} F_i(x, Q^2) \sim 1/Q^2
\end{aligned}
$$

- Bjorken scaling implies that virtual photon is scattered by *pointlike constituents* (partons) — otherwise structure functions would depend on ratio $Q/Q_0$, with $1/Q_0$ a length scale characterizing size of constituents.




up to about W = 1.8 GeV there is structure corresponding to the production of resonances (excited nucleon states);
there is no structure above 1.8 GeV: this is the region of DIS.

Scaling found a natural explanation in the *parton model* (Feynman).

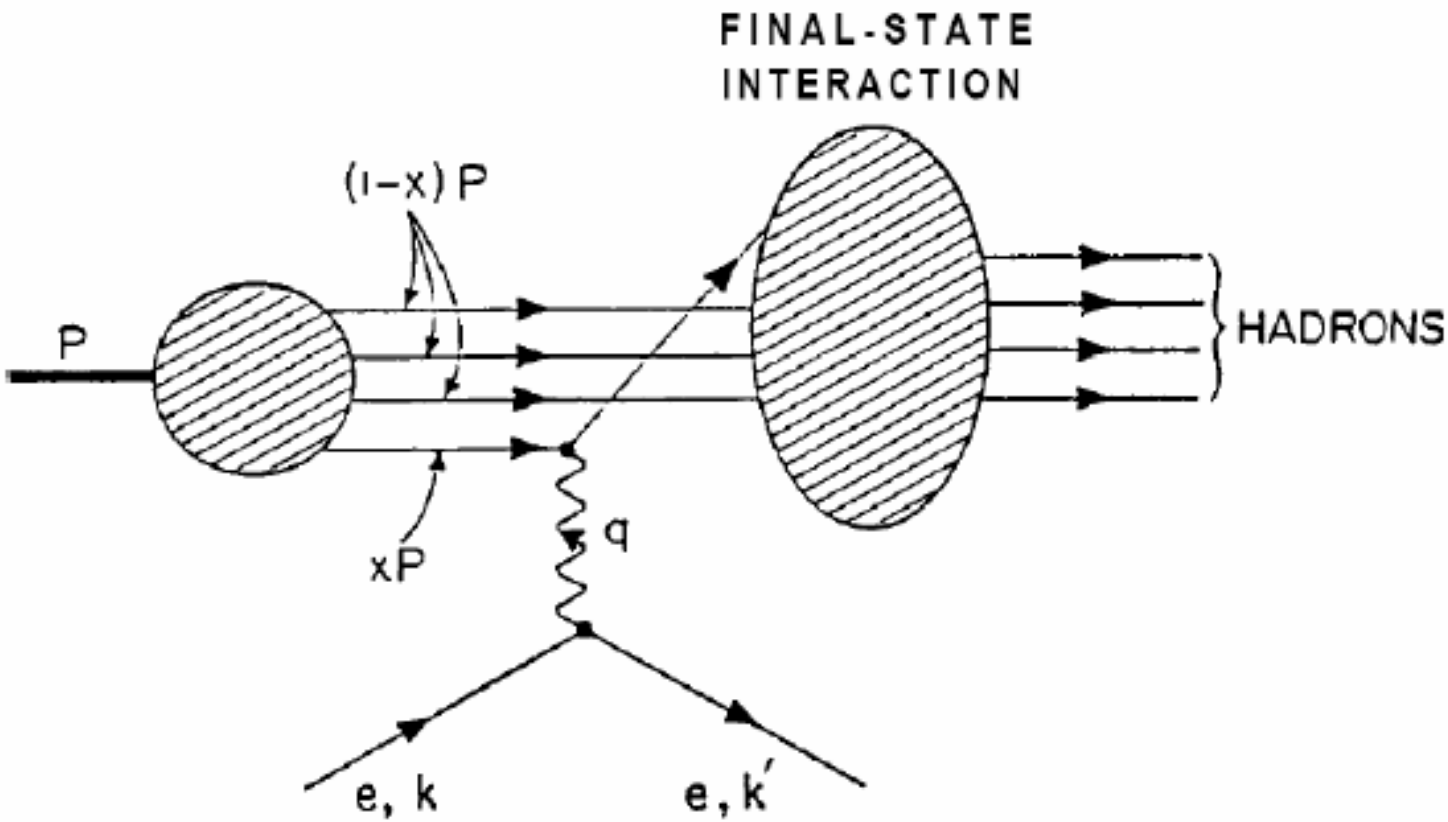
Partons are constituents of the proton (more generally of hadrons).

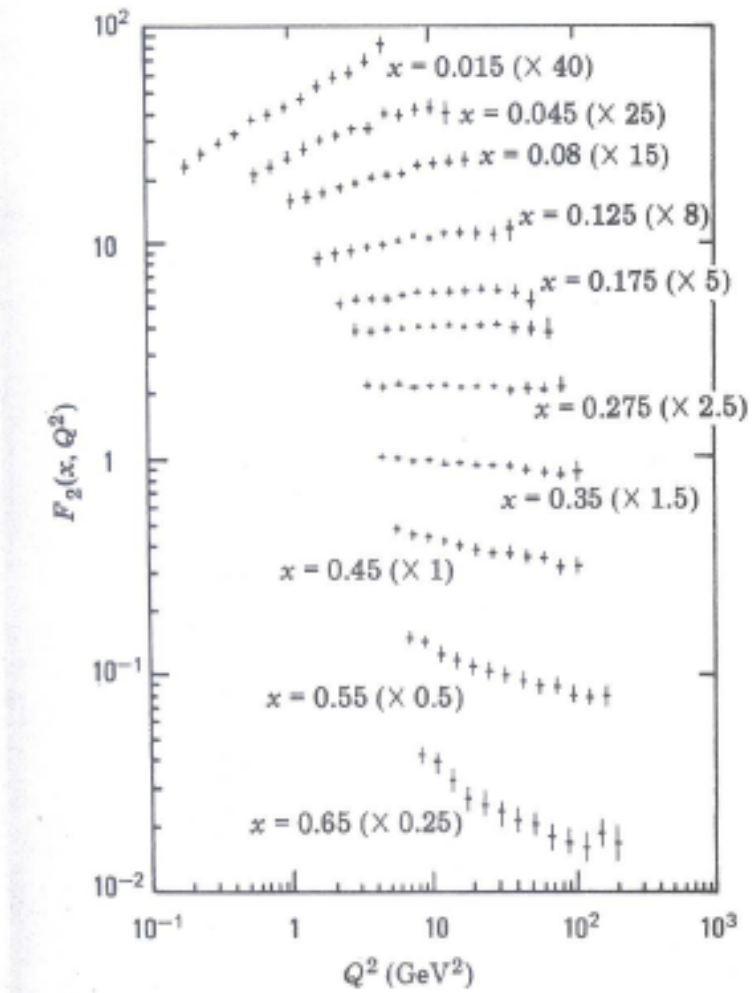
They are quarks and gluons:

***quarks*** are point-like spin-1/2 fermions like the leptons,
***gluons*** are massless spin-1 bosons: they are the carriers of the strong interaction.

Unlike leptons, quarks take part in strong as well as electromagnetic and weak interactions.

## Parton model picture of DIS:



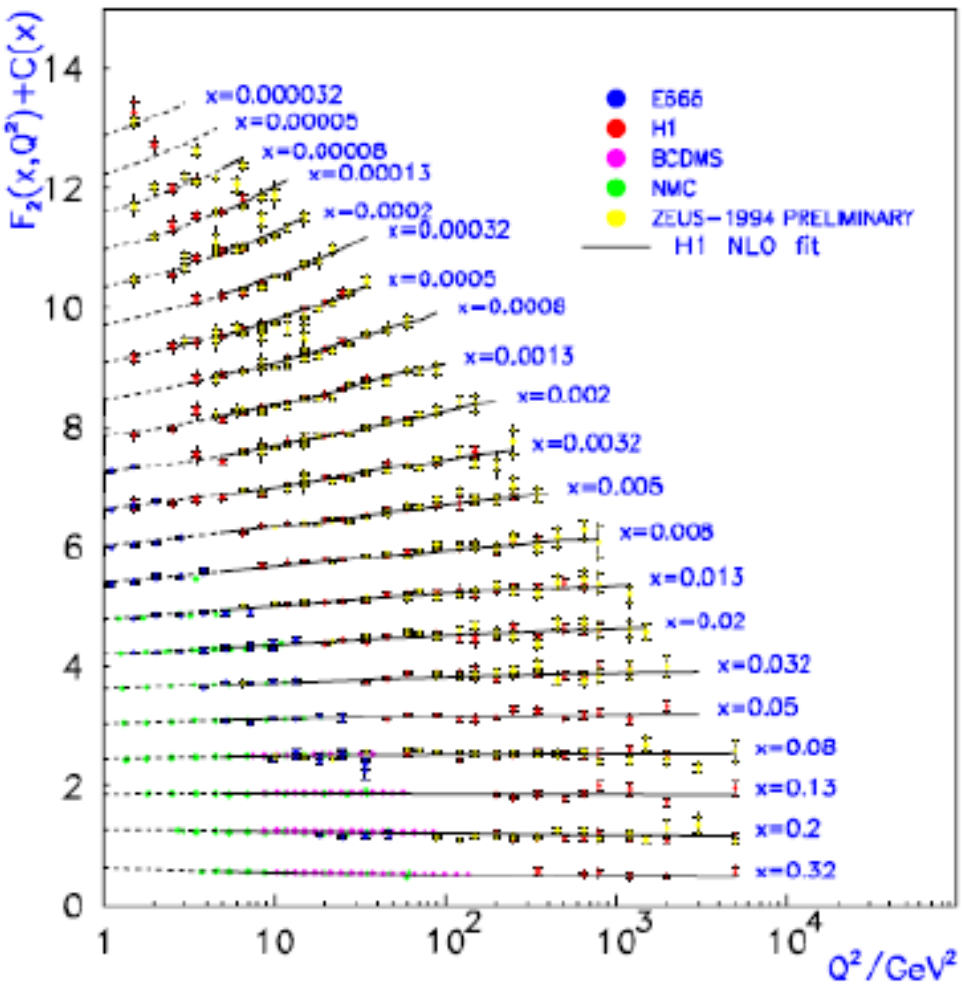



Nucleon structure function $F_2^{\nu N}$ measured in neutrino-nucleon DIS

Data of CDHSW collaboration, 1989

The individual $x$ ranges are scaled by the factors shown to separate them from each other.

Clearly seen is the scaling violation at the lowest and highest $x$ values




Compilation of data on the proton structure function $F_2^p(x,Q^2)$

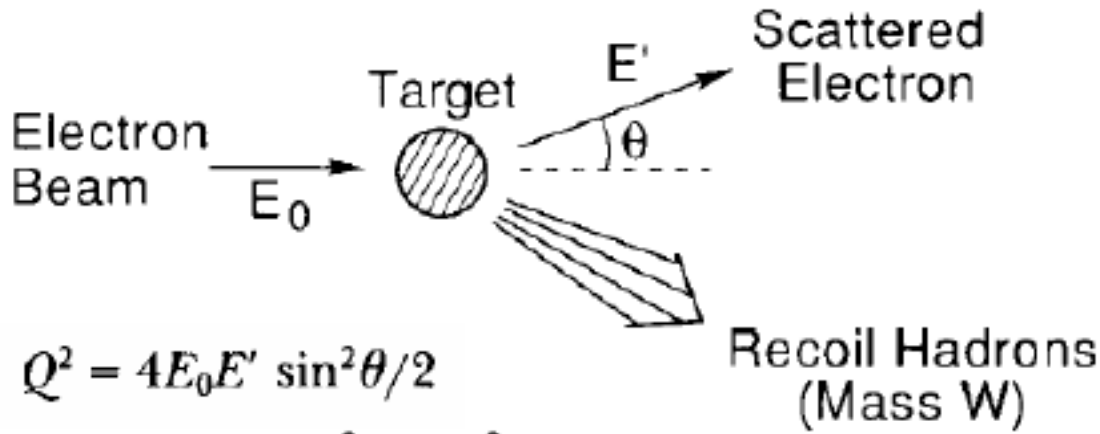
Experiments:

| | |
|---|---|
| E666 | (Fermilab) |
| H1 | (DESY) |
| BCDMS | (CERN) |
| NMC | (CERN) |
| ZEUS | (DESY) |

To $F_2$ of each $x$ range an amount $C(x)$ is added "by hand" to separate the data sets.

Note the scaling violation at the lowest values of $x$

# Kinematics of DIS

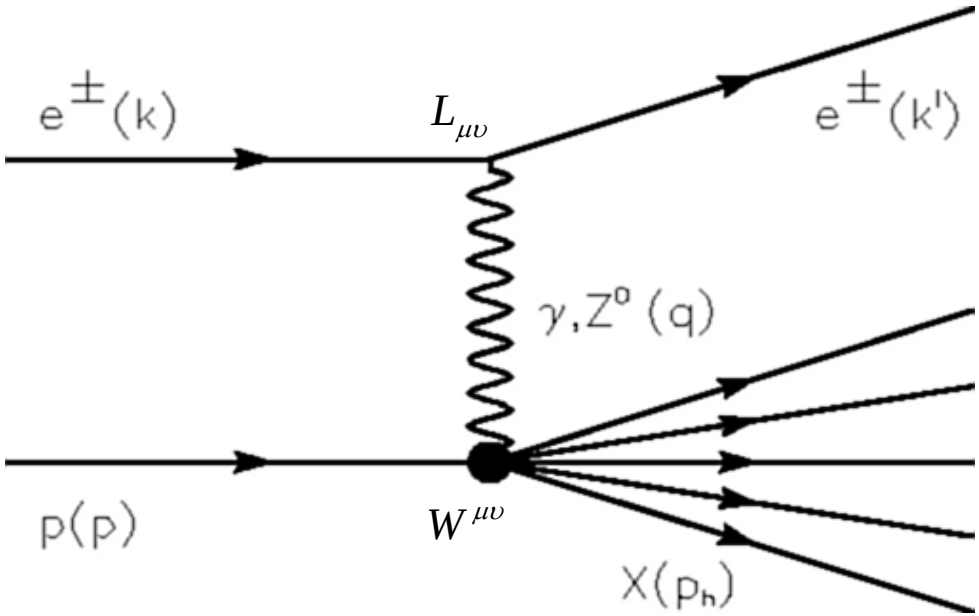



$$Q^2 = 4E_0E' \sin^2\theta/2$$

$$E' = \frac{E_0 - \frac{(W^2 - M^2)}{2M}}{1 + \frac{2E_0}{M}\sin^2\theta/2}$$

or, since $E'$ and $\theta$ are measured:

$$W^2 = M^2 + 2M\left(E_0 - E'\right) - 4E_0E'\sin^2\frac{\theta}{2}$$




Four-momentum transfer squared:

$$Q^2 = -q^2 = -(k-k')^2$$

Bjorken scaling variable:

$$x = -\frac{q^2}{2p.q}$$

Inelasticity scaling variable:

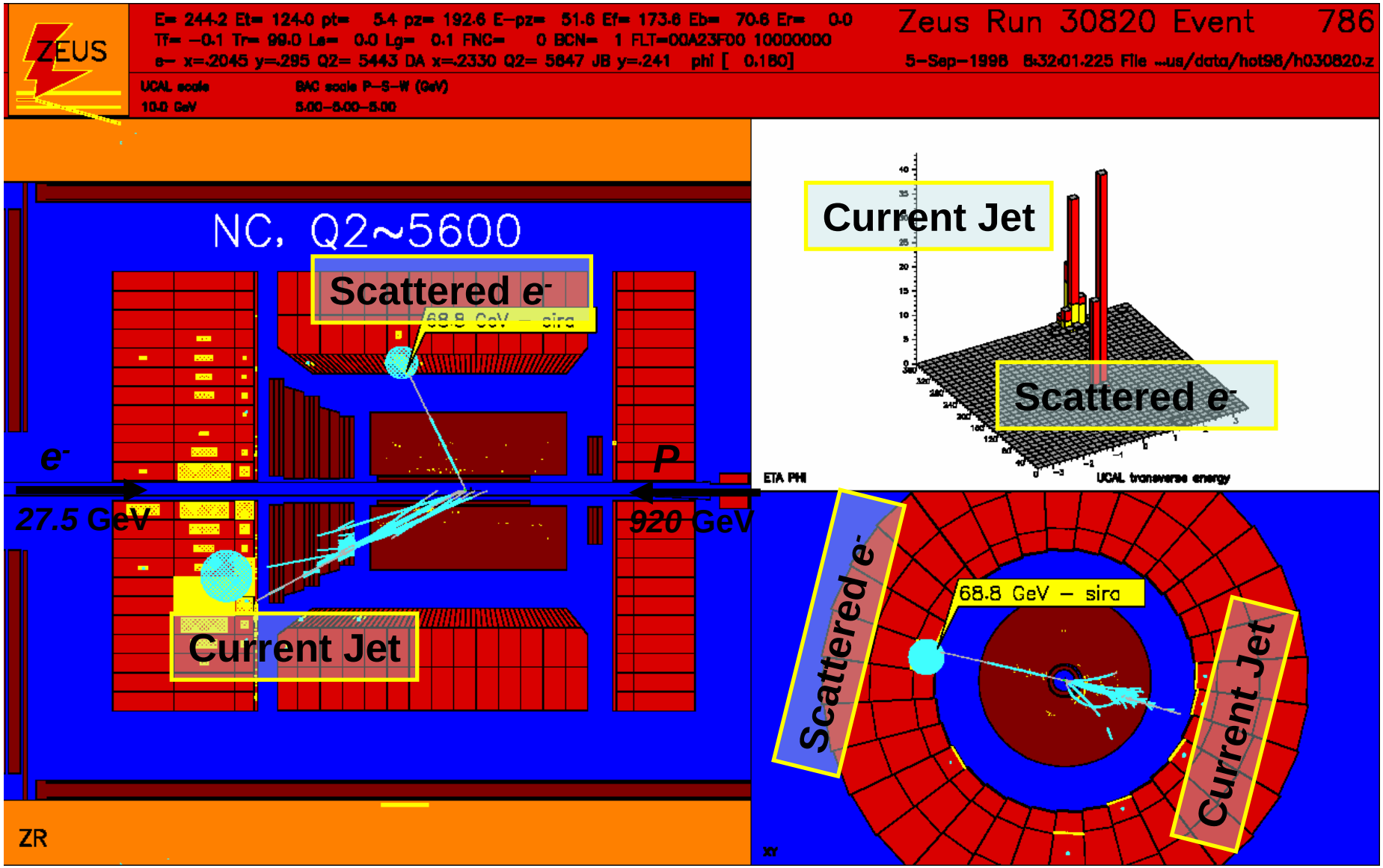
$$y = \frac{p.q}{p.k}$$

Centre of mass energy squared:

$$s = (k+p)^2 \doteq 4E_eE_p$$

$$Q^2 = xys$$

$$W^2 = (q+p)^2 \doteq ys$$


ZEUS
E= 244.2 Et= 124.0 pt= 5.4 pz= 192.6 E−pz= 51.6 Ef= 173.6 Eb= 70.6 Er= 0.0
Tf= −0.1 Tr= 99.0 Le= 0.0 Lg= 0.1 FNC= 0 BCN= 1 FLT=00A23F00 10000000
e− x=.2045 y=.295 Q2= 5443 DA x=.2330 Q2= 5847 JB y=.241 phi [ 0.180]
Zeus Run 30820 Event 786
5−Sep−1998 6:32:01.225 File ...us/data/hot98/h030820.z
UCAL scale
10.0 GeV
BAC scale P−S−W (GeV)
5.00−5.00−5.00
NC, Q2~5600
Scattered e-
68.8 GeV − sira
e-
27.5 GeV
P
920 GeV
Current Jet
ZR
Current Jet
Scattered e-
ETA PHI
UCAL transverse energy
Scattered e-
68.8 GeV − sira
Current Jet
XY

# Reconstruction of kinematic variables $Q^2, x \,\&\, y$

- incoming particles
- electron $E'_e, \theta_e$
- hadronic system $E_h, \gamma_h$
- momentum-conservation

$\Rightarrow$ overconstrained system

$\rightarrow$ choose two quantities for reconstruction

Electron (EL): $E'_e, \theta_e$

$$Q^2_{EL} = 2E_e E'_e(1+\cos\theta_e)$$

$$x_{EL} = \frac{E_e}{E_p}\frac{E'_e(1+\cos\theta_e)}{2E_e - E'_e(1-\cos\theta_e)}$$

$$y_{EL} = 1 - \frac{E'_e}{2E_e}(1-\cos\theta_e)$$

Jacquet-Blondel (JB): only hadronic energies

$$Q^2_{JB} = \frac{P^2_{t,h}}{1-y_{JB}}$$

$$x_{JB} = \frac{Q^2_{JB}}{s \cdot y_{JB}}$$

$$y_{JB} = \frac{(E-p_z)_h}{2E_e}$$

Double-Angle (DA): $\theta, \gamma_h$

$$Q^2_{da} = 4E_e^2 \frac{\sin\gamma_h(1+\cos\theta_e)}{\sin\gamma_h + \sin\theta_e - \sin(\gamma_h+\theta_e)}$$

$$x_{DA} = \frac{E_e}{E_p}\frac{\sin\gamma_h + \sin\theta_e + \sin(\gamma_h+\theta_e)}{\sin\gamma_h + \sin\theta_e - \sin(\gamma_h+\theta_e)}$$

$$y_{DA} = \frac{\sin\theta_e(1-\cos\gamma_h)}{\sin\gamma_h + \sin\theta_e - \sin(\gamma_h+\theta_e)}$$

... and many more mixed methods!

# RC to DIS at SPS (CERN)

**Benvenuti et al. Phys. Lett. B195(87)91**

F2(nucleon) from carbon target at 113.6 GeV
F2(nucleon) from carbon target at 191.7 GeV
F2(nucleon) from carbon target at 270.5 GeV
Measurement of R (=sigL/sigT) with carbon target

**Benvenuti et al. Phys. Lett. B237(87)599**

F2(neutron)/F2(proton) from deuterium target at 280 GeV
F2(proton)-F2(neutron) from deuterium target at 280 GeV

**Benvenuti et al. Phys. Lett. B223(89)485**

F2(proton) at 100 GeV
F2(proton) at 120 GeV
F2(proton) at 200 GeV
F2(proton) at 280 GeV

In inclusive-type experiments when only the final lepton is detected the processes

$$\ell + N \to \ell + \gamma + N, \qquad (1)$$

$$\ell + N \to \ell + \gamma + N^*, \qquad (2)$$

$$\ell + N \to \ell + \gamma + \text{hadrons} \qquad (3)$$

($N^*$ is the nucleon resonance)
cannot be distinguished from the main reaction

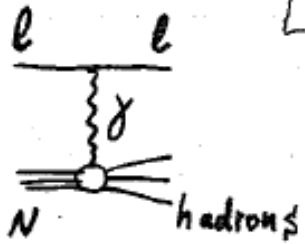
$$\ell + N \to \ell + \text{hadrons}. \qquad (4)$$

Thus, the measured cross section of deep inelastic $\ell N$-scattering is a sum of inclusive cross sections of processes (1), (2), (3) and main process (4)

$$d^2\sigma^{exp} = d^2\sigma^{elt\,tail} + \Sigma d^2\sigma^{Rtail} + d^2\sigma_0(1+\delta), \qquad (5)$$

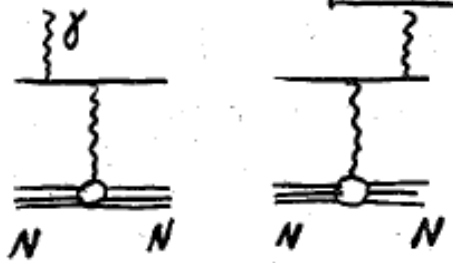
where $d^2\sigma_0$ is the cross section of reaction (4) in the Born approximation and $\delta$ is the EC to the cross section.

$\boxed{\ell + N \to \ell + \text{hadrons}}$



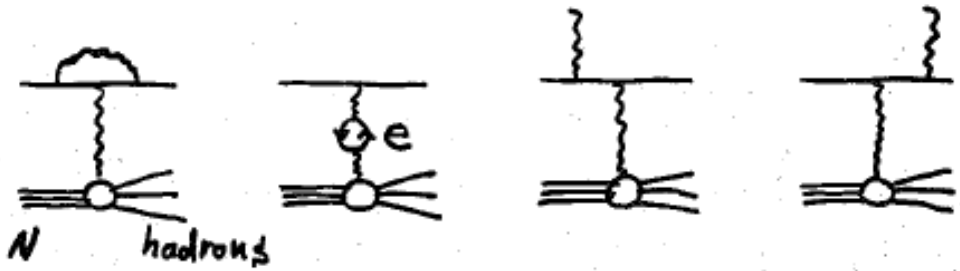

$$\Rightarrow \quad d\sigma_0(\alpha^2) \qquad \boxed{\ell = e, \mu}$$

$\boxed{\ell + N \to \ell + \gamma + N}$




$$\Rightarrow \quad d\sigma^{el}(\alpha^3)$$

Elastic radiative tail

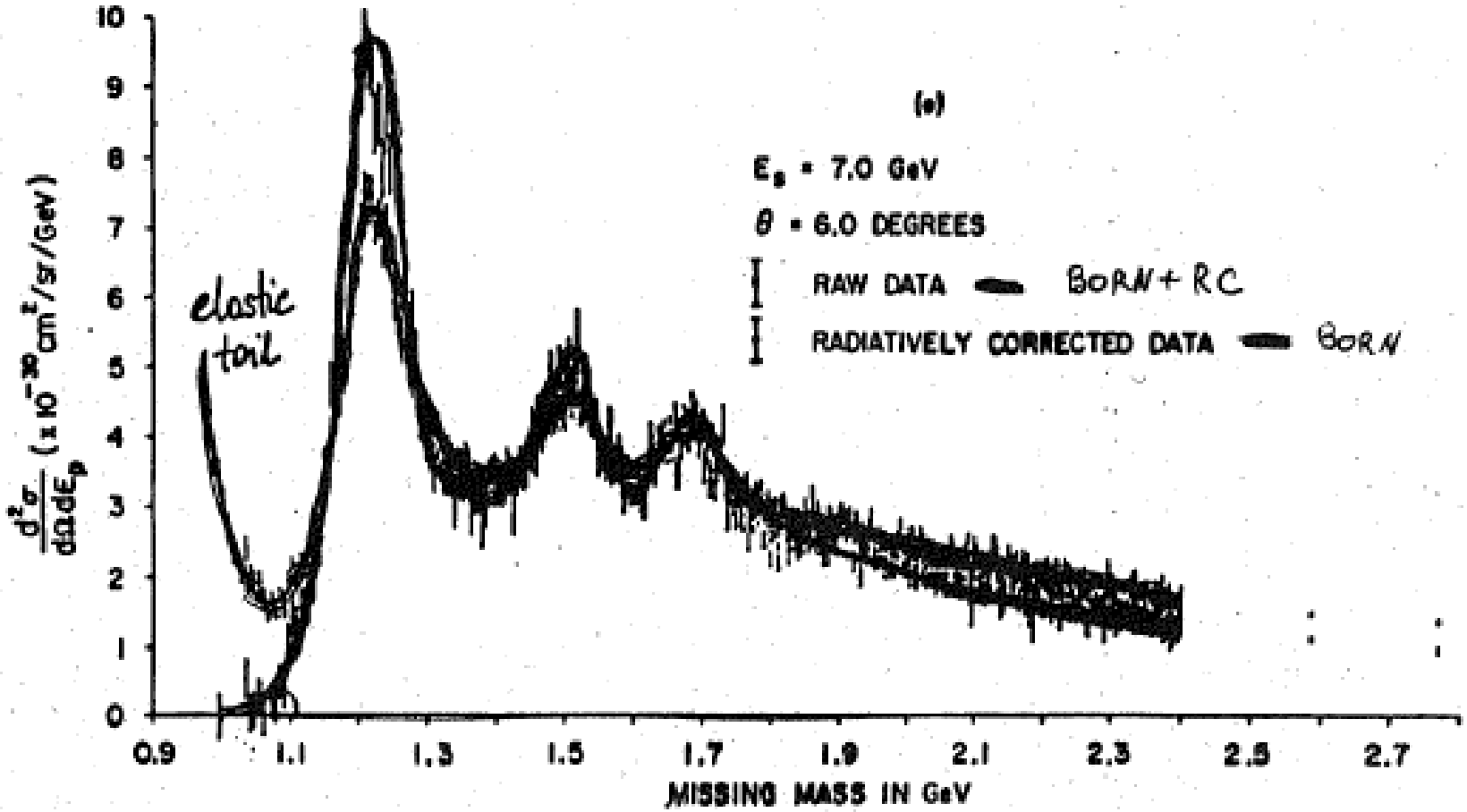



$$\Rightarrow d\sigma^{inel}(\alpha^3)$$

Inelastic radiative tail

$$d\sigma_1(\alpha^3) = d\sigma^{el}(\alpha^3) + d\sigma^{inel}(\alpha^3)$$

$$\boxed{\delta_1(\alpha) = d\sigma_1(\alpha^3) / d\sigma_0(\alpha^2) = \delta^{el}(\alpha) + \delta^{inel}(\alpha)}$$


(e)
$E_e$ = 7.0 GeV
$\theta$ = 6.0 DEGREES
RAW DATA
BORN + RC
RADIATIVELY CORRECTED DATA
BORN
elastic tail
$\frac{d^2\sigma}{d\Omega dE_p}$ ($\times 10^{-30}$ cm$^2$/sr/GeV)
MISSING MASS IN GeV
0
1
2
3
4
5
6
7
8
9
10
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7

REVIEWS OF MODERN PHYSICS VOLUME 41, NUMBER 1 JANUARY 1969

# Radiative Corrections to Elastic and Inelastic $ep$ and $\nu p$ Scattering*

L. W. MO, Y. S. TSAI
*Stanford Linear Accelerator Center, Stanford University, Stanford, California*

$$(d\sigma_r/d\Omega dp)(E_s, E_p) = (d\sigma/d\Omega dp)(E_s, E_p)[1+\delta_r(\Delta)] + (d\sigma_r/d\Omega dp)(w>\Delta),$$

where $d\sigma/d\Omega dp(E_s, E_p)$ is the continuum nonradiative cross section,

$$\delta_r(\Delta) = \frac{-\alpha}{\pi}\left[\tfrac{28}{9} - \tfrac{13}{6}\ln\frac{2(sp)}{m^2} + \left(\ln\frac{E_s}{\Delta} + \ln\frac{E_p}{\Delta}\right)\left(\ln\frac{2(sp)}{m^2} - 1\right) - \Phi\left(-\frac{E_s - E_p}{E_p}\right) - \Phi\left(\frac{E_s - E_p}{E_s}\right)\right],$$

$\Phi(x)$ is the Spence function, and

$$\frac{d\sigma_r}{d\Omega dp}(\omega>\Delta) = \frac{\alpha^3}{2\pi}\frac{E_p}{ME_s}\int_{-1}^{1} d(\cos\theta_k)\int_{\Delta}^{\omega_{max}(\cos\theta_k)}\frac{\omega d\omega}{q^4}\int_0^{2\pi} B_{\mu\nu}{}^c T_{\mu\nu} d\phi_k.$$

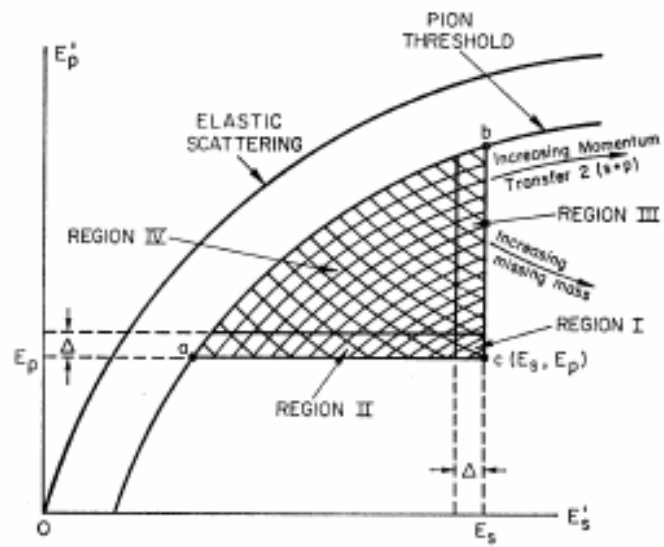



FIG. 3. Kinematic regions necessary for radiative corrections to inelastic electron scattering. $E_s'$ is the incident electron energy and $E_p'$, the scattered electron energy.

1. Some experiments on $\mu p$ -scattering[7] are carried out in the region $Q^2 \simeq m_\mu^2$ ($m_\mu$ - the muon mass). The formulae of Mo and Tsai are inapplicable at such $Q^2$, therefore exact formulae are required.

2. Expressions for the EC in ref.[4] include the "softness" parameter $\Delta$ dividing the contributions of soft and hard photons thus breaking their Lorentz-invariance. However, in inclusive processes these contributions are not separated physically[8] therefore it would be reasonable to obtain formulae independent of the "softness" parameter and covariant in order to make them be applicable directly to the planned experiments on the colliding $ep(\mu p)$ beams.

A revised calculation of electromagnetic radiative corrections was performed in refs.[2,3]. Completely covariant formulae are obtained which contrary to those given by Mo and Tsai, do not contain the unphysical "soft-photon" parameter.

A.A. Akhundov, D.Yu. Bardin and N.M. Shumeiko, *Electromagnetic corrections to deep inelastic lepton-nucleon scattering at high energies. I. Contribution of the radiative tail of elastic peak*, JINR Report, JINR-E2-10147 (1976), 24 pp.
A.A. Akhundov, D.Yu. Bardin and N.M. Shumeiko, *Electromagnetic corrections to deep inelastic $\mu p$ scattering at high energies*, JINR Preprint, JINR-E2-10471 (1977), 18 pp. &
*Yadernaya Fizika, v.26, p.1251-1257 (1977) (in Russian) &*
*Sov. J. Nucl. Phys., v.26, 660 (1977) (in English, USA).*

# RC from Elastic Radiative Tail

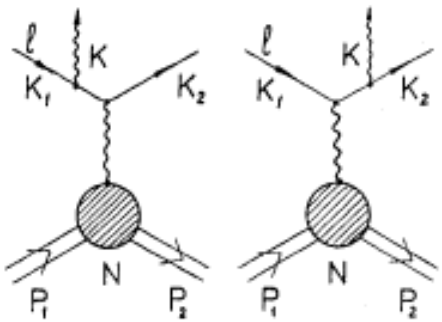
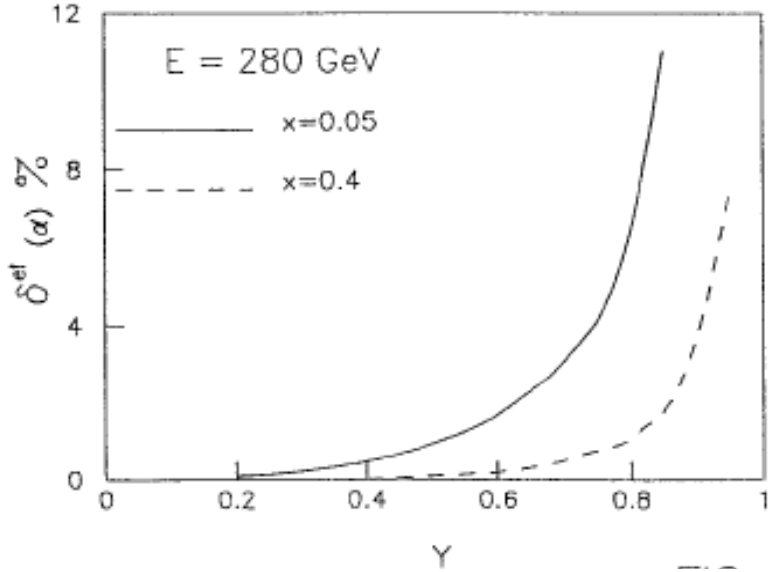



Fig. 1. The diagrams giving the dominant contribution to the cross section of the process (1).



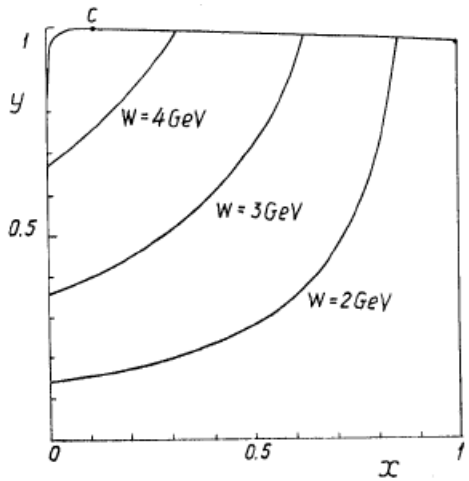
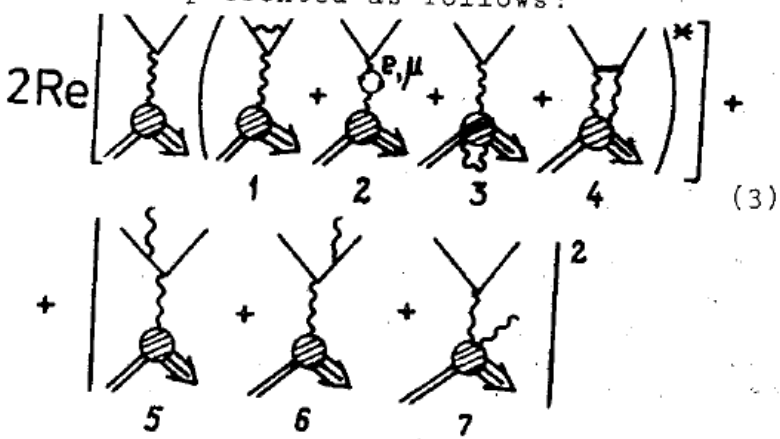
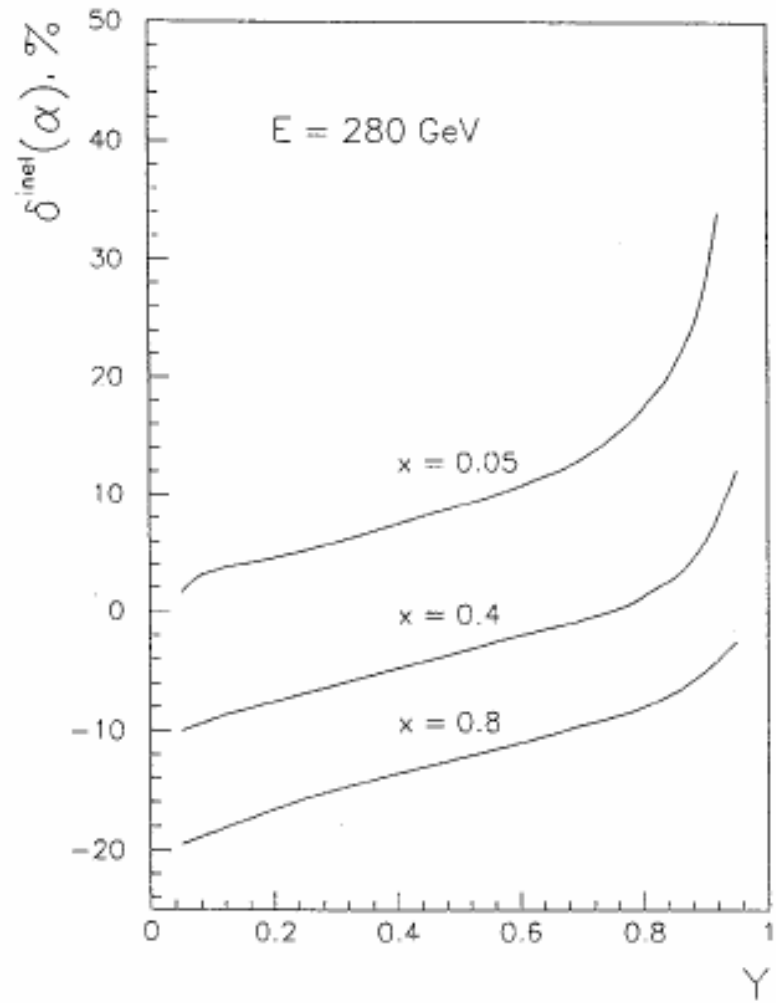




Fig. 3. The physical region of the process (4) in the scaling variables. Numerical values refer to the reaction $\mu p \to \mu X$ at E=12GeV.

# RC from Inelasic Elastic Radiative Tail

In the lowest $\alpha$ order this contribution can be represented as follows:



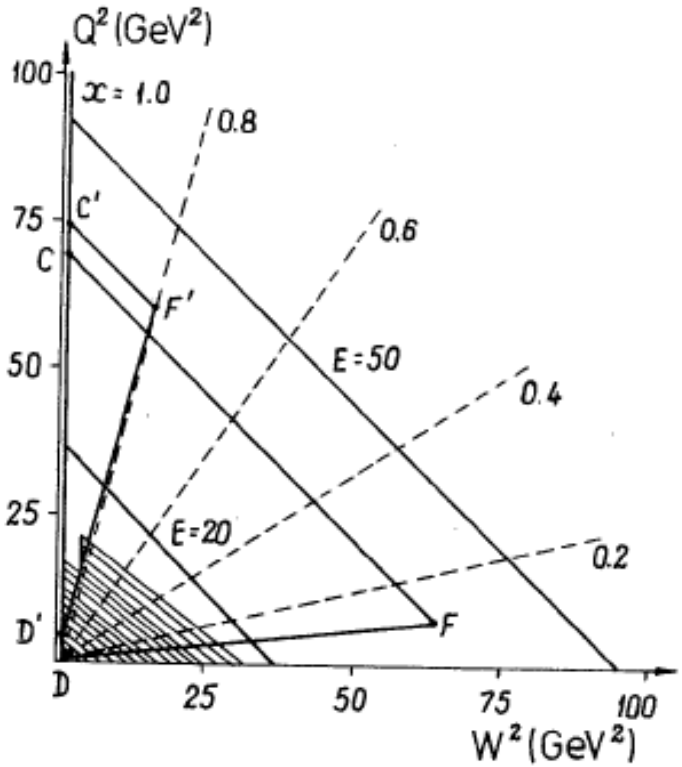
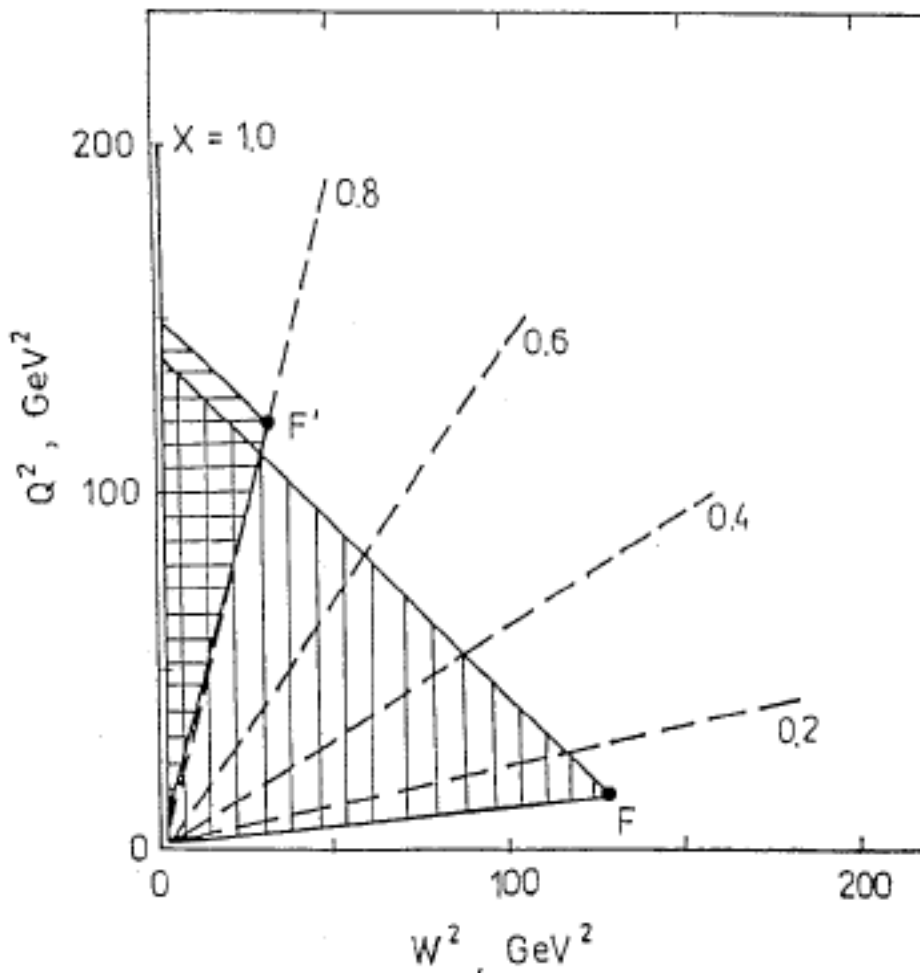
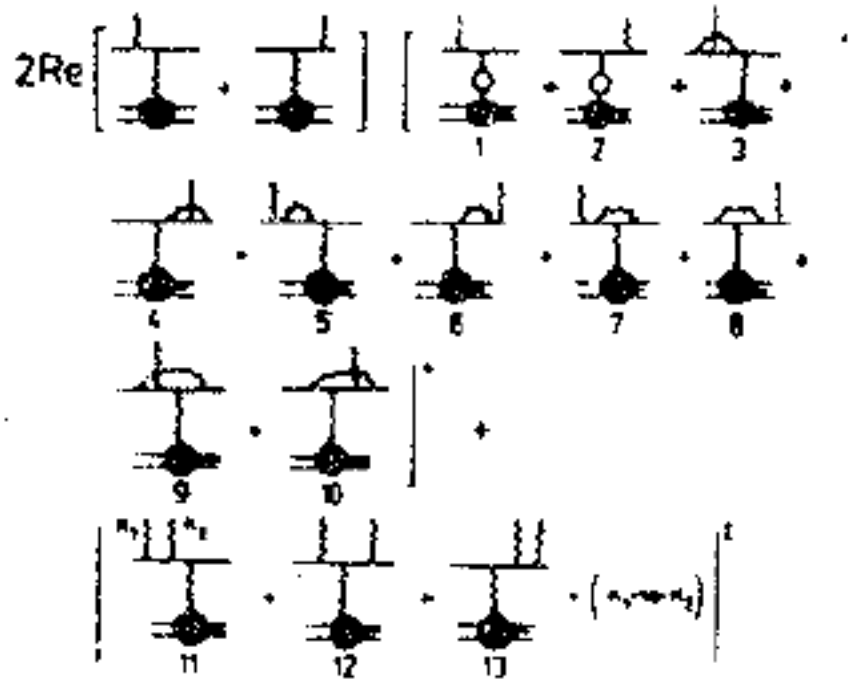
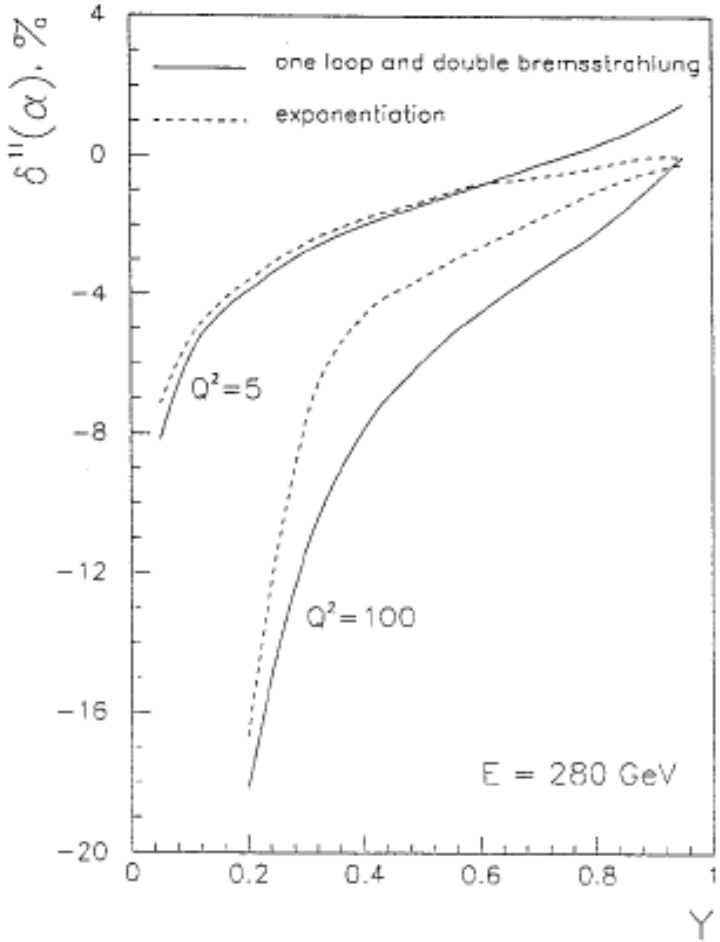



Fig.2. The kinematic region of the deep inelastic fN-scattering in the $(W^2, Q^2)$ plane. The boundary is shown at E=20 and 50 GeV. Dotted lines correspond to x constant. The shaded region shows where the structure functions are studied in detail[13].

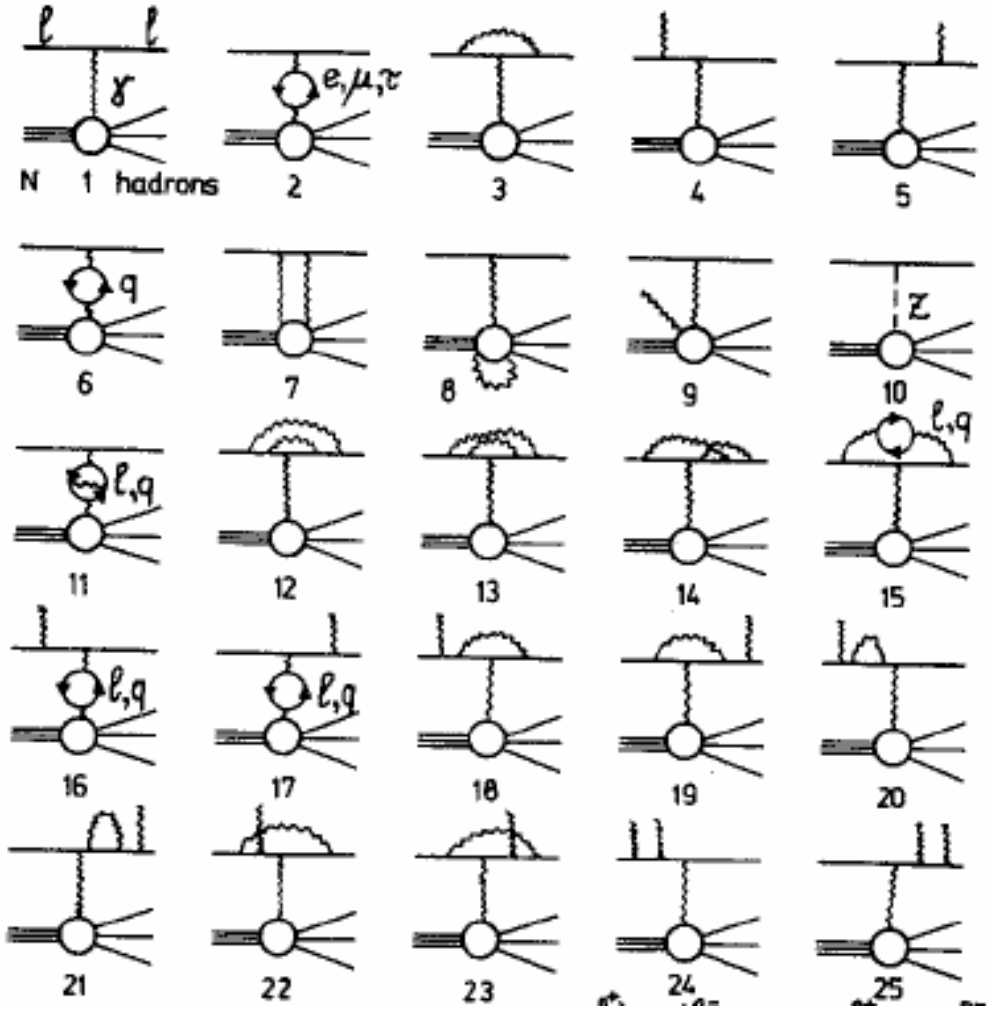


A.A. Akhundov, D.Yu. Bardin and N.M. Shumeiko,
*Electromagnetic corrections to the elastic radiative tail in deep inelastic lepton-nucleon scattering*, JINR Preprint, JINR-P2-85-831 (1985) 12 pp. &
*Yadernaya Fizika, v.44, p.1517-1526 (1986) (in Russian) &*
*Sov. J. Nucl. Phys. v.44, p.988 (1986) (in English, USA).*



# TERAD86 ->BCDMS, NMC



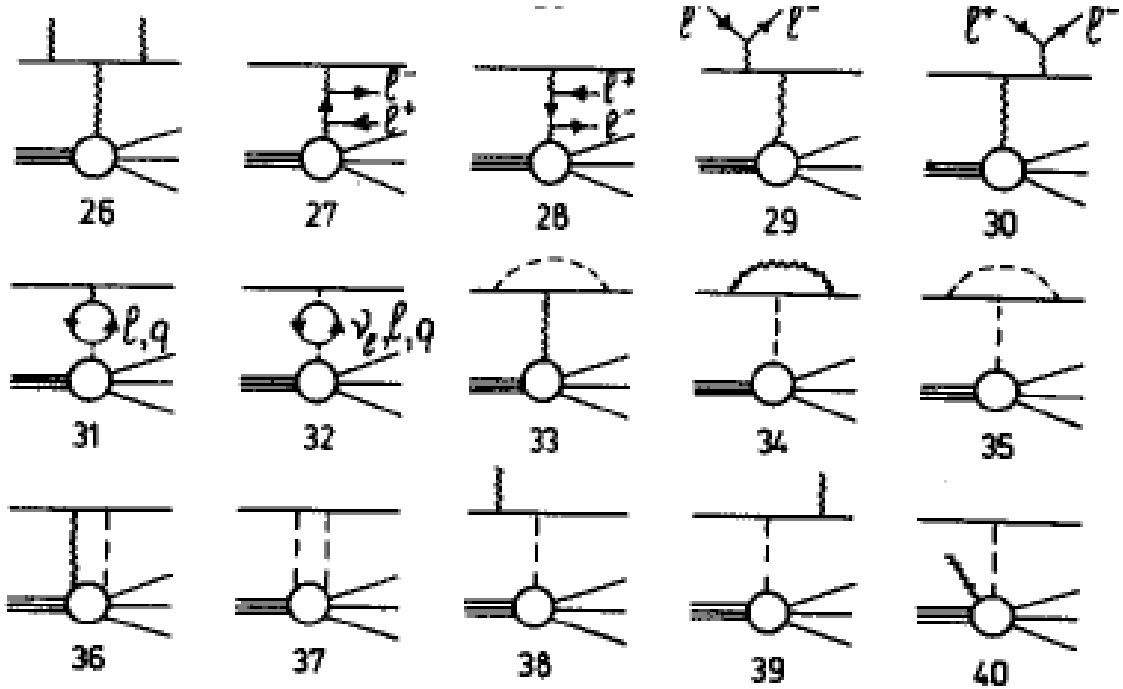



*Fig.1. Lowest- and higher-order electroweak radiative processes contributing to the observed deep inelastic cross section.*

A. Akhundov, D. Bardin and W. Lohmann, Fortran program TERAD86 and JINR Dubna preprint E2-86-104 (1986);
A. Akhundov and W. Lohmann, IfH Zeuthen preprint PHE 90-32 (1990).

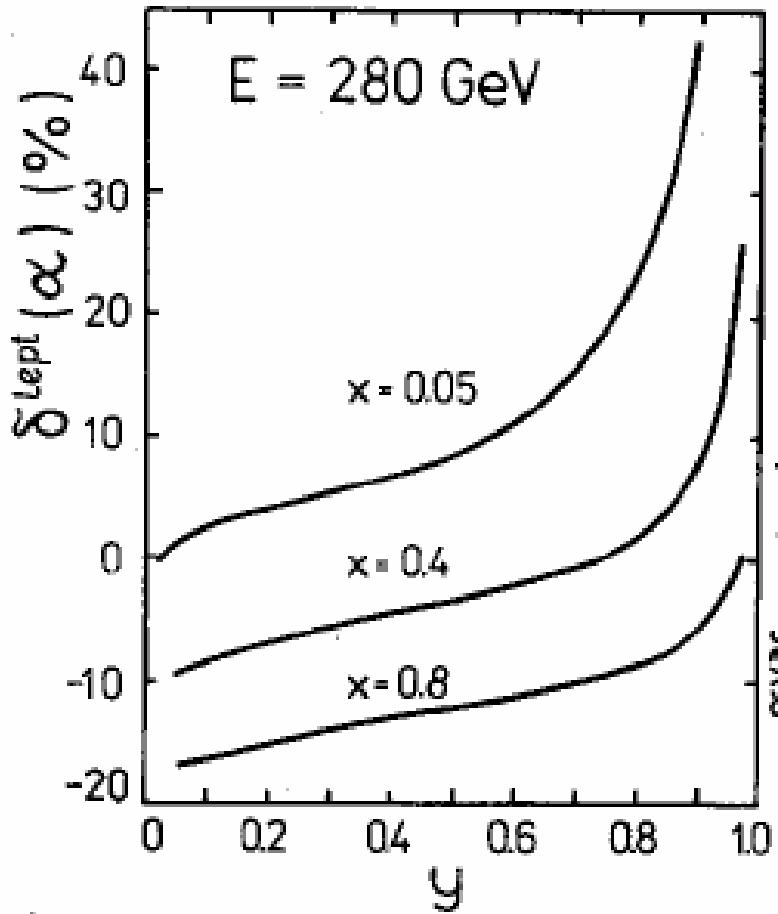



*Fig. 2. The lepton electromagnetic correction $\delta^{lept}(\alpha)$ to deep inelastic $\mu p$ scattering as functions of y for several x at 280 GeV.*

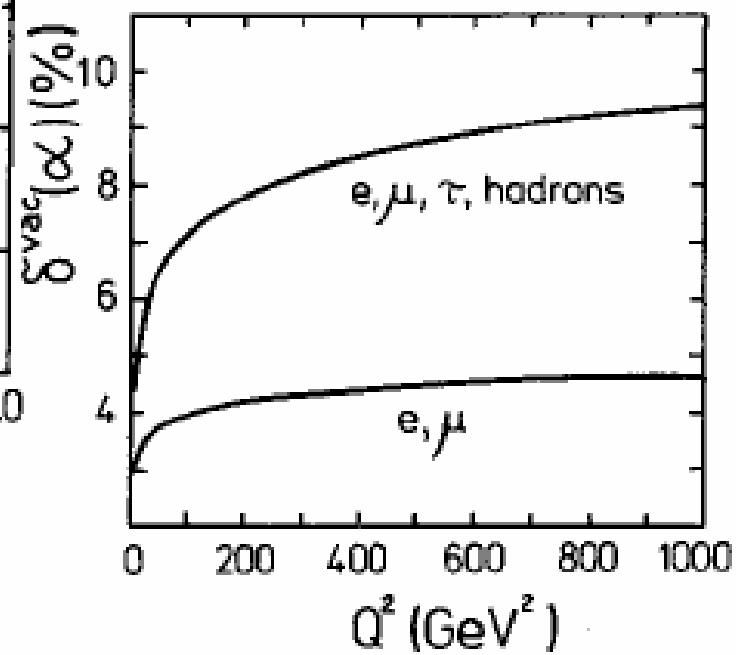



*Fig. 3. The vacuum polarization corrections $\delta^{vac}(\alpha)$ arising from (e, μ) and (e, μ, τ, hadrons) contributions as functions of $Q^2$.*

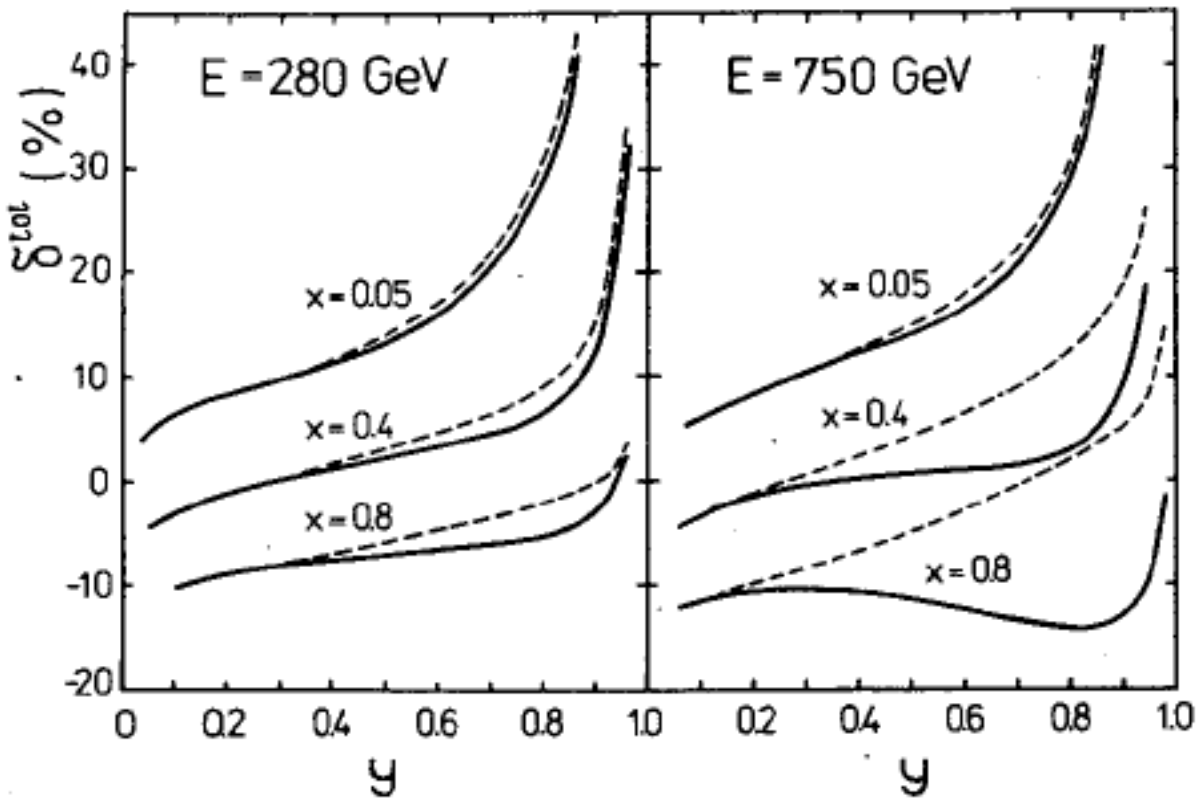
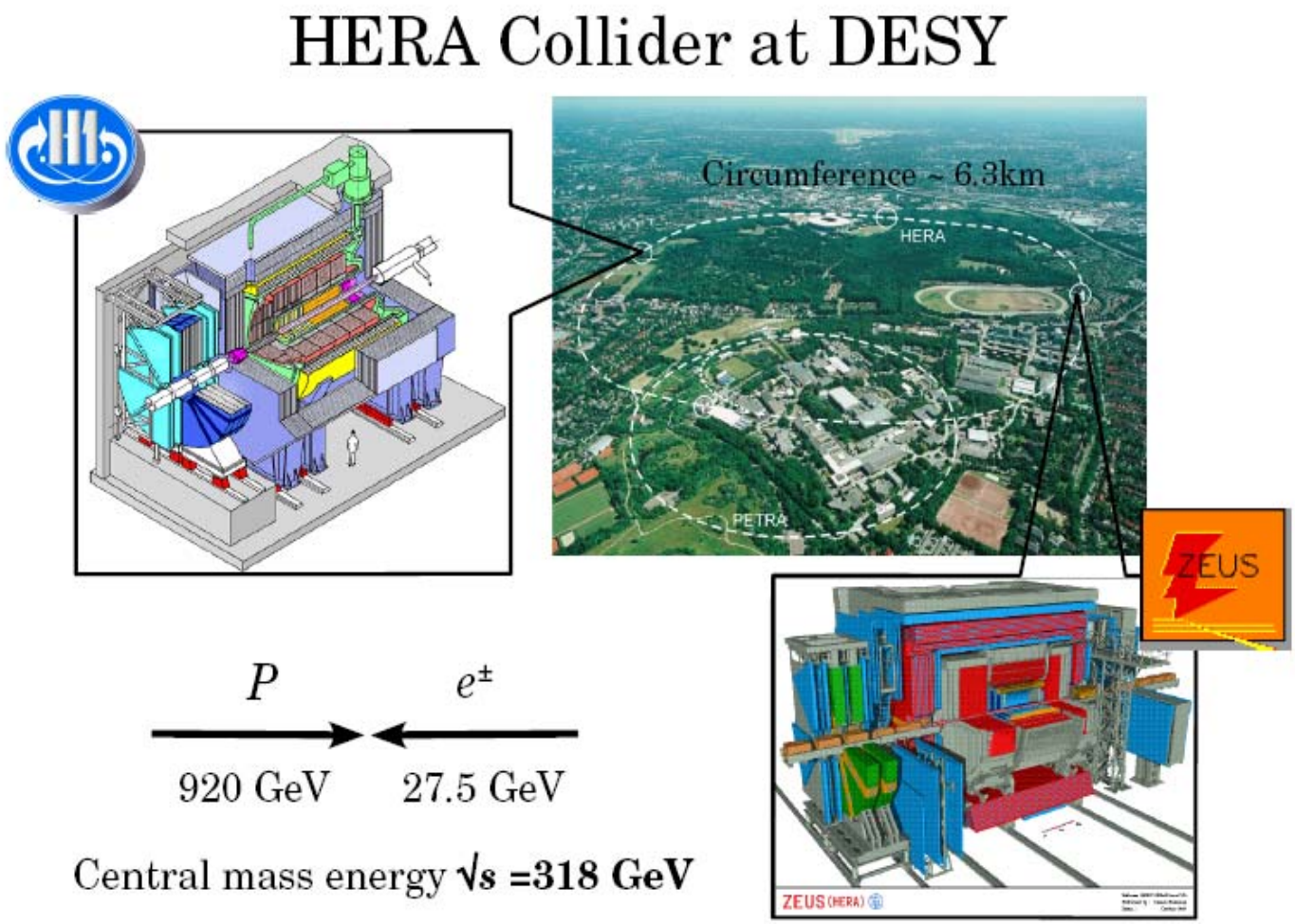


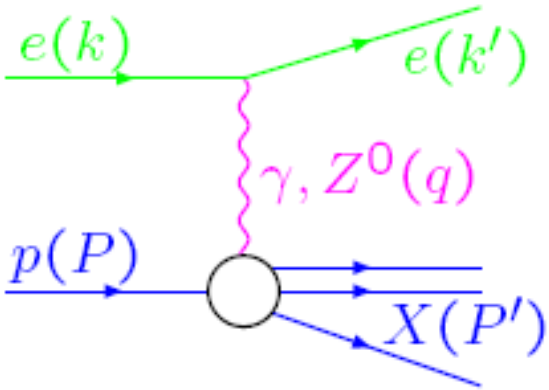

*Fig.5. The total electroweak corrections* $\delta^{tot}_{\pm\lambda}$ *to deep inelastic* $\mu^{\pm}p$ *scattering as functions of* y *for several* x *at 280 and 750 GeV. Solid lines correspond to* $\mu^{+}(\lambda = -0.8)$ *and dotted lines, to* $\mu^{-}$ $(\lambda = +0.8)$.

# RC to DIS at HERA

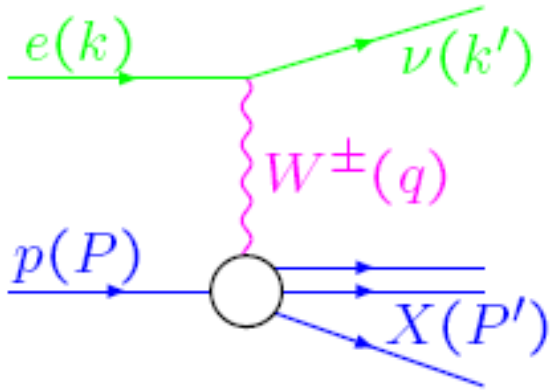


# DIS at HERA

**Neutral Current (NC)**




**Charged Current (CC)**




Invariant kinematic quantities:

| | |
|---|---|
| $Q^2 = -q^2 = -(k-k')^2$ | negative four-momentum transfer squared |
| $x = \frac{Q^2}{2P\cdot q}$ | In proton infinite-momentum frame: fraction of proton momentum |
| $y = \frac{P\cdot q}{P\cdot k}$ | In proto rest-frame: energy-transfer |
| $s = (k+P)^2 = \frac{Q^2}{xy}$ | squared cms energy |

# NC inclusive cross section

mainly QED Propagator at Born-level (leading order (LO) in $\alpha_{\mathrm{QED}}$)

$$\frac{d^2\sigma^{e\pm p}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+ F_2^{\mathrm{NC}} \mp Y_- x F_3^{\mathrm{NC}} - y^2 F_L^{\mathrm{NC}}\right]$$

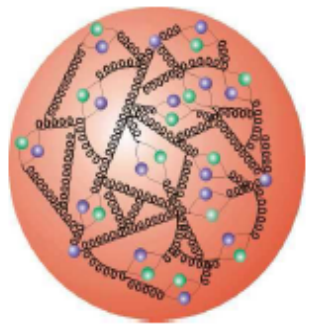
helicity-structure
see CC next week

$$Y_\pm = 1 \pm (1-y)^2$$

$$F_2^{\mathrm{NC}} = x \sum_{q=u\ldots b} A_f\,[q+\bar{q}]$$

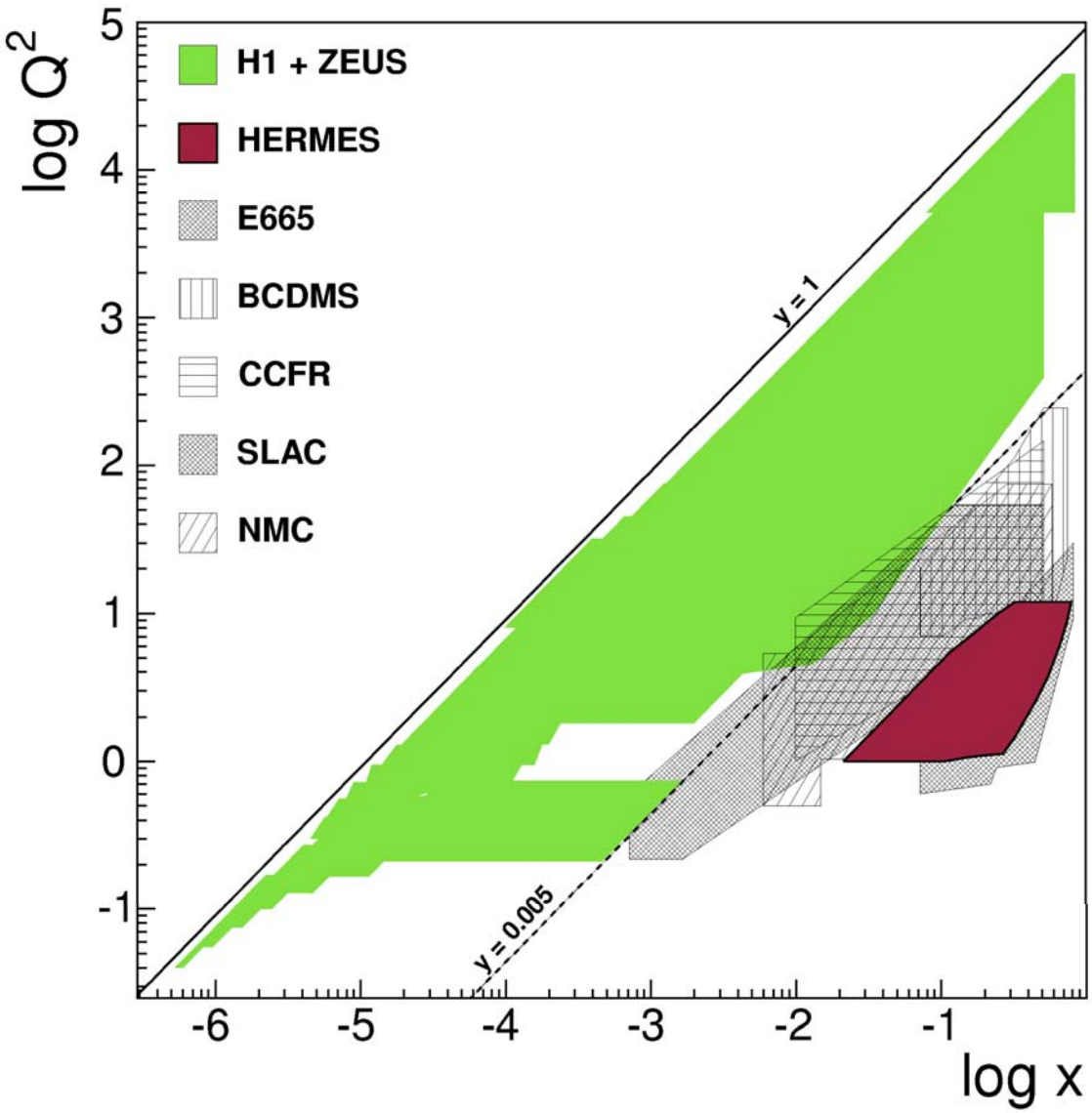
$$xF_3^{\mathrm{NC}} = x \sum_{q=u\ldots b} B_f\,[q-\bar{q}]$$

QCD — evolution of the parton densities, given here in LO



reduced cross section $\approx F_2$

$$\tilde{\sigma} = \frac{xQ^4}{2\pi\alpha^2 Y_+}\frac{d^2\sigma^{\mathrm{NC}}}{dx\,dQ^2}$$


log $Q^2$
5
4
3
2
1
0
-1
H1 + ZEUS
HERMES
E665
BCDMS
CCFR
SLAC
NMC
y = 1
y = 0.005
-6
-5
-4
-3
-2
-1
log x

## 2.1 Kinematics

We consider the processes

$$e(l) + p(p) \to e'(l') + X(p_X), \tag{1}$$

$$e(l) + p(p) \to e'(l') + \gamma(k) + X(p_X), \tag{2}$$

where the momenta are given in parentheses.

The traditional way to define kinematical variables for deep–inelastic lepton scattering is to use the momentum of the final–state lepton:

$$Q_l^2 = -(l - l')^2, \quad x_l = \frac{Q_l^2}{2p \cdot (l - l')}, \quad y_l = \frac{p \cdot (l - l')}{p \cdot l}, \tag{3}$$

with $Q_l^2 = x_l y_l s$ and $s = (p + l)^2$. For the non-radiative process (1) without emission of a photon from the lepton line, $Q_l^2$ determines the momentum transferred to the hadronic system and $x_l$ can be interpreted, in the parton model, as the fraction of the proton momentum entering the hard scattering process.

For the radiative process (2) and when the photon is emitted from the lepton line (see fig. 1) one has, however, to take into account the fact that the emission of momentum leads to a shift of the kinematical variables. The kinematics at the hadronic vertex has to be described rather by

$$Q_h^2 = -(p_X - p)^2, \quad x_h = \frac{Q_h^2}{2p \cdot (p_X - p)}, \quad y_h = \frac{p \cdot (p_X - p)}{p \cdot l}. \tag{4}$$

It is easy to show that

$$x_h \geq x_l, \quad y_h \leq y_l. \tag{5}$$

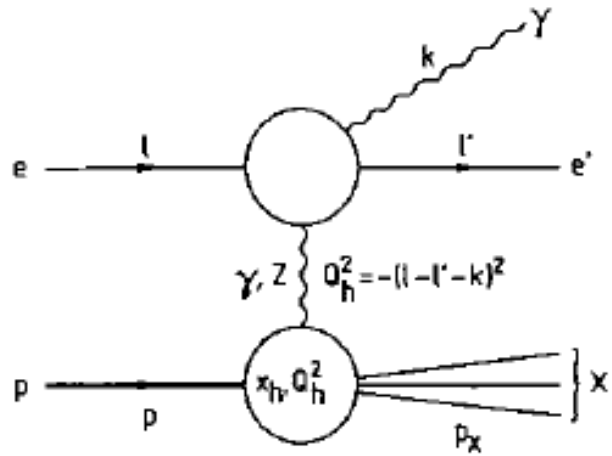



Figure 1: *Notation for radiative lepton–nucleon scattering*

The hadronic momentum transfer $Q_h^2$ can be larger or smaller than the leptonic $Q_l^2$ and it is possible that $Q_h^2$ becomes very small, $Q_h^2 \ll Q_l^2$. Since the magnitude of the neutral-current cross section is determined by the photon propagator $1/Q_h^2$, photon emission with $Q_h^2 \ll Q_l^2$ leads to an enhancement, compared with the photon propagator of the Born cross section $1/Q_l^2$. This is the main reason why radiative corrections to the cross section as a function of leptonic variables can get large, whereas corrections as a function of the true hadronic variables remain of moderate size.

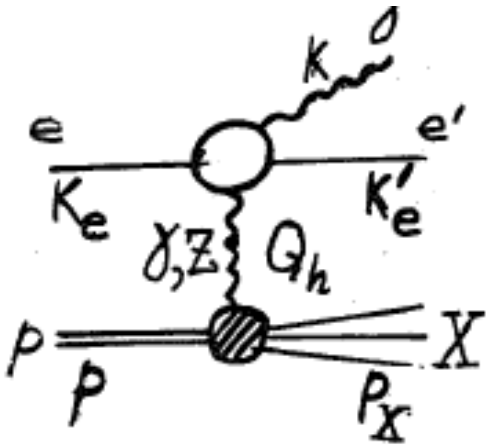
it was proposed to use

$$y_{JB} = \frac{p \cdot (\sum_h p_h - p)}{l \cdot p}, \quad Q_{JB}^2 = \frac{(\sum_h \vec{p}_{T,h})^2}{1 - y_{JB}}, \quad x_{JB} = \frac{Q_{JB}^2}{y_{JB} s},$$

where the sum $\sum_h$ extends over all hadrons in the final-state. The determinatio

so-called mixed variables:

$$Q_m^2 = Q_l^2, \quad y_m = y_h, \quad x_m = \frac{Q_m^2}{y_m s}.$$




$$s = (K_e + p)^2 = 4 E_e E_p$$

- electron variables

$$Q_e^2 = -(K_e - K'_e)^2, \quad y_e = \frac{p(K_e - K'_e)}{p K_e}, \quad x_e = \frac{Q_e^2}{s y_e}$$

- hadron variables

$$Q_h^2 = -(P_X - p)^2, \quad y_h = \frac{p(P_X - p)}{p K_e}, \quad x_h = \frac{Q_h^2}{s y_h}$$

- mixed variables

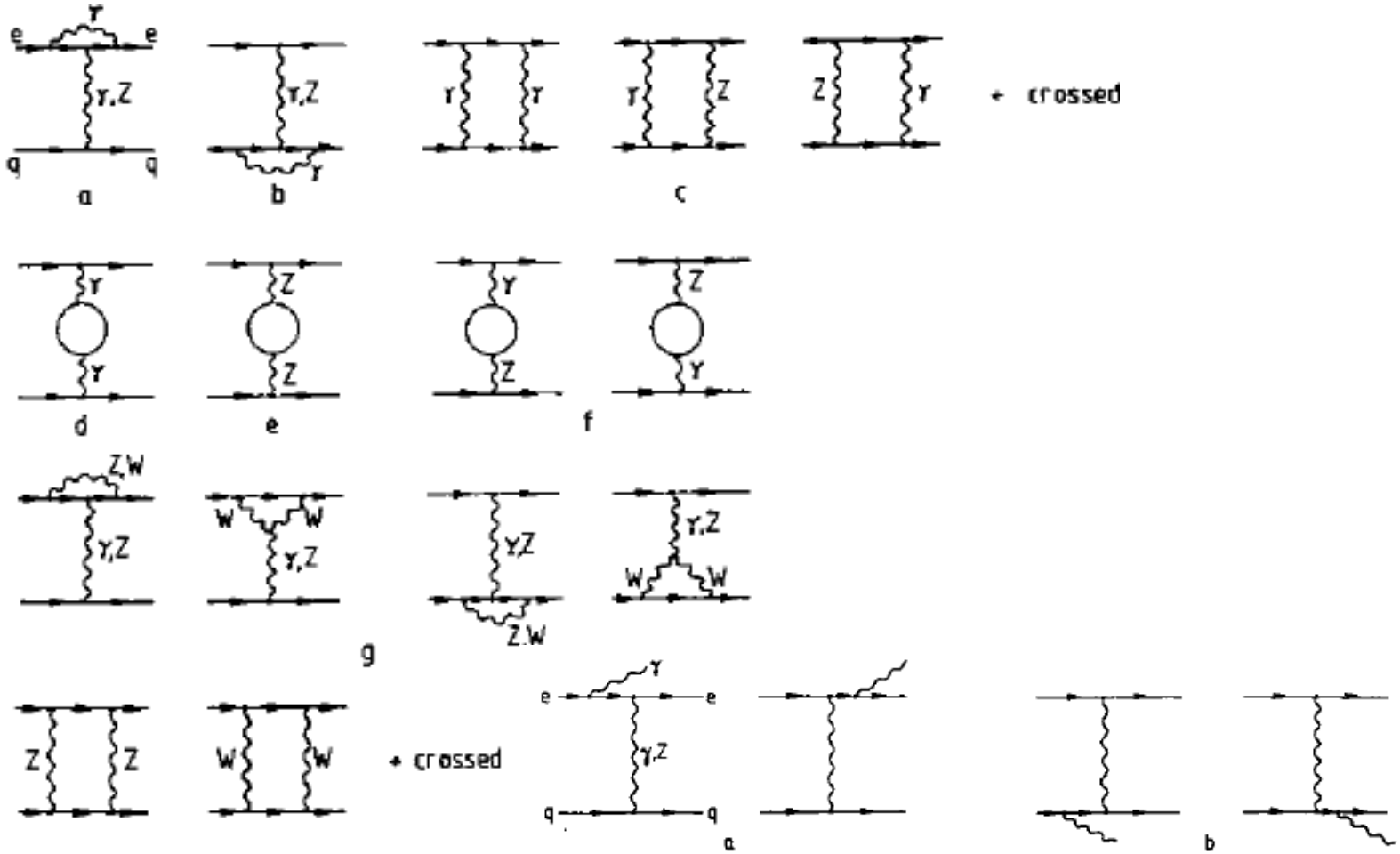
$$Q_m^2 \equiv Q_e^2 , \quad y_m = y_h , \quad x_m = \frac{Q_e^2}{s\, y_h}$$

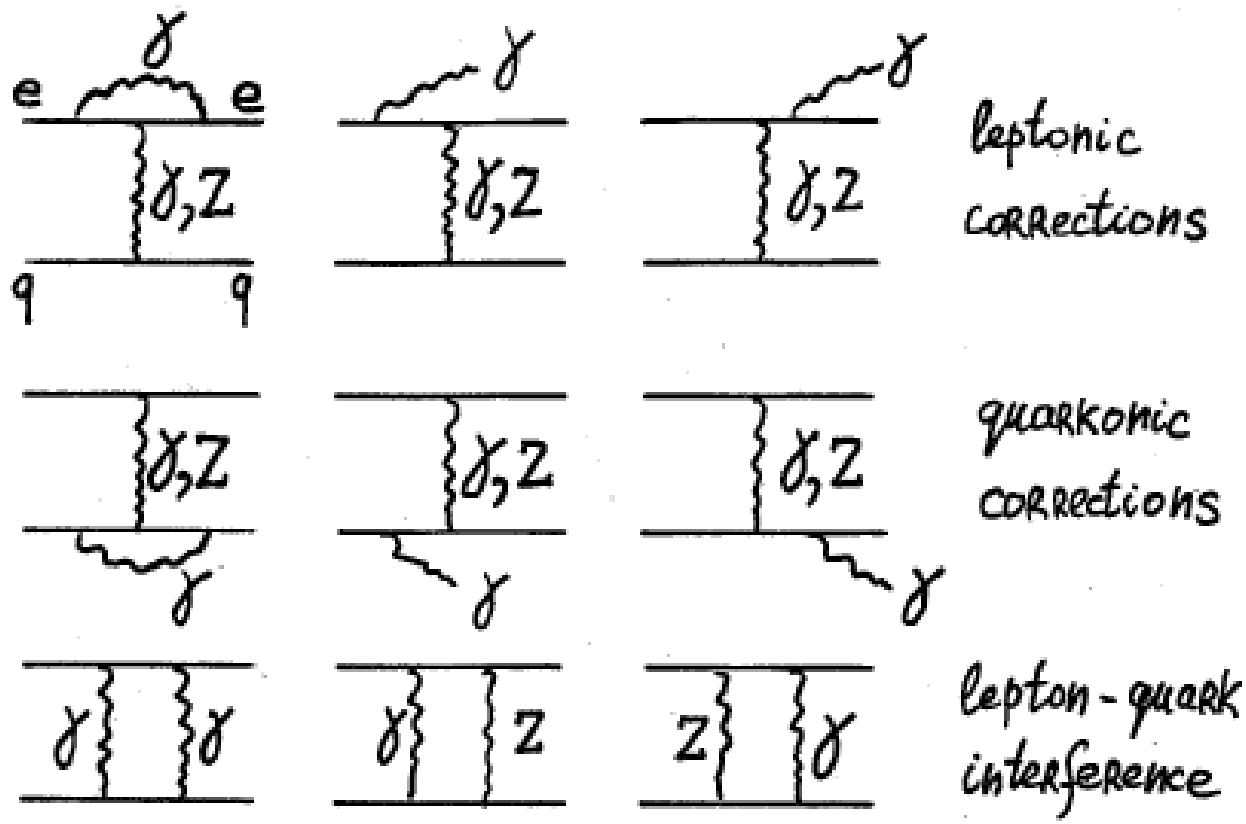
- Jacquet-Blondel variables

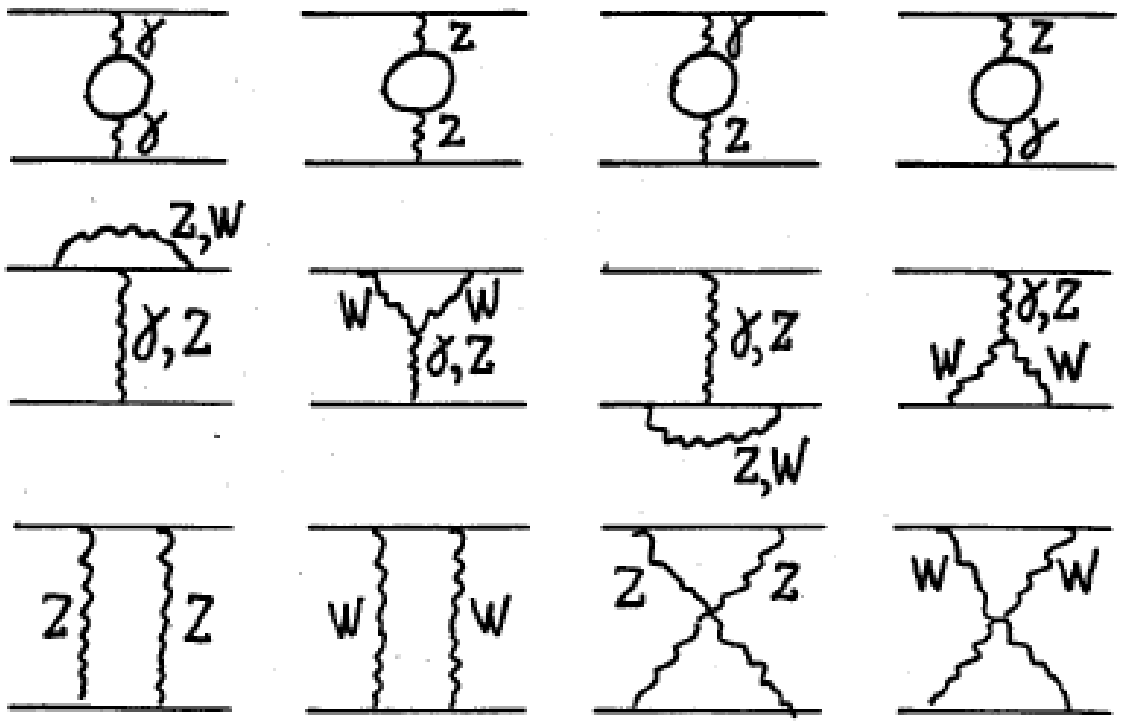
$$Q_{JB}^2 = \frac{\left(\sum \vec{P}_{\perp,h}\right)^2}{1 - y_{JB}} , \quad y_{JB} = \frac{\sum (E_h - P_{z,h})}{2E_e} \simeq y_h ,$$

$$x_{JB} = \frac{Q_{JB}^2}{s\, y_{JB}}$$

# Classificacion of RC

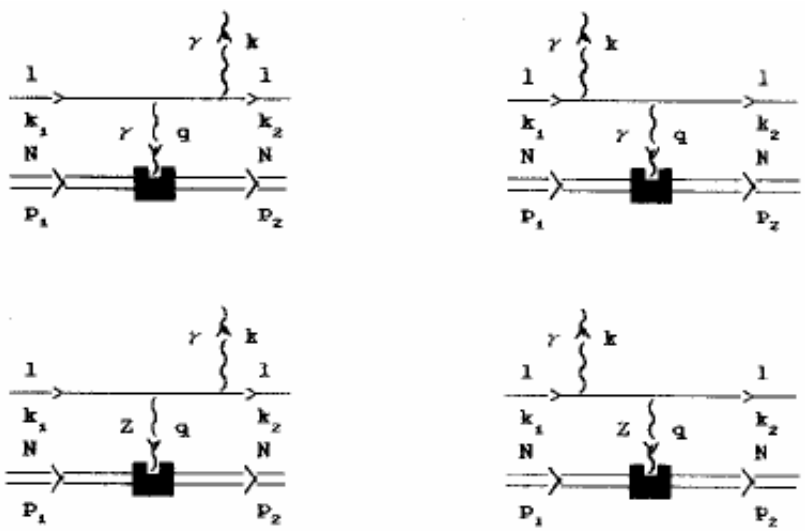



e
e
q
q
γ
γ,Z
leptonic corrections
quarkonic corrections
lepton - quark interference
Z




weak corrections

# Elastic Radiative Tail at HERA



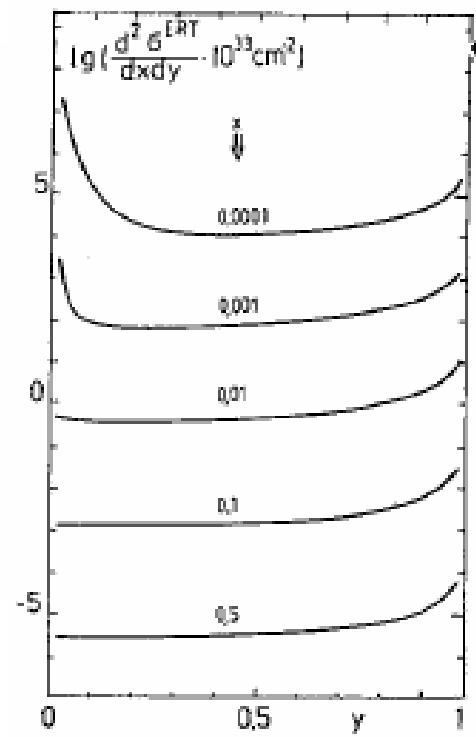




Fig.4. Inclusive cross section of the process e + p ⟶ e + γ + p at S = $10^5$ GeV².

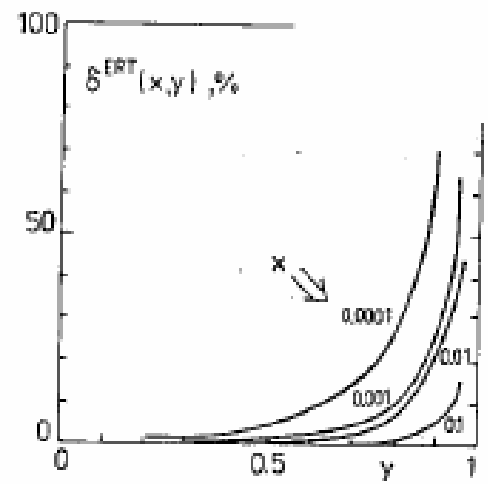
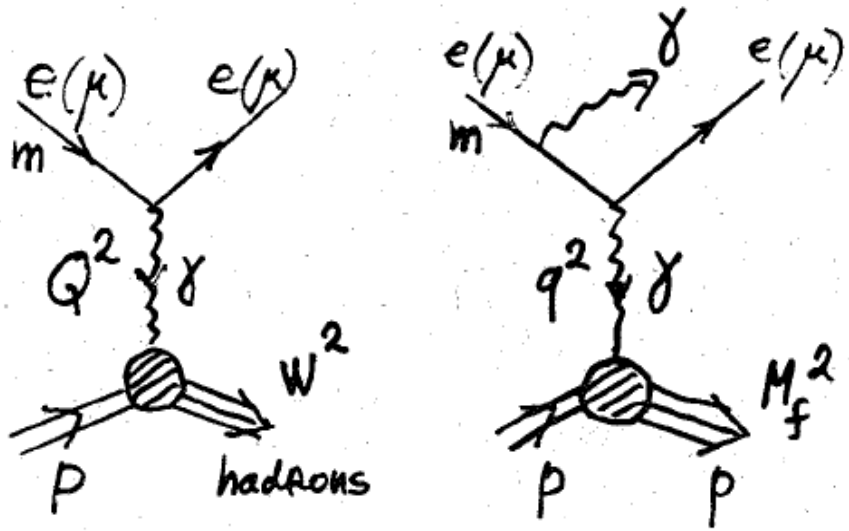
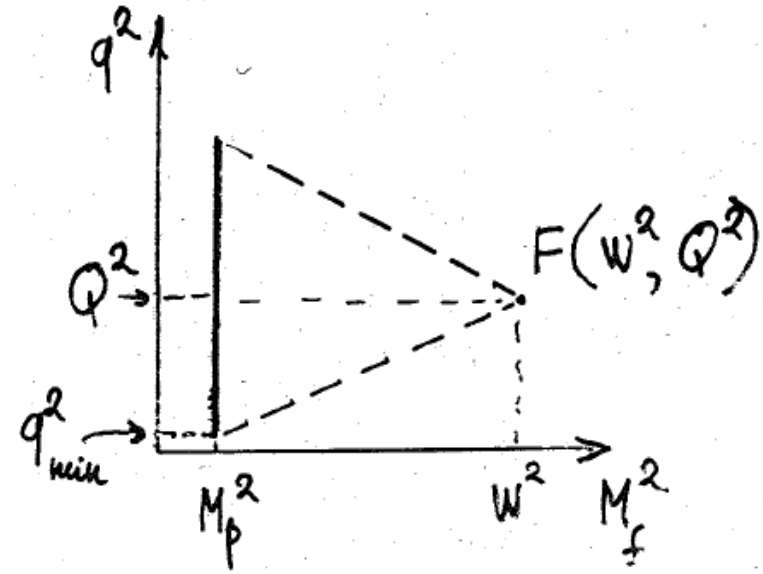



Fig.5. The RRT correction to e + p ⟶ e + X at S = $10^5$ GeV².

## Explanation of the big effects from ERT




$$q^2_{min} \approx m^2 \frac{(W^2 - M_p^2)^2}{s^2}$$

$$\sigma^{BORN} \sim \frac{1}{Q^4}$$

$$\sigma^{ERT} \sim \frac{1}{q^4} \cdot \frac{\alpha}{\pi}$$

**Fortschritte der Physik, v. 44 (1996) p.373-482.**

# Model independent QED corrections to the process $ep \longrightarrow eX$ †

Arif Akhundov [1,2], Dima Bardin [3,4], Lida Kalinovskaya [4], Tord Riemann [5]


## ABSTRACT

We give an exhaustive presentation of the semi-analytical approach to the model independent leptonic QED corrections to deep inelastic neutral current lepton-nucleon scattering. These corrections include photonic bremsstrahlung from and vertex corrections to the lepton current of the order $\mathcal{O}(\alpha)$ with soft photon exponentiation. A common treatment of these radiative corrections in several variables – leptonic, hadronic, mixed, Jaquet-Blondel variables – has been developed and double differential cross-sections are calculated. In all sets of variables we use some structure functions, which depend on the hadronic variables and which do not have to be defined in the quark parton model. The remaining numerical integrations are twofold (for leptonic variables) or onefold (for all other variables). For the case of hadronic variables, all phase space integrals have been performed analytically. Numerical results are presented for a large kinematical range, covering fixed target as well as collider experiments at HERA or LEP$\otimes$LHC, with a special emphasis on HERA physics.

It is well-known from the above mentioned earlier studies, e.g. from [14], that a treatment of the leptonic photonic corrections covers the bulk of the complete corrections to this cross-section. Fortunately, both types of corrections – weak loop insertions and QED corrections related to the hadronic current – are relatively small. For a large part of the kinematical region they are below the experimental accuracy. If the experimental intention is a study of

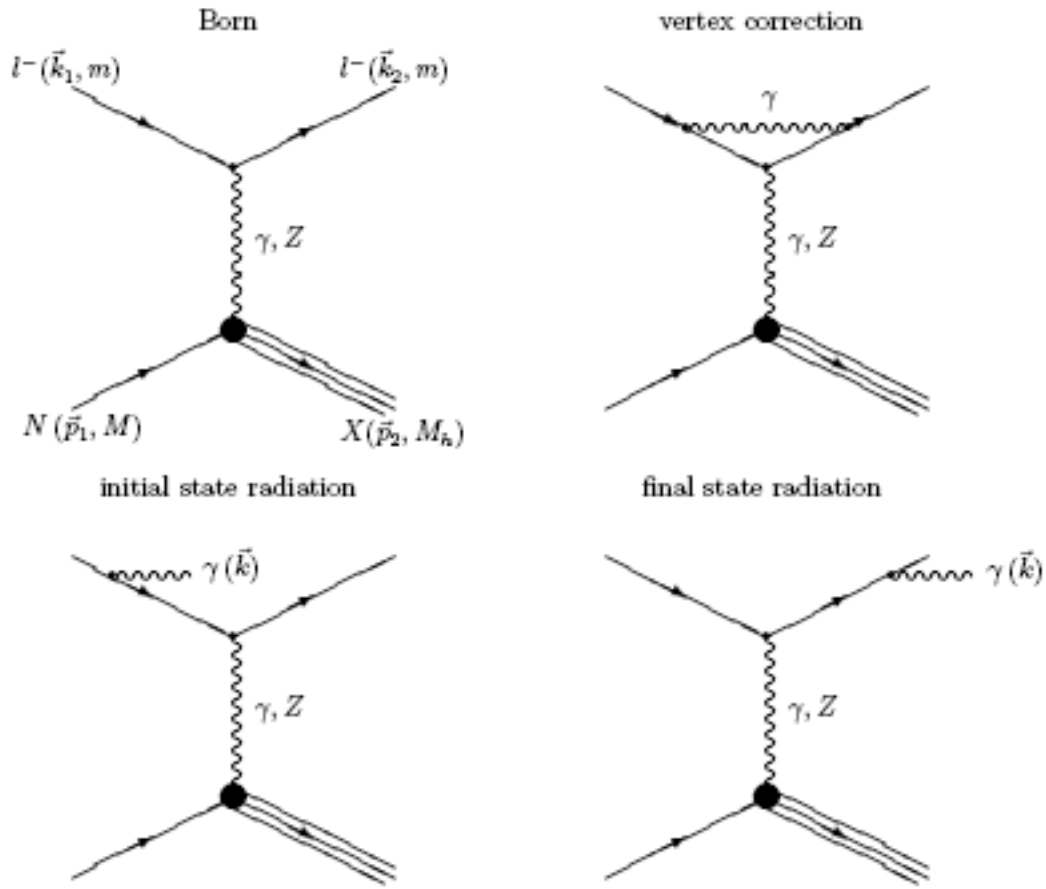
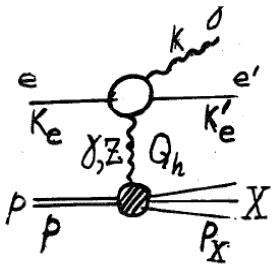



Fig. 1. Feynman graphs contributing in the higher order to deep inelastic $lN$-scattering.

*This article is devoted to complete, model independent, semi-analytical calculations of leptonic corrections to neutral current deep inelastic lepton-nucleon scattering in different kinematical variables.* By *complete* we mean the full $\mathcal{O}(\alpha)$ corrections with soft photon exponentiation which are not restricted to the leading logarithmic approximation. By *semi-analytical* we understand that the Monte-Carlo technique is not used. We perform as many analytical integrations as is possible for a given set of kinematical variables instead. In order to get a double differential cross-section, one has to perform three phase space integrations. In principle, one is interested to describe the cross-sections with structure functions which may have an arbitrary dependence on the variables $x_h, Q_h^2$. Then, for the case of *leptonic* variables, only one analytical integration, for the case of *Jaquet-Blondel* variables or for *mixed* variables – two integrations, and for *hadronic* variables all three phase space integrations may be performed analytically.

In the above discussion, the characteristic elements of the calculation of real bremsstrahlung corrections have been introduced:

- Choice of a reasonable phase space parameterization with a practical set of internal variables which are to be integrated over;
- Choice of the order of integration and complete understanding of the corresponding kinematical boundaries, by necessity without neglect of masses;
- Separation of the infrared singular part of the bremsstrahlung integral with use of a special rest system which has to be chosen appropriately;
- Dedicated performance of the various hard bremsstrahlung integrations which are regulated or finite in the soft photon part of the phase space;
- Calculation of the infrared divergent correction with a reasonable regularization procedure. Consecutively, compensation of the infrared singularity with that of the virtual corrections and elimination of the soft and hard photon separation with establishment of the lorentz invariance of the net correction.




$$s = (K_e + p)^2 = 4E_e E_p$$

- electron variables

$$Q_e^2 = -(K_e - K'_e)^2, \quad y_e = \frac{p(K_e - K'_e)}{pK_e}, \quad x_e = \frac{Q_e^2}{s y_e}$$

- hadron variables

$$Q_h^2 = -(P_X - p)^2, \quad y_h = \frac{p(P_X - p)}{pK_e}, \quad x_h = \frac{Q_h^2}{s y_h}$$

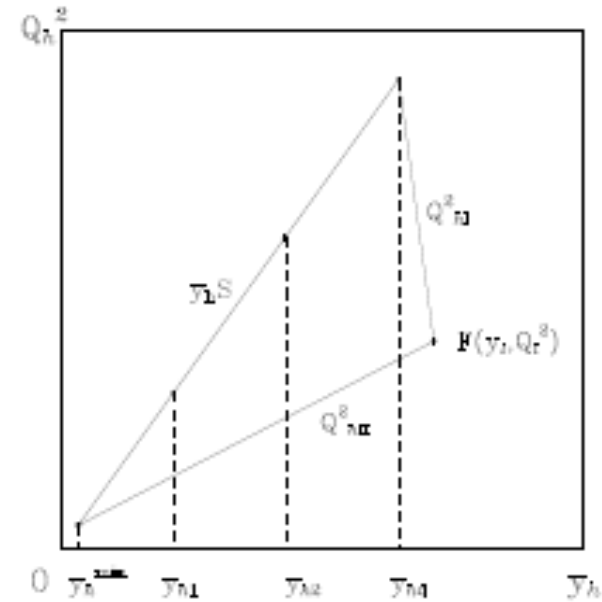
- mixed variables

$$Q_m^2 \equiv Q_e^2, \quad y_m = y_h, \quad x_m = \frac{Q_e^2}{s y_h}$$

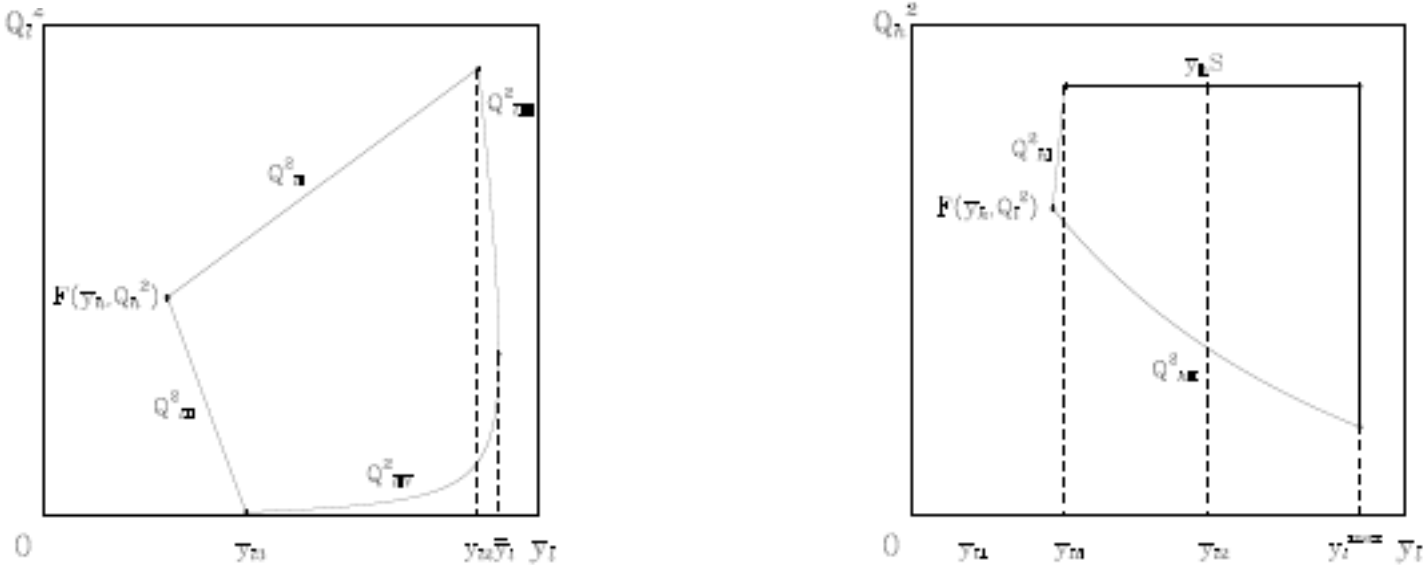
- Jacquet-Blondel variables

$$Q_{JB}^2 = \frac{\left(\sum \vec{P}_{\perp,h}\right)^2}{1 - y_{JB}}, \quad y_{JB} = \frac{\sum (E_h - P_{z,h})}{2E_e} \simeq y_h,$$

$$x_{JB} = \frac{Q_{JB}^2}{s y_{JB}}$$




*Integration region $(y_h, Q_h^2)$ for the cross-section in leptonic variables.*

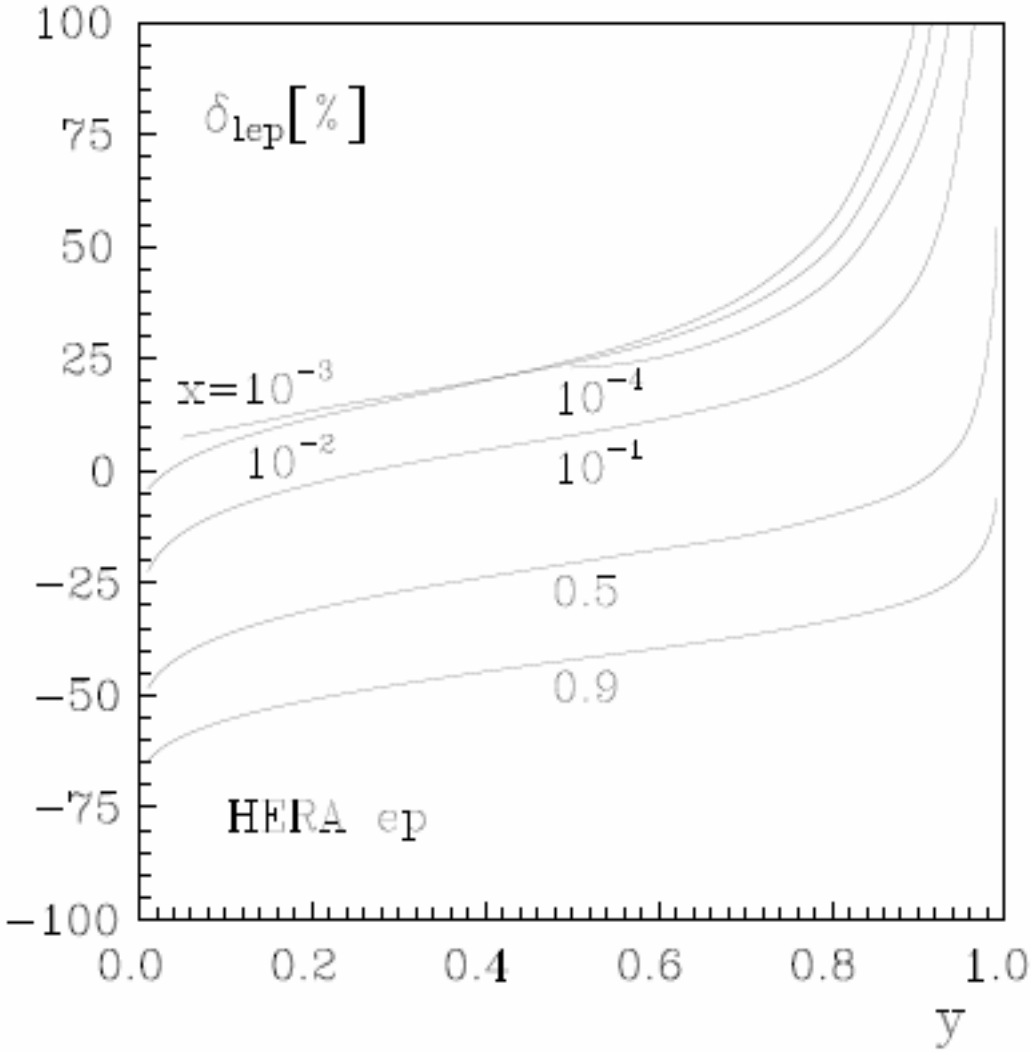


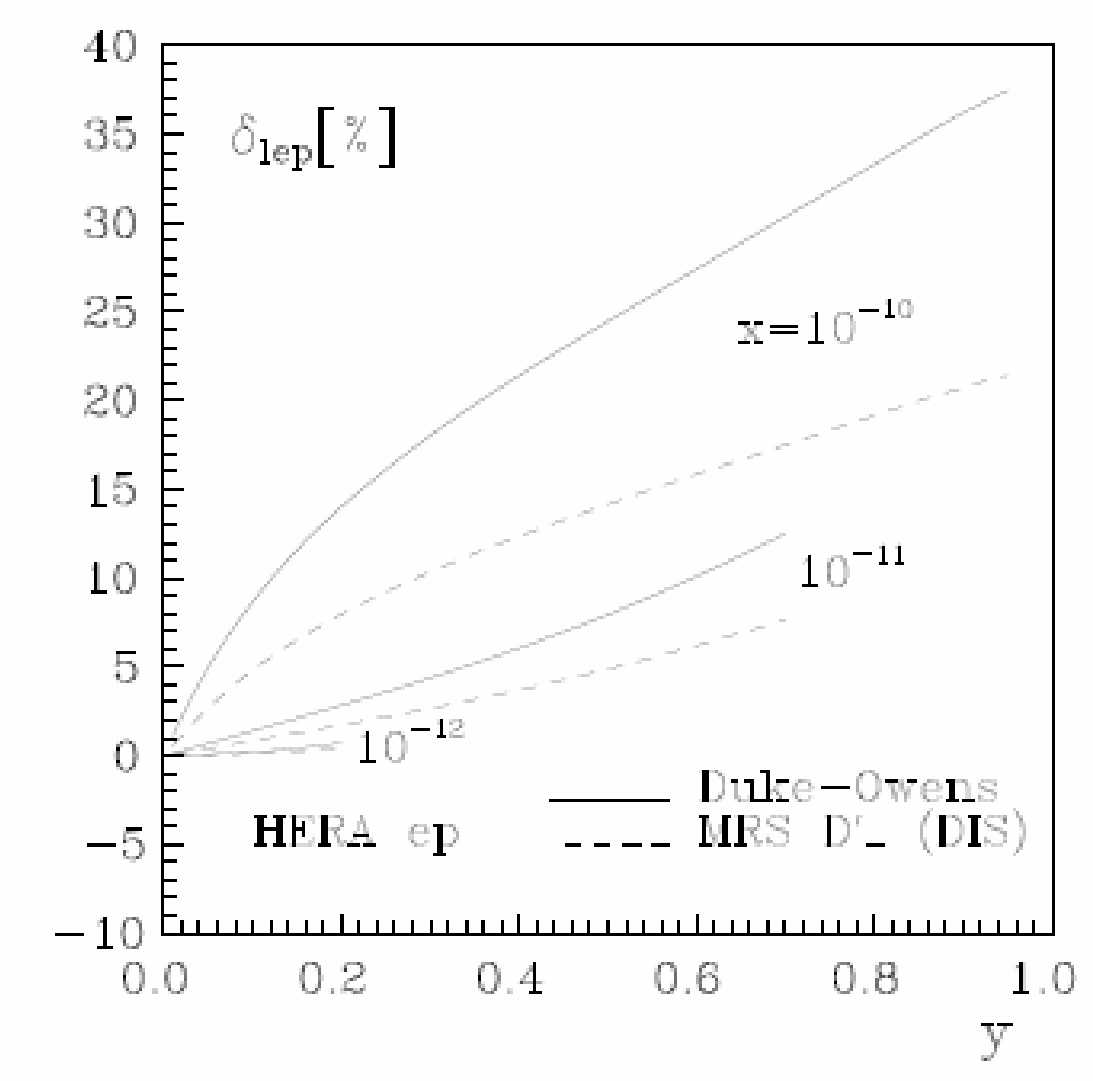

*Integration region $(y_l, Q_l^2)$ for the cross-section in hadronic variable. Integration region $(y_l, Q_h^2)$ for the cross-section in mixed variables, $x_m \leq 1$.*

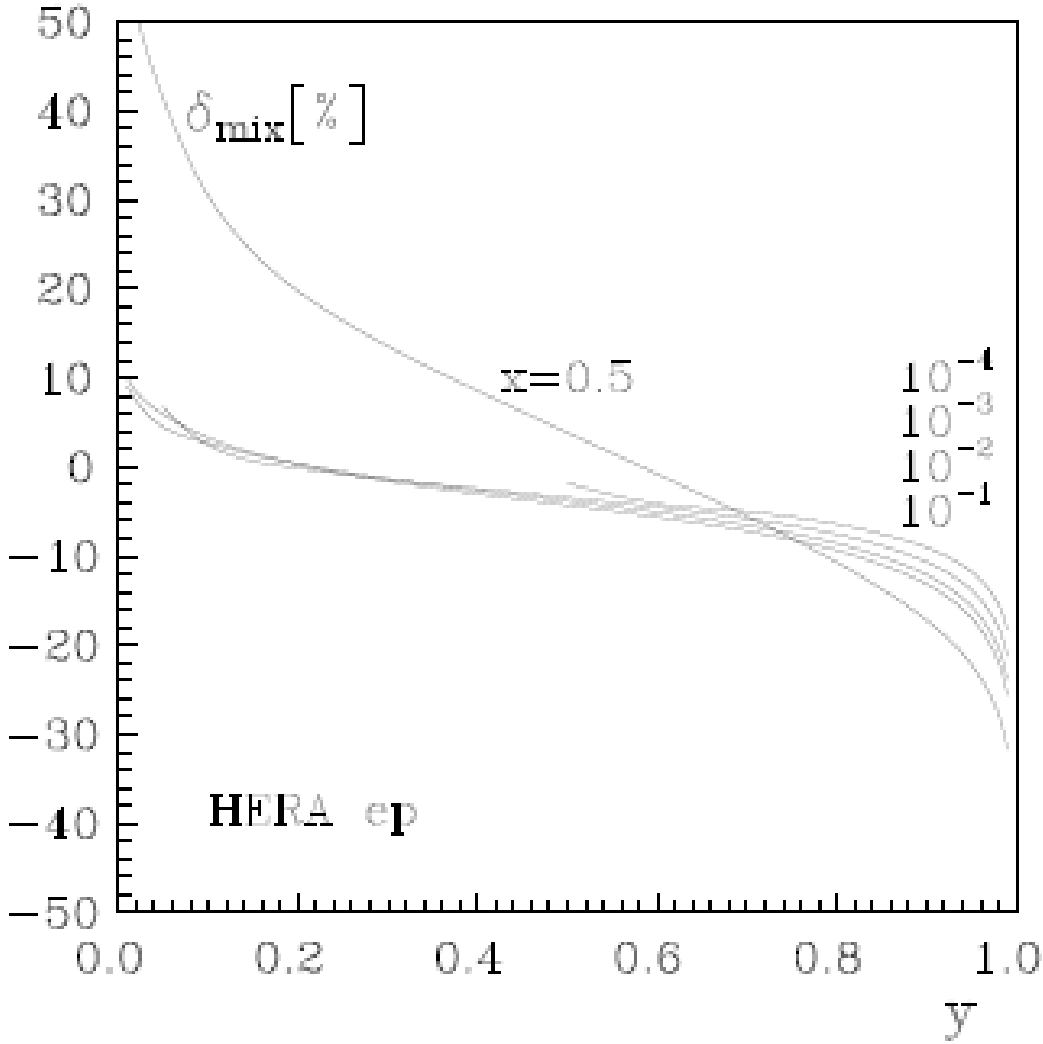
**TERAD91: A Program package for the calculation of the cross-sections of deep inelastic N C and C C scattering at HERA.**

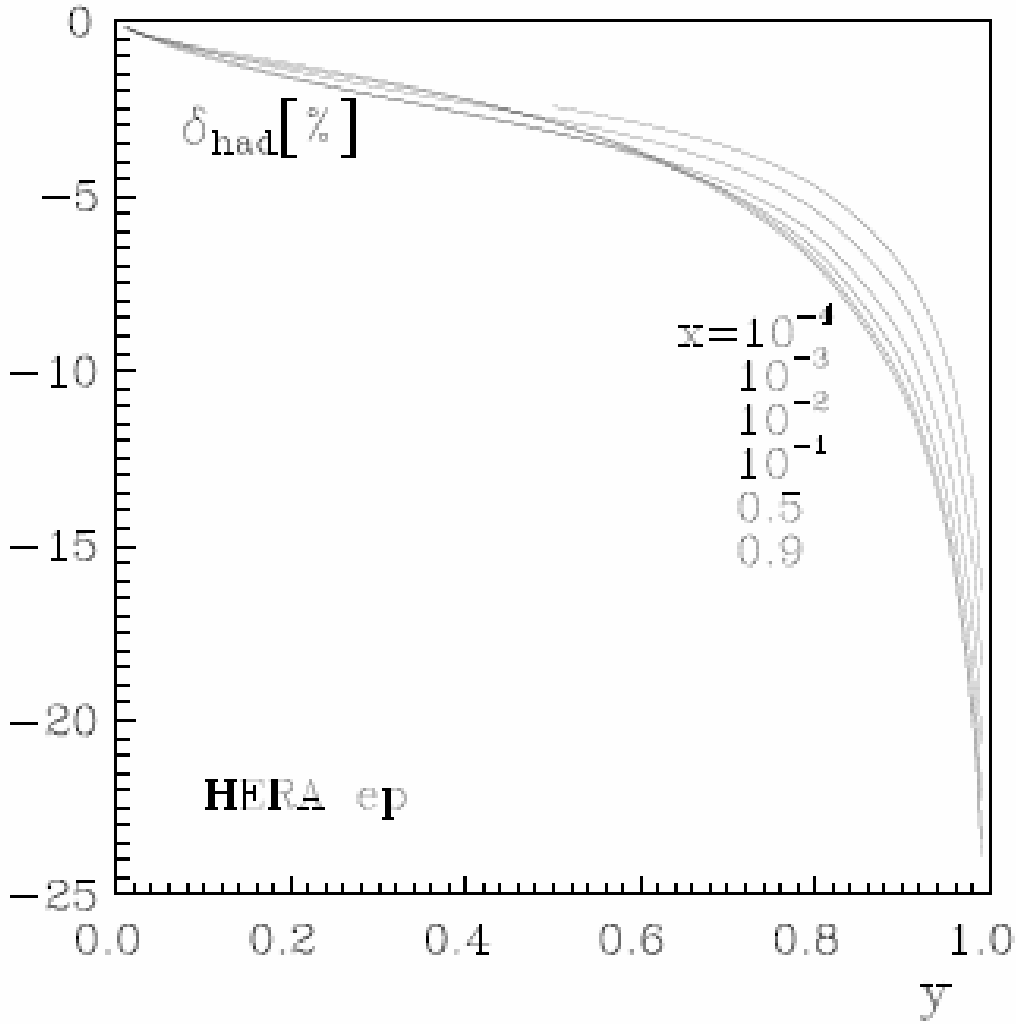
A. Akhundov, D. Bardin, L. Kalinovskaya , T. Riemann ,
In *Hamburg 1991, Proceedings, Physics at HERA, vol. 3, p.1285-1293.

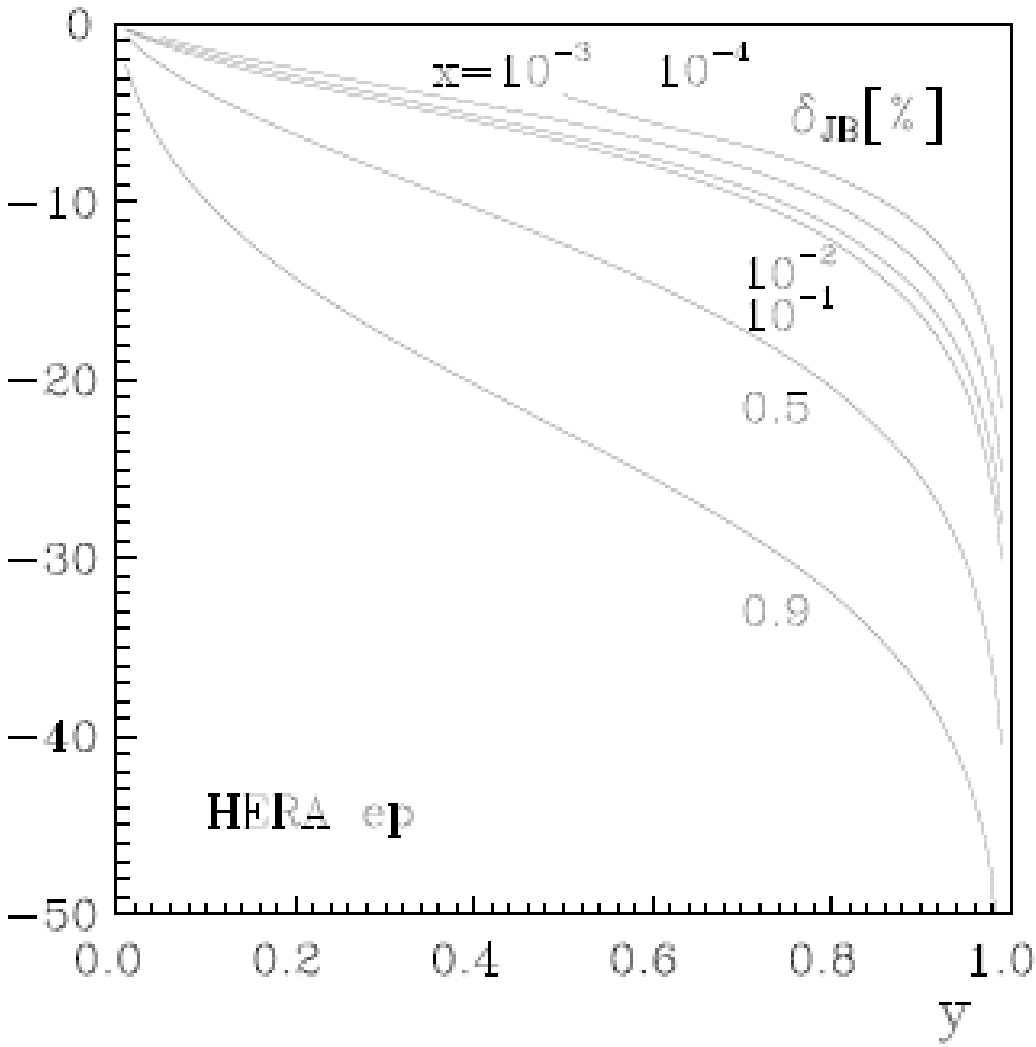
TERAD91 is a semi-analytic code for QED and weak corrections to deep–inelastic NC and CC scattering at HERA. Version 2.10 was released on 3 Oct. 1991. The source of TERAD91 originates from four different codes: TERAD, DISEPNC, DISEPCC, and DIZET, which will be discussed in what follows.


100
75
50
25
0
−25
−50
−75
−100
0.0
0.2
0.4
0.6
0.8
1.0
y
$\delta_{lep}$[%]
x=$10^{-3}$
$10^{-2}$
$10^{-1}$
$10^{-1}$
0.5
0.9
HERA ep


$\delta_{lep}[\%]$
$x=10^{-10}$
$10^{-11}$
$10^{-12}$
Duke−Owens
MRS D'_ (DIS)
HERA ep
40
35
30
25
20
15
10
5
0
−5
−10
0.0
0.2
0.4
0.6
0.8
1.0
y


50
40
30
20
10
0
−10
−20
−30
−40
−50
$\delta_{mix}[\%]$
x=0.5
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
HERA ep
0.0
0.2
0.4
0.6
0.8
1.0
y


0
−5
−10
−15
−20
−25
$\delta_{had}$[%]
x=$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
0.5
0.9
HERA ep
0.0
0.2
0.4
0.6
0.8
1.0
y


0
−10
−20
−30
−40
−50
0.0
0.2
0.4
0.6
0.8
1.0
y
x=10^-3
10^-1
δ_JB[%]
10^-2
10^-1
0.5
0.9
HERA ep

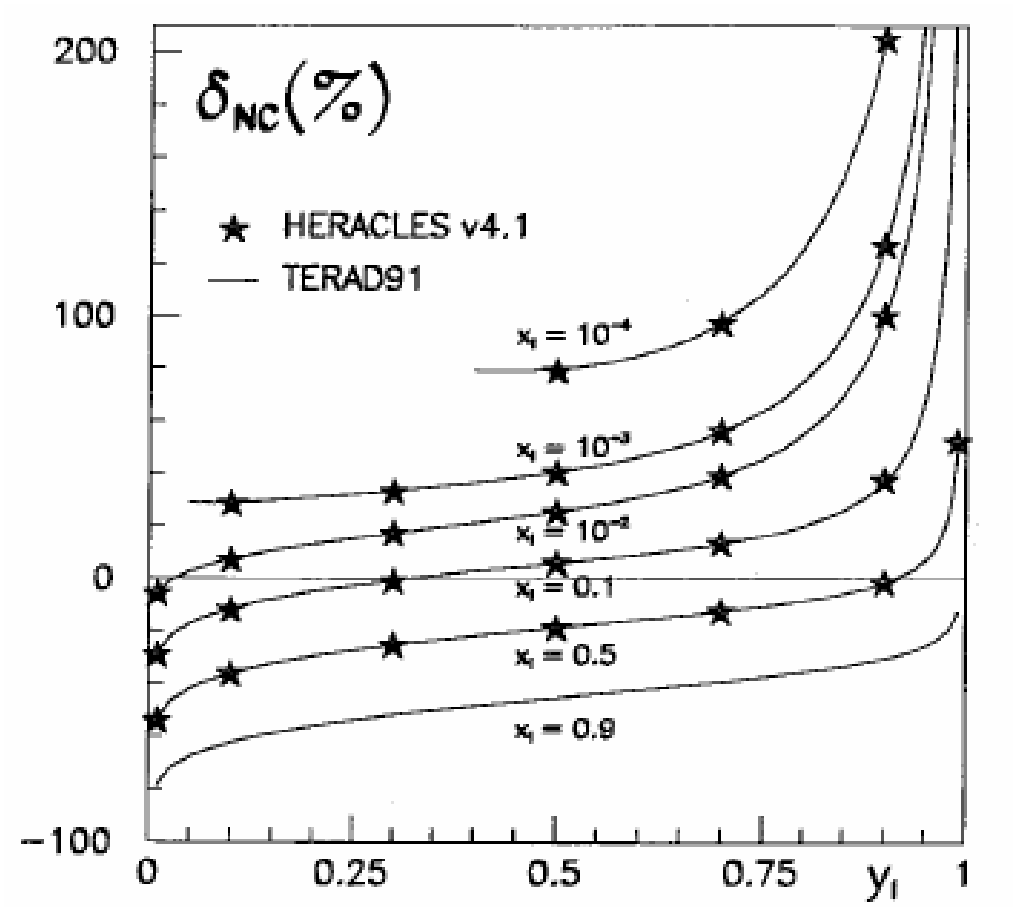



Figure 4: *QED leptonic corrections to $d^2\sigma/dx_l dy_l$ in per cent. The full lines represent results from TERAD91 and the crosses from HERACLES 4.1. There are no points from*

**Comput.Phys.Commun.94:128-184,1996.**

# HECTOR 1.00

## A program for the calculation of QED, QCD and electroweak corrections to $ep$ and $l^{\pm} N$ deep inelastic neutral and charged current scattering *

A. Arbuzov[1#], D. Bardin[1,2], J. Blümlein[2], L. Kalinovskaya[1], T. Riemann[2]

ABSTRACT

A description of the Fortran program HECTOR for a variety of semi-analytical calculations of radiative QED, QCD, and electroweak corrections to the double-differential cross sections of $\mathcal{NC}$ and $\mathcal{CC}$ deep inelastic charged lepton proton (or lepton deuteron) scattering is presented. HECTOR originates from the substantially improved and extended earlier programs HELIOS and TERAD91. It is mainly intended for applications at HERA or LEP$\otimes$LHC, but may be used also for $\mu N$ scattering in fixed target experiments. The QED corrections may be calculated in different sets of variables: leptonic, hadronic, mixed, Jaquet-Blondel, double angle etc. Besides the leading logarithmic approximation up to order $\mathcal{O}(\alpha^2)$, exact $\mathcal{O}(\alpha)$ corrections and inclusive soft photon exponentiation are taken into account. The photoproduction region is also covered.

- Many thanks to the School of Physics for kind invitation to visit the IPM and for hospitality extended me during my stay in Tehran.
- My special thanks to Shiekh Jabbari and Yasaman Farzan.