Question for qualification exam; Sharif U., 2008 Spring

Consider a **complex** scalar field Δ with hypercharge equal to one ($Y = 1$). (Notice that we are using a notation in which $Q = T^3 + Y$ and the hypercharge of the $(\nu_L\ e_L)$ doublet is $-1/2$.) Suppose Δ is a triplet of the electroweak symmetry. This means Δ can be considered a 2×2 complex matrix with vanishing trace (Tr[Δ]=0) that under a SU(2) matrix transforms as

$$\Delta \to U\Delta U^\dagger;$$

a) Verify that the following combination is covariant under SU(2)×U(1)

$$D_\mu\Delta = \partial_\mu\Delta + ig[\Delta, W_\mu] - ig'B_\mu\Delta.$$

b) Explicitly write down the interactions of the different components of this field with $W^\pm$ and Z^0 as well as with photon. Identify the charges of each component of Δ.

c) Suppose the neutral component of Δ acquires a vacuum expectation value: $\langle\Delta^0\rangle \neq 0$. Calculate the contribution of $\langle\Delta^0\rangle$ to the masses of Z^0 and $W^\pm$ then explicitly write down the ratio of the contributions to m_{Z^0} and $m_{W^\pm}$.

d) Considering the answer to the previous question, can the Higgs doublet of the standard model be replaced by triplet Δ?

f) Take the term $L^T c\Delta L$ where $L = (\nu_L\ e_L)$ and

$$c = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{1}$$

Verify that this term is invariant under SU(2)×U(1). Show that if the neutral component of Δ develops a VEV, this term gives a mass to the neutrino. Does the induced mass conserve the lepton number? Can we have a similar term for quarks; *i.e.*, $Q^T c\Delta Q$ where $Q = (u_L\ d_L)$?