Exercise 1

1. Show that

- $$\eta_{\alpha\beta} = \Lambda^{\mu}{}_{\alpha}\eta_{\mu\nu}\Lambda^{\nu}{}_{\beta},$$

  where $\Lambda^{\mu}{}_{\nu}$ is a Lorentz transformation and $\eta_{\alpha\beta}$ is the Minkowski metric.

- For a time-like or light-like four vector, sign of time component doesn't change under Lorentz transformation.

- Lorentz transformations with $\Lambda^{0}{}_{0} \geq 1$ and $\det \Lambda = 1$ make a group.

2. Let $U(\Lambda, a)$ be a unitary Poincare transformation. Show that $J^{\mu\nu}$ and P^{μ} are hermitian.
3. With W^{μ} being the Pauli-Lubanski operator in 3+1 dimension

$$W^{\mu} = \epsilon^{\mu\nu\alpha\beta} P_{\nu} J_{\alpha\beta},$$

show that

$$[W_{\alpha}, P_{\mu}] = 0, P^{\alpha} W_{\alpha} = 0,$$

and compute $\epsilon^{\mu\nu\alpha\beta}[W_{\nu}, J_{\alpha\beta}]$.
4. Compute $[V, P_{\alpha}]$ and $[V, J_{\alpha\beta}]$ where $V = \epsilon^{\mu\nu\alpha\beta} J_{\mu\nu} J_{\alpha\beta}$.
5. Find Gallian algebra by taking Winger-Inünü contraction of Poincare algebra.
6.

- Write Pauli-Lubanski operator in 2+1 dimension and compute its commutator with $J_{\mu\nu}$ and P_{α}.

- Construct massive and massless irreducible representation of 2+1 dimension Poincare group.