Exercise 4

1. If

$$\tilde{\psi}(\Lambda x) = U(\Lambda)\psi(x)$$

where $U(\Lambda) = e^{\frac{i}{2}w.S}$. Show that

- $\bar{\psi}(\Lambda x) = \bar{\psi}U^{-1}(\Lambda)$.
- $\bar{\psi}\gamma^{\mu}\psi$ is a Lorentz vector.
- $\bar{\psi}\psi$ is a scalar.

2. Let $u_r(0)$ and $v_r(0)$ respectively be positive and negative frequently solutions to a massive Dirac equation in the rest frame of particle. Calculate $u_r(p)$ and $v_r(p)$ for generic p, using appropriate Lorentz transformations.
3. Work out $\hat{P}^{\mu}$ and $\hat{J}_{\mu\nu}$ for massless and massive fermionic fields in terms of creation and annihilation operator.
4. If we denote ψ under parity, time reversal and charge conjugation respectively by ψ^P, ψ^T and ψ^C, show that ψ^P, ψ^T and ψ^C transform as a fermion ψ under Lorentz transformation.
5. Work out $\bar{\psi}\gamma^{\mu}\psi$ and $\bar{\psi}\psi$ for Majorana fermions.
6. Study Dirac field operator under CP and CPT.