U(1) gauge symmetry

$$\mathcal{L} = \bar{\psi}(x)\,(i\gamma^\mu \partial_\mu - m)\psi(x)$$

$$\psi(x) \to e^{-i\theta}\psi(x) \qquad (1-i\theta)\psi$$

$$\bar{\psi}(x) \longrightarrow \bar{\psi}(x)\, e^{+i\theta}$$

$$j_\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \overbrace{i\psi}^{\delta\psi} \qquad j_\mu = \bar{\psi}\gamma_\mu \psi$$

$$\theta \to \theta(x)$$ پیمانه‌ای

$$\mathcal{L} \to \mathcal{L} + j^\mu \partial_\mu \theta$$

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - ie A_\mu$$ مشتق هموردا

$$\psi(x) \to e^{-i\theta}\psi(x)$$

$$A_\mu \longrightarrow A_\mu - \frac{1}{e}\partial_\mu \theta$$

$$\bar{\psi}\, i\gamma^\mu D_\mu \psi - \bar{\psi} m \psi$$

$$D_\mu \psi \longrightarrow e^{-i\theta} D_\mu \psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

اتحاد $$[D_\mu, D_\nu]\psi = -ie F_{\mu\nu}\psi$$

$$\cancel{m^2 A_\mu A^\mu}$$

kinetic term

علت بهتر را پایین می بینیم

$$\mathcal{L}_{QED} = \bar{\psi}(x)\,(i\gamma^\mu D_\mu - m)\psi + \left(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

معادلات اویلر-لاگرانژ ← معادلات ماکسول *

جمله‌ی آخر ۳.۲.۱

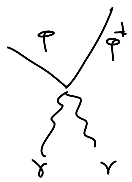
برهمکنش یوکاوا از نوع پیمانه‌ای نیست.

برهمکنش اسکالر

$$(\partial_\mu\varphi)^\dagger\,\partial^\mu\varphi$$

$$\varphi \to e^{-i\theta}\varphi$$

$$\partial^\mu \to D^\mu = \partial^\mu - ieA^\mu$$



$$(D_\mu\varphi)^\dagger D^\mu\varphi = (\partial^\mu\varphi^\dagger + ieA^\mu\varphi^\dagger)(\partial_\mu\varphi - ieA_\mu\varphi) =$$

$$(\partial^\mu\varphi)^\dagger\partial_\mu\varphi + eA^\mu i(\varphi^\dagger\partial_\mu\varphi - (\partial_\mu\varphi)^\dagger\varphi) + e^2A^\mu A_\mu\varphi^\dagger\varphi$$

حد غیرنسبیتی

$$\psi(p) = \frac{1}{\sqrt{2E_p}}\sum_s\left(a_p^s u^s e^{-ip\cdot x} + a_p^{s\dagger} v e^{ip\cdot x}\right)$$

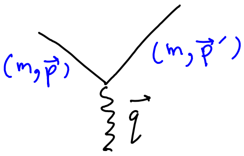
$$u = \begin{bmatrix}\sqrt{p\cdot\sigma}\,\chi \\ \sqrt{p\cdot\bar{\sigma}}\,\chi\end{bmatrix}$$

$$E \simeq m \qquad |\vec{p}| \ll m$$

حد غیرنسبیتی

$$\sqrt{p\cdot\sigma} = \sqrt{E - \vec{p}\cdot\vec{\sigma}} = \sqrt{m}\left(1 - \frac{\vec{\sigma}\cdot\vec{p}}{2m}\right)$$

$$\sqrt{p\cdot\bar{\sigma}} = \sqrt{E + \vec{p}\cdot\vec{\sigma}} = \sqrt{m}\left(1 + \frac{\vec{\sigma}\cdot\vec{p}}{2m}\right)$$



$$\bar{u}\gamma^i u = 2m\ \chi'^{\dagger}\cdot\chi$$

$$\bar{u}\gamma^i u = \left[\chi'^{\dagger}\sqrt{p'\cdot\sigma}\quad \chi'^{\dagger}\sqrt{p'\cdot\bar{\sigma}}\right]\begin{bmatrix}-\sigma^i & 0\\ 0 & \sigma^i\end{bmatrix}\begin{bmatrix}\sqrt{p\cdot\sigma}\,\chi\\ \sqrt{p\cdot\bar{\sigma}}\,\chi\end{bmatrix}$$

$$= m\Big[\chi'^{\dagger}\Big(1-\frac{\vec{\sigma}\cdot\vec{p}\,'}{2m}\Big)\sigma^i\Big(1-\frac{\vec{\sigma}\cdot\vec{p}}{2m}\Big)\chi$$

$$-\chi'^{\dagger}\Big(1+\frac{\vec{\sigma}\cdot\vec{p}\,'}{2m}\Big)\sigma^i\Big(1+\frac{\vec{\sigma}\cdot\vec{p}}{2m}\Big)\chi\Big]$$

$$\vec{p}\,' = \langle\vec{p}\rangle + \frac{\vec{q}}{2}$$

$$\vec{p} = \langle\vec{p}\rangle - \frac{\vec{q}}{2}$$

$$\bar{u}\gamma^i u = -2m\ \chi'^{\dagger}\Big[\frac{\overbrace{\{\langle\vec{p}\rangle\cdot\vec{\sigma},\sigma^i\}}^{2\langle\vec{p}\rangle^i}}{2m} + \frac{\overbrace{[\frac{\vec{q}\cdot\vec{\sigma}}{2},\sigma^i]}^{2i\epsilon^{jik}\frac{q^j\sigma^k}{2}}}{2m}\Big]\chi$$

$$\Rightarrow\quad A_i\,\bar{u}\gamma^i u = -2\vec{A}\cdot\langle\vec{p}\rangle\,\chi'^{\dagger}\chi - \vec{B}\cdot\chi'^{\dagger}\vec{\sigma}\chi$$

$$A_\mu\bar{\psi}\gamma^\mu\psi = A_0\,\overbrace{\chi'^{\dagger}\chi}^{1} + \frac{\vec{A}\cdot\langle\vec{p}\rangle\,\overbrace{\chi'^{\dagger}\chi}^{\text{بستگی به اسپین ندارد}}}{m} + \frac{\vec{B}\,\overbrace{\chi'^{\dagger}\vec{\sigma}\chi}^{2\vec{S}}}{2m}$$

$\chi' = \chi$

تجدید خاطرت

(انرژیان) فصل ۵.۳ ساکورایی

$$A = -\frac{1}{2}\left(By\,\hat{x} - Bx\,\hat{y}\right)$$

~~از موج تخت باید استفاده کنیم~~

$$\vec{A}\cdot\vec{p} = |\vec{B}|\left(-\frac{y}{2}p_x + \frac{xp_y}{2}\right)$$

$$\vec{A}\cdot\vec{p} = \frac{|\vec{B}|\,L_z}{2}$$

از بسته موج باید استفاده کنیم

$$V = -e\,\bar{\psi}\gamma^\mu\psi\,A_\mu = \frac{-eB}{2m}(L+2S)$$

$g=2 \leftarrow$ tree level

$$\mu = \frac{gqB}{2m}\,\vec{S}$$

ذره‌ی باردار با اسپین $\frac{1}{2}$:

$$\frac{eB}{2m_e} = g_B$$ مگنتون بور

$$\frac{eB}{2m_p} = g_N$$ Nuclear magneton

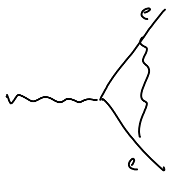
PDG 2006

درمورد الکترون $\mu_e = 1.00115965\,21859 \pm 0.0\ldots\ldots38\,\mu_B$

PDG 2008

Magnetic moment anomaly

$$\frac{g-2}{2} = (1159.65\,21\,811 \pm 0.0000007) \times 10^{-6}$$



ازجمله میون

مهمترین

مشاهده پذیری ذرات

$$\frac{g-2}{2} = (11\,659\,208 \pm 6) \times 10^{-10}$$

OLD SM $\longrightarrow$ $\mu_\nu = 0$ برای

$$\mu_\nu < 0.74 \times 10^{-10}\,\mu_B$$

$\mu_{\pi^\pm}$ μ_{π^0}

Magnetic dipole

Proton

$$\mu = 2.792847351 \pm 0.000000028\,\mu_N$$

neutron

$$\mu = -1.9\,130427 \pm 0.0000005\,\mu_N$$

anomalous

$d \longrightarrow 0$ درمورد همه ی این ذرات

$$\vec{S} \xrightarrow{T} -\vec{S}$$

$$\vec{S} \xrightarrow{T} -\vec{S}$$

$$\vec{B} \xrightarrow{T} -\vec{B}$$

$$\vec{E} \xrightarrow{T} +\vec{E}$$

$$d\,\vec{S}\cdot\vec{E} + \mu\,\vec{S}\cdot\vec{B}$$

$$\Big\downarrow T$$

$$-d\,\vec{S}\cdot\vec{E} + \mu\,\vec{S}\cdot\vec{B}$$

$d_e < 1.4\times 10^{-27}$ e cm ← PDG

$d_\mu < 1.9\times 10^{-19}$ e cm ← arXiv: 0811.1207

$e + \gamma \longrightarrow e\gamma$ برهمکنش کامپتون

$e^- e^- \longrightarrow e^- e^-$ Moller scattering

$e^- e^+ \longrightarrow \gamma\gamma$ pair annihilation

$e^- e^+ \longrightarrow e^- e^+$ BHABHA scattering

$$\mathcal{L}(\varphi, \partial_\mu\varphi)$$

$$\varphi_a \longrightarrow \varphi_a + \delta\varphi_a$$

$$\delta\mathcal{L} = \partial_\mu K^\mu = \frac{\partial\mathcal{L}}{\partial\varphi_a}\delta\varphi_a + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)}\delta\partial_\mu\varphi_a$$

$$= \left(\frac{\partial\mathcal{L}}{\partial\varphi_a} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\varphi_a}\right)\delta\varphi_a + \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)}\delta\varphi_a\right) =$$

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\varphi_a}\delta\varphi_a\right)$$

جریان نوتر $J^\mu = \dfrac{\partial\mathcal{L}}{\partial\partial_\mu\varphi_a}\delta\varphi_a - K^\mu$

در بیشتر مواردی که در فیزیک ذرات ما سروکار داریم $K^\mu = 0$ یعنی

تحت تبدیل نه تنها کنش $\left(\int d^4x\,\mathcal{L}\right)$ بلکه چگالی لاگرانژی

(یعنی $\mathcal{L}$) ناورداست .

تانسور انرژی - تکانه که بیشتر در کیهان‌شناسی با آن سروکار دارید

یک استثنا است.

تقارن انتقال فضا-زمان

$$x^\mu \to x^\mu + a^\mu \qquad \varphi_a(x) \to \varphi_a(x) + a^\mu \partial_\mu \varphi_a(x)$$

$$\mathcal{L} \to \mathcal{L} + a^\mu \partial_\mu \mathcal{L} \qquad T^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \partial_\nu \varphi - \mathcal{L}\, \delta^\mu{}_\nu$$

SU(n)

$$\varphi_a \to \varphi_a + \delta \varphi_a$$

$$\delta \varphi_a = -i \theta^i T^i_{ab} \varphi_b \qquad \varphi_a \to \varphi'_a = e^{-i\theta^i T^i_{ab}} \varphi_b$$

$$a, b = 1 \ldots n$$

$$j^{i\mu}(x) \equiv -i \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi_a} T^i_{ab} \varphi_b$$

$$Q^i(t) \equiv \int d^3x \; j^{i0}(\vec{x}, t)$$

Canonical quantization

$$\pi_a = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_a)}$$

$$[\varphi_a(\vec{x}, t), \varphi_b(\vec{x}, t)] = [\pi_a(\vec{x}, t), \pi_b(\vec{x}, t)] = 0$$

$$[\varphi_a(\vec{x}, t), \pi_b(\vec{x}, t)] = i\delta_{ab}\, \delta^3(\vec{x} - \vec{y})$$

اشتباه پایین کتاب رابطه ۳.۱۴

کوانتی

$$\partial_\mu j^{i\nu} = i[P_\mu, j^{i\nu}]$$

عملگر چهار-تکانه

$$\frac{dQ^i}{dt} = i[H, Q^i] = 0$$

$$[Q^i, Q^j] = i f_{ijk} Q^k \qquad *$$

$$\boxed{[Q^i, \varphi] = -T^i \varphi} \quad *$$

$$\varphi \longrightarrow \varphi' = e^{i\theta^i Q^i} \varphi \, e^{-i\theta^i Q^i} = e^{-i\theta^i T^i} \varphi$$

تئوری پیمانه‌ای Yang-Mills

SU(2)

Yang Mills 1954

$$U(1) \longrightarrow SU(2)$$

ایزواسپین سرتاسری است.

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} \qquad \psi \rightarrow \psi'(x) = U \psi(x)$$

$$U = e^{-ig \frac{\tau^i}{2} \theta^i(x)}$$

$$\tau^i = \sigma^i \qquad \tau^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \tau^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \tau^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

ایده‌ی اولیه Yang-Mills

یَنگ و میلز برای طرح کردن ایده‌ی اولیه‌ی خود از دوتایی $\begin{pmatrix} p \\ n \end{pmatrix}$ استفاده کرده بودند

$$D_\mu = \partial_\mu - ig \vec{A}_\mu$$

$$\vec{A}_\mu = \sum_{i=1}^{3} \frac{\tau^i}{2} A_\mu^i = \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu$$

مشابه U(1) می‌خواهیم داشته باشیم

$$D_\mu \psi \longrightarrow (D_\mu \psi)' = D_\mu' \psi' = U (D_\mu \psi)$$

در این صورت $\bar{\psi} D_\mu \gamma^\mu \psi$ ناوردا خواهد ماند.

$D_\mu \gamma^\mu \overset{?}{=} \gamma^\mu D_\mu$ آیا ترتیب مهم است؟

$$(D_\mu\psi)' = D'_\mu U\psi = U(D_\mu\psi)$$

$$\Downarrow$$

$$D'_\mu = U D_\mu U^{-1}$$

$$\downarrow$$

$$\vec{A}'_\mu = U\vec{A}_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}$$

تبدیل‌های بی‌نهایت کوچک

$$U = 1 - i\theta + \mathcal{O}(\theta^2)$$

$$U^{-1} = 1 + i\theta + \mathcal{O}(\theta^2)$$

$$\theta = \sum_{i=1}^{3} \frac{\tau^i}{2}\theta^i = \frac{\vec{\tau}}{2}\cdot\vec{\theta}$$

$$\vec{A}'_\mu = \vec{A}_\mu - i[\theta, \vec{A}_\mu] - \frac{\partial_\mu\theta}{g}$$

$$\delta A^i_\mu = \varepsilon_{ijk}\theta^j A^k_\mu - \frac{\partial_\mu\theta^i}{g}$$

$$[D_\mu, D_\nu]\psi = -ig\vec{F}_{\mu\nu}\psi$$

$$\vec{F}_{\mu\nu} = \sum_{i=1}^{3}\frac{\tau^i}{2}F^i_{\mu\nu} = \frac{\vec{\tau}}{2}\cdot\vec{F}_{\mu\nu}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\varepsilon_{ijk}A^j_\mu A^k_\nu$$

$$\vec{F}'_{\mu\nu} = U\vec{F}_{\mu\nu}U^{-1}$$

$$\mathrm{Tr}[\vec{F}'_{\mu\nu}\cdot\vec{F}^{\mu\nu\prime}] = \mathrm{Tr}[\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}]$$

$$-\frac{1}{2}\mathrm{Tr}[\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}] = -\frac{1}{2}\sum_{i,j=1}^{3}\mathrm{Tr}\left[\frac{\tau^i}{2}F^i_{\mu\nu}\frac{\tau^j}{2}F^{j\mu\nu}\right]$$

$$= -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu}$$

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_G$$

$$\mathcal{L}_F = \bar{\psi}(i\not{\partial} - m)\psi$$

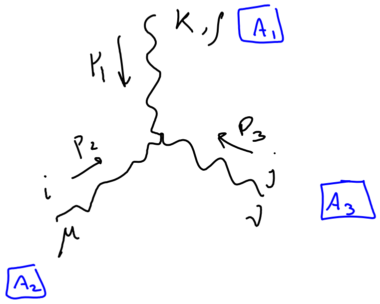
$$\mathcal{L}_G = \underbrace{-}_{\text{علامت منفی}} \frac{1}{2} \operatorname{Tr}[\vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu}]$$

$$F_{0i} = -E A^i \qquad F^{0i} = E A^i$$

$$\cancel{m A^i_\mu A^{i\mu}}$$

$$\mathcal{L}_G = -\frac{1}{2} \partial_\mu A^i_\nu \left(\partial^\mu A^{i\nu} - \partial^\nu A^{i\mu}\right)$$

$$-g\, \varepsilon_{ijk} A^i_\mu A^j_\nu \partial^\mu A^{k\nu} - \frac{g^2}{4} \varepsilon_{ijk} \varepsilon_{ilm} A^j_\mu A^k_\nu A^{l\mu} A^{m\nu}$$



*

$$f_{i_1 j_1 k_1} A^{i_1}_{\mu_1} A^{j_1}_{\nu_1} \partial^{\mu_1} A^{k_1\nu_1} \,|\, A_{1\rho,k} A_{2\mu,i} A_{3\nu,j} \rangle$$

$$= f_{i_1 j_1 k_1} A^{i_1}_{\mu_1} A^{j_1}_{\nu_1} \partial^{\mu_1} A^{k_1\nu_1} \,|\, A_{1\rho,k} A_{2\mu,i} A_{3\nu,j} \rangle$$

$$+ \begin{pmatrix} A_1 \leftrightarrow A_3 \\ P_3 \leftrightarrow P_1 \\ k \leftrightarrow j \end{pmatrix} + \begin{pmatrix} A_2 \leftrightarrow A_3 \\ P_3 \leftrightarrow P_2 \\ j \leftrightarrow i \end{pmatrix}$$

$$= i\Big[\, P_3 \cdot A_1^k \; A_2^i \cdot A_3^j \quad f_{kij}$$

$$+ P_3 \cdot A_2^i \; A_1^k \cdot A_3^j \quad f_{ikj}$$

$$+ P_1 \cdot A_3^j \; A_2^i \cdot A_1^k \quad f_{jik}$$

$$+ P_1 \cdot A_2^i \; A_1^j \cdot A_3^k \quad f_{ijk}$$

$$\cdot \; D \cdot A^k \quad A^j \cdot A^i \quad f_{..} +$$

$$+ P_2 \cdot A_1^k \quad A_3^j \cdot A_2^i \quad f_{kji} +$$
$$P_2 \cdot A_3^j \quad A_1^k \cdot A_2^i \quad f_{jki} \Big] = K$$

$$V = g f_{ijk} A_\mu^i A_\nu^j \partial^\mu A^{k\nu}$$

رأس $\quad -iV = g\, K$

فرقی بین چپ-دست و راست-دست نمی‌گذارد.

جرم A_μ^i نداریم.

$$\mathcal{L} = (\partial_\mu \varphi)^\dagger \partial^\mu \varphi - V(\varphi^* \varphi)$$

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}} \qquad \varphi^* = \frac{\varphi_1 - i\varphi_2}{\sqrt{2}}$$

$$V(\varphi^*\varphi) = m^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2$$

$\lambda > 0 \longrightarrow$ Unbounded from below

$$\langle 0|\varphi_1|0\rangle = \varphi_{01} \qquad \langle 0|\varphi_2|0\rangle = \varphi_{02}$$

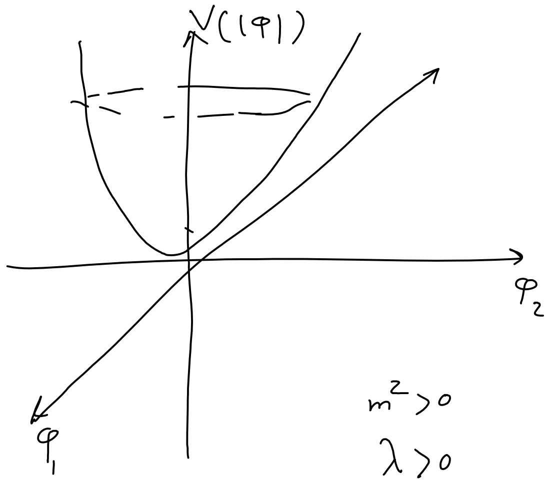
$$V(\varphi_1, \varphi_2) = V(\varphi_{01}, \varphi_{02}) + \sum_{a=1,2} \left(\frac{\partial V}{\partial \varphi_a}\right)_0 (\varphi_a - \varphi_{0a})$$
$$+ \frac{1}{2} \sum_{\substack{a,b=\\1,2}} \left(\frac{\partial^2 V}{\partial \varphi_a \partial \varphi_b}\right) (\varphi_a - \varphi_{0a})(\varphi_b - \varphi_{0b}) + \cdots$$

V در $\langle \varphi \rangle = (\varphi_{01}, \varphi_{02})$ کمینه است یعنی جمله‌ی دوم صفر می‌شود.

$$m_{ab}^2 = \left(\frac{\partial^2 V}{\partial \varphi_a \partial \varphi_b}\right)_0$$

فاز ویگنر

Wigner phase



$m^2 > 0$

$\lambda > 0$

$$\left(\frac{\partial V}{\partial \varphi_1}\right)_0 = m^2 \varphi_{01} + \lambda \varphi_{01}\left(\varphi_{01}^2 + \varphi_{02}^2\right) = 0$$

$$\left(\frac{\partial V}{\partial \varphi_2}\right)_0 = m^2 \varphi_{02} + \lambda \varphi_{02}\left(\varphi_{01}^2 + \varphi_{02}^2\right) = 0$$

$$\varphi_{01} = \varphi_{02} = 0$$

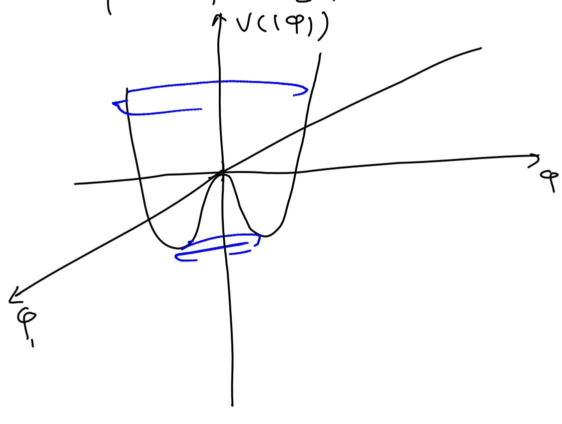
$$m_{ab}^2 = \begin{bmatrix} m^2 & 0 \\ 0 & m^2 \end{bmatrix}$$

فاز نمبو- گلدستون

Nambu-Goldstone phase

$$V(\varphi_1^2 + \varphi_2^2) = -\frac{\mu^2}{2}\left(\varphi_1^2 + \varphi_2^2\right) +$$

$$\frac{\lambda}{4}\left(\varphi_1^2 + \varphi_2^2\right)^2$$



$$\left(\frac{\partial V}{\partial \varphi_1}\right)_0 = -\mu^2 \varphi_{01} + \lambda \varphi_{01} (\varphi_{01}^2 + \varphi_{02}^2) = 0$$

$$\left(\frac{\partial V}{\partial \varphi_2}\right)_0 = -\mu^2 \varphi_{02} + \lambda \varphi_{02} (\varphi_{01}^2 + \varphi_{02}^2) = 0$$

$$\varphi_{01}^2 + \varphi_{02}^2 = v^2 = \frac{\mu^2}{\lambda}$$

$$\frac{\partial^2 V}{\partial \varphi_1^2} = (-\mu^2 + \lambda(\varphi_1^2 + \varphi_2^2)) + 2\lambda \varphi_1^2$$

$$\frac{\partial^2 V}{\partial \varphi_2^2} = (-\mu^2 + \lambda(\varphi_1^2 + \varphi_2^2)) + 2\lambda \varphi_2^2$$

$$\frac{\partial^2 V}{\partial \varphi_1 \partial \varphi_2} = 2\lambda \varphi_1 \varphi_2$$

$$m_{ab}^2 = \begin{bmatrix} 2\lambda v^2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\varphi_1' = \varphi_1 - v$$

$$\varphi_2' = \varphi_2$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_1')^2 + \frac{1}{2}(\partial_\mu \varphi_2')^2 - \frac{(2\lambda v^2)}{2} \varphi_1'^2$$

$$+ \lambda v \varphi_1' (\varphi_1'^2 + \varphi_2'^2) - \frac{\lambda}{4} (\varphi_1'^2 + \varphi_2'^2)^2$$

تقارن پنهان شده

$2\lambda v^2 = \varphi_1' = \varphi_1 - v$ جرم

$0 = \varphi_2'$ جرم

$$\varphi = \frac{\rho}{\sqrt{2}} e^{i\theta/v}$$

$$\partial_\mu \varphi = \frac{1}{\sqrt{2}} e^{i\frac{\theta}{v}} \left(\partial_\mu \rho + \frac{i\rho}{v} \partial_\mu \theta \right)$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{1}{2v^2} \rho^2 (\partial_\mu \theta)^2 - V(\rho^2)$$

$$\rho(x) = v + \eta(x)$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \eta)^2 + \frac{1}{2} (\partial_\mu \theta)^2 + \frac{\eta}{v} (\partial_\mu \theta)^2 + \frac{\eta^2}{2v^2} (\partial_\mu \theta)^2$$

$$- V(\rho^2)$$

$$V(\rho^2) = \frac{1}{2} (2\mu^2) \eta^2 + \lambda v \eta^3 + \frac{\lambda}{4} \eta^4 - \frac{1}{2} \mu^2 v^2$$

$$m_\eta = \sqrt{2\mu^2}$$

تعمیم به مدل $SU(2)$

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$$\mathcal{L} = (D_\mu \varphi)^\dagger (D^\mu \varphi) - \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - V(\varphi^\dagger \varphi)$$

$$D_\mu \varphi = \left(\partial_\mu - ig \frac{\tau^i}{2} A^i_\mu \right) \varphi \qquad i = 1, 2, 3$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g \varepsilon_{ijk} A^j_\mu A^k_\nu$$

$$V(\varphi^\dagger \varphi) = -\mu^2 \varphi^\dagger \varphi + \lambda \varphi^\dagger \varphi$$

باز- پارامتری کردن با میدان های حقیقی $H(x)$ و $\xi^i(x)$

$$\varphi(x) = \frac{e^{\frac{i\tau^i \xi^i}{2v}}}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

پیمانه‌ی یکانی

Unitary gauge

$$\varphi(x) \longrightarrow \varphi'(x) = U(x)\,\varphi(x) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ v+H(x) \end{bmatrix}$$

$$\vec{A}_\mu \longrightarrow \vec{B}_\mu = U(x)\,\vec{A}_\mu\,U^{-1}(x) - \frac{i}{g}(\partial_\mu U)U^{-1}$$

$$U(x) = e^{-i\tau^i \frac{\xi^i(x)}{2v}}$$

گلدستون

ξ ها پریدند! کجا رفتند؟

$$\left[(D_\mu\varphi)'\right]^{+a}(D^\mu\varphi)'_a = \frac{1}{2}\partial_\mu H\,\partial^\mu H + g^2 B^i_\mu B^{j\mu}\left(\frac{\tau^i}{2}\right)^a_b\left(\frac{\tau^j}{2}\right)^c_a \varphi'^b\varphi'_c$$

$$= \frac{1}{2}\partial_\mu H\,\partial^\mu H + \frac{g^2}{8}B^i_\mu B^{i\mu}(v+H)^2$$

$$\mathcal{L} = \frac{\partial_\mu H\,\partial^\mu H}{2} - \mu^2 H^2 - \frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + \frac{g^2 v^2}{8}B^i_\mu B^{i\mu}$$

$$+\frac{g^2}{8}B^i_\mu B^{i\mu}H(2v+H) - \lambda v H^3 - \frac{\lambda}{4}H^4 - \frac{v^4}{4}$$

$$m_B = \frac{gv}{2} \qquad m_H = \sqrt{2\mu^2}$$

* معادله اویلر لاگرانژ را برای مولفه‌ی طولی، مولفه‌ی عرضی و مولفه‌زمانی B_μ به دست آورید (جملات برهمکنش را وارد نکنید میدان آزاد درنظر بگیرید.