۲۰۱۰/۱۰/۳۰
۰۱:۲۷ ب.ظ

$$J_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu}$$

$$\vec{K}^i = J^{0i} = L^{0i} + S^{0i}$$

$$\vec{J}^i = \varepsilon^{ijk} J_{jk}$$

$$[S_{\mu\nu}, P_\alpha] = 0$$

$$[S_{\mu\nu}, L_{\alpha\beta}] = 0$$

$$[S_{\mu\nu}, S_{\alpha\beta}] \neq 0$$

$$\left.\begin{array}{l} [S^{0i}, \vec{S}^k] \neq 0 \\ [K^i, \vec{S}^k] \neq 0 \end{array}\right\} \text{در حالت کلی}$$

اما اگر جهت خیز با جهت $\vec{J}$ یکی باشد J تغییر نمی‌کند.

$$[Q, \varphi] = i\delta\varphi \qquad \left([Q, \varphi] = i\delta\varphi\right)^\dagger$$

$$Q\varphi - \varphi Q = i\delta\varphi \xrightarrow{\text{Conjugate}} \varphi^\dagger Q^\dagger - Q^\dagger \varphi^\dagger = -i\delta\varphi^\dagger$$

$$[Q^\dagger, \varphi^\dagger] = +i\delta\varphi^\dagger$$

$Q = Q^\dagger \rightarrow$ در مورد بارهای معمولی که سراغ داریم. اما باز هم این

ابهام در ترتیب را حل نمی‌کند.

کلاسیک $\quad AB \xrightarrow{\text{کوانتمی}}$

$$\frac{AB + B^\dagger A^\dagger}{2}$$

$$\frac{BA + A^\dagger B^\dagger}{2}$$

$$\cos\alpha\, AB + \sin\alpha\, BA + H.C.$$

مدل استاندارد

$SU(3) \times SU(2)_L \times U(1)_Y$

Glashow - Weinberg - Salam

(GWS)

Glashow, 1961; Weinberg, 1967; Salam 1968

در برهمکنش های ضعیف

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu \\ \pi^- \to \mu^- \bar{\nu}_\mu \\ n \to p + e^- + \bar{\nu}_e \end{cases}$$

left-handed leptons

right-handed anti leptons

$$J_\mu(x) \equiv J_\mu^+(x) = \bar{\nu}_{eL}(x)\, \gamma_\mu\, e_L(x) = \bar{\nu}_e(x)\, \gamma_\mu \frac{1-\gamma_5}{2} e(x)$$

$$J_\mu^\dagger(x) \equiv J_\mu^-(x) = \bar{e}_L\, \gamma_\mu\, \nu_{eL} = \frac{1}{2}\, \bar{e}\, \gamma_\mu (1-\gamma_5)\, \nu_e$$

$$L = \frac{1-\gamma_5}{2} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

$$\tau^+ = \frac{\tau^1 + i\tau^2}{2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\tau^- = \frac{\tau^1 - i\tau^2}{2} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$J_\mu^+ = \bar{L}\, \gamma_\mu\, \tau^+ L$$

$$J_\mu^- = \bar{L}\, \gamma_\mu\, \tau^- L$$

مشابه موارد بالا و با به کارگیری ماتریس τ^3 می توان نوشت

$$J_\mu^3 = \bar{L}\gamma_\mu \frac{\tau^3}{2} L = \frac{1}{2}\bar{\nu}_e \gamma_\mu \nu_e - \frac{1}{2}\bar{e}_L \gamma_\mu e_L$$

$$W_\mu^{\pm} \qquad W_\mu^3$$

$$J_\mu^i(x) = \bar{L}\gamma_\mu \frac{\tau^i}{2} L$$

$$T^i = \int J_0^i(x)\, d^3x$$

$$[T^i, T^j] = i\,\varepsilon^{ijk}\, T^k$$

Levi - Civita تانسور

$$\varepsilon^{123} = 1$$

$$R = \frac{1+\gamma_5}{2} e = e_R$$

$$J_\mu^3 \neq J_\mu^{em}$$ وسوسه

$$J_\mu^{em} = -\bar{e}\gamma^\mu e = -\bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R$$

$$J^{em}_\mu = q\bar{\psi}\gamma_\mu\psi$$

$$Q = \int J^{em}_0 d^3x = -\int \bar{e}\gamma_0 e\, d^3x =$$

$$-\int (e_L^\dagger e_L + e_R^\dagger e_R)\, d^3x$$

$$[Q, T^3] = ? \qquad [Q, T^1] = ? \qquad [Q, T^2] = ?$$

$SU(2)_L$ و $U(1)_{em}$ همزمان نمی‌توانند تقارن های مدل باشند.

$$SU(2)_L : \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \qquad\qquad U(1)_{em} : \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

$U(1)_Y$

$$\frac{Y}{2} = Q - T^3 = \int d^3x \left(\frac{-1}{2} \nu_{eL}^\dagger \nu_{eL} - \frac{1}{2} e_L^\dagger e_L - e_R^\dagger e_R \right)$$

$$[Q - T^3, T^i] = 0$$

$U(1)_Y$ و $SU(2)$ تقارن های همزمان مدل می‌توانند باشند.

$$Q = T^3 + \frac{Y}{2}$$

ذرات	Q	(T, T^3)	Y
ν_e, ν_μ, ν_τ	0	$(\frac{1}{2}, +\frac{1}{2})$	-1
e_L, μ_L, τ_L	-1	$(\frac{1}{2}, -\frac{1}{2})$	-1
e_R, μ_R, τ_R	-1	0	-2

نکته ی تاریخی (دقت زیاد تا اشتباه نشود) !!!

رابطه ی Nakano - Nishijima - Gell-Mann

$$\begin{cases} \text{Nakano \& Nishijima}, & 1953 \\ \text{Gell-Mann} & 1953 \end{cases}$$

$$Y = B + S$$

p در مورد $= ?$

K^+ در مورد $= ?$

$$K^+ = |u\bar{s}\rangle$$

$SU(2)$ ایزواسپین با $SU(2)$ پیمانه‌ای فرق دارد.

به دیدگاه مدرن بازگردیم.

$$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad R = e_R$$

$$SU(2)_L : \quad L \to L' = e^{-i\frac{\alpha^i \tau^i}{2}} L \;, \quad R \to R' = R$$

$$U(1)_Y : \quad L \to L' = e^{i\beta/2} L \;, \quad R \to R' = e^{i\beta} R$$

$$\alpha^i = \alpha^i(x) \qquad \beta = \beta(x)$$

$SU(2)_L \times U(1)_Y$ لاگرانژی ناوردا تحت

$$\mathcal{L}_F = \bar{L}\, i\gamma^\mu \left(\partial_\mu - ig \frac{\vec{\tau}}{2}\cdot\vec{A}_\mu + \frac{i}{2} g' B_\mu\right) L$$
$$+ \bar{R}\, i\gamma^\mu \left(\partial_\mu + ig' B_\mu\right) R$$

$$D_\mu = \partial_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{A}_\mu - ig'\frac{Y}{2} B_\mu$$

$$\cancel{m\,\bar{e}e = m\,\bar{e}_R e_L + m\,\bar{e}_L e_R}$$

matter field $\quad L, R$
gauge field $\quad \vec{A}_\mu, B_\mu$

$$\mathcal{L}_G = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\,\varepsilon_{ijk} A^j_\mu A^k_\nu$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

No mass term for B_μ or $\vec{A}_\mu$

در دنیای واقعی این میدان‌ها جرم‌دار هستند.

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$$

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad \leftarrow \quad Q = T_3 + \frac{Y}{2} \rightarrow Y_\varphi = ?$$

$$\mathcal{L}_s = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi^\dagger \phi)$$

گردآن

$$D_\mu \phi = \left(\partial_\mu - i g \frac{\vec{\tau} \cdot \vec{A}_\mu}{2} - i g' B_\mu\right) \phi$$

$$V(\phi^\dagger \phi) = m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$\lambda > 0$$

کتاب یک اشتباه تایپی دارد!

$$\mathcal{L}_y = -G_e (\bar{L} \phi R + \bar{R} \phi^\dagger L) + h.c.$$

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_G + \mathcal{L}_s + \mathcal{L}_y$$

$SU(2)_L \times U(1)_Y$ شکست خود به خود تقارن

$$m^2 = -\mu^2 \qquad \mu^2 > 0$$

$$\varphi_0 = \langle 0 | \varphi | 0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\frac{\tau^3}{2} \varphi_0 = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = -\frac{\varphi_0}{2}$$

$$Y \varphi_0 = \varphi_0$$

بنابراین

$$e^{-i\alpha^3 \frac{\tau^3}{2}} \varphi_0 \neq \varphi_0 \qquad e^{-i\beta \frac{Y}{2}} \varphi_0 \neq \varphi_0$$

اما

$$Q = T^3 + \frac{Y}{2}$$

$$Q\varphi_0 = \left(T^3 + \frac{Y}{2}\right)\varphi_0 = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{bmatrix}0\\ \frac{v}{\sqrt{2}}\end{bmatrix} = 0$$

$$e^{-i\varepsilon Q}\varphi_0 = \varphi_0$$

نتیجه تقارن متناظر با T^3 و Y شکسته اما تقارن متناظر با Q حفظ.

به زبان دیگر

$$\varphi \longrightarrow \varphi' = e^{iT^3\alpha}\,\varphi\, e^{-iT^3\alpha}$$

$$\begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix} \longrightarrow \begin{bmatrix}e^{i\alpha}\varphi_1\\ e^{-i\alpha}\varphi_2\end{bmatrix}$$

$$\langle 0|\varphi|0\rangle \neq \langle 0|\varphi'|0\rangle$$

$$T^3|0\rangle \neq 0$$

اما

$$Q|0\rangle = 0$$

$$\phi = \begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} = e^{i\frac{\vec{\tau}\cdot\vec{\xi}}{2v}} \begin{bmatrix}0\\ \frac{v+H}{\sqrt{2}}\end{bmatrix}$$

$$U(\xi) = e^{-i\frac{\vec{\tau}\cdot\vec{\xi}}{2v}}$$

پیمانه‌ی یکانی

$$\varphi' = U(\xi)\varphi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

$$L' = U(\xi) L$$

$$\vec{A}'_\mu = U(\xi)\vec{A}_\mu U(\xi)^{-1} - \frac{i}{g}\left(\partial_\mu U(\xi)\right) U^\dagger(\xi)$$

$$R' = R$$

$$B'_\mu = B_\mu$$

$$\mathcal{L}_F = \bar{L}' i\gamma^\mu\left(\partial_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{A}'_\mu + \frac{i}{2}g' B'_\mu\right)L'$$

$$+ \cdots$$

$$\mathcal{L}_s = (D_\mu\varphi)'^\dagger (D^\mu\varphi)'$$

$$(D_\mu\varphi)' = \left(\partial_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{A}'_\mu - \frac{ig' B'_\mu}{2}\right)\begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

جملات درجه ی ۲

$$\mathcal{L}_{mass} = \frac{v^2}{2}\begin{pmatrix} 0 & 1 \end{pmatrix}\left(g\frac{\vec{\tau}}{2}\cdot\vec{A}'_\mu + \frac{g'}{2}B'_\mu\right)\left(g\frac{\vec{\tau}}{2}\cdot\vec{A}'^\mu + \frac{g'}{2}B'^\mu\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$\frac{v^2}{8}\left(g^2 A'^1_\mu A'^{1\mu} + g^2 A'^2_\mu A'^{2\mu} + \left(g A'^3_\mu - g' B'_\mu\right)^2\right)$$

$$W^\pm_\mu = \frac{A^1_\mu \mp i A^2_\mu}{\sqrt{2}}$$

$$\frac{v^2}{8} g^2 \left(A'^{1}_{\mu} A'^{1\mu} + A'^{2}_{\mu} A'^{2\mu} \right) = \frac{m_W^2}{2} W^+_\mu W^{-\mu}$$

$$m_W = \frac{g v}{2} \qquad\qquad (W^+_\mu)^\dagger = W^-_\mu$$

$$\frac{v^2}{8} \left(A'^{3}_{\mu} \quad B'_\mu \right) \begin{pmatrix} g^2 & -g g' \\ -g g' & g'^2 \end{pmatrix} \begin{pmatrix} A'^{3\mu} \\ B'^{\mu} \end{pmatrix}$$

$$\Big\downarrow \text{ قطری کردن}$$

$$\frac{v^2}{8} \left(Z_\mu \quad A_\mu \right) \begin{bmatrix} g^2 + g'^2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Z^\mu \\ A^\mu \end{bmatrix}$$

$$= \frac{v^2}{8} (g^2 + g'^2) \, Z_\mu Z^\mu + 0 \, A_\mu A^\mu$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A'^{3}_{\mu} \\ B'_\mu \end{pmatrix}$$

$\theta_W =$ زاویه‌ی واینبرگ یا زاویه‌ی اختلاط ضعیف

$$\operatorname{tg}\theta_W = \frac{g'}{g}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \qquad\qquad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$m_Z = \frac{v}{2} \sqrt{g^2 + g'^2}$$

$$m_Z = \frac{m_W}{\cos\theta_W} \quad \longleftarrow$$! پیش‌بینی

در پیمانه یکانی

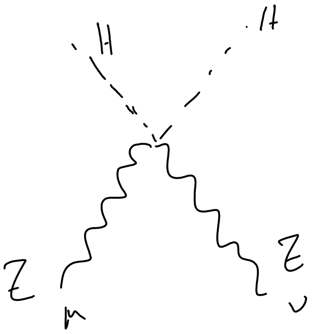
$$V(\varphi^{\dagger}\varphi) = -\frac{\mu^2 v^2}{4} + \frac{1}{2}(2\mu^2)H^2 + \lambda v H^3$$

$$+\frac{\lambda}{4}H^4 \qquad m_H = \sqrt{2\mu^2}$$

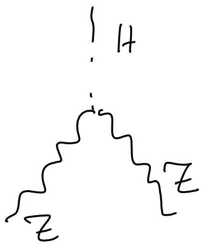
$$\mathcal{L}_s = (D_\mu\phi)(D^\mu\phi) - V(\varphi^{\dagger}\phi) =$$

$$\frac{1}{2}\partial_\mu H \partial^\mu H - \frac{M_H^2}{2}H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4$$

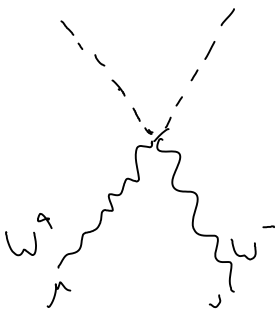
$$+\frac{g^2}{8}(H^2 + 2Hv)\left[\frac{Z_\mu Z^\mu}{\cos^2\theta_W} + 2W_\mu^+ W^{\mu -}\right]$$



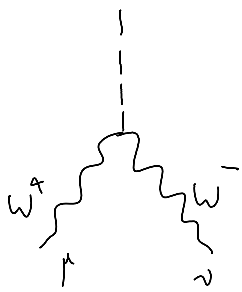
$$i\frac{g^2}{2}\frac{\eta_{\mu\nu}}{\cos^2\theta_W}$$



$$\frac{ig^2\,\eta_{\mu\nu}\,v}{2\cos^2\theta_W}$$



$$\frac{ig^2}{2}\eta_{\mu\nu}$$

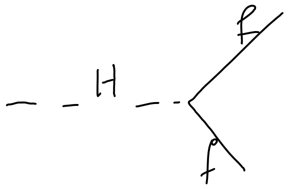


$$\frac{ig^2 v}{2}$$

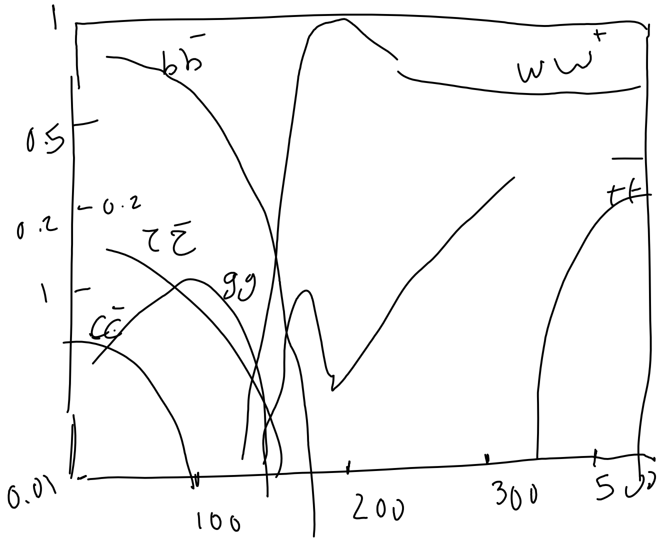
$$\mathcal{L}_Y = -G_e(\bar{L}\varphi R + \bar{R}\varphi^{\dagger}L)$$

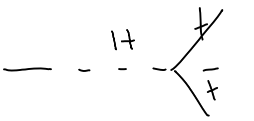
$$= -\frac{G_e v}{\sqrt{2}}\,\bar{e}e - \frac{G_e}{\sqrt{2}}\,H\bar{e}e$$

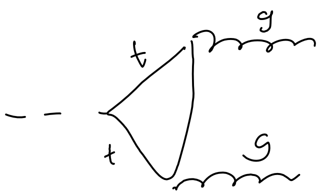
$$m_e = \frac{G_e v}{\sqrt{2}}$$

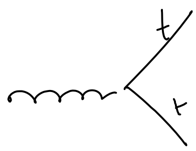


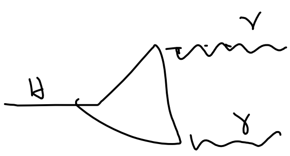
$$-i\,\frac{m_f}{v}$$











نکتهٔ اول شکست خودبه‌خود تقارن

تقدم و تأخر زمانی

انرژی پایین $U(1)_{em}$ و β-decay
weak interactions

$E \ll m_{EW} \simeq m_W$

$E > m_{EW}$ W^{+}
W^{-}
$U(1)_{em}$

در کیهان‌شناسی اول و آخر فرق دارد.

$\mu^2(\underline{T})$

همه‌ی بحث‌های ما در دمای صفر است.
دقت کنید که دمای صفر و انرژی صفر یکی نیستند.
در LHC دما ؟
انرژی ؟

نکته‌ی دوم

$SU(2) \times U(1)$ یا $U(2)$

$e^{i\beta}$ $e^{i\frac{\tau^i}{2}\alpha^i}$ $\begin{bmatrix} - \\ - \end{bmatrix}$

دو جور ضریب جفت‌شدگی داریم

g, g'

! نه چهار جور؟!   و نه یک جورا!

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \longrightarrow$$ یک Hypercharge دارند

اما e_R و e_L هایپرچارج متفاوت دارند.

نکته‌ی سوم

بالای ص ۶۱

Ghost   در پیمانه‌ی یکانی لزوم فیزیکی نیست!

CC and NC

CC: یا مخفف Charged current

NC: یا مخفف Neutral current

kinetic

$$\mathcal{L} = \bar{L}\, i\gamma_\mu \partial^\mu L + \bar{R}\, i\gamma_\mu \partial^\mu R =$$

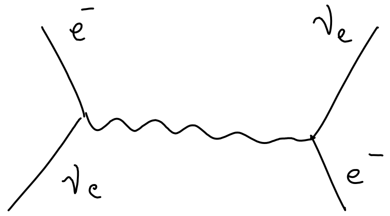
$$\bar{e}\, i\gamma^\mu \partial_\mu e + \bar{\nu}_{eL}\, i\gamma^\mu \partial_\mu \nu_{eL}$$

$$\mathcal{L}_{CC} = g\left(J_\mu^1 A^{1\mu} + J_\mu^2 A^{2\mu} \right) =$$

$$\frac{g}{\sqrt{2}}\left(J_\mu^+ W_\mu^- + J_\mu^+ W^{+\mu} \right)$$

$$J_\mu^{\pm} = \bar{L}\, \gamma_\mu\, \tau^{\pm} L \qquad \tau^{\pm} = \frac{\tau^1 \pm i \tau^2}{2}$$

$$J_\mu^{+} = \frac{1}{2}\, \bar{\nu}_e \gamma_\mu (1-\gamma_5)\, e$$



$$M = (-) \frac{g^2}{(2)} J^{+\mu} \frac{i\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2 + i\varepsilon} J^{-\nu}$$

از کجا آمد؟

$$q^2 \ll m_W^2$$

$$M = -i \frac{g^2}{2 m_W^2} J^{+\mu} J^{-}_{\mu}$$

$$\mathcal{L}_{cc}^{eff} = -\frac{G_F}{\sqrt{2}}\, 4 J^{+\mu} J_\mu^{-} = -\frac{G_F}{\sqrt{2}} \left(\bar{\nu}_e \gamma^\mu (1-\gamma_5) e\right)$$

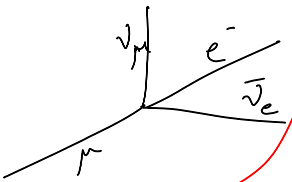
$$\left(\bar{e} \gamma_\mu (1-\gamma_5) \nu_e\right)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2} \qquad G_F = 1.16639 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$$m_W = \frac{g v}{2} \qquad v = \frac{1}{(\sqrt{2} G_F)^{\frac{1}{2}}} = 246\ \mathrm{GeV}$$

$$\frac{v}{\sqrt{2}} \simeq 170 \text{ GeV} \simeq m_t$$

$$\mathcal{L}^{eff}_{CC} = -\frac{G_F}{\sqrt{2}} \left(\bar{\nu}_e \gamma^\mu (1-\gamma_5) e\right)\left(\bar{\mu} \gamma_\mu (1-\gamma_5) \nu_\mu\right) \text{ [illegible]} ,$$



$$\mathcal{L}_{NC} = g J^3_\mu A^{3\mu} + \frac{g'}{2} J^Y_\mu B^\mu =$$

$$\left(g\sin\theta_W \, J^3_\mu + g'\cos\theta_W \, \frac{J^Y_\mu}{2}\right) A^\mu$$

$$+ \underbrace{\left(g\cos\theta_W \, J^3_\mu - g'\sin\theta_W \, \frac{J^Y_\mu}{2}\right)}_{\frac{g}{\cos\theta_W} J^Z_\mu} Z^\mu$$

$$J^{em}_\mu = J^3_\mu + \frac{J^Y_\mu}{2}$$

$$g\sin\theta_W \, J^3_\mu + g'\cos\theta_W \, \frac{J^Y_\mu}{2} = g'\cos\theta_W \, J^{em}_\mu$$

$$+ \left(g\sin\theta_W - g'\cos\theta_W\right) J^3_\mu$$

$$e = g'\cos\theta_W = g\sin\theta_W$$

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \qquad e = \frac{g g'}{\sqrt{g^2 + g'^2}}$$

$$\frac{g}{\cos\theta_W} J^Z_\mu = g\cos\theta_W \, J^3_\mu - g'\sin\theta_W \, \frac{J^Y_\mu}{2} = \frac{g}{\cos\theta_W}\left(J^3_\mu - \sin^2\theta_W \, J^{em}_\mu\right)$$

$$L = \begin{pmatrix} f \\ f' \end{pmatrix}_L \qquad R^f = f_R \qquad R^{f'} = f'_R$$

$$J_\mu^Z = J_\mu^3 - \sin^2\theta_W \, J_\mu^{em} =$$

$$\bar{L} \gamma_\mu \frac{\tau^3}{2} L - \sin^2\theta_W \left(Q_f \bar{f} \gamma_\mu f + Q_{f'} \bar{f}' \gamma_\mu f' \right)$$

$$= a_L^f \bar{f}_L \gamma_\mu f_L + a_R^f \bar{f}_R \gamma_\mu f_R + a_L^{f'} \bar{f}'_L \gamma_\mu f'_L + a_R^{f'} \bar{f}'_R \gamma_\mu f'_R$$

$$a_L^f = \frac{1}{2} - q_f \sin^2\theta_W \qquad a_L^{f'} = -\frac{1}{2} - q_{f'} \sin^2\theta_W$$

$$a_R^f = -q_f \sin^2\theta_W \qquad a_R^{f'} = -q_{f'} \sin^2\theta_W$$

$$J_\mu^Z = \bar{f} \gamma_\mu (C_V^f - C_A^f \gamma_5) f + \bar{f}' \gamma_\mu (C_V^{f'} - C_A^{f'} \gamma_5) f'$$

$$C_V^f = \frac{1}{2}(a_L^f + a_R^f) = \frac{1}{4} - Q_f \sin^2\theta_W$$

$$C_A^f = \frac{1}{2}(a_L^f - a_R^f) = \frac{1}{4}$$

$$C_V^{f'} = \frac{1}{2}(a_L^{f'} + a_R^{f'}) = -\frac{1}{4} - Q_{f'} \sin^2\theta_W$$

$$C_A^{f'} = \frac{1}{2}(a_L^{f'} - a_R^{f'}) = -\frac{1}{4}$$