۲۰۱۰/۱۱/۲۰
۰۱:۰۳ ب.ظ

فیزیک نوترینو

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{bmatrix} \qquad \sigma^\mu = (\mathbb{1}, \vec\sigma) \qquad \bar\sigma^\mu = (1, -\vec\sigma)$$

قرارداد notation Morii etal

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{bmatrix}$$

مستقل از نوتاسیون

در نوتاسیون کتاب (۱-)

$m=0$ → هلیسیتی

$$\psi_R \equiv \frac{1+\gamma_5}{2}\psi \qquad \psi_L = \frac{1-\gamma_5}{2}\psi$$

مستقل از نوتاسیون

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] = \frac{i}{2}\begin{bmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar\sigma^{\mu\nu} \end{bmatrix}$$

$$\sigma^{\mu\nu} = \sigma^\mu\bar\sigma^\nu \qquad \bar\sigma^{\mu\nu} = \bar\sigma^\mu\sigma^\nu \qquad \mu \neq \nu$$

$$\psi_R = \begin{pmatrix} 0 \\ \eta_{\dot\alpha} \end{pmatrix} \qquad \psi_L = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix} \qquad \alpha, \dot\alpha = 1, 2$$

ماتریس های ۲×۲ مختلط

$SL(2,C)$

$SU(2,C)$ ← مولدهای ...!؟؟

کتاب ۲

$$[\gamma_5, \Sigma^{\mu\nu}] = 0$$

We easily know that each of chiral fermion forms an irreducible representation of Lorentz transformation $SL(2,c)$, as $[\gamma_5, \Sigma^{\mu\nu}] = 0$

$$\text{as} \quad [\gamma_5, \Sigma^{\mu\nu}] = 0$$

$$m(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)$$

$$\psi_D = \psi_R + \psi_L = \begin{pmatrix} \xi_\alpha \\ \eta_\alpha \end{pmatrix}$$

$$\psi_R = R\psi_D \qquad \psi_L = L\psi_D \qquad R = \frac{1+\gamma_5}{2} \quad L = \frac{1-\gamma_5}{2}$$

$$\psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\bar{\psi}_D (i\not{\partial} - m) = \bar{\psi}_R i\not{\partial}\psi_R + \bar{\psi}_L i\not{\partial}\psi_L - m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

$$(\psi_R)^c = -i\gamma^2 \left(\frac{1+\gamma_5}{2}\psi\right)^*$$

$$R(\psi_R^c) = \frac{1+\gamma_5}{2}\psi_R^c = 0$$

$$L\psi_R^c = \psi_R^c$$

$$\psi_R = \begin{pmatrix} 0 \\ \eta_\alpha \end{pmatrix} \qquad \psi_R^c = \begin{bmatrix} -i\sigma_2 \eta^* \\ 0 \end{bmatrix} = \begin{bmatrix} -\eta_2^* \\ +\eta_1^* \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{\eta}^{\dot\alpha} = \varepsilon^{\dot\alpha\beta}\bar{\eta}_{\dot\beta} \qquad \bar{\eta}_{\dot j} = (\eta_\alpha)^*$$ نمادگذاری نقطه

$$\psi_L = \begin{bmatrix} \xi^\alpha \\ 0 \end{bmatrix} \qquad \psi_L^c = \begin{bmatrix} 0 \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \xi_2^* \\ -\xi_1^* \end{bmatrix}$$

$$\psi_{M_1} = \psi_R + \psi_R^c = \begin{bmatrix} -i\sigma_2 \eta^* \\ \eta_\alpha \end{bmatrix}$$

$$\psi_{M_2} = \psi_L + \psi_L^c = \begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix}$$

$$\psi_{M_1}^c = \psi_{M_1} \qquad \psi_{M_2}^c = \psi_{M_2}$$

اسپینور های مایورانا

$$\begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} -i\sigma_2 \eta^* \\ \eta \end{bmatrix}$$

$$\bar\psi_D i\cancel{\partial}\psi_D = \bar\psi_R i\cancel{\partial}\psi_R + \bar\psi_L i\cancel{\partial}\psi_L =$$

$$\frac{1}{2}\left(\bar\psi_{M_1} i\cancel{\partial}\psi_{M_1} + \bar\psi_{M_2} i\cancel{\partial}\psi_{M_2}\right)$$

جزو تمرینات هست.

تعداد درجات آزادی

جرم صفر ← انتشارگرها

جرم دیراک - جرم مایورانا

جرم دیراک:

$$-m_D \bar\psi_D \psi_D = -m_D(\bar\psi_L\psi_R + H.c.) = m_D(\xi^\alpha \eta_\alpha + H.c.)$$

جرم مایورانا:

$$-\frac{1}{2} m_R \left(\overline{\psi_R^c}\, \psi_R + H.c. \right) = \frac{m_R}{2} \left(\eta^\alpha \eta_\alpha + H.c. \right)$$

$$-\frac{m_L}{2} \left(\overline{\psi_L^c}\, \psi_L + H.c. \right) = \frac{m_L}{2} \left(\xi^\alpha \xi_\alpha + H.c. \right)$$

$$\bar{\psi}_R\, i\not{\partial}\, \psi_R - \frac{m_R}{2} \left(\overline{\psi_R^c}\, \psi_R + H.c. \right) = \frac{1}{2} \bar{\psi}_{M_1} \left(i\not{\partial} - m_R \right) \psi_{M_1}$$

$$\bar{\psi}_L\, i\not{\partial}\, \psi_L - \frac{m_L}{2} \left(\overline{\psi_L^c}\, \psi_L + H.c. \right) = \frac{1}{2} \bar{\psi}_{M_2} \left(i\not{\partial} - m_L \right) \psi_{M_2}$$

انواع ممکن برای جرم های نوترینو

ذرات باردار نمی‌توانند جرم مایورانا داشته باشند.

جستاری به ابرتقارن

کلی‌ترین حالت

$$\mathcal{L}_m = -\frac{1}{2} m_R\, \overline{\nu_R^c}\, \nu_R - \frac{m_L}{2}\, \overline{\nu_L^c}\, \nu_L - m_D\, \bar{\nu}_R \nu_L + H.c.$$

$$\mathcal{L}_m = -\frac{1}{2} \left(\overline{\nu_L^c} \quad \bar{\nu}_R \right) \overbrace{\begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}}^{\mathcal{M}} \begin{bmatrix} \nu_L \\ \nu_R^c \end{bmatrix} + H.c.$$

* نشان دهید که $\bar{\nu}_R \nu_L = \overline{\nu_L^c}\, \nu_R^c$

	ν_L	ν_R^c
$L=$	1	-1

	ν_L	ν_R^c
$L=$	1	-1

ماتریس مختلط و متقارن M

$$M_\nu = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}$$

$$\begin{cases} m_s = \frac{1}{2}\left\{ (m_L + m_R) + \sqrt{(m_R - m_L)^2 + 4m_D^2} \right. \\ m_a = \frac{1}{2}\left\{ -(m_L + m_R) + \sqrt{(m_R - m_L)^2 + 4m_D^2} \right. \\ \nu_s = \sin\theta_\nu \, \nu_L + \cos\theta_\nu \, \nu_R^c \\ \nu_a = i\left(\cos\theta_\nu \, \nu_L - \sin\theta_L \, \nu_R^c\right) \end{cases}$$

حذف i

$$tg\, 2\theta_\nu = \frac{2m_D}{m_R - m_L}$$

$m_R = m_L$ $\leftarrow$ pure Dirac $m_a = m_s$

$$\mathcal{L}_m = -\frac{1}{2} m_s \overline{\nu_s^c} \, \nu_s - \frac{1}{2} m_a \overline{\nu_a^c} \, \nu_a + H.c.$$

$$N_s = \nu_s + \nu_s^c \qquad N_a = \nu_a + \nu_a^c$$

$$\mathcal{L}_\nu = \frac{1}{2}\left\{ \overline{N}_s (i\partial\!\!\!/ - m_s) N_s + \overline{N}_a (i\partial\!\!\!/ - m_a) N_a \right\}$$

تناظر با میدان اسکالر

مختلط

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$$

$$\mu^2 |\varphi|^2 + \frac{1}{2}\left(m^2 \varphi^2 + m^{*2} \varphi^{*2}\right)$$

$$\mathcal{L}_m = -\frac{1}{2}\begin{pmatrix}\varphi_1 & \varphi_2\end{pmatrix}\begin{bmatrix} m_1^2 & m_{12}^2 \\ m_{12}^2 & m_2^2 \end{bmatrix}\begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix}$$

$m^2 = 0 \qquad m_1^2 = m_2^2 \quad , \; m_{12}^2 = 0$

$m_L = m_R = 0 \qquad m_s = m_a$

$$\nu_D = \frac{N_s - i N_a}{\sqrt{2}} = \nu_R + \nu_L$$

$$\mathcal{L}_\nu = \bar{\nu}_D (i\not{\partial} - m_D)\nu_D$$

$$\langle 0 | \psi_m \bar{\psi}_m | 0 \rangle = \frac{i}{\not{p} - m} \qquad \leftarrow (6.40)$$

(6.41)

$$\langle 0 | \psi_m \psi_m^{Tr} | 0 \rangle = \frac{i}{\not{p} - m} C^{-1} \qquad C = i\gamma^0\gamma^2$$

* تمرین تا شنبه 6.40 را حل کنید.

معادله حرکت (اویلر لاگرانژ) را از لاگرانژی زیر به دست آورید.

$$\bar{\psi}\, i\not{\partial}\, \psi \qquad \left(M\, \overline{\psi^c}\, \psi + H.c.\right)$$

* تمرین تا سه شنبه 6.40 را حل کنید.

معادله حرکت (اویلر لاگرانژ) را از لاگرانژی زیر بدست آورید.

$$\bar{\psi}\, i\not{\partial}\, \psi - \left(\frac{M}{2}\,\overline{\psi^c}\,\psi + H.c.\right)$$

مکانیزم های تولید جرم نوترینو

Majorana ?? $0\nu\beta\beta$

Dirac

دیراک خالص

N_a N_s $m_a = m_s = m_D$

$Y\, \bar{\nu}_R\, H^\dagger i\sigma_2 L \longrightarrow$ جرم دیراک خالص

شبه دیراک

pseudo-Dirac

Wolfenstein, 1981

$m_R, m_L \ll m_D$ $\theta_\nu \simeq \frac{\pi}{4}$

oscillation

$\nu_L \to \nu_R^c$

(Kobayash Lim, 2001)

$$m_s^2 - m_a^2 \simeq 2 m_D (m_R + m_L)$$

$$P(\nu_L \to \nu_R^c) = \sin^2\left(\frac{m_s^2 - m_a^2}{2E} t\right)$$

زمان

See saw

(Yanagida, 1979 ; Gell-Mann-Ramond-Slansky, 1979)

Peter Minkowski

$$m_R \ \bar{\nu}_R^c \ \nu_R$$

$$\underbrace{\cancel{m \ \nu_L \ \nu_L}}_{I_3 = 1}$$

$$\bar{\nu}_L^c \ H_T \ \nu_L$$

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$$

$$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}$$

ویژه‌حالات

$$m_a \simeq -\frac{m_D^2}{m_R} \ll m_D$$

$$m_s = m_R \qquad \theta_\nu \simeq \frac{m_D}{m_R} \ll 1$$

$$N_s \simeq \nu_R + \nu_R^c \qquad N_a \simeq i(\nu_L - \nu_L^c)$$

N decouples

N_S decouples

$$m_R \bar{\nu}_R^c \nu_R + Y \bar{\nu}_R \phi \epsilon L$$

$$m_0 = Y \langle \varphi^0 \rangle \qquad Y^2 \varphi \epsilon L \ \frac{1}{M_R} \ \varphi \epsilon L$$

آنچه که در کتاب آمده است.

charge conjugation

$$\sum_{a=1}^{3} \frac{C}{M} \varphi^{tr} \epsilon \sigma_a \varphi (\nu_L^{tr} \ e_L^{tr}) \epsilon \sigma^a C \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

آیا عبارت فوق تحت $U(1)_Y$ ناوردا است؟

〃 〃 〃 〃 $SU(2)$ 〃 〃 ؟

آیا پس از شکست تقارن الکتروضعیف عبارت فوق به $U(1)_{em}$

احترام می‌گذارد؟ آیا به ν_L جرم می‌دهد؟

$$\sigma^\mu = (1, \vec{\sigma})$$

راهنمای

$$(\sigma^\mu)_{\alpha\beta} (\sigma_\mu)_{\gamma\delta} = 2 \varepsilon_{\alpha\gamma} \varepsilon_{\beta\delta}$$ ← لم

$$(\sigma^a)_{\alpha\beta} (\sigma_a)_{\gamma\delta} = ?$$

Lepton sector

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \text{ و } \nu_{eR} \ \nu_{\mu R} \ \nu_{\tau R} \ e_R \ \mu_R \ \tau_R$$

$m_D \to$ diagonal دیراک فرمیون

$\bar{\nu}_{eR} \; \nu_{eL} \; \ldots \; \leftarrow L_e$
ثابت

$\nu_e \to \nu_\mu$

All three neutrino masses are degenerate
$\Downarrow$
$U(3)$ symmetry

$m_\nu \leftarrow$ nondegerate $\longleftrightarrow$ GIM mechanism
mixing

Flavor Mixing

Dirac scenario

$$\mathcal{L}_m = (m_D)_{\alpha\beta} \, \bar{\nu}_{\alpha L} \, \nu_{\beta R} + \text{H.c.}$$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = U^\dagger m_D V$$

$$\begin{bmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{bmatrix} = U \begin{bmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{bmatrix}$$

$\lceil \nu_{1L} \; \backslash$

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left(\bar{e}_L \quad \bar{\mu}_L \quad \bar{\tau}_L \right) U \gamma_\mu \begin{pmatrix} \nu_{2L} \\ \nu_{3L} \end{pmatrix} W^{-}$$

$U \rightarrow$ Maki - Nakagawa - Sakat, 1962

$$(\nu_L)_\alpha \longrightarrow (\nu_L)_\beta$$

$$m_D m_D^{\dagger} = U \, diag(m_1^2, m_2^2, m_3^2)\, U^{\dagger}$$

Seesaw

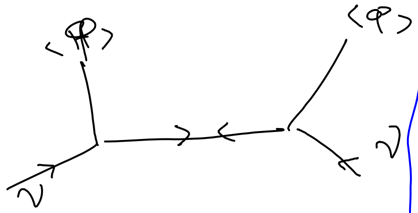
$$\Psi = \left(\nu_{eL} \quad \nu_{\mu L} \quad \nu_{\tau L} \quad (\nu_{eR})^c \quad (\nu_{\mu R})^c \quad (\nu_{\tau R})^c \right)$$

$$m_\nu = \begin{pmatrix} m_L & m_D^{Tr} \\ m_D & m_R \end{pmatrix}$$

$$m_L^{Tr} = m_L \qquad m_R^{Tr} = m_R$$

$$-f^D_{\alpha\beta} \left(\bar{\nu}_{\alpha L} \quad \bar{l}_{\alpha L} \right) \tilde{\varphi}\, \nu_{\beta R} - (m_R)_{\alpha\beta}\, \bar{\nu}^c_{\alpha R}\, \nu_{\beta R}$$

$$(m_D)_{\alpha\beta} = f^{D*}_{\alpha\beta} \times \frac{v}{\sqrt{2}}$$



$$\begin{bmatrix} iI & -i m_D^{tr} m_R^{*-1} \\ m_R^{-1} m_D^{*} & I \end{bmatrix} m_\nu \begin{bmatrix} iI & m_D^{\dagger} m_R^{-1} \\ -i m_R^{-1} m_D & I \end{bmatrix}$$

$$\simeq \begin{bmatrix} m_D^{Tr} (m_R^{*})^{-1} m_D & 0 \\ 0 & m_R^{*} \end{bmatrix}$$

$$m_{\nu L} = m_D^{Tr} \left(m_R^*\right)^{-1} m_D$$

$$U^{Tr} m_{\nu L} U = \mathrm{diag}(m_1, m_2, m_3)$$

$$m_{\nu_L}^{+} m_{\nu L} = U \,\mathrm{diag}(m_1^2, m_2^2, m_3^2)\, U^{+}$$

magnetic dipole moment

$$(\Box + m_i^2)\nu_i = 0 \qquad E_i = \sqrt{\vec{p}^{\,2} + m_i^2} \simeq |\vec{p}|^2 + \frac{m_i^2}{2E}$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = \begin{bmatrix}\frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E_2} & \\ & & \frac{m_3^2}{2E_3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = U\begin{bmatrix}\frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E}\end{bmatrix}U^{+}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

$$\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\\ \nu_\tau(t)\end{bmatrix} = e^{-\frac{i}{2E} m_\nu m_\nu^{+} t}\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{bmatrix}$$

$$= U\begin{pmatrix} e^{-\frac{i m_1^2 t}{2E}} & 0 & 0\\ 0 & e^{-\frac{i m_2^2 t}{2E}} & 0\\ 0 & 0 & e^{-\frac{i m_3^2 t}{2E}}\end{pmatrix}U^{+}\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{bmatrix}$$

$i m_i^2$

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U_{\beta i}\, e^{\frac{-i m_i^2}{2E} t}\, U^*_{\alpha i} \right|^2$$

$$= \left| \sum_i U_{\beta i}\, e^{\frac{-i \Delta m_{i1}^2}{2E} t}\, U^*_{\alpha i} \right|^2$$

$$\Delta m_{i1}^2 = m_i^2 - m_1^2$$

$\therefore$ Dirac mass ten

$$\mathcal{L}_c = \frac{g}{\sqrt{2}}\, \bar{\ell}_{iL}\, \gamma_\mu\, \nu_{jL}\, U^{PMNS}_{ij}\, W^{-\mu}$$

non Unitary

چند فاز

ج: $\frac{(n-1)(n-2)}{2}$

$\bar{\nu}^c\, m_\nu\, \nu$

$\hookrightarrow$ non symmetric

$$\frac{n(n-1)}{2}$$

$$\begin{cases} \frac{(n-1)(n-2)}{2} \\ n-1 \quad \leftarrow \text{ماجورانا} \end{cases}$$

$$m_\nu = U_{PMNS}\, \underbrace{m_\nu^{diag}}\, U^T_{PMNS}$$

$$\text{dia}\,(m_1 e^{i\alpha_1},\ m_2\, e^{i\alpha_2}\ \ m_3 e^{i\alpha_3})$$

مثال

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m_{21}^2}{4E} t\right)$$

$$P(\nu_e \longrightarrow \nu_e) = 1 - \sin^2 2\theta \, \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

CPT

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \longrightarrow \bar{\nu}_\alpha)$$

CP

$$P(\nu_\alpha \longrightarrow \nu_\beta) = P(\bar{\nu}_\alpha \longrightarrow \bar{\nu}_\beta)$$

T

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

در دو نوترینو $\quad P(\nu_\mu \to \nu_\mu) = P(\nu_e \longrightarrow \nu_e)$

Averaging

$$\bar{P}(\nu_e \to \nu_\mu) = \frac{1}{2}\sin^2 2\theta$$

$$\bar{P}(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta$$

$$\bar{P}(\nu_e \to \nu_e) \geq \frac{1}{2}$$

$$\bar{P}(\nu_e \to \nu_e) \geq \frac{1}{n}$$ نشان دهیم

$$\bar{P}(\nu_\alpha \to \nu_\beta) = \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2$$

$$\bar{P}(\nu_\alpha \to \nu_\alpha) = \sum_i |U_{\alpha i}|^4$$

$$\sum_i |U_{\alpha i}|^2 = 1$$

$$\sum_i |U_{\alpha i}|^4 + 2\sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 = 1$$

$$\sum_{ij} \left(|U_{\alpha i}|^2 - |U_{\alpha j}|^2\right)^2 \geqslant 0$$

$$(n-1)\left(\sum_i |U_{\alpha i}|^4\right) - 2\sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \geqslant 0$$

$$\sum_i |U_{\alpha i}|^4 \geq \frac{1}{n}$$

$$\sum_i |U_{\alpha i}|^4 \geq \frac{1}{n}$$