۲۰۱۰/۱۱/۲۰
۰۱:۰۳ ب.ظ

فیزیک نوترینو

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix} \qquad \sigma^\mu = (1, \vec{\sigma}) \qquad \bar{\sigma}^\mu = (1, -\vec{\sigma})$$

فرق دارد notation Morii et al

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{در نوتاسیون کتاب } (-1)}$$

مستقل از نوتاسیون

$m = 0 \rightarrow$ هلیسیتی

$$\psi_R \equiv \frac{1+\gamma_5}{2}\psi \qquad \psi_L = \frac{1-\gamma_5}{2}\psi$$

مستقل از نوتاسیون

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] = \frac{i}{2}\begin{bmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{bmatrix}$$

$$\sigma^{\mu\nu} = \sigma^\mu \bar{\sigma}^\nu \qquad \bar{\sigma}^{\mu\nu} = \bar{\sigma}^\mu \sigma^\nu \qquad \mu \neq \nu$$

$$\psi_R = \begin{pmatrix} 0 \\ \eta^{\dot{\alpha}} \end{pmatrix} \qquad \psi_L = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix} \qquad \alpha, \dot{\alpha} = 1, 2$$

ماتریس های ۲×۲ مختلط

$SL(2,C)$

$SU(2,C)$ ← ؟؟؟ مولدهای

کتاب!

$$[\gamma_5, \Sigma^{\mu\nu}] = 0$$

We easily know that each of chiral fermion forms an irreducible representation of Lorentz transformation $SL(2,c)$, as $[\gamma_5, \Sigma^{\mu\nu}] = 0$

$$m(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)$$

$\psi \quad \psi_L, \psi \quad (\xi_\alpha$

$$\psi_D = \psi_R + \psi_L = \begin{pmatrix} \xi_\alpha \\ \eta_\alpha \end{pmatrix}$$

$$\psi_R = R\psi_D \qquad \psi_L = L\psi_D \qquad R = \frac{1+\gamma_5}{2} \quad L = \frac{1-\gamma_5}{2}$$

$$\psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\bar\psi_D (i\not\partial - m) = \bar\psi_R i\not\partial \psi_R + \bar\psi_L i\not\partial \psi_L - m(\bar\psi_R \psi_L + \bar\psi_L \psi_R)$$

$$(\psi_R)^c = -i\gamma^2 \left(\frac{1+\gamma_5}{2}\psi\right)^*$$

$$R(\psi_R^c) = \frac{1+\gamma_5}{2}\psi_R^c = 0$$

$$L\psi_R^c = \psi_R^c$$

$$\psi_R = \begin{pmatrix} 0 \\ \eta_\alpha \end{pmatrix} \qquad \psi_R^c = \begin{bmatrix} -i\sigma_2 \eta^* \\ 0 \end{bmatrix} = \begin{bmatrix} -\eta_2^* \\ +\eta_1^* \\ 0 \\ 0 \end{bmatrix}$$

$$\bar\eta^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\eta_{\dot\beta} \qquad \bar\eta_{\dot{j}} = (\eta_\alpha)^*$$ نمادگذاری نقطه

$$\psi_L = \begin{bmatrix} \xi^\alpha \\ 0 \end{bmatrix} \qquad \psi_L^c = \begin{bmatrix} 0 \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \xi_2^* \\ -\xi_1^* \end{bmatrix}$$

$$\psi_{M_1} = \psi_R + \psi_R^c = \begin{bmatrix} -i\sigma_2 \eta \\ \eta_\alpha \end{bmatrix}$$

$$\psi_{M_2} = \psi_L + \psi_L^c = \begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix}$$

$$\psi_{M_1}^c = \psi_{M_1} \qquad \psi_{M_2}^c = \psi_{M_2}$$

اسپینور های مایورانا

$[\ \xi \quad |$ $\quad [-i\sigma_2 \eta^* \,]$

$$\begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} -i\sigma_2 \eta^* \\ \eta \end{bmatrix}$$

$$\bar{\psi}_D i\not{\partial} \psi_D = \bar{\psi}_R i\not{\partial} \psi_R + \bar{\psi}_L i\not{\partial} \psi_L =$$

$$\frac{1}{2}\left(\bar{\psi}_{M_1} i\not{\partial} \psi_{M_1} + \bar{\psi}_{M_2} i\not{\partial} \psi_{M_2} \right)$$

جزو تمرینات هست.

تعداد درجات آزادی

جرم صفر ← انشارگرها

جرم دیراک - جرم مایورانا

جرم دیراک :

$$-m_D \bar{\psi}_D \psi_D = -m_D (\bar{\psi}_L \psi_R + H.c.) = m_D(\xi^\dagger \eta_\alpha + H.c.)$$

جرم مایورانا :

$$-\frac{1}{2} m_R \left(\overline{\psi_R^c} \psi_R + H.c. \right) = \frac{m_R}{2} \left(\eta^\dagger \eta_\alpha + H.c. \right)$$

$$-\frac{m_L}{2} \left(\overline{\psi_L^c} \psi_L + H.c. \right) = \frac{m_L}{2} \left(\xi^\dagger \xi_\alpha + H.c. \right)$$

$$\Downarrow$$

$$\bar{\psi}_R i\not{\partial} \psi_R - \frac{m_R}{2} \left(\overline{\psi_R^c} \psi_R + H.c. \right) = \frac{1}{2} \bar{\psi}_{M_1} (i\not{\partial} - m_R) \psi_{M_1}$$

$$\bar{\psi}_L i\not{\partial} \psi_L - \frac{m_L}{2} \left(\overline{\psi_L^c} \psi_L + H.c. \right) = \frac{1}{2} \bar{\psi}_{M_2} (i\not{\partial} - m_L) \psi_{M_2}$$

انواع ممکن برای جرم های نوترینو

ذرات باردار نمی‌توانند جرم مایورانا داشته باشند.

جستاری به ابرتقارن

کلی‌ترین حالت

$$\mathcal{L}_m = -\frac{1}{2} m_R \, \bar{\nu}^c_R \nu_R - \frac{m_L}{2} \bar{\nu}^c_L \nu_L - m_D \bar{\nu}_R \nu_L + H.c.$$

$$\mathcal{L}_m = -\frac{1}{2} \left(\bar{\nu}^c_L \quad \bar{\nu}_R \right) \overbrace{\begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}}^{M} \begin{bmatrix} \nu_L \\ \nu^c_R \end{bmatrix} + H.c.$$

$\bar{\nu}_R \nu_L = \bar{\nu}^c_L \nu^c_R$ نشان دهید که *

	ν_L	ν^c_R
$L=$	1	-1

M ماتریس مختلط و متقارن

$$M_\nu = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}$$

$$\begin{cases} m_s = \frac{1}{2} \left\{ (m_L + m_R) + \sqrt{(m_R - m_L)^2 + 4 m_D^2} \right. \\ m_a = \frac{1}{2} \left\{ -(m_L + m_R) + \sqrt{(m_R - m_L)^2 + 4 m_D^2} \right. \\ \nu_s = \sin\theta_\nu \, \nu_L + \cos\theta_\nu \, \nu^c_R \\ \nu_a = \underline{\underline{i}} \left(\cos\theta_\nu \, \nu_L - \sin\theta_L \, \nu^c_R \right) \end{cases}$$

i حلت

$$tg\, 2\theta_\nu = \frac{2 m_D}{m_R - m_L}$$

$m_R = m_L \quad \leftarrow$ pure Dirac $\quad m_a = m_s$

$$\mathcal{L}_m = -\frac{1}{2} m_s \overline{\nu_s^c} \nu_s - \frac{1}{2} m_a \overline{\nu_a^c} \nu_a + H.c.$$

$$N_s = \nu_s + \nu_s^c \qquad N_a = \nu_a + \nu_a^c$$

$$\mathcal{L}_\nu = \frac{1}{2} \left\{ \overline{N}_s (i\cancel{\partial} - m_s) N_s + \overline{N}_a (i\cancel{\partial} - m_a) N_a \right\}$$

تناظر با میدان اسکالر

مختلط

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$$

$$M^2 |\varphi|^2 + \frac{1}{2} (m^2 \varphi^2 + m^{*2} \varphi^{*2})$$

$$\mathcal{L}_m = -\frac{1}{2} (\varphi_1 \;\; \varphi_2) \begin{bmatrix} m_1^2 & m_{12}^2 \\ m_{12}^2 & m_2^2 \end{bmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$$m^2 = 0 \qquad m_1^2 = m_2^2 \;, \; m_{12}^2 = 0$$

$$m_L = m_R = 0 \qquad m_s = m_a$$

$$\nu_D = \frac{N_s - i N_a}{\sqrt{2}} = \nu_R + \nu_L$$

$$\mathcal{L}_\nu = \bar{\nu}_D (i\cancel{\partial} - m_D) \nu_D$$

$$\langle 0 | \psi_m \bar{\psi}_m | 0 \rangle = \frac{i}{\cancel{p} - m} \quad \text{(6.40)}$$

$$\langle 0 | \psi_m \psi_m^{Tr} | 0 \rangle = \frac{i}{\cancel{p} - m} C^{-1} \qquad C = i\gamma^0\gamma^2 \quad \text{(6.41)}$$

* تمرین تا شنبه 6.40 را حل کنید.

* تمرین تا سه‌شنبه 6.40 را حل کنید.
معادله حرکت (اویلر لاگرانژ) را از لاگرانژی زیر بدست آورید.

$$\bar{\psi}\, i\partial\!\!\!/\, \psi - \left(\frac{M}{2}\,\overline{\psi^c}\,\psi + H.c.\right)$$

مکانیزم های تولید جرم نوترینو

Majorana ?? $0\nu\beta\beta$
Dirac

دیراک خالص

$N_a \quad N_s \qquad m_a = m_s = m_D$

$Y\, \bar{\nu}_R\, H^{\dagger}_{1/2} L \longrightarrow$ جرم دیراک خالص

شبه دیراک

pseudo – Dirac

Wolfenstein, 1981

$m_R, m_L \ll m_D \qquad \theta_\nu \simeq \frac{\pi}{4}$

oscillation

$\nu_L \rightarrow \nu_R^c$

(Kobayash Lim, 2001)

$$m_s^2 - m_a^2 \simeq 2 m_D (m_R + m_L)$$

$$P(\nu_L \to \nu_R^c) = \sin^2\left(\frac{m_s^2 - m_a^2}{2E}\, t\right)$$

زمان

See saw

(Yanagida, 1979 ; Gell-Mann-Ramond-Slansky, 1979)

Peter Minkowski

$$m_R\, \bar{\nu}_R^c\, \nu_R \qquad \underbrace{\cancel{m\, \bar{\nu}_L\, \nu_L}}_{I_3 = 1}$$

$$\bar{\nu}_L^c\, H_T\, \nu_L$$

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$$

$$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}$$

ویژه‌حالات

$$m_a \simeq - \frac{m_D^2}{m_R} \ll m_D$$

$$m_s = m_R \qquad \theta_\nu \simeq \frac{m_D}{m_R} \ll 1$$

$$N_s \simeq \nu_R + \nu_R^c \qquad N_a \simeq i(\nu_L - \nu_L^c)$$

N_s decouples

$$m_R\, \bar{\nu}_R^c\, \nu_R + Y\, \bar{\nu}_R\, \phi\, \epsilon\, L$$

$$m_D = Y\langle \varphi^0 \rangle \qquad Y^2\, \varphi\, \epsilon\, L\, \frac{1}{M_R}\, \varphi\, \epsilon\, L$$

آنچه که در کتاب آمده است.

charge conjugation

$$\sum_{a=1}^{3} \frac{c}{M}\, \varphi^{tr}\, \epsilon\, \sigma_a\, \varphi \,(\nu_L^{tr} \quad e_L^{tr})\, \epsilon\, \sigma^a\, C \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

* آیا عبارت فوق تحت $U(1)_Y$ ناورداست؟

~ ~ ~ ~ $SU(2)$ ~ ~ ؟

آیا پس از شکست تقارن الکتروضعیف عبارت فوق به $U(1)_{em}$

احترام می‌گذارد؟ آیا به ν_L جرم می‌دهد؟

$$\sigma^\mu = (1, \vec{\sigma})$$

راهنمای

$$(\sigma^\mu)_{\alpha\beta}(\sigma_\mu)_{\gamma\delta} = 2\,\varepsilon_{\alpha\gamma}\,\varepsilon_{\beta\delta} \leftarrow \text{لم}$$

$$(\sigma^a)_{\alpha\beta}\,(\sigma_a)_{\gamma\delta} = ?$$

Lepton sector

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} ;\ \nu_{eR}\ \nu_{\mu R}\ \nu_{\tau R}\ e_R\ \mu_R\ \tau_R$$

$m_D \to$ diagonal دیراک فرمیون

$\bar{\nu}_{eR}\ \nu_{eL}$ .... $\leftarrow L_e$ ثابت

$\nu_e \to \nu_\mu$

All three neutrino masses are degenerate

$\Downarrow$

$U(3)$ symmetry

m non degenerate → C ...

$m_\nu \leftarrow$ nondegerate $\longleftrightarrow$ GIM mechanism

mixing

Flavor Mixing

Dirac scenario

$$\mathcal{L}_m = (m_D)_{\alpha\beta}\ \overline{\nu}_{\alpha L}\ \nu_{\beta R} + \text{H.c.}$$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = U^\dagger m_D V$$

$$\begin{bmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{bmatrix} = U \begin{bmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{bmatrix}$$

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left(\overline{e}_L \quad \overline{\mu}_L \quad \overline{\tau}_L \right) U \gamma_\mu \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} W^{-\mu}$$

$U \rightarrow$ Maki - Nakagawa - Sakat, 1962

$$(\nu_L)_\alpha \longrightarrow (\nu_L)_\beta$$

$$m_D m_D^\dagger = U\, \text{diag}(m_1^2, m_2^2, m_3^2)\, U^\dagger$$

Seesaw

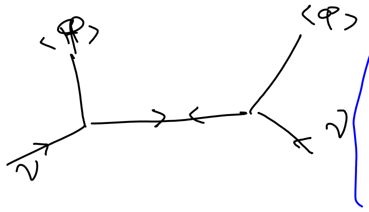
$$\Psi = \left(\nu_{eL}\ \ \nu_{\mu L}\ \ \nu_{\tau L}\ \ (\nu_{eR})^c\ \ (\nu_{\mu R})^c\ \ (\nu_{\tau R})^c\right)$$

$$m_\nu = \begin{pmatrix} m_L & m_D^{Tr} \\ m_D & m_R \end{pmatrix}$$

$$m_L^{Tr} = m_L \qquad m_R^{Tr} = m_R$$

$$- f^D_{\alpha\beta} \left(\overline{\nu}_{\alpha L} \quad \overline{l}_{\alpha L} \right) \tilde{\varphi}\, \nu_{\beta R} - (m_R)_{\alpha\beta}\, \overline{\nu^c_{\alpha R}}\, \nu_{\beta R}$$

$$(m_D)_{\alpha\beta} = f^{D*}_{\alpha\beta} \times \frac{v}{\sqrt{2}}$$



$$\begin{bmatrix} iI & -i\, m_D^{tr}\, m_R^{*-1} \\ m_R^{-1} m_D^* & I \end{bmatrix} m_\nu \begin{bmatrix} iI & m_D^\dagger\, m_R^{-1} \\ -i\, m_R^{-1} m_D & I \end{bmatrix}$$

$$= \begin{bmatrix} m_D^{Tr}\,(m_R^*)^{-1}\, m_D & 0 \\ 0 & m_R^* \end{bmatrix}$$

$$m_{\nu L} = m_D^{Tr}\,(m_R^*)^{-1}\, m_D$$

$$U^{Tr}\, m_{\nu L}\, U = \mathrm{diag}(m_1, m_2, m_3)$$

$$m_{\nu_L}^\dagger m_{\nu_L} = U\, \mathrm{diag}(m_1^2, m_2^2, m_3^2)\, U^\dagger$$

magnetic dipole moment

$$(\Box + m_i^2)\,\nu_i = 0 \qquad E_i = \sqrt{\vec{p}^{\,2} + m_i^2} \simeq |\vec{p}|^2 + \frac{m_i^2}{2E}$$

$$i\frac{d}{dt}\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E_2} & \\ & & \frac{m_3^2}{2E_3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$i\frac{d}{dt}\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E} \end{bmatrix} U^\dagger \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

$$\begin{bmatrix} \nu_e(t) \\ \nu_\mu(t) \end{bmatrix} \quad e^{-\frac{i}{2E} m_\nu m_\nu^\dagger t} \begin{bmatrix} \nu_e(0) \\ \nu_\mu(0) \end{bmatrix}$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = \begin{bmatrix}\frac{i}{2E} & & \\ & \frac{m_2^2}{2E_2} & \\ & & \frac{m_3^2}{2E_3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = U\begin{bmatrix}\frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E}\end{bmatrix}U^\dagger\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

$$\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\\ \nu_\tau(t)\end{bmatrix} = e^{-\frac{i}{2E} m_\nu m_\nu^\dagger t}\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{bmatrix}$$

$$= U\begin{pmatrix}e^{-\frac{im_1^2 t}{2E}} & 0 & 0\\ 0 & e^{-\frac{im_2^2 t}{2E}} & 0\\ 0 & 0 & e^{-\frac{im_3^2 t}{2E}}\end{pmatrix}U^\dagger\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{bmatrix}$$

$$P(\nu_\alpha \to \nu_\beta) = \left|\sum_i U_{\beta i}\, e^{-\frac{im_i^2}{2E}t}\, U^*_{\alpha i}\right|^2$$

$$= \left|\sum_i U_{\beta i}\, e^{-\frac{i\Delta m_{i1}^2}{2E}t}\, U^*_{\alpha i}\right|^2$$

$$\Delta m_{i1}^2 = m_i^2 - m_1^2$$

∵ Dirac mass ten

$$\mathcal{L}_c = \frac{g}{\sqrt{2}}\,\bar{\ell}_{iL}\,\gamma_\mu\,\nu_{jL}\,U^{PMNS}_{ij}\,W^{-\mu}$$

non Unitary

جند فاز

ج: $\frac{(n-1)(n-2)}{2}$

$\bar{\nu}^c\, m_\nu\, \nu$

↳ non symmetric

$$\frac{n(n-1)}{2}$$

$$\begin{cases} \dfrac{(n-1)(n-2)}{2} \\ n-1 \leftarrow \text{ماورانا} \end{cases}$$

$$m_\nu = U_{PMNS} \underbrace{m_\nu^{diag}}_{} U_{PMNS}^T$$

$$dia\,(m_1 e^{i\alpha_1}, m_2 e^{i\alpha_2} \quad m_3 e^{i\alpha_3})$$

مثال

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2}{4E} t\right)$$

$$P(\nu_e \longrightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

CPT

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

CP

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

T

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

دردو نوترینو $$P(\nu_\mu \to \nu_\mu) = P(\nu_e \to \nu_e)$$

Averaging

$$\bar{P}(\nu_e \to \nu_\mu) = \frac{1}{2} \sin^2 2\theta$$

$$\bar{P}(\nu_e \to \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta$$

$$\bar{P}(\nu_e \to \nu_e) \geq \frac{1}{2}$$

$$\bar{P}(\nu_e \to \nu_e) \geq \frac{1}{n}$$ نشان دهید

$$\bar{P}(\nu_\alpha \to \nu_\beta) = \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2$$

$$\bar{P}(\nu_\alpha \to \nu_\alpha) = \sum_i |U_{\alpha i}|^4$$

$$\sum_i |U_{\alpha i}|^2 = 1$$

$$\sum_i |U_{\alpha i}|^4 + 2 \sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 = 1$$

$$\sum_{ij} (|U_{\alpha i}|^2 - |U_{\alpha j}|^2)^2 \geq 0$$

$$(n-1)\left(\sum_i |U_{\alpha i}|^4\right) - 2 \sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \geq 0$$

$$\sum_i |U_{\alpha i}|^4 \geq \frac{1}{n}$$

معادله‌ی اویلر_لاگرانژ   معادله‌ی حرکت

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - \frac{1}{2}\left(m \bar{\psi}^c \psi + H.c. \right)$$

$$\psi^c = -i\gamma^2 \psi^* \qquad \bar{\psi}^c = -i \psi^T \gamma^2 \gamma^0$$

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + \frac{i}{2}\left(m \psi^T \gamma^2 \gamma^0 \psi + m^* \psi^\dagger \gamma^0 \gamma^2 \psi^* \right)$$

$$\not{\partial} \psi + m^* \gamma^2 \psi^* = 0$$

$$\psi = \int \frac{d^3 p}{(2\pi)^3} \left(e^{ip\cdot x} u\, a + e^{-ip\cdot x} v\, a^{c\dagger} \right)$$

$$a = a^c$$

Boost

$$u_s = \begin{bmatrix} \sqrt{p\cdot\sigma}\, \xi_s \\ \sqrt{p\cdot\bar{\sigma}}\, \eta_s \end{bmatrix} \qquad m^* v_s^* = i\gamma^2 \begin{bmatrix} 0 & p\cdot\sigma \\ p\cdot\bar{\sigma} & 0 \end{bmatrix} u$$

$$m v_s = i\gamma^2 \begin{bmatrix} 0 & p\cdot\sigma^* \\ p\cdot\bar{\sigma}^* & 0 \end{bmatrix} \begin{bmatrix} \sqrt{p\cdot\sigma}^*\, \xi^* \\ \sqrt{p\cdot\bar{\sigma}}^*\, \eta^* \end{bmatrix} = i m \gamma^2 \begin{bmatrix} \sqrt{p\cdot\sigma}^*\, \eta_s^* \\ \sqrt{p\cdot\bar{\sigma}}^*\, \xi_s^* \end{bmatrix}$$

$$v_s = \begin{bmatrix} i\sigma_2 \sqrt{p\cdot\bar{\sigma}}^*\, \xi_s^* \\ -i\sigma_2 \sqrt{p\cdot\sigma}^*\, \eta_s^* \end{bmatrix} = \begin{bmatrix} \sqrt{p\cdot\sigma}\, (i\sigma_2 \xi_s^*) \\ -\sqrt{p\cdot\bar{\sigma}}\, (i\sigma_2 \eta_s^*) \end{bmatrix}$$

$$\xi = \eta \times \alpha$$ اسپین مشخص

فرمیون مایورانا

$$\psi^c = e^{i\beta}\psi \Rightarrow \eta^* = e^{i\beta}\xi^*$$

$$\xi^* = e^{i\beta}\eta^*$$

$$\Rightarrow |\eta^*|^2 = |\xi|^2 \quad , \quad \eta^* = e^{i\beta}\xi$$

$$\Rightarrow |\eta^*|^2 = |\xi|^2 \quad , \quad \eta^* = e^{i\beta}\xi$$

$$\sum_s \xi_s \xi_s^\dagger = 1 \qquad \sum_s \eta_s \eta_s^\dagger = 1 \qquad \sum_s \xi_s \eta_s^\dagger = e^{i\beta}$$

$$\sum_s \eta_s \xi_s^\dagger = e^{-i\beta}$$

$$u\bar{u} = \begin{bmatrix} m e^{i\beta} & p\cdot\sigma \\ p\cdot\bar{\sigma} & m e^{-i\beta} \end{bmatrix} \qquad v\bar{v} = \begin{bmatrix} -m e^{-i\beta} & p\cdot\sigma \\ p\cdot\bar{\sigma} & -m e^{i\beta} \end{bmatrix}$$

$$u v^T = \begin{bmatrix} -im\sigma_2 & e^{i\beta}\, p\cdot\sigma\, i\sigma_2 \\ p\cdot\bar{\sigma}(-i\sigma_2)e^{-i\beta} & i\sigma_2 m \end{bmatrix}$$

$$\langle T(\psi\bar{\psi})\rangle = \frac{\begin{bmatrix} m e^{i\beta} & p\cdot\sigma \\ p\cdot\bar{\sigma} & m e^{-i\beta} \end{bmatrix}}{p^2 - m^2} \qquad \psi = \psi^c \qquad \psi = -\psi^c$$

$$\langle T(\psi\psi^T)\rangle = \frac{\begin{bmatrix} m & p\cdot\sigma\, e^{i\beta} \\ p\cdot\bar{\sigma}\, e^{-i\beta} & m \end{bmatrix}}{p^2 - m^2}\, \underset{\downarrow\; i\gamma\gamma^2}{C}$$

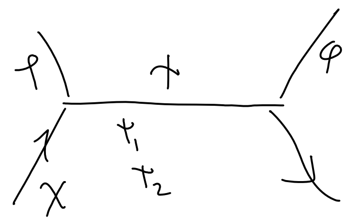
$$\psi = \psi^c \longrightarrow \beta = 0 \qquad\qquad \mathcal{L} = \bar{\psi}\, i\partial\!\!\!/\, \psi - \frac{m\,\overline{\psi^c}\psi + \text{H.c.}}{2}$$

$$\psi = -\psi^c \longrightarrow \beta = \pi$$

$$\psi^c = C\bar{\psi}^T \qquad\qquad \psi^T = \pm\psi^{c\,T} = \pm\bar{\psi}\, C^T$$

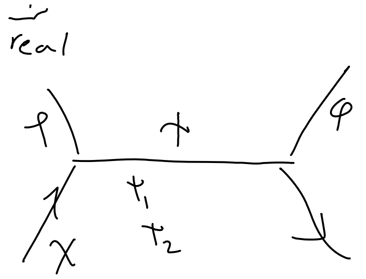
آیا فاز معنای فیزیکی دارد؟ ـ بله !

$$\underbrace{\varphi}_{\text{real}}\, \bar{\chi}\, \psi$$

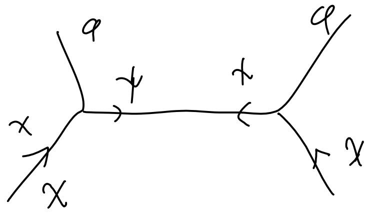


$$\frac{\bar{\chi}\begin{bmatrix} 0 & p\cdot\sigma \\ p\cdot\bar{\sigma} & 0 \end{bmatrix}\chi}{p^2 - m^2}$$

real

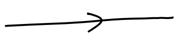


$$\frac{\bar{\chi}\begin{bmatrix} 0 & p\cdot\sigma \\ p\cdot\bar{\sigma} & 0 \end{bmatrix}\chi}{p^2-m^2}$$



$$\frac{\chi^T \; m C \; \chi}{p^2-m^2}$$

می‌توانستیم برحسب چپ و راست بسط دهیم.

 $\frac{i\cancel{p}}{p^2}$

 $\frac{-i\cancel{p}}{p^2} \; imC \; \frac{i\cancel{p}^T}{p^2}$

 $\frac{-i\cancel{p}}{p^2}(imC)\frac{i\cancel{p}^T}{p^2}(-imC)\frac{i\cancel{p}}{p^2}$

$\vdots$

 $\frac{i\cancel{p}}{p^2-m^2} = \frac{i\cancel{p}}{p^2} + \frac{i\cancel{p}}{p^2}\frac{m^2}{p^2} + \cdots$

 $\frac{imC}{p^2-m^2} = \text{-----}$

$\frac{c}{M} \; \varphi^{Tr} \varepsilon \sigma_a \varphi \quad L^T \varepsilon \sigma_a C L$

$\varphi = \begin{bmatrix} \varphi^+ \\ \varphi^0 \end{bmatrix} \qquad L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$

$(\sigma^\mu)_{\alpha\beta} \; (\sigma_\mu)_{\gamma\delta} = 2\, \varepsilon_{\alpha\gamma} \, \varepsilon_{\beta\delta}$

$(\sigma^\mu)\;(\sigma\;) = \delta \;\delta \; - (\sigma^a)\;(\sigma^a)$

$$(\sigma^{\mu})_{\alpha\beta}(\sigma_{\mu})_{\gamma\delta} = \delta_{\alpha\beta}\,\delta_{\gamma\delta} - (\sigma^{a})_{\alpha\beta}(\sigma^{a})_{\gamma\delta}$$

$$\sum_a \chi_1^T \sigma_a \chi_2 \;\; \chi_3^T \sigma_a \chi_4 = \chi_1^T\chi_2\;\chi_3^T\chi_4$$

$$-2\;\chi_1^T \varepsilon \chi_3 \quad \chi_2^T \varepsilon \chi_4$$

$$-4\,\chi_3^T \varepsilon \chi_1$$

$$\chi_1^T = \varphi^T\varepsilon \qquad \chi_2 = \varphi \qquad \chi_3^T = L^T\varepsilon \qquad \chi_4 = CL$$

$$= \frac{C}{M}\Big[\underbrace{\varphi^T\epsilon\varphi}_{\text{singlet}} \;\; \underbrace{L^T\varepsilon CL}_{\text{singlet}} + 2\,\underbrace{L^T\varepsilon\varphi}_{\text{singlet}} \;\; \underbrace{\varphi^T\varepsilon CL}_{\text{singlet}}\Big]$$

unitary gauge $\quad \varphi = (0 \quad \frac{H+v}{\sqrt{2}})$

همان چیزی که من نوشته بودم.

چندتا اصطلاح

$P(\nu_\alpha \to \nu_\beta)$ appearance probability $(\alpha \neq \beta)$

$P(\nu_\alpha \to \nu_\alpha)$ survival probability

$1-P(\nu_\alpha \to \nu_\alpha)$ disappearance probability

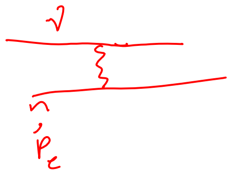
پاراگراف اول بخش ۴-۶

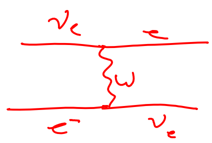
$\overline{P(\nu_e \to \nu_e)} > \frac{1}{n}$ $\qquad n=3 \leftarrow SM$

R. Davis $\qquad |U_{e1}|^2 = |U_{e2}|^2 = |U_{e3}|^2 = \frac{1}{3}$

$V(x)$

از خالده





Only ν_e ???

Only ν_e ??? ν_e ?

Elastic forward scattering

$$\frac{G_F}{\sqrt{2}}\ \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \quad \bar{e}\, \gamma^\mu (1-\gamma_5) \nu_e$$

$$\Big\Downarrow \ \text{Fierz transformation} \quad \text{anti- Commuting}$$

$$\frac{G_F}{\sqrt{2}}\ \bar{\nu}_e \gamma_\mu (1-\gamma_5) \nu_e \quad \bar{e}\, \gamma^\mu (1-\gamma_5) e$$

* تمرین ۱ــ۸ جزوه‌ی نوترینوی س

همان تمرین را برای حالتی که محیط قطبیده است حل کنید.

سه‌شنبه هفته‌ی بعد صبح حل می‌کنیم.

$$\bar{\nu}_L \gamma_0 \nu_L V_c(x)$$

$$V_c(x) = \sqrt{2}\, G_F N_e(x)$$

Dispersion relation — معادله‌ی پاشندگی

$$\mathcal{L} = \bar{\nu}_L i \cancel{\partial} \nu_L - \bar{\nu}_L \gamma_0 \nu_L V_c(x)$$

↓ معادله‌ی اویلر-لاگرانژ

$$\cancel{p}\, \nu_L - V_c \gamma_0 \nu_L = 0$$

$$\left((E - V_c)\gamma^0 - \vec{p}\cdot\vec{\sigma}\right)\nu_L = 0$$

$$\left((E - V_c)\gamma^0 + \vec{p}\cdot\vec{\sigma}\right)\lambda$$

$$\left[(E-V_c)^2 - |\vec{p}|^2\right]\nu_L = 0$$

$E = V_c + |\vec{p}|$ ← معادله‌ی پاشندگی

* نشان دهید که در حضور حلمه جری (دیراک - مایورانا)

معادله ی پاشندگی به صورت زیر تبدیل می شود .

$$E = V_c + \sqrt{\vec{P}^2 + m^2}$$

نکته ی لفظی

Majorana مایورانا

Jarlskog یارلسکوگ

تقریب فرانسبیتی

$$E \simeq V_c + |\vec{P}| + \frac{m^2}{2|\vec{P}|} \, .$$

$$i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \left\{ U \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{bmatrix} \cdot U^{\dagger} + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$a = \sqrt{2}\, G_F N_e$$

فاز کلی

تقلیل به آنالیز دو نوترینو

$$\frac{\Delta m_{31}^2}{2E} \gg \frac{\Delta m_{21}^2}{2E}$$

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U_{PMNS} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$U_{PMNS} =$

$$\begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \end{bmatrix}$$

$$\begin{bmatrix} -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$$\times \operatorname{diag}\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1\right)$$

$$\theta_{13} \ll 1 \qquad s_{23} \simeq c_{23} \simeq \frac{\pi}{4}$$

$$\nu_e \overset{\overset{=}{?}}{\simeq} c_{12}\nu_1 + \sin\theta_{12}\,\nu_2$$

$$\nu' = -\sin\theta_{12}\,\nu_1 + \cos\theta_{12}\,\nu_2$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_e \\ \nu'\end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{2}\,G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta_{12} \\ \frac{\Delta m^2}{4E}\sin 2\theta_{12} & \frac{\Delta m^2}{2E}\cos 2\theta_{12} \end{bmatrix}}_{H(t)} \begin{bmatrix}\nu_e \\ \nu'\end{bmatrix}$$

mild time dependence $\longleftrightarrow$ adiabacity

$$U_m(t)^\dagger H(t)\, U_m(t) = \begin{bmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{bmatrix}$$

$$U_m(t) = \begin{bmatrix} \cos\theta_m(t) & \sin\theta_m(t) \\ -\sin\theta_m(t) & \cos\theta_m(t) \end{bmatrix}$$

$$\begin{bmatrix}\nu_e \\ \nu'\end{bmatrix} = U_m(t)\begin{bmatrix}\nu_{m1} \\ \nu_{m2}\end{bmatrix}$$

$$E_{1,2}(t) = \frac{1}{2}\left(\sqrt{2}\,G_F N_e(t) + \frac{\Delta m^2}{2E}\cos 2\theta \pm \sqrt{\left(\sqrt{2}\,G_F N_e(t) - \frac{\Delta m^2}{2E}\cos 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E}\sin 2\theta\right)^2}\right)$$

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}\,G_F N_e(t)}$$

$$N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E G_F \sqrt{2}} \longrightarrow \theta_m = \frac{\pi}{4}$$

$$i \frac{d}{dt}\begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix} = \left\{ \begin{pmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{pmatrix} + \begin{pmatrix} 0 & i\dot{\theta}_m \\ -i\dot{\theta}_m & 0 \end{pmatrix} \right\} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}$$

adiabatic condition $|\dot{\theta}_m| \ll |E_2 - E_1|$

resonance : لحظه خطرناک

منطقه رزنانس

$$\tan 2\theta_m > 1$$

پهنای منطقه‌ی رزنانس $= \Delta X$

$$\sqrt{2} G_F \frac{dN_x}{dx} \Delta X = \frac{\Delta m^2}{2E} \sin 2\theta$$

$$\boxed{N|_{resonance} = \frac{\Delta m^2 \cos 2\theta}{2E \sqrt{2} G_F}}$$

$$\Delta x = \frac{\tan 2\theta}{\frac{1}{N}\frac{dN}{dx}} = \frac{\tan 2\theta}{\frac{d \ln N}{dx}}$$

شرط آدیاباتیک $\Delta x (E_2 - E_1)\big|_{resonance} \gg 1$

$$\frac{\tan 2\theta}{\frac{d \ln N_e}{dx}\Big|_{res}} \gg \frac{E}{\Delta m^2 \sin 2\theta}$$

resonance = تشدید

$$\nu_{m1}(t) = T e^{-i\int_0^t E_1(t')\,dt'}\ \nu_{m1}(0)$$

$$\nu_{m2}(t) = T e^{-i\int_0^t E_2(t')\,dt'}\ \nu_{m2}(0)$$

$$P(\nu_e \to \nu_e) = \left|\langle \nu_e |\, T \exp\left(-i\int_0^t H(t')\,dt'\right) |\nu_e\rangle\right|^2 =$$

زمنی

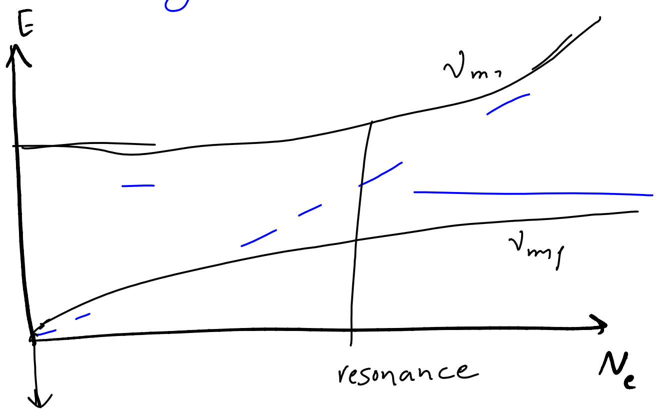
$$\left| \begin{pmatrix} \cos\theta_m(t) & \sin\theta_m(t) \end{pmatrix} \begin{bmatrix} e^{-i\int_0^t E_1(t')\,dt'} & 0 \\ 0 & e^{-i\int_0^t E_2(t')\,dt'} \end{bmatrix} \begin{bmatrix} \cos\theta_m(0) \\ \sin\theta_m(0) \end{bmatrix} \right|^2 =$$

$$\overline{P}(\nu_e \to \nu_e) = \cos^2\theta_m(t)\cos^2\theta_m(0) + \sin^2\theta_m(t)\sin^2\theta_m(0)$$

MSW

Wolfenstein, 1978

Mikheyev & Smirnov, 1985



[illegible]

$$\overline{P}(\nu_e \to \nu_e) = (1 - P_{jump})\left(\cos^2\theta_m(t)\cos^2\theta_m(0) + \sin^2\theta_m(t)\sin^2\theta_m(0)\right) + P_{jump}\left(\sin^2\theta_m(t)\cos^2\theta_m(0)\right)$$

$$+ \cos^2\theta_m(t) \sin^2\theta_m(0) \Big)$$

$\theta_m(t) = \theta \longleftarrow$ خلاء

$\theta_m(0) = \frac{\pi}{2} \longleftarrow$ بسیار غلیظ

$$\bar{P}(\nu_e \to \nu_e) \simeq \sin^2\theta + P_{jump} \cos^2\theta$$

پرش تنها در منطقه تشدیدی شود :

$$N_e = N_e^{res} + \overbrace{\frac{dN_e}{dx} x}^{\text{خطی}}$$

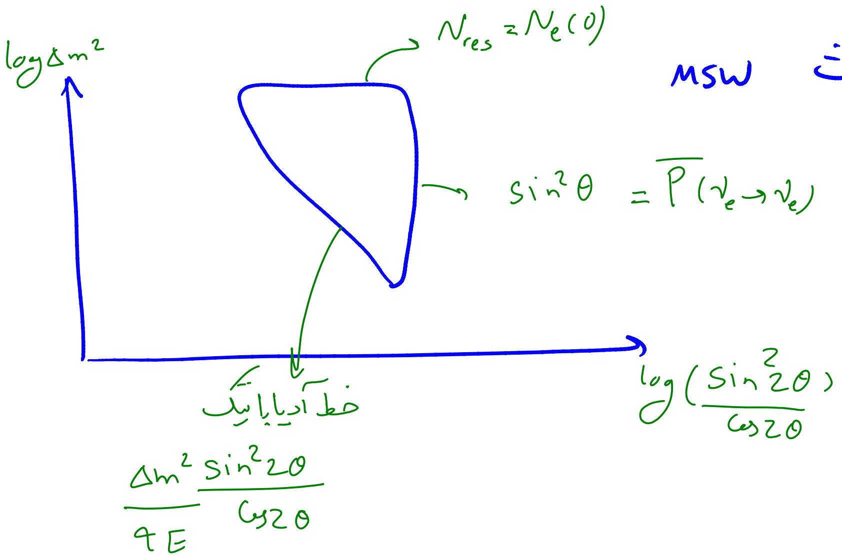
$$\underbrace{P_{jump} = \exp\left(-\frac{\pi}{4} \frac{\Delta m^2 \sin^2 2\theta}{4E \cos 2\theta \left. \frac{d \ln N_e}{dx} \right|_{res}}\right)}$$

Landau - Zener formula

Landau, 1932

Zener, 1932

MSW مثلث



ψ_1 و ψ_2 فرمیون وایل

انتی دنگ

$$\underbrace{\left(\psi_1^T C \psi_2\right)^T} = -\psi_1^T C \psi_2$$

$$= -\psi_2^T C \psi_1$$

$$(m_\nu)_{\alpha\beta} = \underbrace{\overbrace{\frac{(m_\nu)_{\alpha\beta} + (m_\nu)_{\beta\alpha}}{2}}^{\text{symmetric}}}_{(M_\nu)_{\alpha\beta}} + \underbrace{\overbrace{\frac{(m_\nu)_{\alpha\beta} - (m_\nu)_{\beta\alpha}}{2}}^{\text{anti-symmetric}}}_{(A_\nu)_{\alpha\beta}}$$

$$(m_\nu)_{\alpha\beta} \; \nu_{L\alpha}^T C \nu_{L\beta} = (M_\nu)_{\alpha\beta} \; \nu_{L\alpha}^T C \nu_{L\beta}$$

ماتریس جرمی مایورانا متقارن است.

$$m_\nu = U_{PMNS} \begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix} U_{PMNS}^T$$

$$U_{PMNS} = V_{23} V_{13} V_{12} \Phi$$

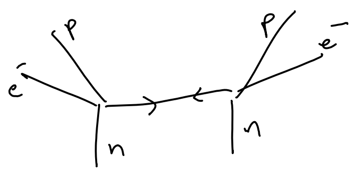
$$V_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \quad V_{13} = \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}$$

$$V_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \Phi = \mathrm{diag}(1, e^{i\alpha_2}, e^{i\alpha_3})$$

۹ پارامتر

Oscillation $\begin{cases} \Delta m_{21}^2, \; \Delta m_{31}^2 \\ \theta_{12}, \theta_{13}, \theta_{23} \end{cases}$

δ



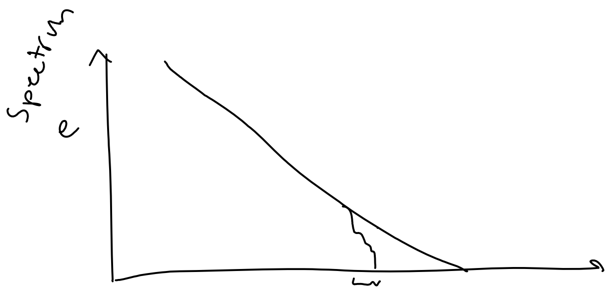
$M_\alpha \qquad (m_\nu)_{ee}$

$$= m_1 e^{i\alpha_1} U_{e1}^2 + m_2 e^{i\alpha_2} U_{e2}^2 + m_3 e^{i\alpha_3} U_{e3}^2$$

$| \; , \; , \; |^2$

$|M|^2$

Kurie plot



$m_{eff} = m_1$

Katrin $\longrightarrow m_1 \longrightarrow$ فاز

$\left\{\begin{array}{l}\text{Solar neutrino} \\ \text{KamLAND}\end{array}\right.$ Δm^2_{12} θ_{12}

atmospheric Δm^2_{31} $\theta_{23} \approx \frac{\pi}{4}$

K2K
MINOS

$\left\{\begin{array}{l}\text{CERN - Gransasso} \\ \\ \text{T2K}\end{array}\right.$ $\nwarrow \nu_\tau$

Nova

CHOOZ $\longrightarrow$ $\left\{\begin{array}{l}\text{Daya Bay} \\ \text{Double - CHOOZ} \\ \text{Reno}\end{array}\right.$ $\longleftarrow \theta_{13} < 10^\circ$

$\theta_{13} = ?$

$\delta = ??$

منابع نوترینو

؟انرژی $\longrightarrow$ $\bar{\nu}_e$ راکتور

؟سد $\longrightarrow$ ν_e خورشیدی

آتمسفری ν_e و ν_μ 〃 ؟

ابرنواختر

relic neutrino

AGN - GRBs

نوترینوی زمینی

آشکارسازها

Super Kamiokande SK

↑

Kamiokande - IMP

SNO ← D_2O

ICECUBE
BAICAL
ANTARES

ANITA

نوترینوهای خورشیدی را چگونه آشکارسازی می‌کنند؟

〃 راکتوری 〃 〃 〃 〃 〃 ؟

$\bar{\nu}_e \longrightarrow e^+$