۲۰۱۰/۱۱/۲۰
۰۱:۰۳ ب.ظ

فیزیک نوترینو

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{bmatrix} \qquad \sigma^\mu = (1, \vec\sigma) \qquad \bar\sigma^\mu = (1, -\vec\sigma)$$

قرارداد notation Morii et al

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

مستقل از نوتاسیون

در نوتاسیون کتاب (۱-)

$m = 0 \rightarrow$ هلیسیتی

$$\psi_R \equiv \frac{1+\gamma_5}{2}\psi \qquad \psi_L = \frac{1-\gamma_5}{2}\psi$$

مستقل از نوتاسیون

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] = \frac{i}{2}\begin{bmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar\sigma^{\mu\nu} \end{bmatrix}$$

$$\sigma^{\mu\nu} = \sigma^\mu\bar\sigma^\nu \qquad \bar\sigma^{\mu\nu} = \bar\sigma^\mu\sigma^\nu \qquad \mu \neq \nu$$

$$\psi_R = \begin{pmatrix} 0 \\ \eta_{\dot\alpha} \end{pmatrix} \qquad \psi_L = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix} \qquad \alpha, \dot\alpha = 1, 2$$

ماتریس های ۲×۲ مختلط

$SL(2,C)$

$SU(2,C) \leftarrow$ ؟! مولدهای

کتاب ؟

$$[\gamma_5, \Sigma^{\mu\nu}] = 0$$

We easily know that each of chiral fermion forms an irreducible representation of Lorentz transformation $SL(2,c)$, as $[\gamma_5, \Sigma^{\mu\nu}] = 0$

$$m(\bar\psi_R\psi_L + \bar\psi_L\psi_R)$$

$$\psi_D = \psi_R + \psi_L = \begin{pmatrix} \xi_\alpha \\ \eta_{\dot\alpha} \end{pmatrix}$$

$$\psi_R = R\psi_D \qquad \psi_L = L\psi_D \qquad R = \frac{1+\gamma_5}{2} \quad L = \frac{1-\gamma_5}{2}$$

$$\psi_R = R\,\psi_D \qquad \psi_L = L\,\psi_D \qquad R = \frac{1+\gamma_5}{2} \quad L = \frac{1-\gamma_5}{2}$$

$$\psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\bar{\psi}_D (i\not\partial - m) = \bar{\psi}_R i\not\partial \psi_R + \bar{\psi}_L i\not\partial \psi_L - m(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)$$

$$(\psi_R)^c = -i\gamma^2 \left(\frac{1+\gamma_5}{2}\psi\right)^*$$

$$R(\psi_R^c) = \frac{1+\gamma_5}{2}\psi_R^c = 0$$

$$L\,\psi_R^c = \psi_R^c$$

$$\psi_R = \begin{pmatrix} 0 \\ \eta_\alpha \end{pmatrix} \qquad \psi_R^c = \begin{bmatrix} -i\sigma_2 \eta^* \\ 0 \end{bmatrix} = \begin{bmatrix} -\eta_2^* \\ +\eta_1^* \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{\eta}^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\,\bar{\eta}_{\dot\beta} \qquad \bar{\eta}_{\dot{j}} = (\eta_\alpha)^*$$ نمادگذاری نقطه

$$\psi_L = \begin{bmatrix} \xi^\alpha \\ 0 \end{bmatrix} \qquad \psi_L^c = \begin{bmatrix} 0 \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \xi_2^* \\ -\xi_1^* \end{bmatrix}$$

$$\psi_{M_1} = \psi_R + \psi_R^c = \begin{bmatrix} -i\sigma_2 \eta \\ \eta_\alpha \end{bmatrix}$$

$$\psi_{M_2} = \psi_L + \psi_L^c = \begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix}$$

$$\psi_{M_1}^c = \psi_{M_1} \qquad \psi_{M_2}^c = \psi_{M_2}$$

اسپینور های مایورانا

$$\begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} -i\sigma_2 \eta^* \\ \eta \end{bmatrix}$$

$$\bar{\psi}_D i\not\partial \psi_D = \bar{\psi}_R i\not\partial \psi_R + \bar{\psi}_L i\not\partial \psi_L =$$

$$\frac{1}{2}\left(\bar\psi_{M_1} i\not\partial \psi_{M_1} + \bar\psi_{M_2} i\not\partial \psi_{M_2}\right)$$

جزو تمرینات هست.

تعداد درجات آزادی

جرم صفر $\rightarrow$ انتشارگرها

جرم دیراک - جرم مایورانا

جرم دیراک :

$$-m_D \bar\psi_D \psi_D = -m_D\left(\bar\psi_L \psi_R + H.c.\right) = m_D\left(\xi^\alpha \eta_\alpha + H.c.\right)$$

جرم مایورانا :

$$-\frac{1}{2} m_R \left(\overline{\psi_R^c}\psi_R + H.c.\right) = \frac{m_R}{2}\left(\eta^\alpha \eta_\alpha + H.c.\right)$$

$$-\frac{m_L}{2}\left(\overline{\psi_L^c}\psi_L + H.c.\right) = \frac{m_L}{2}\left(\xi^\alpha \xi_\alpha + H.c.\right)$$

$$\Downarrow$$

$$\bar\psi_R i\not\partial \psi_R - \frac{m_R}{2}\left(\overline{\psi_R^c}\psi_R + H.c.\right) = \frac{1}{2}\bar\psi_{M_1}\left(i\not\partial - m_R\right)\psi_{M_1}$$

$$\bar\psi_L i\not\partial \psi_L - \frac{m_L}{2}\left(\overline{\psi_L^c}\psi_L + H.c.\right) = \frac{1}{2}\bar\psi_{M_2}\left(i\not\partial - m_L\right)\psi_{M_2}$$

انواع ممکن برای جرم های نوترینو

ذرات باردار نمی‌توانند جرم مایورانا داشته باشند.

جستاری به ارتقادن

کلی‌ترین حالت

$$\mathcal{L}_m = -\frac{1}{2} m_R \overline{\nu_R^c}\nu_R - \frac{m_L}{2}\overline{\nu_L^c}\nu_L - m_D \bar\nu_R \nu_L + H.c.$$

$$\mathcal{L}_m = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \bar\nu_R\end{pmatrix}\overbrace{\begin{bmatrix} m_L & m_D \\ m_D & m_R\end{bmatrix}}^{M}\begin{bmatrix}\nu_L \\ \nu_R^c\end{bmatrix} + H.c.$$

* نشان دهید که $\overline{\nu_R}\,\nu_L = \overline{\nu_L^c}\,\nu_R^c$

	ν_L	ν_R^c
$L=$	1	-1

ماتریس مختلط و متقارن M

$$M_\nu = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}$$

$$\begin{cases} m_s = \frac{1}{2}\left\{(m_L+m_R) + \sqrt{(m_R-m_L)^2+4m_D^2}\right. \\ m_a = \frac{1}{2}\left\{-(m_L+m_R) + \sqrt{(m_R-m_L)^2+4m_D^2}\right. \\ \nu_s = \sin\theta_\nu\,\nu_L + \cos\theta_\nu\,\nu_R^c \\ \nu_a = i\left(\cos\theta_\nu\,\nu_L - \sin\theta_L\,\nu_R^c\right) \end{cases}$$

حذف i

$$tg\,2\theta_\nu = \frac{2m_D}{m_R-m_L}$$

$m_R = m_L$ $\leftarrow$ pure Dirac $m_a = m_s$

$$\mathcal{L}_m = -\frac{1}{2}m_s\,\overline{\nu_s^c}\,\nu_s - \frac{1}{2}m_a\,\overline{\nu_a^c}\,\nu_a + H.c.$$

$$N_s = \nu_s + \nu_s^c \qquad N_a = \nu_a + \nu_a^c$$

$$\mathcal{L}_\nu = \frac{1}{2}\left\{\overline{N_s}(i\not\partial - m_s)N_s + \overline{N_a}(i\not\partial - m_a)N_a\right\}$$

تناظر با میدان اسکالر

مختلط

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$$

$$M^2|\varphi|^2 + \frac{1}{2}(m^2\varphi^2 + m^{*2}\varphi^{*2})$$

$$\mathcal{L}_m = -\frac{1}{2}(\varphi_1 \ \varphi_2)\begin{bmatrix} m_1^2 & m_{12}^2 \\ m_{12}^2 & m_2^2 \end{bmatrix}\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$m^2 = 0 \qquad m_1^2 = m_2^2 \ , \ m_{12}^2 = 0$

$m_L = m_R = 0 \qquad m_s = m_a$

$$\nu_D = \frac{N_s - iN_a}{\sqrt{2}} = \nu_R + \nu_L$$

$$\mathcal{L}_\nu = \bar{\nu}_D (i\not{\partial} - m_D)\nu_D$$

$$\langle 0|\psi_M \bar{\psi}_M|0\rangle = \frac{i}{\not{p} - m} \qquad (6.40)$$

(6.41)

$$\langle 0|\psi_M \psi_M^T|0\rangle = \frac{i}{\not{p} - m} C^{-1} \qquad C = i\gamma^0\gamma^2$$

* تمرین تا شنبه 6.40 را حل کنید.

معادله حرکت (اویلر لاگرانژ) را از لاگرانژی زیر بدست آورید.

$$\bar{\psi} i\not{\partial}\psi - \left(\frac{M}{2}\overline{\psi^c}\psi + H.c.\right)$$

مکانیزم های تولید جرم نوترینو

Majorana ?? $\qquad 0\nu\beta\beta$

Dirac

دیراک خالص

$N_a \qquad N_s \qquad m_a = m_s = m_n$

$N_a \qquad N_s \qquad m_a = m_s = m_b$

$Y \, \bar{\nu}_R \, H^T i\tau_2 L \longrightarrow$ جرم دیراک خالص

شبه دیراک

pseudo – Dirac

Wolfenstein, 1981

$m_R \, , \, m_L \lll m_D \qquad \theta_\nu \simeq \frac{\pi}{4}$

oscillation

$\nu_L \to \nu_R^c$

(Kobayash Lim, 2001)

$m_s^2 - m_a^2 \simeq 2 m_D (m_R + m_L)$

$P(\nu_L \to \nu_R^c) = \sin^2\left(\frac{m_s^2 - m_a^2}{2E} t\right)$

نشان

See saw

(Yanagida, 1979 ; Gell-Mann - Ramond - Slansky, 1979)

Peter Minkowski

$m_R \, \bar{\nu}_R^c \, \nu_R \qquad \underbrace{\cancel{m \, \bar{\nu}_L^c \, \nu_L}}_{I_3 = 1}$

$\bar{\nu}_L^c \, H_T \, \nu_L$

$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$

$$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}$$

ویژه‌حالات

$$m_a \cong (-)\frac{m_D^2}{m_R} \ll m_D$$

$$m_s = m_R \qquad \theta_\nu \cong \frac{m_D}{m_R} \ll 1$$

$$N_s \simeq \nu_R + \nu_R^c \qquad N_a \cong i(\nu_L - \nu_L^c)$$

N_s decouples

$$m_R\, \bar{\nu}_R^c \nu_R + Y \bar{\nu}_R \phi \epsilon L$$

$$m_D = Y\langle \varphi^0 \rangle \qquad Y^2 \varphi \epsilon L \frac{1}{M_R} \varphi \epsilon L$$

آنچه که در کتاب آمده است.

charge conjugation

$$\sum_{a=1}^{3} \frac{c}{M} \varphi^{tr} \epsilon\, \sigma_a \varphi \left(\nu_L^{tr} \quad e_L^{tr}\right) \epsilon\, \sigma^a C \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

* آیا عبارت فوق تحت $U(1)_Y$ ناورداست؟

" " " " $SU(2)$ " " ؟

آیا پس از شکست تقارن الکتروضعیف عبارت فوق به $U(1)_{em}$

احترام می‌گذارد؟ آیا به ν_L جرم می‌دهد؟

$$\sigma^\mu = (1, \vec{\sigma})$$

راهنمایی

$$(\sigma^\mu)_{\alpha\beta} (\sigma_\mu)_{\gamma\delta} = 2\, \varepsilon_{\alpha\gamma}\, \varepsilon_{\beta\delta} \leftarrow \text{لم}$$

$$(\sigma^a)_{\alpha\beta} (\sigma_a)_{\gamma\delta} = ?$$

Lepton sector

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix};\ \nu_{eR}\ \nu_{\mu R}\ \nu_{\tau R}\ e_R\ \mu_R\ \tau_R$$

$m_D \to$ diagonal دیراک فرمیون

$\bar{\nu}_{eR} \; \nu_{eL} \; \ldots \leftarrow \underset{\text{ثابت}}{L_e}$

$\nu_e \to \nu_\mu$

All three neutrino masses are degenerate

$\Downarrow$

$U(3)$ symmetry

$m_\nu \leftarrow$ nondegerate $\longleftrightarrow$ GIM mechanism

mixing

Flavor Mixing

Dirac scenario

$$\mathcal{L}_m = (m_D)_{\alpha\beta} \, \overline{\nu}_{\alpha L} \, \nu_{\beta R} + \text{H.c.}$$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = U^\dagger m_D V$$

$$\begin{bmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{bmatrix} = U \begin{bmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{bmatrix}$$

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left(\bar{e}_L \;\; \bar{\mu}_L \;\; \bar{\tau}_L \right) U \gamma_\mu \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} W^{-\mu}$$

$U \to$ Maki - Nakagawa - Sakata, 1962

$$(\nu_L)_\alpha \longrightarrow (\nu_L)_\beta$$

$$m_D m_D^\dagger = U \,\text{diag}(m_1^2, m_2^2, m_3^2)\, U^\dagger$$

Seesaw

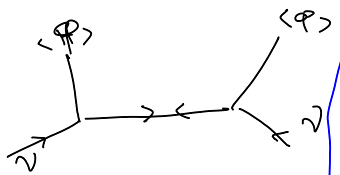
$$\Psi = (\nu_{eL}\ \ \nu_{\mu L}\ \ \nu_{\tau L}\ \ (\nu_{eR})^c\ \ (\nu_{\mu R})^c\ \ (\nu_{\tau R})^c)$$

$$m_\nu = \begin{pmatrix} m_L & m_D^{Tr} \\ m_D & m_R \end{pmatrix}$$

$$m_L^{Tr} = m_L \qquad m_R^{Tr} = m_R$$

$$-f^D_{\alpha\beta}\,(\bar\nu_{\alpha L}\ \ \bar l_{\alpha L})\,\tilde\varphi\,\nu_{\beta R} - (m_R)_{\alpha\beta}\,\overline{\nu^c_{\alpha R}}\,\nu_{\beta R}$$

$$(m_D)_{\alpha\beta} = f^{D*}_{\alpha\beta} \times \frac{v}{\sqrt{2}}$$



$$\begin{bmatrix} iI & -i\,m_D^{tr}\,m_R^{*-1} \\ m_R^{-1} m_D^* & I \end{bmatrix} m_\nu \begin{bmatrix} iI & m_D^\dagger\, m_R^{-1} \\ -i\,m_R^{-1} m_D & I \end{bmatrix}$$

$$\simeq \begin{bmatrix} m_D^{Tr}\,(m_R^*)^{-1}\,m_D & 0 \\ 0 & m_R^* \end{bmatrix}$$

$$m_{\nu L} = m_D^{Tr}\,(m_R^*)^{-1}\,m_D$$

$$U^{Tr}\,m_{\nu L}\,U = diag(m_1, m_2, m_3)$$

$$m_{\nu L}^\dagger m_{\nu L} = U\,diag(m_1^2, m_2^2, m_3^2)\,U^\dagger$$

magnetic dipble moment

$$(\Box + m_i^2)\,\nu_i = 0 \qquad E_i = \sqrt{\vec{p}^{\,2} + m_i^2} \simeq |\vec{p}|^2 + \frac{m_i^2}{2E}$$

$$i\frac{d}{dt}\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E_2} & \\ & & \frac{m_3^2}{2E_3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = \begin{bmatrix}\frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E_2} & \\ & & \frac{m_3^2}{2E_3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = U\begin{bmatrix}\frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E}\end{bmatrix}U^\dagger\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

$$\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\\ \nu_\tau(t)\end{bmatrix} = e^{-\frac{i}{2E} m_\nu m_\nu^\dagger t}\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{bmatrix}$$

$$= U\begin{pmatrix}e^{-\frac{im_1^2 t}{2E}} & 0 & 0\\ 0 & e^{-\frac{im_2^2 t}{2E}} & 0\\ 0 & 0 & e^{-\frac{im_3^2 t}{2E}}\end{pmatrix}U^\dagger\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{bmatrix}$$

$$P(\nu_\alpha \to \nu_\beta) = \left|\sum_i U_{\beta i}\, e^{-\frac{im_i^2}{2E}t}\, U_{\alpha i}^*\right|^2$$

$$= \left|\sum_i U_{\beta i}\, e^{-\frac{i\Delta m_{i1}^2}{2E}t}\, U_{\alpha i}^*\right|^2$$

$$\Delta m_{i1}^2 = m_i^2 - m_1^2$$

∵ Dirac mass ten

$$\mathcal{L}_c = \frac{g}{\sqrt{2}}\,\bar{\ell}_{iL}\,\gamma_\mu\,\nu_{jL}\quad U_{ij}^{PMNS}\,W^{-\mu}$$

non Unitary

جند فاز

$$\text{ج}: \quad \frac{(n-1)(n-2)}{2}$$

$$\bar{\nu}^c\, m_\nu\, \nu$$

↳ non symmetric

$$\frac{n(n-1)}{2}$$

$$\begin{cases} \dfrac{(n-1)(n-2)}{2} \\ n-1 \quad \leftarrow \text{ماجورانا} \end{cases}$$

$$m_\nu = U_{PMNS} \, \underbrace{m_\nu^{diag}}_{dia(m_1 e^{i\alpha_1},\ m_2 e^{i\alpha_2}\ m_3 e^{i\alpha_3})} \, U_{PMNS}^T$$

مثال

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m_{21}^2}{4E} t\right)$$

$$P(\nu_e \longrightarrow \nu_e) = 1 - \sin^2 2\theta \, \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

CPT

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

CP

$$P(\nu_\alpha \longrightarrow \nu_\beta) = P(\bar{\nu}_\alpha \longrightarrow \bar{\nu}_\beta)$$

T

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

در دو نوترینو $\qquad P(\nu_\mu \to \nu_\mu) = P(\nu_e \longrightarrow \nu_e)$

Averaging

$$\bar{P}(\nu_e \to \nu_\mu) = \frac{1}{2} \sin^2 2\theta$$

$$\bar{P}(\nu_e \to \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta$$

$$\bar{P}(\nu_e \to \nu_e) \geq \frac{1}{2}$$

$$\bar{P}(\nu_e \to \nu_e) \geq \frac{1}{n}$$ نکته ی دهم

$$\bar{P}(\nu_\alpha \to \nu_\beta) = \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2$$

$$\bar{P}(\nu_\alpha \to \nu_\alpha) = \sum_i |U_{\alpha i}|^4$$

$$\sum_i |U_{\alpha i}|^2 = 1$$

$$\sum_i |U_{\alpha i}|^4 + 2 \sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 = 1$$

$$\sum_{ij} \left(|U_{\alpha i}|^2 - |U_{\alpha j}|^2 \right)^2 \geq 0$$

$$(n-1)\left(\sum_i |U_{\alpha i}|^4 \right) - 2 \sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \geq 0$$

$$\sum_i |U_{\alpha i}|^4 \geq \frac{1}{n}$$

معادله‌ی اویلر_لاگرانژ   معادله‌ی حرکت

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - \frac{1}{2} \left(m \bar{\psi^c} \psi + H.c. \right)$$

$$\psi^c = -i\gamma^2 \psi^* \qquad \bar{\psi^c} = -i \psi^T \gamma^2 \gamma^0$$

$$\psi^c = -i\gamma^2\psi^* \qquad \overline{\psi^c} = -i\psi^T\gamma^2\gamma^0$$

$$\mathcal{L} = \bar{\psi} i \not{\partial}\psi + \frac{i}{2}\left(m\psi^T\gamma^2\gamma^0\psi + m^*\psi^\dagger\gamma^0\gamma^2\psi^*\right)$$

$$\not{\partial}\psi + m^*\gamma^2\psi^* = 0$$

$$\psi = \int \frac{d^3p}{(2\pi)^3}\left(e^{ip\cdot x} u\, a + e^{-ip\cdot x} v\, a^{c\dagger}\right)$$

$$a = a^c$$

Boost

$$u_s = \begin{bmatrix} \sqrt{p\cdot\sigma}\ \xi_s \\ \sqrt{p\cdot\bar\sigma}\ \eta_s \end{bmatrix} \qquad m^* v_s^* = i\gamma^2 \begin{bmatrix} 0 & p\cdot\sigma \\ p\cdot\bar\sigma & 0 \end{bmatrix} u$$

$$m v_s = i\gamma^2 \begin{bmatrix} 0 & p\cdot\sigma^* \\ p\cdot\bar\sigma^* & 0 \end{bmatrix} \begin{bmatrix} \sqrt{p\cdot\sigma}^*\ \xi^* \\ \sqrt{p\cdot\bar\sigma}^*\ \eta^* \end{bmatrix} = i m\gamma^2 \begin{bmatrix} \sqrt{p\cdot\sigma}^*\ \eta_s^* \\ \sqrt{p\cdot\bar\sigma}^*\ \xi_s^* \end{bmatrix}$$

$$v_s = \begin{bmatrix} i\sigma_2\sqrt{p\cdot\bar\sigma}^*\ \xi_s^* \\ -i\sigma_2\sqrt{p\cdot\sigma}^*\ \eta_s^* \end{bmatrix} = \begin{pmatrix} \sqrt{p\cdot\sigma}\,(i\sigma_2\xi_s^*) \\ -\sqrt{p\cdot\bar\sigma}\,(i\sigma_2\eta_s^*) \end{pmatrix}$$

$$\xi = \eta \times \alpha$$ اسپین شخص

فرمیون مایورانا $\qquad \psi^c = e^{i\beta}\psi \Rightarrow \eta^* = e^{i\beta}\xi^*$

$$\xi^* = e^{i\beta}\eta^*$$

$$\Rightarrow |\eta^*|^2 = |\xi|^2 , \quad \eta^* = e^{i\beta}\xi$$

$$\sum_s \xi_s\xi_s^\dagger = 1 \qquad \sum_s \eta_s\eta_s^\dagger = 1 \qquad \sum_s \xi_s\eta_s^\dagger = e^{i\beta}$$

$$\sum_s \eta_s\xi_s^\dagger = e^{-i\beta}$$

$$u\bar{u} = \begin{bmatrix} m e^{i\beta} & p\cdot\sigma \\ p\cdot\bar\sigma & m e^{-i\beta} \end{bmatrix} \qquad v\bar{v} = \begin{bmatrix} -m e^{-i\beta} & p\cdot\sigma \\ p\cdot\bar\sigma & -m e^{i\beta} \end{bmatrix}$$

$$u v^T = \begin{bmatrix} -im\sigma_2 & e^{i\beta}\, p\cdot\sigma\, i\sigma_2 \\ p\cdot\bar\sigma(-i\sigma_2)e^{-i\beta} & i\sigma_2 m \end{bmatrix}$$

$$u v^T = \begin{bmatrix} -im\sigma_2 & e^{i\beta}\, p\cdot\sigma\, i\sigma_2 \\ p\cdot\bar\sigma(-i\sigma_2)e^{-i\beta} & i\sigma_2 m \end{bmatrix}$$

$$\langle T(\psi\bar\psi)\rangle = \frac{\begin{bmatrix} m e^{i\beta} & p\cdot\sigma \\ p\cdot\bar\sigma & m e^{-i\beta}\end{bmatrix}}{p^2-m^2} \qquad \psi = \psi^c \qquad \psi = -\psi^c$$

$$\langle T(\psi\psi^T)\rangle = \frac{\begin{bmatrix} m & p\cdot\sigma\, e^{i\beta} \\ p\cdot\bar\sigma\, e^{-i\beta} & m\end{bmatrix}}{p^2-m^2}\, \underset{i\gamma^0\gamma^2}{C}$$

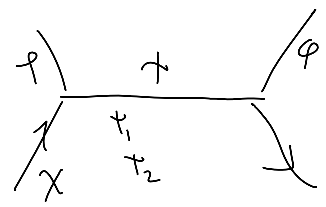
$$\psi = \psi^c \longrightarrow \beta = 0 \qquad \mathcal{L} = \bar\psi i\partial\!\!\!/\psi - \frac{m\,\bar\psi^c\psi + \text{H.c.}}{2}$$

$$\psi = -\psi^c \longrightarrow \beta = \pi$$

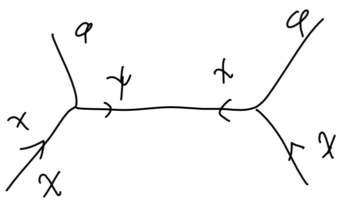
$$\psi^c = C\bar\psi^T \qquad \psi^T = \pm\psi^{c\,T} = \pm\bar\psi\, C^T$$

آیا فاز معنای فیزیکی دارد؟ بله !

$$\underbrace{\varphi}_{\text{real}}\,\bar\chi\,\psi$$



$$\bar\chi\,\frac{\begin{bmatrix} 0 & p\cdot\sigma \\ p\cdot\bar\sigma & 0\end{bmatrix}}{p^2-m^2}\,\chi$$



$$\chi^T\,\frac{m\,C}{p^2-m^2}\,\chi$$

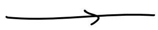
می‌توانستیم برحسب چپ و راست بسط دهیم.

$$\longrightarrow \qquad \frac{i\,p\!\!\!/}{p^2}$$

$$\rightarrow\!\times\!\leftarrow \qquad \frac{-i\,p\!\!\!/}{p^2}\; imC\; \frac{i\,p\!\!\!/^T}{p^2}$$

$$\frac{-i\not p}{p^2}(imC)\frac{i\not p^T}{p^2}(-imC)\frac{i\not p}{p^2}$$

$$\vdots$$



$$\frac{i\not p}{p^2-m^2} = \frac{i\not p}{p^2} + \frac{i\not p}{p^2}\frac{m^2}{p^2} + \dots$$

$$\frac{imC}{p^2-m^2} = \dots$$

$$\frac{c}{M}\,\varphi^{T_r}\varepsilon\sigma_a\varphi \quad L^T\varepsilon\sigma_a CL$$

$$\varphi = \begin{bmatrix}\varphi^+\\ \varphi^0\end{bmatrix} \qquad L = \begin{bmatrix}\nu_L\\ e_L\end{bmatrix}$$

$$(\sigma^\mu)_{\alpha\beta}(\sigma_\mu)_{\gamma\delta} = 2\,\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$$

$$(\sigma^\mu)_{\alpha\beta}(\sigma_\mu)_{\gamma\delta} = \delta_{\alpha\beta}\delta_{\gamma\delta} - (\sigma^a)_{\alpha\beta}(\sigma^a)_{\gamma\delta}$$

$$\sum_a \chi_1^T\sigma_a\chi_2\;\chi_3^T\sigma_a\chi_4 = \chi_1^T\chi_2\;\chi_3^T\chi_4$$

$$-2\,\underbrace{\chi_1^T\varepsilon\chi_3}_{-(4)\,\chi_3^T\varepsilon\chi_1}\;\chi_2^T\varepsilon\chi_4$$

$$\chi_1^T = \varphi^T\varepsilon \qquad \chi_2 = \varphi \qquad \chi_3^T = L^T\varepsilon \qquad \chi_4 = CL$$

$$= \frac{c}{M}\left[\underbrace{\varphi^T\epsilon\varphi}_{\text{singlet}}\;\underbrace{L^T\varepsilon CL}_{\text{singlet}} + 2\,\underbrace{L^T\varepsilon\varphi}_{\text{singlet}}\;\underbrace{\varphi^T\varepsilon CL}_{\text{singlet}}\right]$$

unitary gauge $\varphi = (0 \;\; \frac{H+v}{\sqrt{2}})$

0

همان چیزی که من نوشته بودم.

چندتا اصطلاح

$P(\nu_\alpha \to \nu_\beta)$ appearance probability $(\alpha \neq \beta)$

$P(\nu_\alpha \to \nu_\alpha)$ survival probability

$P(\nu_\alpha \longrightarrow \nu_\alpha)$ survival probability

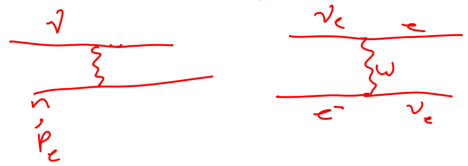
$1 - P(\nu_\alpha \to \nu_\alpha)$ disappearance probability

پاراگراف اول بخش ۴-۶

$\overline{P}(\nu_e \longrightarrow \nu_e) > \frac{1}{n}$ $\quad n=3 \leftarrow SM$

R. Davis $\quad |U_{e1}|^2 = |U_{e2}|^2 = |U_{e3}|^2 = \frac{1}{3}$

$V(x)$



Only ν_e ???

Elastic forward scattering

$$\frac{G_F}{\sqrt{2}}\, \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \;\; \bar{e}\, \gamma^\mu (1-\gamma_5) \nu_e$$

$\Downarrow$ Fierz transformation

anti-Commuting

$$\frac{G_F}{\sqrt{2}}\, \bar{\nu}_e \gamma_\mu (1-\gamma_5) \nu_e \;\; \bar{e}\, \gamma^\mu (1-\gamma_5) e$$

* تمرین ۱-۸ جزوه‌ی نوترینوی‌س
همان تمرین را برای حالتی که محیط قطبیده است حل کنید.

سه‌شنبه هفته‌ی بعد صبح حل می‌کنیم.

$$\bar{\nu}_L \gamma_0 \nu_L V_c(x)$$

$$V_c(x) = \sqrt{2}\, G_F N_e(x)$$

Dispersion relation $\quad$ معادله‌ی پاشندگی

$$\mathcal{L} = \bar{\nu}_L i \not{\partial} \nu_L - \bar{\nu}_L \gamma_0 \nu_L V(x)$$

$$\mathcal{L} = \bar{\nu}_L i \not{\partial} \nu_L - \bar{\nu}_L \gamma_0 \nu_L V_c(x)$$

معادله‌ی اویلر-لاگرانژ ↓

$$\not{p}\, \nu_L - V_c \gamma_0 \nu_L = 0$$

$$\left((E - V_c)\gamma^0 - \vec{p}\cdot\vec{\sigma}\right) \nu_L = 0$$

$$\left((E - V_c)\gamma^0 + \vec{p}\cdot\vec{\sigma}\right) \times$$

$$\left[(E - V_c)^2 - |\vec{p}|^2\right] \nu_L = 0$$

$E = V_c + |\vec{p}|$ ← معادله‌ی پاشندگی

* نشان دهید که در حضور جمله جرم (دیراک - مایورانا)

معادله‌ی پاشندگی به صورت زیر تبدیل می‌شود.

$$E = V_c + \sqrt{\vec{p}^2 + m^2}$$

نکته‌ی لفظی

Majorana مایورانا

Jarlskog یارلسکوگ

تقریب فرانسبیتی

$$E \simeq V_c + |\vec{p}| + \frac{m^2}{2|\vec{p}|}.$$

$$i \frac{d}{dt}\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \left\{ U \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{bmatrix} \cdot U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$a = \sqrt{2}\, G_F N_e$$

فاز کلی

تقلیل به آنالیز دو نوترینو

$$\frac{\Delta m_{31}^2}{2E} \gg \frac{\Delta m_{21}^2}{2E}$$

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U_{PMNS} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$U_{PMNS} =$

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$$\times \, \mathrm{diag}\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1\right)$$

$\theta_{13} \ll 1$ $\qquad$ $s_{23} \simeq c_{23} = \frac{\pi}{4}$

$$\nu_e \overset{?}{\simeq} c_{12}\nu_1 + \sin\theta_{12}\,\nu_2$$

$$\nu' = -\sin\theta_{12}\nu_1 + \cos\theta_{12}\nu_2$$

$$i\frac{d}{dt}\begin{bmatrix} \nu_e \\ \nu' \end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{2}\,G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta_{12} \\ \frac{\Delta m^2}{4E}\sin 2\theta_{12} & \frac{\Delta m^2}{2E}\cos 2\theta_{12} \end{bmatrix}}_{H(t)} \begin{bmatrix} \nu_e \\ \nu' \end{bmatrix}$$

mild time dependence $\longleftrightarrow$ adiabacity

$$U_m(t)^\dagger\, H(t)\, U_m(t) = \begin{bmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{bmatrix}$$

$$U_m(t) = \begin{bmatrix} \cos\theta_m(t) & \sin\theta_m(t) \\ -\sin\theta_m(t) & \cos\theta_m(t) \end{bmatrix}$$

$$\begin{bmatrix} \nu_e \\ \nu' \end{bmatrix} = U_m(t)\begin{bmatrix} \nu_{m1} \\ \nu_{m2} \end{bmatrix}$$

$$E_{1,2}(t) = \frac{1}{2}\left(\sqrt{2}\,G_F N_e(t) + \frac{\Delta m^2}{2E}\cos 2\theta \pm\right.$$

$$\sqrt{\left(\sqrt{2}\, G_F N_e(t) - \frac{\Delta m^2}{2E}\cos 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E}\sin 2\theta\right)^2}$$

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}\, G_F N_e(t)}$$

$$N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E\, G_F \sqrt{2}} \longrightarrow \theta_m = \frac{\pi}{4}$$

$$i\frac{d}{dt}\begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix} = \left\{ \begin{pmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{pmatrix} + \begin{pmatrix} 0 & i\dot{\theta}_m \\ -i\dot{\theta}_m & 0 \end{pmatrix} \right\} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}$$

adiabatic condition $|\dot{\theta}_m| \ll |E_2 - E_1|$

resonance : له خطرناک

منطقه رزنانس

$$\tan 2\theta_m > 1$$

پهنای منطقه‌ی رزنانس $= \Delta X$

$$\sqrt{2}\, G_F \frac{dN_x}{dx}\Delta X = \frac{\Delta m^2}{2E}\sin 2\theta$$

$$\boxed{N\big|_{resonance} = \frac{\Delta m^2 \cos 2\theta}{2E\, \sqrt{2} G_F}}$$

$$\Delta X = \frac{\tan 2\theta}{\frac{1}{N}\frac{dN}{dx}} = \frac{\tan 2\theta}{\frac{d \ln N}{dx}}$$

شرط آدیاباتیک $\Delta X\,(E_2 - E_1)\big|_{resonance} \gg 1$

$$\frac{\tan 2\theta}{\left.\frac{d \ln N_e}{dx}\right|_{res}} \gg \frac{E}{\Delta m^2 \sin 2\theta}$$

بقای انرژی چه؟

resonance = تشدید

$$\nu_{m_1}(t) = T e^{-i\int_0^t E_1(t')dt'} \nu_{m_1}(0)$$

$$\nu_{m_2}(t) = T e^{-i\int_0^t E_2(t')dt'} \nu_{m_2}(0)$$

$$P(\nu_e \to \nu_e) = \left|\langle \nu_e |\, T \exp\left(-i\int_0^t H(t')\,dt'\right) |\nu_e\rangle\right|^2 =$$

من

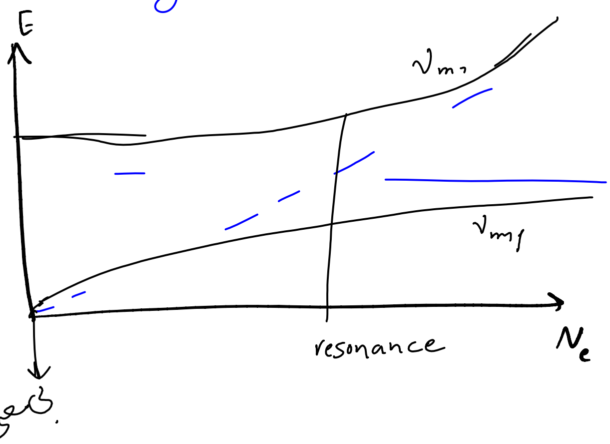
$$\left|\begin{pmatrix}\cos\theta_m(t) & \sin\theta_m(t)\end{pmatrix} \begin{bmatrix} e^{-i\int_0^t E_1(t')dt'} & 0 \\ 0 & e^{-i\int_0^t E_2(t')dt'} \end{bmatrix} \begin{bmatrix}\cos\theta_m(0) \\ \sin\theta_m(0)\end{bmatrix}\right|^2 =$$

$$\bar{P}(\nu_e \to \nu_e) = \cos^2\theta_m(t)\cos^2\theta_m(0) + \sin^2\theta_m(t)\sin^2\theta_m(0)$$

MSW

Wolfenstein, 1978

Mikheyev & Smirnov, 1985



$$\bar{P}(\nu_e \to \nu_e) = (1 - P_{jump})\left(\cos^2\theta_m(t)\cos^2\theta_m(0) + \sin^2\theta_m(t)\sin^2\theta_m(0)\right) + P_{jump}\left(\sin^2\theta_m(t)\cos^2\theta_m(0)\right)$$

$$+ \cos^2\theta_m(t)\ \sin^2\theta_m(0) \Big)$$

$\theta_m(t) = \theta \quad \leftarrow$ خلاء

$\theta_m(0) = \frac{\pi}{2} \quad \leftarrow$ بسیار غلیظ

$$\bar{P}(\nu_e \to \nu_e) \simeq \sin^2\theta + P_{jump} \cos^2\theta$$

پرش تنها در منطقه تشدیدی شود :

خطی

$$N_e = N_e^{res} + \frac{dN_e}{dx} x$$

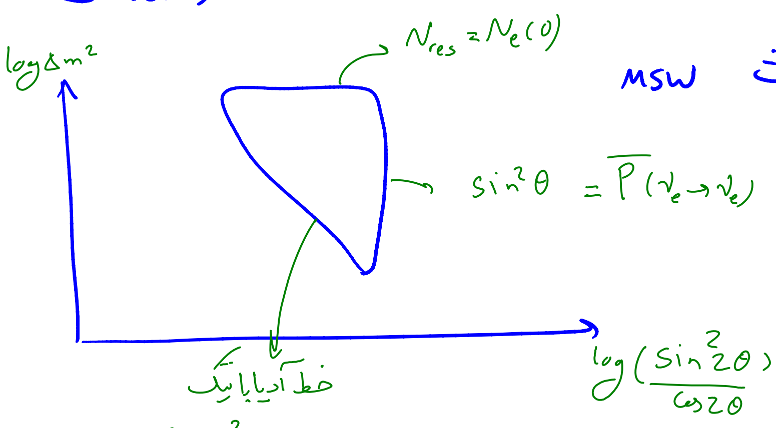
$$\underbrace{P_{jump} = \exp\left(- \frac{\pi}{4} \frac{\Delta m^2 \sin^2 2\theta}{4E \cos 2\theta \left. \frac{d \ln N_e}{dx} \right|_{res}} \right)}$$

Landau-Zener formula

Landau, 1932

Zener, 1932

MSW مثلث



$$\frac{\Delta m^2}{4E} \frac{\sin^2 2\theta}{\cos 2\theta} = cte$$

پاد و نوترینوی وایل

آنتی دیراک

$$\underbrace{(\psi_1^T C \psi_2)^T} = - \psi_1^T C \psi_2$$

$$= - \psi_2^T C \psi_1$$

symmetric

anti-symmetric

$$(m_\nu)_{\alpha\beta} = \underbrace{\frac{(m_\nu)_{\alpha\beta} + (m_\nu)_{\beta\alpha}}{2}}_{(M_\nu)_{\alpha\beta}} + \underbrace{\frac{(m_\nu)_{\alpha\beta} - (m_\nu)_{\beta\alpha}}{2}}_{(A_\nu)_{\alpha\beta}}$$

$$(m_\nu)_{\alpha\beta} \; \nu_{L\alpha}^T C \nu_{L\beta} = (M_\nu)_{\alpha\beta} \; \nu_{L\alpha}^T C \nu_{L\beta}$$

ماتریس جرمی مایورانا متقارن است.

$$m_\nu = U_{PMNS} \begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix} U_{PMNS}^T$$

$$U_{PMNS} = V_{23} V_{13} V_{12} \Phi$$

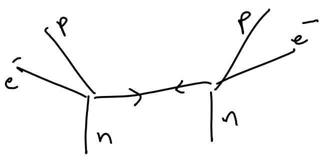
$$V_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \quad V_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{bmatrix}$$

$$V_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \Phi = \mathrm{diag}(1, e^{i\alpha_2}, e^{i\alpha_3})$$

۹ پارامتر

Oscillation $\begin{cases} \Delta m_{21}^2 , \Delta m_{31}^2 \\ \theta_{12}, \theta_{13}, \theta_{23} \end{cases}$

δ

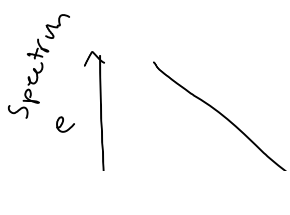


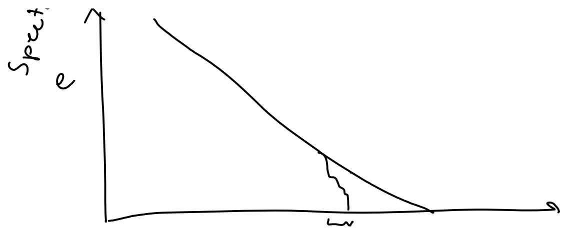
M_α $(m_\nu)_{ee}$

$$= m_1 e^{i\alpha_1} U_{e1}^2 + m_2 e^{i\alpha_2} U_{e2}^2 + m_3 e^{i\alpha_3} U_{e3}^2$$

$|M|^2$

Kurie plot





$m_{eff} = m_1$

Katrin $\longrightarrow m_1$ $\longrightarrow$ فاز

$\left\{\begin{array}{l}\text{Solar neutrino} \\ \text{KamLAND}\end{array}\right.$ Δm^2_{12}, θ_{12}

atmospheric Δm^2_{31}, $\theta_{23} \approx \frac{\pi}{4}$

K2K
MINOS

$\left\{\begin{array}{l}\text{CERN – Gransasso} \\ \text{T2K}\end{array}\right.$ $\nwarrow \nu_\tau$

Nova

CHOOZ $\longrightarrow$ $\left\{\begin{array}{l}\text{Daya Bay} \\ \text{Double-CHOOZ} \\ \text{Reno}\end{array}\right.$ $\longleftarrow$ $\theta_{13} < 10^\circ$

$\theta_{13} = ?$

$\delta = ??$

منابع نوترینو

راکتور $\bar{\nu}_e$ $\longrightarrow$ انرژی؟

خورشیدی ν_e $\longrightarrow$ 〃؟

اتمسفری ν_μ, ν_e 〃؟

ابرنواختر

relic neutrino

AGN – GRBs

نوترینوی زمینی

آشکارسازها

Super Kamiokande SK

↑

Kamiokande . IMP

SNO ← D_2O

ICECUBE
BAICAL
ANTARES

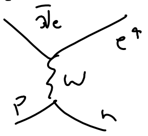
ANITA

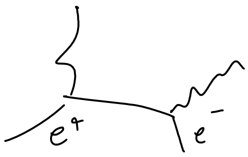
نوترینوهای خورشیدی را چه گونه آشکارسازی می‌کنند ؟

در رآکتوری 〃 〃 〃 〃 〃 〃 ؟

KamLAND

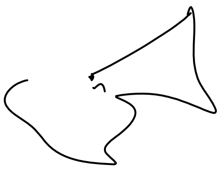
$\bar{\nu}_e \longrightarrow e^+$





$\bar{\nu}_e$ در روز

۳۰ در ثانیه



200 μsec $\quad n + p \to d + \underline{\underline{\gamma}}$

SNO in Canada $\underline{\underline{D_2O}}$

SK مشترک با

ES : $\nu_e + e^- \longrightarrow \nu_e + e^-$ دیاگرام‌ها را بکشید

NC : $\nu + D \longrightarrow \nu + n + p$

$n + D \leftarrow \gamma + \dots$

خیلی کم $\quad$ 6 Mev

$n + H \to D$

خارج

CC : $\nu_e + D \longrightarrow e^- + p + p$

Seesaw

type I ??

type II ?

type III ?

Neutrino Oscillation in Three Generation Scheme

Δm^2_{21}, Δm^2_{31}

θ_{12} θ_{13} θ_{23}, δ

$$\Delta m^2_{31} = \Delta m^2_{atm} \simeq 2 \times 10^{-3} \ eV^2$$

$$\Delta m^2_{21} = \Delta m^2_{solar} \simeq 7 \times 10^{-5} \ eV^2$$

Vacuum neutrino oscillation due to Δm^2_{31}

$$\frac{\Delta m^2_{21}}{E} L = 3.6 \times 10^{-2} \frac{\Delta m^2_{21} / 7 \times 10^{-5} eV^2}{E / 1 GeV} \frac{L}{100 km} \ll 1$$

$$\frac{\Delta m^2_{31}}{E} L = 1.0 \times \frac{\frac{\Delta m^2_{31}}{2 \times 10^{-3} eV^2}}{\frac{E}{1 GeV}} \frac{L}{100 km} \gtrsim 1$$

K2K KEK to Kamiokande

$\begin{cases} \text{OPERA} \\ \text{ICARUS} \end{cases}$

MINOS

$$P(\nu_\alpha \to \nu_\beta) = \left| \underbrace{U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2}}_{\delta_{\alpha\beta} - U^*_{\alpha 3} U_{\beta 3}} + U^*_{\alpha 3} U_{\beta 3} e^{\frac{-i \Delta m^2_{31}}{2E} t} \right|^2$$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4 (1 - |U_{\mu 3}|^2) |U_{\mu 3}|^2 \sin^2(\frac{\Delta m^2_{31}}{4E} t)$$

$$= 1 - 4 (1 - \sin^2\theta_{23} \cos^2\theta_{13}) \sin^2\theta_{23} \cos^2\theta_{13} \sin^2 \frac{\Delta m^2_{31} t}{4E}$$

$$\simeq 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31} t}{4E}$$

$$P(\nu_e \to \nu_e) = 1 - 4\,(1-|U_{e3}|^2)\,|U_{e3}|^2 \sin^2\!\left(\frac{\Delta m^2_{31}}{4E}\, t\right)$$

$$= 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31}}{4E} t \simeq 1$$

CHOOZ $\quad E \sim$ few MeV $\quad P(\nu_e \to \nu_e) \simeq 1$

$L \sim 1\,\mathrm{km}$

$$\sin^2 2\theta_{13} \leq 0.2$$

$$P(\nu_\mu \longrightarrow \nu_\tau) = 4\,|U_{\mu 3}|^2 |U_{\tau 3}|^2 \sin^2\!\left(\frac{\Delta m^2_{31}}{4E} t\right)$$

$$= \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2 \frac{\Delta m^2_{31}}{4E} t$$

$$\simeq \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31}}{4E} t$$

$$P(\nu_\mu \to \nu_e) = 4|U_{e3}|^2 |U_{\mu 3}|^2 \sin^2 \frac{\Delta m^2_{31} t}{4E}$$

$$\simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \frac{\Delta m^2_{31}}{4E} t \simeq 0$$

نوترینوهای جوی Δm^2_{31} و θ_{13} $\quad \left\{ \begin{array}{l} \nu_e \\ \nu_\mu \to \nu_\tau \end{array} \right.$

اثر δ ؟؟

Vacuum oscillation due to Δm^2_{21}

$$\frac{\Delta m^2_{31} L}{2E} \gg 1 \quad \leftarrow \text{averaging}$$

$$\frac{\Delta m^2_{21}}{E} L = 7.2 \, \frac{\Delta m^2_{31} / 7\times 10^{-5}\,\mathrm{eV}^2}{\frac{E}{5\,\mathrm{MeV}}} \left(\frac{L}{100\,\mathrm{km}}\right) \simeq 1$$

$$\frac{\Delta m^2_{31}}{E} L = 2\times 10^2 \, \frac{\frac{\Delta m^2_{31}}{2\times 10^{-3}\,\mathrm{eV}^2}}{\frac{E}{5\,\mathrm{MeV}}} \, \frac{L}{100\,\mathrm{km}} \gg 1$$

KamLAND

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\beta 1} U_{\alpha 1}^* + U_{\beta 2} U_{\alpha 2}^* e^{-i\frac{\Delta m_{21}^2 t}{2E}} \right|^2$$

$$+ |U_{\beta 3}|^2 |U_{\alpha 3}|^2$$

$$P(\nu_\alpha \to \nu_\alpha) = (1 - |U_{\alpha 3}|^2)^2 \, P_{eff}(\nu_\alpha \to \nu_\alpha) + |U_{\alpha 3}|^4$$

$$P_{eff}(\nu_\alpha \to \nu_\alpha) = 1 - 4 \frac{|U_{\alpha 2}|^2 (1 - |U_{\alpha 2}|^2 - |U_{\alpha 3}|^2)}{(1 - |U_{\alpha 3}|^2)^2} \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

$$S = \cos^4\theta_{13} \, S_{eff}(\theta_{12}, \Delta m_{21}^2) + \sin^4\theta_{13}$$

$$\simeq \cos 2\theta_{13} \, S_{eff}(\theta_{12}, \Delta m_{21}^2)$$

$$S_{eff}(\theta_{12}, \Delta m_{21}^2) = 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

برای $\bar{\nu}_e$ هم همینطور

CP violation in neutrino oscillation

$$A_{\alpha\beta}^{CP} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

$$A_{\alpha\beta}^{T} = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$

$$A_{\alpha\beta}^{CP} = A_{\alpha\beta}^{T}$$

CP: $U_{\alpha i} \longrightarrow U_{\alpha i}^*$

$$A_{\alpha\beta}^{T} = -A_{\beta\alpha}^{T}$$

$$A_{\alpha\beta}^{CP} = -A_{\beta\alpha}^{CP} \quad \Longrightarrow \quad A_{\alpha\alpha}^{CP} = 0$$

$$P(\nu_\alpha \to \nu_\alpha) = P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$$

$$\sum_{\beta=e,\mu,\tau} P(\nu_\alpha \to \nu_\beta) = \sum_{\beta=e,\mu,\tau} P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 1$$

$$A^{CP}_{e\mu} = A^{CP}_{\tau e} \qquad A^{CP}_{\mu\tau} = A^{CP}_{e\mu}$$

$$A^{CP}_{e\mu} = A^{CP}_{\mu\tau} = A^{CP}_{\tau e} \equiv A^{CP}$$

δ

$$A^{CP}_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = -4\sum_{i<j}$$

$$\mathrm{Im}\left[U_{\alpha i} U^*_{\beta i} U_{\beta j} U^*_{\alpha j}\right] \sin \frac{\Delta m^2_{ij} t}{2E}$$

$$J_{\alpha\beta,ij} = \mathrm{Im}\left[U_{\alpha i} U^*_{\beta i} U_{\beta j} U^*_{\alpha j}\right]$$

$$J_{\alpha\beta,ij} = -J_{\beta\alpha,ij} \qquad J_{\alpha\beta,ji} = -J_{\alpha\beta,ij}$$

$$J_{\alpha\beta,12} + J_{\alpha\beta,22} + J_{\alpha\beta,32} = J_{\alpha\beta,12} + J_{\alpha\beta,32} = 0$$

$$J_{\alpha\beta,13} = J_{\alpha\beta,32}$$

همین طور

همین طور

$$J_{e\mu,ij} = J_{\mu\tau,ij} = J_{\tau e,ij}$$

Jarlskog پارامتر

$$J \equiv J_{e\mu,12}$$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta$$

$$A^{CP} = -4J\left\{\sin\frac{\Delta m^2_{12} t}{2E} + \sin\frac{\Delta m^2_{23} t}{2E} + \sin\frac{\Delta m^2_{31} t}{2E}\right\}$$

$$+\sin\frac{\Delta m_{31}^2 t}{2E}\Big\}$$

Re-phasing invariant

تغییر پایه

$$\nu_\alpha \longrightarrow U_{\alpha\beta}\,\nu_\beta \qquad \longrightarrow \bar{\ell}_\alpha \gamma^\mu \frac{1-\gamma_5}{2}\nu_\alpha W_\mu$$

$$l_{L\alpha} \longrightarrow U_{\alpha\beta}\, l_{L\beta}$$

$$m_\nu^\dagger m_\nu \longrightarrow U^\dagger m_\nu^\dagger m_\nu U$$

$$m_l^\dagger m_l \longrightarrow U^\dagger m_l^\dagger m_l U$$

$$\bar{l}_{R\beta}\, m_l\, l_{L\beta}$$

$$[m_\nu^\dagger m_\nu, m_l^\dagger m_l] \longrightarrow U^\dagger [m_\nu^\dagger m_\nu, m_l^\dagger m_l] U$$

$$\mathrm{Det}[[m_\nu^\dagger m_\nu, m_l^\dagger m_l]] \longleftarrow$$ ناوردا

ناوردا تحت چه؟

* تمرین : به کار گرفتن پارامتریزاسیون استاندارد $\mathrm{Det}[[m_\nu^\dagger m_\nu, m_l^\dagger m_l]]$ را محاسبه کنید.

Observing CP-violation

1) $\Delta m_{21}^2 \neq 0$ یا $\Delta m_{31}^2 \neq 0$

2) $\theta_{ij} \neq 0$ or $\sin\delta \neq 0$

3) no time averaging

Resonant Matter Oscillation

اثر ماده + سه نسل $\rightarrow$ حل تحلیلی ندارد

$$S = P(\nu_e \to \nu_e) = \cos^4\theta_{13}\, S_{2\nu}(\theta_{12}, \Delta m_{21}^2; a_{ee}) + \sin^4\theta_{13}$$

$$S = P(\nu_e \to \nu_e) = \cos^4\theta_{13}\, S_{eff}(\theta_{12}, \Delta m^2_{21}; a_{eff}) + \sin^4\theta_{13}$$

$$a_{eff} \equiv \cos^2\theta_{13}\, a(x) = \cos^2\theta_{13}\, \sqrt{2}\, G_F N_e(x)$$

Lim, 1987; Smirnov, 1992; Shi and Schramm 1992

$$\frac{\Delta m^2_{21}}{2E}, \quad \sqrt{2}\, G_F N_e \ll \frac{\Delta m^2_{31}}{2E}$$

$$H \simeq \mathrm{diag}\left(0, 0, \frac{\Delta m^2_{31}}{2E}\right)$$

$$\nu_e' \equiv \cos\theta_{12}\,\nu_1 + \sin\theta_{12}\,\nu_2$$

$$\nu_\mu' = -\sin\theta_{12}\,\nu_1 + \cos\theta_{12}\,\nu_2$$

$$\xi \equiv \begin{pmatrix} \nu_e' \\ \nu_\mu' \\ \nu_3 \end{pmatrix} = \begin{pmatrix} \cos\theta_{12}\,\nu_1 + \sin\theta_{12}\,\nu_2 \\ -\sin\theta_{12}\,\nu_1 + \cos\theta_{12}\,\nu_2 \\ \nu_3 \end{pmatrix}$$

$$\xi = \begin{pmatrix} \nu_e' \\ \nu_\mu' \\ \nu_\tau' \end{pmatrix} = \begin{pmatrix} \cos\theta_{13}\,\nu_e - \sin\theta_{13}\, e^{-i\delta}\,\tilde{\nu}_\tau \\ \tilde{\nu}_\mu \\ \sin\theta_{13}\, e^{i\delta}\,\nu_e + \cos\theta_{13}\,\tilde{\nu}_\tau \end{pmatrix}$$

$$\tilde{\nu}_\mu \equiv \cos\theta_{23}\,\nu_\mu - \sin\theta_{23}\,\nu_\tau \simeq \frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)$$

$$\tilde{\nu}_\tau \equiv \sin\theta_{23}\,\nu_\mu + \cos\theta_{23}\,\nu_\tau \simeq \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau)$$

$$\theta_{23} \simeq \frac{\pi}{4}$$

فقط ν_e' اثر ماده را حس می‌کند.

$$i\,\frac{d\xi}{dt} = \left\{ V_{12} \begin{pmatrix} 0 & & \\ & \Delta_{12} & \\ & & \Delta_{13} \end{pmatrix} V_{12}^\dagger + V_{13}^\dagger \begin{pmatrix} a(t) & & \\ & 0 & \\ & & 0 \end{pmatrix} V_{13} \right\} \xi$$

$$= \left\{ \begin{pmatrix} \Delta_{12} S_{12}^2 + a\, c_{13}^2 & \Delta_{12} S_{12} C_{12} & 0 \\ \Delta_{12} S_{12} C_{12} & \Delta_{12} C_{12}^2 & 0 \end{pmatrix} \right. \xi$$

$$0 \qquad\qquad 0 \qquad\qquad \Delta_{13} + a s_{13}^2 \Big)$$

$$\begin{bmatrix} 0 & 0 & a s_{13} c_{13} \\ 0 & 0 & 0 \\ a s_{13} c_{13} & 0 & 0 \end{bmatrix} \Big\} \xi$$

$$\Delta_{12} = \frac{\Delta m_{21}^2}{2E} \qquad \Delta_{13} = \frac{\Delta m_{31}^2}{2E}$$

$$c_{ij} = \cos\theta_{ij} \qquad s_{ij} = \sin\theta_{ij} \qquad \frac{\Delta m_{21}^2}{2E}, \sqrt{2} G_F N_e \ll \frac{\Delta m_{31}^2}{2E}$$

$$i \frac{d\xi}{dt} \simeq \begin{pmatrix} \Delta_{12} s_{12}^2 + a c_{13}^2 & \Delta_{12} s_{12} c_{12} & 0 \\ \Delta_{12} s_{12} c_{12} & \Delta_{12} c_{12}^2 & 0 \\ 0 & 0 & \Delta_{13} + a s_{13}^2 \end{pmatrix} \xi$$

$$a_{eff} = \cos^2\theta_{13}\, a(t)$$

$$S = \left| c_{13}^2 A_{eff}(\nu_e \to \nu_e) + s_{13}^2 e^{-i(\Delta_{13} + a s_{13}^2)t} \right|^2$$

$$\simeq \cos 2\theta_{13} \; S_{eff}(\theta_{12}, \Delta m_{21}^2; a_{eff})$$

$$e^{-i\Delta_{13} t} \longrightarrow 0$$

$$\frac{a \sin\theta_{13}}{\frac{\Delta m_{31}^2}{2E}} < 10^{-2}$$ خطای ترتیب

$$\pi^+ \to \mu^+ + \nu_\mu$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

$$R = \frac{(\nu_\mu + \bar{\nu}_\mu)_{obs} / (\nu_e + \bar{\nu}_e)_{obs}}{(\nu_\mu + \bar{\nu}_\mu)_{pred} / (\nu_e + \bar{\nu}_e)_{pred}} \sim 0.6$$

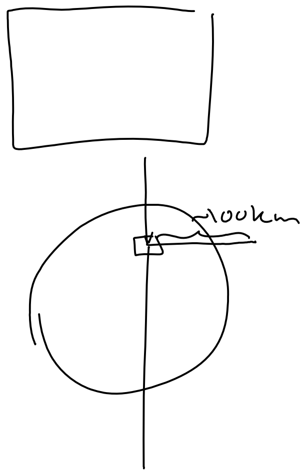
ν_e خیلی کم سازگار نیست. $\qquad$ ν_μ کمتر کم سازگار با مشاهده

$$|\Delta m_{31}^2| = (1.3 - 3.0) \times 10^{-3}\ eV^2$$

$$|\Delta m^2_{31}| = (1.3-3.0)\times 10^{-3}\ eV^2$$

$$\sin^2 2\theta_{23} > 0.9$$

Sub-GeV Multi-GeV



Solar neutrino oscillation

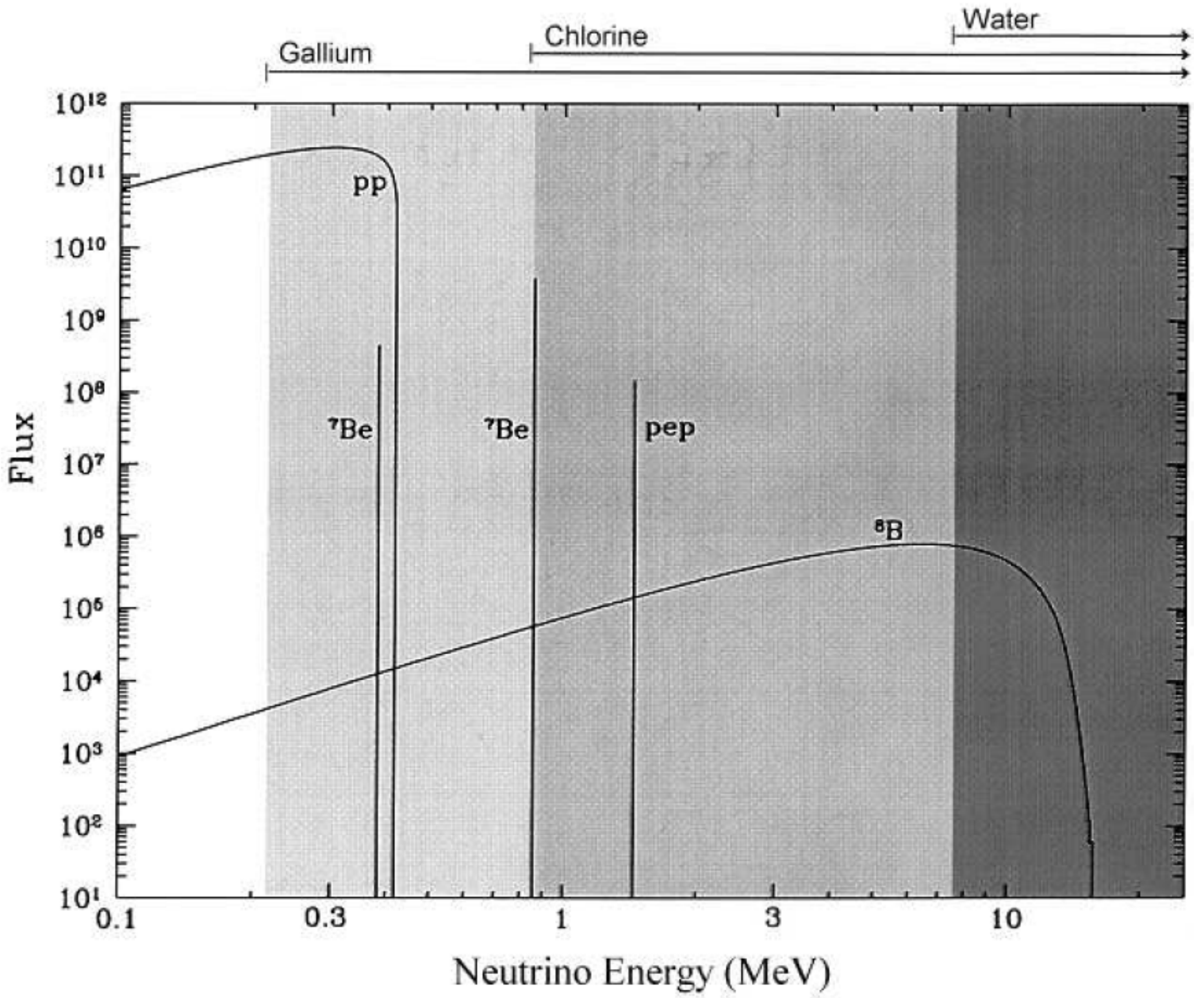
$$p + p \longrightarrow D + e^+ + \nu_e \qquad \langle E_\nu \rangle = 0.26\ MeV$$

$$\vdots$$

$$e^- + {}^7Be \longrightarrow {}^7Li + \nu_e \qquad E_\nu = 0.86\ MeV$$

$$\vdots$$

$${}^8B \longrightarrow {}^8Be + e^+ + \nu_e \qquad \langle E_\nu \rangle = 7.2\ MeV$$



$$4e^- + 4p \rightarrow {}^4He + 2\nu_e + \gamma \quad (26.73\,MeV)$$

R. Davis Cl experiment

C_2Cl_4 Homestake mine

(۱

Cl experiment

$$\nu_e + {}^{37}Cl \longrightarrow {}^{37}Ar + e^-$$

$E_{th} = 0.81$ MeV

B, Be

SK (۲

$\nu + e^- \rightarrow \nu + e^-$ $E_{th} = 5$ MeV

(۳

Ga experiment SAGE GNO

Ga experiment SAGE, GNO ''

$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$

pp Be B

SNO

$\nu_e + D \rightarrow e^- + p + p$

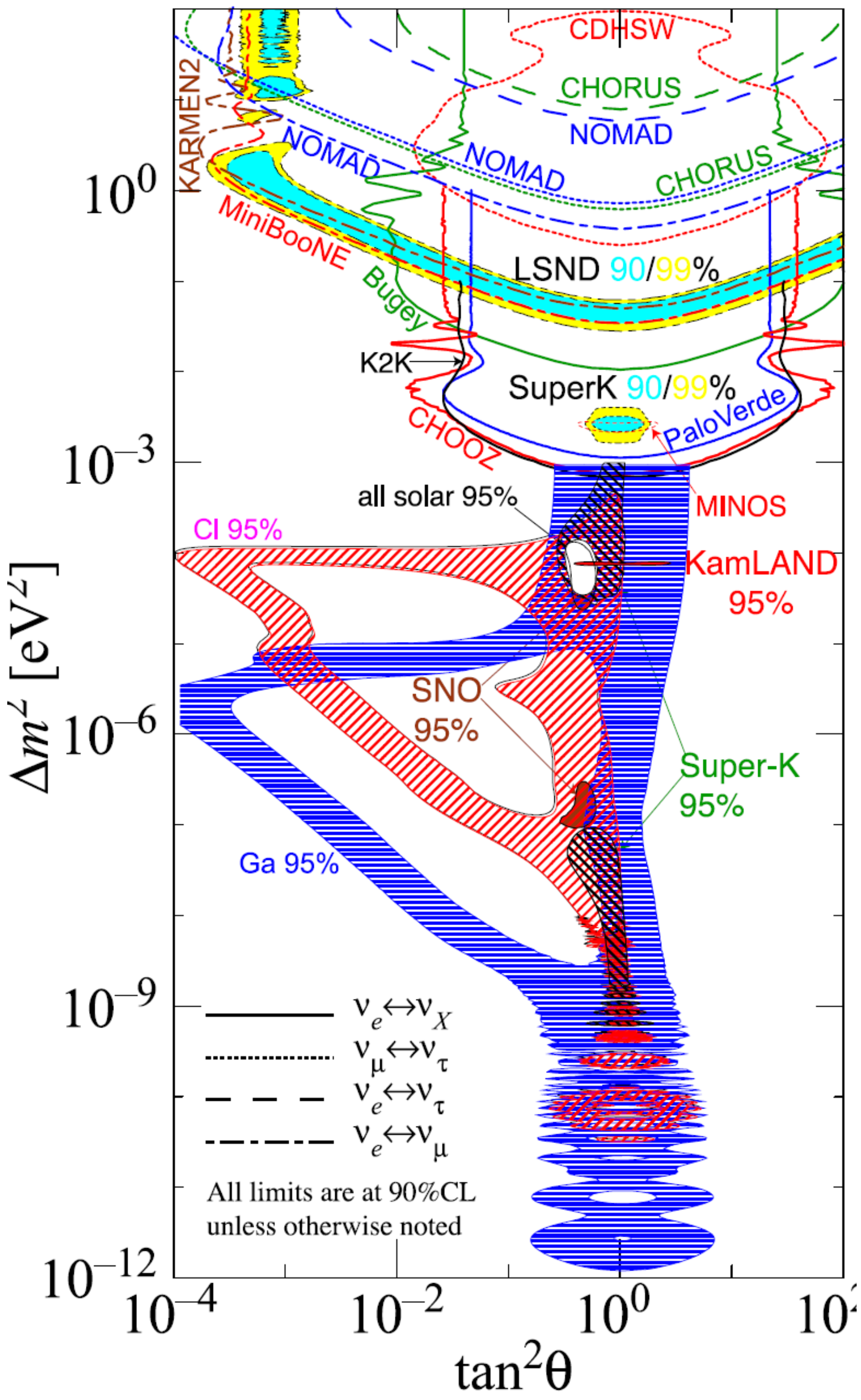
$\nu + D \rightarrow \nu + n + p$

$\uparrow$
B neutrinos

radio-chemical Cl & Ga experiments

observed / predicted < 1

NC SNO لمبجز



$$\varphi_{SK} = \varphi_{SSM}\{S + (1-S)\underbrace{R_\sigma}\}$$

$R_\sigma \sim \frac{1}{6}$

$$\varphi_{SNO}(CC) = \varphi_{SSM} \times S$$

ν_μ, ν_e

ES ?

$$\varphi_{SK} \simeq 2.4 \times 10^{6} \ cm^{-2}\, sec^{-1}$$

$$\varphi_{SNO}(CC) \simeq 1.8 \times 10^{6} \ cm^{-1}$$

$$\varphi_{SSM} = 5.4 \times 10^{6} \quad cm^{-2}\ sec^{-1}$$

$\cancel{\nu_e \rightarrow \nu_s}$

Δm^2_{21} و Δm^2_{31} علامت