Final exam of field theory course (1/23/2016)

1) Show that the mass of a particle and its antiparticle are exactly equal regardless of whether they are elementary or composed bound states. (Notice that a consequence of this is that anti-proton and anti-hydrogen have masses exactly equal to those of proton and hydrogen).

2) Show that the total decay rate of a particle and anti-particle are exactly equal. Discuss whether we can repeat the same claim about their partial decay rates.

3) Consider a complex scalar Φ with the following Lagrangian:

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - M^2 \Phi^\dagger \Phi - (m^2 \Phi^2 + \text{H.c.}) + (Y \Phi \bar{\psi} \frac{1+\gamma^5}{2} \psi + \text{H.c.})$$

where ψ is a Dirac field and Y is the Yukawa coupling.
3-a) What is the necessary and sufficient condition for CP-conservation of this Lagrangian.
3-b) Suppose CP is conserved. Decompose Φ as $\Phi = (\phi_1 + i\phi_2)/\sqrt{2}$ where ϕ_1 and ϕ_2 are real fields. Are ϕ_1 and ϕ_2 scalars or pseudoscalars?

4) Consider interaction of form

$$\sum_{ij} (\lambda_{ij} \Phi \bar{\chi}_i \psi_j + \text{H.c.})$$

between Dirac (four-component) spinors ψ_j (i.e., $\psi_j^T = (\psi_{Lj}^T \ \psi_{Rj}^T)$) with mass m_j and Chiral two component spinor χ_i with mass M_i of form

$$M_i \chi_i^T C \chi_i + H.c.$$

[in which $C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$] and a complex scalar field Φ with mass of m_ϕ. Assume $M_i > m_\phi + m_j$.
4-a) Calculate the total decay rate of $\chi_i \to \psi_j \Phi$ at tree level. Use this formula to calculate the total decay rate of χ_i.
4-b) Draw the loop diagrams contributing to $\chi_i \to \psi_j \Phi$ and calculate the amplitude corresponding to each.
4-c) Calculate $\Gamma(\chi_i \to \Phi \psi_j) - \Gamma(\chi_i \to \bar{\Phi} \bar{\psi}_j)$. Under what condition this difference is nonzero? Can we conclude this difference should equal to zero from discussion of Eq. (2)?

5-a) Repeat the proof of Ward identity in section 7 of Peskin when the model includes a charged scalar described by Φ.
5-b) Repeat the proof of Ward identity in section 7 of Peskin when the model includes two charged scalars described by Φ_1 and Φ_2 with charges equal respectively to $+1$ and $+2$ and a coupling of form $(A \Phi_2^\dagger \Phi_1^2 / 2 + H.c)$ in which A is a constant with dimension of mass.