Course on "Statistical Physics of Fields In and Out of Equilibrium"

Code (IPM): 113558 / Spring Semester 2016 (1394-95)

Course information and outline

February 6, 2016

- **Time:**
  Sundays, 14:00 - 15:40 and 16:30 - 18:10 (four-unit course)
  Starting from Sunday February 14, 2014 (Bahman 25, 1394)

- **Place:**
  School of Physics, Institute for Research in Fundamental Sciences (IPM),
  Farmanieh Ave, Next to Kouh-e-Nour Building, Tehran, Iran
  Classroom A, Ground floor, Farmanieh Central Building (IPM)

- **Website: http://www.physics.ipm.ac.ir/~naji/fields_SS2016.html**

- **Tutorial sessions:**
  The course will be tutored by Dr. Bahman Roostaei (IPM); time and place of the tutorial sessions will be arranged in coordination with the students.

- **Intended audience and pre-requisites:**
  This course is intended primarily for Ph.D. students in Physics. It may nevertheless be useful for and attended by advanced M.Sc/B.Sc. students with a background in Physics. A comprehensive knowledge of **thermodynamics** and **elementary statistical mechanics** (comparable to the level of standard B.Sc./M.Sc. textbooks) will be necessary to sign up for this course and attend the lectures. **All students including those who sign up for the course and those who merely audit the course will be required to attend the lectures and tutorial sessions on a regular basis and do the homework assignments** (see the exception for auditing students).

- **Scope & summary:**
  Starting with a short review of the basics of thermodynamics and statistical mechanics, I shall give an overview of statistical fields in and out of (thermal) equilibrium with emphasis made (by giving various examples) on their applications in the context of soft matter and condensed matter physics. This will be followed by discussing different methods of constructing field theories either phenomenologically or based on microscopic, particle-based Hamiltonians. The essential methodology of statistical field theory shall be covered in several lectures throughout the course, ranging from elements of the calculus of variations, functional integrals (and path integrals) to perturbative expansions, Feynman diagrams and the renormalization group techniques. The first part of the course will be focused on equilibrium physics of fields with the bulk of the material in this part dedicated to the theory of critical phenomena; nevertheless,

ample time will be given to other modern examples of equilibrium fields such as Coulomb fluids (or gases) and fluctuation-induced phenomena (or Casimir effect). The rest of the course shall include non-equilibrium statistical physics of fields, starting from kinetic theory of many-particle systems to hydrodynamics of fluids, hydrodynamic fluctuations, dissipative dynamics of fields near and far from equilibrium accompanied by a few lectures on the Kardar-Parisi-Zhang equation, dynamic RG and preliminaries of the Martin-Siggia-Rose approach to stochastic, dissipative dynamics of fields. **See below for a more comprehensive list of topics to be covered in this course.**

- **Homework, exams & term essay:**

Homework assignments will be given on a regular basis and in the form of problem sets. The homework solutions will be graded, making altogether 40% of the final score with the rest coming from the mid-term and final exams as well as an evaluation by the tutor of the active participation and contribution of the students to the discussions in the tutorial sessions (20% each).

As an alternative to the mid-term and final exams, the students may choose to **write** and **present** a term essay on a subject they select from a list of suggested topics; the list will be handed out to students in the first few weeks. Students can propose other topics for their term essay but they need to discuss their choice of the term-essay topic with myself or the tutor first. Essays must be prepared in the form of standard scientific reports or review articles using a Latex template file and other specifications to be circulated with the list of topics. Terms essays must be submitted before a deadline at around four weeks before the end of the course (exact date shall be announced later).

Exception for auditing students: As noted above, students who will audit the course will be required to attend the lectures and the tutorial sessions on a regular basis and do the homework assignments just like the students who will officially sign up for the course. However, the auditing students may elect to opt out from doing the homework assignments at any time **during the course of the first 8 lectures**. In this case, and as a form of contribution to the class, they will be required to do a term essay as described above **before the mid-term exam**.

- **References:**

There are many valuable books covering the topics of this course at different levels. I will not follow any particular textbook in general and rely mostly on self-designed lectures, especially for my lectures on **field theory for Coulomb fluids**, **fluctuation-induced phenomena**, and **statistical physics of fields out of equilibrium**, for which there are much fewer textbooks or other references that can be suitable for this course. For the lectures on **methods of statistical field theory** and **critical phenomena**, I recommend (any of) the four textbooks by Kardar (2007), Brezin (2010), Uzunov (2010) and/or Goldenfeld (1992) , while other references can be used as well (see below).

Statistical field theory, phase transitions & critical phenomena:

Key textbooks for this part of the course:

- M. Kardar, *Statistical Physics of Fields* (2007)
- E. Brezin, *Introduction to Statistical Field Theory* (2010)
- D.I. Uzunov, *Introduction to the Theory of Critical Phenomena* (2010)
- N. Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group* (1992)

Other excellent textbooks (in reverse chronological order):

- H. Nishimori & G. Ortiz, *Elements of Phase Transitions & Critical Phenomena* (2010)
- G. Mussardo, *Statistical Field Theory* (2010)
- A. Altland & B. Simons, *Condensed Matter Field Theory* (2010)
- J. Zinn-Justin, *Phase Transitions and the Renormalization Group* (2007)
- W.D. McComb, *Renormalization Methods: A Guide for Beginners* (2004)
- J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (2002)
- H. Kleinert & Schulte-Frohlinde, *Critical Properties of ϕ^4-Theories* (2001)
- P.M. Chaikin & T.C. Lubensky, *Principles of Condensed Matter Physics* (1995)
- J.J. Binney et al., *The Theory of Critical Phenomena* (1992)
- J. Yeomans, *Statistical Mechanics of Phase Transitions* (1992)
- H. Kleinert, *Gauge Fields in Condensed Matter: Vol. 1* (1989)
- C. Itzykson & J.-M. Drouffe, *Statistical Field Theory: Vols. 1* & 2 (1989)
- G. Parisi, *Statistical Field Theory* (1988)
- S.-K. Ma, *Modern Theory of Critical Phenomena* (1976)
- F. Ravndal, *Scaling and Renormalization Groups* (1976)
- H.E. Stanley, *Introduction to Phase Transitions & Critical Phenomena* (1971)

Statistical physics of fields out of equilibrium:

- **Self-designed lectures**; the following references of varying levels of difficulty may also be consulted by the interested students:
- A. Kamenev, *Field Theory of Non-Equilibrium Systems* (2011), Ch. 8
- M. Kardar, *Statistical Physics of Fields* (2007), Ch. 9
- G.F. Mazenko, *Nonequilibrium Statistical Mechanics* (2006), Ch. 8
- P.M. Chaikin & T.C. Lubensky, *Principles of Condensed Matter Physics* (1995), Ch. 8
- N. Goldenfeld, *Lectures on Phase Transitions & the Renormalization Group* (1992), Ch. 8
- S.-K. Ma, *Modern Theory of Critical Phenomena* (1976), Ch. XI- Ch. XIV
- D. Forster, *Hydrodynamic Fluctuations, Broken Symmetry & Correlation Functions* (1975)

Field theory for Coulomb fluids:

- **Self-designed lectures**; a few book chapters & review papers will also be introduced as possible additional references during the course.

Fluctuation-induced phenomena (Casimir effect):

- **Self-designed lectures**; the following books may also be consulted by the more advanced students:
- M. Bordag et al., *Advances in the Casimir Effect* (2009)
- V.A. Parsegian, *Van der Waals Forces* (2005)

◉ Outline of the course:

Statistical Physics of Fields In Equilibrium

✤ **Part I — Preliminaries**

- **Statistical fields: A scenery** *(slides presentation)*
  - Modern applications in soft & condensed matter
- **Statistical physics of particles**
  - Equilibrium statistical mechanics & thermodynamics: A brief review
  - Standard microscopic (particle and spin) models
- **Statistical field theory: Basic methods**
  - Construction of field actions: Coarse-graining — from particles to fields
  - Effective field actions: Phenomenological models
  - Calculus of variations and the least action principle
    - Continuous symmetries, currents & conservation laws
  - Functional integrals & the partition function
  - Other routes to construction of exact or effective field theories
    - Hubbard-Stratonovich transformation
    - Edwards-Gupta method
  - Generating functionals, correlation functions & cumulant expansion
  - Gaussian integrals & Wick's theorem
  - Perturbative expansion & Feynman diagrams
  - Saddle-point method: Mean field & fluctuation corrections

✤ **Part II — Field theory for Coulomb fluids**

- **Coulomb interactions in soft matter and biology** *(slides presentation)*
- **Coulomb fluids: The primitive model**
  - Exact field action
  - Mean-field theory: The Poisson-Boltzmann equation
  - Debye-Huckel ("Yukawa") approximation
- **One-component Coulomb fluid in confinement: From weak to strong coupling**
  - Mean-field theory: The Gouy-Chapman solution & effective interactions
  - Gaussian fluctuations (one-loop approximation)
  - Strong-coupling theory (virial expansion)
  - Comparison with numerical simulations *(slides presentation)*
  - Bridging the gap: Variational methods

- ✤ **Part III — Fluctuation-induced phenomena (Casimir effect)**
  - Quantum vacuum: Electromagnetic Casimir force between perfect ``mirrors”
  - Massless free field in confinement: Thermal (classical) Casimir effect
  - Lifshitz-Casimir-van der Waals interaction between dielectric bodies
  - Massive fluctuations, Debye screening & Casimir effect in Coulomb fluids
  - A plethora of fluctuation-induced phenomena: From superfluid films to liquid crystals *(slides presentation)*

- ✤ **Part IV — Phase transitions and critical phenomena**
  - **Generalities, brief history and modern examples** *(slides presentation)*
    - Bulk phases and phase transitions: Simple fluids & ferromagnets
    - Criticality, scaling & universality: Experiments and simulations
    - Classifications of (bulk) phase transitions
    - Multicritical points & other features of phase diagrams
    - Order parameter & broken symmetry
    - Modern examples from liquid crystals to superfluids & superconductors
  - **Widom scaling hypothesis**
    - Homogeneous functions
    - Scale invariance & scaling exponents
  - **Ising model**
    - Exact solutions
    - Mean-field theory: Weiss molecular field & Bragg-Williams approximation
    - Kadanoff block spins & Widom scaling
  - **Real-space renormalization group (RG)**
    - Wilson block spins, renormalization & fixed points
    - Ising model revisited: RG in one and two dimensions
  - **Landau mean-field theory**
    - Landau-Ginzburg phenomenology
    - ϕ^4 theory: Spontaneous symmetry breaking
    - Thermodynamic limit and ergodicity breaking
    - Discontinuous *vs* continuous transition (first *vs* second order)
    - ϕ^6 theory: Tricritical point & other aspects
  - **Gaussian fluctuations**
    - Correlation functions & susceptibilities
    - Fluctuation corrections to mean field & the upper critical dimension
    - Ginzburg criterion
    - Discrete symmetry breaking: Domain walls
    - Continuous symmetry breaking: Goldstone modes
    - Lower critical dimension & the Mermin-Wagner theorem

- **Momentum-space RG**
  - RG scheme revisited: Coarse grain, rescale & renormalize
  - The Gaussian model: Exact solution & RG analysis
- **ϕ^4 theory**
  - Perturbative RG & Feynman diagrams
  - ϵ-expansion & critical exponents
- **Special topics**
  - Nonlinear σ model: Critical behavior near dimension two
  - XY model, topological defects & the Kosterlitz–Thouless transition
  - RG for the two-dimensional Coulomb gas
  - Two-dimensional solids and melting

Statistical Physics of Fields Out of Equilibrium

✤ Part I — Kinetic Theory & Hydrodynamics: From particles to fields

- **Boltzmann equation**
- **Hydrodynamic limit: Navier-Stokes equations**
- **Hydrodynamic fluctuations near equilibrium**

✤ Part II — Dissipative dynamics of particles

- **Random walks and Brownian motion**
  - Theory of stochastic processes: A brief review
  - Langevin & Fokker-Planck equations
- **Path-integral approach to Brownian motion**

✤ Part III — Dissipative dynamics of fields near equilibrium

- **Model A: Non-conserved dynamics**
  - Time-dependent, stochastic Landau-Ginzburg equation
  - Edwards-Wilkinson model for stochastic surface relaxation
  - Dynamic scale invariance
- **Model B: Conserved dynamics**

✤ Part IV — Dissipative dynamics of fields far from equilibrium

- **Kardar-Parisi-Zhang (KPZ) equation**
  - Nonlinear stochastic surface growth
  - KPZ universality class: Dynamic RG
- **Field theory for stochastic partial differential equations**
  - Langevin-type dynamics & generalized Fokker-Planck equation
  - Functional-integral representation & Martin-Siggia-Rose action