July 16, 2016

Final Exam of QFT-II course

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This exam is take-home and is to be returned by 9am Thursday 28 July.
Total mark: 100.

1) Axial gauge fixing and Faddeev-Popov ghosts. **30/100**

Consider a gauge theory with gauge group G and fix the axial gauge:

$$n \cdot A = 0,$$

where n_μ is a (unit) vector which could be null, time-like or space-like.

1-1) Perform all steps of the gauge fixing in the path integral formulation and write the full gauge fixed action.

1-2) Draw all the basic Feynman graphs and write the expressions of them in the axial gauge.

1-3) Consider adding fermionic matter field in the fundamental representation of G and perform complete one-loop analysis of

– Gauge field two point function;

– Ghost two point function;

– Ghost-gauge field vertex.

1-4) Using the above, compute the β-function of the theory. Is β-function gauge dependent? Does it depend on the choice of n and how does is compare with the Lorenz gauge results?

2) Gauge coupling and its effect on running of other couplings. **30/100**

Consider the gauge-Yukawa theory with the action:

$$L = -\frac{1}{2} Tr F_{\mu\nu}^2 + \frac{1}{2}(D_\mu \Phi)^2 - \bar{\psi}(i\not{D} - m)\psi + \lambda \bar{\psi}\Phi\psi.$$

2-1) If the fermion fields are in the fundamental representation of the gauge group G. What are the allowed representations for the scalar Φ field to make the above action gauge invariant?

2-2) Use background field method to compute the β-function of the Yukawa coupling λ.

2-3) Use the background field method to compute the one-loop β-function of the theory for the gauge coupling g.

2-4) Compute the running of the mass parameter m.

2-5) Analyze the RG flow and fixed points of the theory in the $\lambda - g$ plane.

3) On BRST charge. **25/100**

Consider a general gauge fixing function $\mathcal{G}(A) = 0$, leading to the gauge fixed action

$$\mathcal{L} = -\frac{1}{2} Tr F_{\mu\nu}^2 + Tr(\omega.\mathcal{G}) - \frac{1}{2} Tr\omega^2 + \mathcal{L}_{matter} + \mathcal{L}_{ghost}.$$

where $\mathcal{L}_{matter}$ contains generic matter fields φ_i in given representations $\mathcal{R}_i$ of the gauge group G.

3-1) Is BRST an on-shell or off-shell symmetry?

3-2) Work out BRST transformations for the axial gauge fixing, i.e. for $\mathcal{G}(A) = n \cdot A - \omega = 0$.

3-3) Show that the BRST charge in this gauge Q_n squares to zero, $Q_n^2 = 0$, leading to a BRST cohomology.

3-4) Show that the BRST charge in the Lorenz gauge $Q_{L.G.}$ is given by the following *operator on the fields*:

$$Q_{L.G.} = (\frac{\delta}{\delta \lambda^d} + \omega_d - \frac{g}{2} f_{abd} c^a \bar{c}^b) c^d, \qquad (*)$$

where $\frac{\delta}{\delta \lambda^d}$ is the operator generating infinitesimal gauge transformation (with gauge parameter λ^d) and acts on the gauge fields and all the other matter fields.

HINTS: How does the ghost transform under gauge transformations? Recall the commutation of ghost and anti-ghost fields.

3-5) Is the expression $(*)$ gauge-fixing dependent? Work out the form of the BTRST charge operator (similar to the one given above) in the axial gauge.

4) $2d$ Yang-Mills Theory. **15/100**

Consider the Yang-Mills Theory in two dimensions

$$S = \int d^2x \left[-\frac{1}{2} Tr F_{\mu\nu}^2 + i\bar{\psi} \not{D} \psi \right].$$

Compute its β-function and compare it with the $4d$ case.