May 17, 2016

Midterm Exam of QFT-II course

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This exam is take-home and is to be returned by 9am Monday May 23rd.
Total mark: 100.

1) Formalities with path integral formulation.

1-1) Show that for any theory with fields $\phi_\alpha(x)$ and action $S[\phi_\alpha]$

$$\langle \frac{\delta S}{\delta\phi_\alpha(x)}\frac{\delta S}{\delta\phi_\beta(y)}\rangle = i\langle\frac{\delta^2 S}{\delta\phi_\alpha(x)\delta\phi_\beta(y)}\rangle$$

1-2) Use the above for a $\lambda\phi^4$ theory to compute the two-point function $\langle\phi^3(x)\phi^3(y)\rangle$.

1-3) Use the above in QED to compute $\langle J_\mu(x)J_\nu(y)\rangle$ where $J^\mu(x)=\bar\psi\gamma^\mu\psi(x)$ is the current.

2) $\lambda\phi^4$ at two loops. Consider the massless 4d $\lambda\phi^4$ theory:

$$L=\frac{1}{2}(\partial\phi)^2-\frac{1}{4!}\lambda\phi^4.$$

2-1) Compute the counter-term for two and four point functions in leading and subleading divergence levels.

2-2) Compute two-loop γ and β-functions.

2-3) Compute the anomalous scaling dimension for the scalar field ϕ.

2-4) Compute the energy (scale) dependence of renormalized coupling λ at two loops.

2-5) Does this theory have a non-trivial stable IR/UV fixed points?

2-6) Compute the two-loop γ-function for the mass operator $\phi^2(x)$.

3) Marginal deformations of QED. Consider QED deformed by marginal operators δL:

$$L=-\frac{1}{4}F_{\mu\nu}^2-i\bar\psi\gamma^\mu\partial_\mu\psi-m\bar\psi\psi-e\bar\psi\gamma^\mu\psi A_\mu+\delta L.$$

3-1) For

$$\delta L=\frac{1}{\xi}(\partial^\mu A_\mu)^2,$$

3-1-1) compute the one-loop γ-function of the above operator.
3-1-2) Find running of the ξ parameter with energy. Discuss the physical meaning of this γ-function.

3-2) For

$$\delta L=\lambda(A^\mu A_\mu)^2,$$

3-2-1) compute the one-loop γ-function of the above operator.
3-2-2) Find running of the λ parameter with energy.

Note: *Neither of these two operators are gauge invariant. How does this affect the results or computations?*

4) $\lambda\phi^6$ theory in 3d. Consider the Lagrangian

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{1}{4!}\lambda\phi^4 - \frac{1}{6!}g\phi^6.$$

4-1) Show that this theory is power-counting renormalizable.

4-2) Write down the Callan-Symanzik equation for 2, 4, 6 and 8 point functions.

4-3) Compute the one-loop γ and β-functions for λ and g couplings.

4-4) Does the theory have non-trivial stable fixed points?

4-5) Let us suppose that we perturb the theory by irrelevant coupling $\kappa\phi^8$ at energy scale Λ. Compute the one-loop β function for coupling κ and find its running with energy scale μ.

5) Coleman-Weinberg potential in Yukawa-$\lambda\phi^4$ theory. Consider the Lagrangian

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{1}{4!}\lambda(\phi^2 - \phi_0^2)^2 + \bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + g\bar{\psi}\psi\phi.$$

5-1) Compute one-loop effective potential for the scalar field (obtained through integrating out ϕ and ψ field fluctuations). Use the $\overline{\text{MS}}$ regularization scheme.

5-2) Compute the one loop β-functions for the couplings g and λ and write the scale-dependent coupling.

5-3) Rewrite the effective potential in terms of renormalized couplings.

5-4) Compute the one loop γ-function for ϕ and ψ fields.

5-5) Compute one-loop γ-function for bosonic and fermionic mass operators $\phi^2(x)$ and $\bar{\psi}\psi(x)$.