فیزیک نوترینو

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix} \qquad \sigma^\mu = (1, \vec{\sigma}) \qquad \bar{\sigma}^\mu = (1, -\vec{\sigma})$$

قرارداد notation Moril etal

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{در نوتاسیون کتاب } (-1)}$$

مستقل از نوتاسیون

$m=0 \rightarrow$ هلیسیتی

$$\psi_R = \frac{1+\gamma_5}{2}\psi \qquad \psi_L = \frac{1-\gamma_5}{2}\psi$$

مستقل از نوتاسیون

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] = \frac{i}{2}\begin{bmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{bmatrix}$$

$$\sigma^{\mu\nu} = \sigma^\mu \bar{\sigma}^\nu \qquad \bar{\sigma}^{\mu\nu} = \bar{\sigma}^\mu \sigma^\nu \qquad \mu \neq \nu$$

$$\psi_R = \begin{pmatrix} 0 \\ \eta_{\dot\alpha} \end{pmatrix} \qquad \psi_L = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix} \qquad \alpha, \dot\alpha = 1, 2$$

ماتریس های ۲×۲ مختلط

$SL(2,C)$

$SU(2,C)$ ← مولدهای ..!!؟

کتاب!

$$[\gamma_5, \Sigma^{\mu\nu}] = 0$$

We easily know that each of chiral fermion forms an irreducible representation of Lorentz transformation $SL(2,c)$, as $[\gamma_5, \Sigma^{\mu\nu}] = 0$

$$m(\bar\psi_R\psi_L + \bar\psi_L\psi_R)$$

$$\psi_D = \psi_R + \psi_L = \begin{pmatrix} \xi_\alpha \\ \eta_{\dot\alpha} \end{pmatrix}$$

$$\psi_R = R\psi_D \qquad \psi_L = L\psi_D \qquad R = \frac{1+\gamma_5}{2} \quad L = \frac{1-\gamma_5}{2}$$

$$\psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\bar\psi_D(i\not\partial - m) = \bar\psi_R i\not\partial\psi_R + \bar\psi_L i\not\partial\psi_L - m(\bar\psi_R\psi_L + \bar\psi_L\psi_R)$$

$$(\psi_R)^c = -i\gamma^2 \left(\frac{1+\gamma_5}{2} \psi \right)^*$$

$$R(\psi_R^c) = \frac{1+\gamma_5}{2} \psi_R^c = 0$$

$$L \psi_R^c = \psi_R^c$$

$$\psi_R = \begin{pmatrix} 0 \\ \eta_\lambda \end{pmatrix} \qquad \psi_R^c = \begin{bmatrix} -i\sigma_2 \eta^* \\ 0 \end{bmatrix} = \begin{bmatrix} -\eta_2^* \\ +\eta_1^* \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{\eta}^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta} \bar{\eta}_{\dot\beta} \qquad \bar{\eta}_{\dot\alpha} = (\eta_\alpha)^*$$ نمادگذاری نقطه

$$\psi_L = \begin{bmatrix} \xi^\alpha \\ 0 \end{bmatrix} \qquad \psi_L^c = \begin{bmatrix} 0 \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \xi_2^* \\ -\xi_1^* \end{bmatrix}$$

$$\psi_{M_1} = \psi_R + \psi_R^c = \begin{bmatrix} -i\sigma_2 \eta \\ \eta_\alpha \end{bmatrix}$$

$$\psi_{M_2} = \psi_L + \psi_L^c = \begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix}$$

$$\psi_{M_1}^c = \psi_{M_1} \qquad \psi_{M_2}^c = \psi_{M_2}$$

اسپینور های مایورانا

$$\begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} -i\sigma_2 \eta^* \\ \eta \end{bmatrix}$$

$$\bar{\psi}_D i\cancel{\partial} \psi_D = \bar{\psi}_R i\cancel{\partial} \psi_R + \bar{\psi}_L i\cancel{\partial} \psi_L =$$

$$\frac{1}{2} \left(\bar{\psi}_{M_1} i\cancel{\partial} \psi_{M_1} + \bar{\psi}_{M_2} i\cancel{\partial} \psi_{M_2} \right)$$

جزو تمرینات هست.

تعداد درجات آزادی

جرم صفر $\rightarrow$ انشارد‌ها

جرم دیراک - جرم مایورانا

جرم دیراک :

$$-m_D \bar{\psi}_D \psi_D = -m_D (\bar{\psi}_L \psi_R + H.c.) = m_D \xi^\alpha \eta_\alpha + H.c.)$$

$$-m_D \bar{\psi}_D \psi_D = -m_D(\bar{\psi}_L \psi_R + H.c.) = m_D(\xi^\alpha \eta_\alpha + H.c.)$$

جرم مایورانا:

$$-\frac{1}{2} m_R (\overline{\psi_R^c} \psi_R + H.c.) = \frac{m_R}{2}(\eta^\alpha \eta_\alpha + H.c.)$$

$$-\frac{m_L}{2}(\overline{\psi_L^c} \psi_L + H.c.) = \frac{m_L}{2}(\xi^\alpha \xi_\alpha + H.c.)$$

$$\bar{\psi}_R i\partial\!\!\!/ \psi_R - \frac{m_R}{2}(\overline{\psi_R^c} \psi_R + H.c.) = \frac{1}{2}\bar{\psi}_{M_1}(i\partial\!\!\!/ - m_R)\psi_{M_1}$$

$$\bar{\psi}_L i\partial\!\!\!/ \psi_L - \frac{m_L}{2}(\overline{\psi_L^c} \psi_L + H.c.) = \frac{1}{2}\bar{\psi}_{M_2}(i\partial\!\!\!/ - m_L)\psi_{M_2}$$

انواع ممکن برای جرم‌های نوترینو

ذرات باردار نمی‌توانند جرم مایورانا داشته باشند.

کلی‌ترین حالت

جستاری به ارتباطات

$$\mathcal{L}_m = -\frac{1}{2} m_R \overline{\nu_R^c} \nu_R - \frac{m_L}{2} \overline{\nu_L^c} \nu_L - m_D \bar{\nu}_R \nu_L + H.c.$$

$$\mathcal{L}_m = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \bar{\nu}_R\end{pmatrix} \overbrace{\begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}}^{M} \begin{bmatrix}\nu_L \\ \nu_R^c\end{bmatrix} + H.c.$$

* نشان دهید که $\bar{\nu}_R \nu_L = \overline{\nu_L^c} \nu_R^c$

	ν_L	ν_R^c
$L=$	1	-1

M ماتریس مختلط و متقارن

$$M_\nu = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}$$

$$\begin{cases} m_s = \frac{1}{2}\left\{(m_L + m_R) + \sqrt{(m_R - m_L)^2 + 4m_D^2}\right. \\ m_a = \frac{1}{2}\left\{-(m_L + m_R) + \sqrt{(m_R - m_L)^2 + 4m_D^2}\right. \\ \nu = \sin\theta_\nu \nu_L + \cos\theta_\nu \nu_R^c \end{cases}$$

$$\begin{cases} \nu_s = \sin\theta_\nu\, \nu_L + \cos\theta_\nu\, \nu_R^c \\ \nu_\alpha = \underset{=}{i} \left(\cos\theta_\nu\, \nu_L - \sin\theta_L\, \nu_R^c \right) \end{cases}$$

حذف i

$$tg\, 2\theta_\nu = \frac{2m_D}{m_R - m_L}$$

$m_R = m_L = 0 \quad \leftarrow$ pure Dirac $\qquad m_a = m_s$

$$\mathcal{L}_m = -\frac{1}{2} m_s \overline{\nu_s^c}\, \nu_s - \frac{1}{2} m_a \overline{\nu_a^c}\, \nu_a + H.c.$$

$$N_s = \nu_s + \nu_s^c \qquad N_a = \nu_a + \nu_a^c$$

$$\mathcal{L}_\nu = \frac{1}{2} \left\{ \bar{N}_s (i\cancel{\partial} - m_s) N_s + \bar{N}_a (i\cancel{\partial} - m_a) N_a \right\}$$

تناظر با میدان اسکالر

مختلط

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$$

$$M^2 |\varphi|^2 + \frac{1}{2} \left(m^2 \varphi^2 + m^{*2} \varphi^{*2} \right)$$

$$\mathcal{L}_m = -\frac{1}{2} (\varphi_1 \;\; \varphi_2) \begin{bmatrix} m_1^2 & m_{12}^2 \\ m_{12}^2 & m_2^2 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$$

$$m^2 = 0 \qquad m_1^2 = m_2^2 \;, \quad m_{12}^2 = 0$$

$$m_L = m_R = 0 \qquad m_s = m_a$$

$$\nu_D = \frac{N_s - i N_a}{\sqrt{2}} = \nu_R + \nu_L$$

$$\mathcal{L}_\nu = \bar{\nu}_D (i\cancel{\partial} - m_D) \nu_D$$

$$\langle 0 | \psi_m \bar{\psi}_m | 0 \rangle = \frac{i}{\cancel{p} - m} \quad \leftarrow (6.40)$$

(6.41)

$$\langle 0 | \psi_m \psi_m^{Tr} | 0 \rangle = \frac{i}{\;} C^{-1} \qquad C = i\gamma^0\gamma^2$$

$\partial\!\!\!/ - m$

تمرین * 6.40 را حل کنید.

معادله حرکت (اویلر لاگرانژ) را از لاگرانژی زیر بدست آورید.

$$\bar\psi\, i\partial\!\!\!/\, \psi - \left(\frac{M}{2}\overline{\psi^c}\psi + H.c.\right)$$

مکانیزم های تولید جرم نوترینو

Majorana ?? $0\nu\beta\beta$
Dirac

دیراک خالص

$N_a \quad N_s \qquad m_a = m_s = m_b$

$Y\, \bar\nu_R H^T i\sigma_2 L \longrightarrow$ جرم دیراک خالص

شبه دیراک

pseudo-Dirac

Wolfenstein, 1981

$m_R, m_L \ll m_D \qquad \theta_\nu \simeq \frac{\pi}{4}$

oscillation

$\nu_L \to \nu_R^c$

(Kobayash Lim, 2001)

$$m_s^2 - m_a^2 \simeq 2 m_D (m_R + m_L)$$

$$P(\nu_L \to \nu_R^c) = \sin^2\left(\frac{m_s^2 - m_a^2}{2E}\, t\right)$$

زمان

See saw

(Yanagida, 1979 ; Gell-Mann - Ramond - Slansky, 1979)

Peter Minkowski

$m_R\, \overline{\nu_R^c}\, \nu_R \qquad \cancel{m\, \bar\nu_L \nu_L}$

$\bar{I}_3 = 1$

$\bar{\nu}_L^c \, H_T \, \nu_L$

$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$

$$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}$$

ویژه‌حالات

$m_a \simeq - \frac{m_D^2}{m_R} \ll m_D$

$m_s = m_R \qquad \theta_\nu \simeq \frac{m_D}{m_R} \ll 1$

$N_s \simeq \nu_R + \nu_R^c \qquad N_a \simeq i(\nu_L - \nu_L^c)$

N_s decouples

$m_R \bar{\nu}_R^c \nu_R + Y \bar{\nu}_R \phi \epsilon L$

$m_D = Y\langle \varphi^0 \rangle \qquad Y^2 \varphi \epsilon L \, \frac{1}{M_R} \, \varphi \epsilon L$

آنچه که در کتاب آمده است.

charge conjugation

$$\sum_{a=1}^{3} \frac{c}{M} \varphi^{tr} \epsilon \sigma_a \varphi \begin{pmatrix} \nu_L^{tr} & e_L^{tr} \end{pmatrix} \epsilon \sigma^a C \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

* آیا عبارت فوق تحت $U(1)_Y$ ناورداست؟

~ ~ ~ ~ $SU(2)$ ~ ~ ~ ؟

آیا پس از شکست تقارن الکتروضعیف عبارت فوق به $U(1)_{em}$

احترام می‌گذارد؟ آیا به ν_L جرم می‌دهد؟

$\sigma^\mu = (1, \vec{\sigma})$ راهنمایی

$(\sigma^\mu)_{\alpha\beta} (\sigma_\mu)_{\gamma\delta} = 2 \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$ ← لم

$(\sigma^a)_{\alpha\beta} (\sigma_a)_{\gamma\delta} = ?$

Lepton sector

$\begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix}$ $\begin{pmatrix}\nu_{\mu L}\\ \mu_L\end{pmatrix}$ $\begin{pmatrix}\nu_{\tau L}\\ \tau_L\end{pmatrix}$; $\nu_{eR}\ \nu_{\mu R}\ \nu_{\tau R}\ \ e_R\ \mu_R\ \tau_R$

$m_D \to$ diagonal   دیراک فرمیون

$\bar{\nu}_{eR}\ \nu_{eL}$ .... $\leftarrow L_e$ ثابت

$\nu_e \to \nu_\mu$

All three neutrino masses are degenerate

$\Downarrow$

$U(3)$ symmetry

$m_\nu \sim$ nondegerate $\longleftrightarrow$ GIM mechanism

mixing

Flavor Mixing

Dirac scenario

$$\mathcal{L}_m = (m_D)_{\alpha\beta}\ \bar{\nu}_{\alpha L}\ \nu_{\beta R} + \text{H.c.}$$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = U^\dagger m_D V$$

$$\begin{bmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{bmatrix} = U \begin{bmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{bmatrix}$$

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left(\bar{e}_L \quad \bar{\mu}_L \quad \bar{\tau}_L \right) U \gamma_\mu \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} W^{-\mu}$$

$U \to$ Maki - Nakagawa - Sakata, 1962

$$(\nu_L)_\alpha \longrightarrow (\nu_L)_\beta$$

$$m_D m_D^\dagger = U\, \text{diag}(m_1^2, m_2^2, m_3^2)\, U^\dagger$$

Seesaw

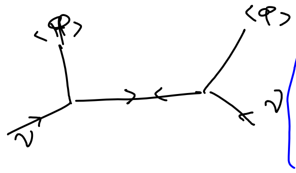
$$\Psi = (\nu_{eL}\ \ \nu_{\mu L}\ \ \nu_{\tau L}\ \ (\nu_{eR})^c\ \ (\nu_{\mu R})^c\ \ (\nu_{\tau R})^c)$$

$$m_\nu = \begin{pmatrix} m_L & m_D^{Tr} \\ m_D & m_R \end{pmatrix}$$

$$m_L^{Tr} = m_L \qquad m_R^{Tr} = m_R$$

$$-f^D_{\alpha\beta} \left(\overline{\nu}_{\alpha L} \quad \overline{l}_{\alpha L} \right) \tilde{\varphi}\, \nu_{\beta R} - (m_R)_{\alpha\beta}\, \overline{\nu^c_{\alpha R}}\, \nu_{\beta R}$$

$$(m_D)_{\alpha\beta} = f^{D*}_{\alpha\beta} \times \frac{v}{\sqrt{2}}$$



$$\begin{bmatrix} iI & -i m_D^{tr} m_R^{*-1} \\ m_R^{-1} m_D^* & I \end{bmatrix} m_\nu \begin{bmatrix} iI & m_D^\dagger m_R^{-1} \\ -i m_R^{-1} m_D & I \end{bmatrix}$$

$$= \begin{bmatrix} m_D^{Tr} (m_R^*)^{-1} m_D & 0 \\ 0 & m_R^* \end{bmatrix}$$

$$m_{\nu L} = m_D^{Tr} (m_R^*)^{-1} m_D$$

$$U^{Tr} m_{\nu L} U = \mathrm{diag}(m_1, m_2, m_3)$$

$$m_{\nu L}^\dagger m_{\nu L} = U\, \mathrm{diag}(m_1^2, m_2^2, m_3^2)\, U^\dagger$$

magnetic dipole moment

$$(\Box + m_i^2)\nu_i = 0 \qquad E_i = \sqrt{\vec{p}^{\,2} + m_i^2} \simeq |\vec{p}|^2 + \frac{m_i^2}{2E}$$

$$i \frac{d}{dt} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E_2} & \\ & & \frac{m_3^2}{2E_3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E} \end{bmatrix} U^\dagger \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

$$\begin{bmatrix} \nu_e(t) \\ \nu_\mu(t) \\ \nu_\tau(t) \end{bmatrix} = e^{-\frac{i}{2E} m_\nu m_\nu^\dagger t} \begin{bmatrix} \nu_e(0) \\ \nu_\mu(0) \\ \nu_\tau(0) \end{bmatrix}$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = \begin{bmatrix}\frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E_2} & \\ & & \frac{m_3^2}{2E_3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

$$i\frac{d}{dt}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = U\begin{bmatrix}\frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E}\end{bmatrix}U^\dagger\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

$$\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\\ \nu_\tau(t)\end{bmatrix} = e^{-\frac{i}{2E} m_\nu m_\nu^\dagger t}\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{bmatrix}$$

$$= U\begin{pmatrix} e^{-\frac{im_1^2 t}{2E}} & 0 & 0\\ 0 & e^{-\frac{im_2^2 t}{2E}} & 0\\ 0 & 0 & e^{-\frac{im_3^2 t}{2E}}\end{pmatrix}U^\dagger\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{bmatrix}$$

$$P(\nu_\alpha\to\nu_\beta) = \left|\sum_i U_{\beta i}\, e^{-\frac{im_i^2}{2E}t}\, U_{\alpha i}^*\right|^2$$

$$= \left|\sum_i U_{\beta i}\, e^{-\frac{i\Delta m_{i1}^2}{2E}t}\, U_{\alpha i}^*\right|^2$$

$$\Delta m_{i1}^2 = m_i^2 - m_1^2$$

∴ Dirac mass ten

$$\mathcal{L}_c = \frac{g}{\sqrt{2}}\,\bar{\ell}_{iL}\,\gamma_\mu\,\nu_{jL}\,U_{ij}^{PMNS}\,W^{-\mu}$$

non Unitary

چند فاز

$$Z:\ \frac{(n-1)(n-2)}{2}$$

$\bar{\nu}^c\, m_\nu\, \nu$

↳ symmetric

$$\frac{n(n-1)}{2}$$

$$\begin{cases}\frac{(n-1)(n-2)}{2}\\ n-1 \quad \leftarrow \text{ماجورانا}\end{cases}$$

$$m_\nu = U_{PMNS} \underbrace{m_\nu^{diag}} U_{PMNS}^T$$

$$\text{dia}\left(m_1 e^{i\alpha_1},\ m_2 e^{i\alpha_2}\ \ m_3 e^{i\alpha_3}\right)$$

[illegible]

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2}{4E} t\right)$$

$$P(\nu_e \longrightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

CPT

$$P(\nu_\alpha \to \nu_\beta) = P(\bar\nu_\beta \to \bar\nu_\alpha)$$

CP

$$P(\nu_\alpha \longrightarrow \nu_\beta) = P(\bar\nu_\alpha \longrightarrow \bar\nu_\beta)$$

T

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

در دو نوترینو $\qquad P(\nu_\mu \to \nu_\mu) = P(\nu_e \longrightarrow \nu_e)$

Averaging

$$\overline{P}(\nu_e \to \nu_\mu) = \frac{1}{2} \sin^2 2\theta$$

$$\overline{P}(\nu_e \longrightarrow \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta$$

$$\bar{P}(\nu_e \to \nu_e) \geq \frac{1}{2}$$

$$\bar{P}(\nu_e \to \nu_e) \geq \frac{1}{n}$$ تمرین دهم

$$\bar{P}(\nu_\alpha \to \nu_\beta) = \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2$$

$$\bar{P}(\nu_\alpha \to \nu_\alpha) = \sum_i |U_{\alpha i}|^4$$

$$\sum_i |U_{\alpha i}|^2 = 1$$

$$\sum_i |U_{\alpha i}|^4 + 2 \sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 = 1$$

$$\sum_{ij} \left(|U_{\alpha i}|^2 - |U_{\alpha j}|^2\right)^2 \geqslant 0$$

$$(n-1)\left(\sum_i |U_{\alpha i}|^4\right) - 2 \sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \geq 0$$

$$\sum_i |U_{\alpha i}|^4 \geq \frac{1}{n}$$

معادله‌ی حرکت　　　معادله‌ی اویلر_لاگرانژ

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - \frac{1}{2}\left(m \bar{\psi}^c \psi + H.c.\right)$$

$$\psi^c = -i\gamma^2 \psi^* \qquad \bar{\psi}^c = -i \psi^T \gamma^2 \gamma^0$$

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + \frac{i}{2}\left(m \psi^T \gamma^2 \gamma^0 \psi + m^* \psi^\dagger \gamma^0 \gamma^2 \psi^*\right)$$

$$\not{\partial}\psi + m^* \gamma^2 \psi^* = 0$$

$$\psi = \int \frac{d^3p}{(2\pi)^3} \left(e^{ip\cdot x} u\, a + e^{-ip\cdot x} v\, a^{c\dagger}\right)$$

$$a = a^c$$

Boost

$$2 \begin{bmatrix} 0 & p\cdot\sigma \end{bmatrix}$$

$$u_s = \begin{bmatrix} \sqrt{p\cdot\sigma}\,\xi_s \\ \sqrt{p\cdot\bar\sigma}\,\eta_s \end{bmatrix} \qquad m^* v_s = i\gamma \begin{bmatrix} p\cdot\bar\sigma & 0 \end{bmatrix} u$$

$$m v_s = i\gamma^2 \begin{bmatrix} 0 & p\cdot\sigma^* \\ p\cdot\bar\sigma^* & 0 \end{bmatrix} \begin{bmatrix} \sqrt{p\cdot\sigma}^*\,\xi^* \\ \sqrt{p\cdot\bar\sigma}^*\,\eta^* \end{bmatrix} = i m \gamma^2 \begin{bmatrix} \sqrt{p\cdot\sigma}^*\,\eta_s^* \\ \sqrt{p\cdot\bar\sigma}^*\,\xi_s^* \end{bmatrix}$$

$$v_s = \begin{bmatrix} i\sigma_2 \sqrt{p\cdot\bar\sigma}^*\,\xi_s^* \\ -i\sigma_2 \sqrt{p\cdot\sigma}^*\,\eta_s^* \end{bmatrix} = \begin{pmatrix} \sqrt{p\cdot\sigma}\,(i\sigma_2\,\xi_s^*) \\ -\sqrt{p\cdot\bar\sigma}\,(i\sigma_2\,\eta_s^*) \end{pmatrix}$$

$$\xi = \eta \times \alpha$$ اسپین مشخص

فرمیون مایورانا

$$\psi^c = e^{i\beta}\psi \Rightarrow \eta^* = e^{i\beta}\xi$$
$$\xi^* = e^{i\beta}\eta$$

$$\Rightarrow |\eta^*|^2 = |\xi|^2 \quad , \quad \eta^* = e^{i\beta}\xi$$

$$\sum_s \xi_s \xi_s^\dagger = 1 \qquad \sum_s \eta_s \eta_s^\dagger = 1 \qquad \sum \xi_s \eta_s^\dagger = e^{i\beta}$$

$$\sum_s \eta_s \xi_s^\dagger = e^{-i\beta}$$

$$u\bar u = \begin{bmatrix} m e^{i\beta} & p\cdot\sigma \\ p\cdot\bar\sigma & m e^{-i\beta} \end{bmatrix} \qquad v\bar v = \begin{bmatrix} -m e^{-i\beta} & p\cdot\sigma \\ p\cdot\bar\sigma & -m e^{i\beta} \end{bmatrix}$$

$$u v^T = \begin{bmatrix} -i m\sigma_2 & e^{i\beta}\, p\cdot\sigma\, i\sigma_2 \\ p\cdot\bar\sigma(-i\sigma_2)e^{-i\beta} & i\sigma_2 m \end{bmatrix}$$

$$\langle T(\psi\bar\psi)\rangle = \frac{\begin{bmatrix} m e^{i\beta} & p\cdot\sigma \\ p\cdot\bar\sigma & m e^{-i\beta} \end{bmatrix}}{p^2 - m^2} \qquad \psi = \psi^c \qquad \psi = -\psi^c$$

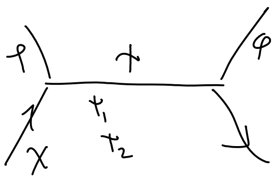
$$\langle T(\psi\psi^T)\rangle = \frac{\begin{bmatrix} m & p\cdot\sigma\, e^{i\beta} \\ p\cdot\bar\sigma\, e^{-i\beta} & m \end{bmatrix}}{p^2 - m^2}\, C$$
$$C \to i\gamma^0\gamma^2$$

$$\psi = \psi^c \longrightarrow \beta = 0 \qquad \mathcal{L} = \bar\psi i\partial\!\!\!/\psi - \frac{m\,\bar\psi^c\psi + \text{H.c.}}{2}$$
$$\psi = -\psi^c \longrightarrow \beta = \pi$$

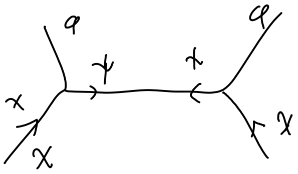
$$\psi^c = C\bar\psi^T \qquad \psi^T = \pm\psi^{c\,T} \Rightarrow \pm\bar\psi\, C^T$$

آیا فاز معنای فیزیکی دارد؟ ـ بله!

$$\underbrace{\varphi}_{\text{real}} \; \bar{\chi} \; \psi$$

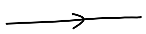


$$\frac{\bar{\chi}\begin{bmatrix} 0 & p\cdot\sigma \\ p\cdot\bar{\sigma} & 0 \end{bmatrix}\chi}{p^2-m^2}$$



$$\chi^T \frac{m\,C}{p^2-m^2} \chi$$

می‌توانیم برحسب چپ وراست بسط دهیم.

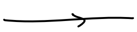


$$\frac{i\not{p}}{p^2}$$

$$\frac{-i\not{p}}{p^2} \quad imC \quad \frac{i\not{p}^T}{p^2}$$

$$\frac{-i\not{p}}{p^2}(imC)\frac{i\not{p}^T}{p^2}(-imC)\frac{i\not{p}}{p^2}$$

$$\vdots$$



$$\frac{i\not{p}}{p^2-m^2} = \frac{i\not{p}}{p^2} + \frac{i\not{p}}{p^2}\frac{m^2}{p^2} + \dots$$

$$\frac{imC}{p^2-m^2} = \text{-----}$$

$$\frac{c}{M} \quad \varphi^{Tr}\varepsilon\,\sigma_a\varphi \quad L^T\varepsilon\,\sigma_a C L$$

$$\varphi = \begin{bmatrix}\varphi^+ \\ \varphi^0\end{bmatrix} \qquad L = \begin{bmatrix}\nu_L \\ e_L\end{bmatrix}$$

$$(\sigma^\mu)_{\alpha\beta}\,(\sigma_\mu)_{\gamma\delta} = 2\,\varepsilon_{\alpha\gamma}\,\varepsilon_{\beta\delta}$$

$$(\sigma^\mu)_{\alpha\beta}\,(\sigma_\mu)_{\gamma\delta} = \delta_{\alpha\beta}\,\delta_{\gamma\delta} - (\sigma^a)_{\alpha\beta}\,(\sigma^a)_{\gamma\delta}$$

$$\sum_a \chi_1^T\sigma_a\chi_2 \;\; \chi_3^T\sigma_a\chi_4 = \chi_1^T\chi_2\;\chi_3^T\chi_4$$

$$-2\,\underbrace{\chi_1^T\varepsilon\chi_3}_{-(4)\,\chi_3^T\varepsilon\chi_1} \quad \chi_2^T\varepsilon\chi_4$$

$$\chi_1^T = \varphi^T\varepsilon \qquad \chi_2 = \varphi \qquad \chi_3^T = L^T\varepsilon \qquad \chi_4 = CL$$

$$= \frac{c}{M}\left[\underbrace{\varphi^T \epsilon \varphi}_{\text{singlet}} \; \underbrace{L^T \epsilon C L}_{\text{singlet}} + 2 \underbrace{L^T \epsilon \varphi}_{\text{singlet}} \; \underbrace{\varphi^T \epsilon C L}_{\text{singlet}} \right]$$

unitary gauge $\varphi = \left(0 \quad \frac{H+v}{\sqrt{2}}\right)$

0

همان چیزی که من نوشته بودم.

چندتا اصطلاح

$P(\nu_\alpha \to \nu_\beta)$ appearance probability $(\alpha \neq \beta)$

$P(\nu_\alpha \to \nu_\alpha)$ survival probability

$1 - P(\nu_\alpha \to \nu_\alpha)$ disappearance probability

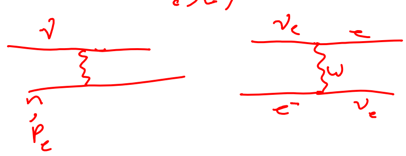
پاراگراف اول بخش ۶-۴

$\overline{P(\nu_e \to \nu_e)} > \frac{1}{n}$ $\quad n=3 \leftarrow SM$

R. Davis $\quad |U_{e1}|^2 = |U_{e2}|^2 = |U_{e3}|^2 = \frac{1}{3}$

$V(x)$

اثر ماده



Only ν_e ???

Elastic forward scattering

$$\frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \; \bar{e} \gamma^\mu (1-\gamma_5) \nu_e$$

Fierz transformation

anti-Commuting

$$\frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma_\mu (1-\gamma_5) \nu_e \; \bar{e} \gamma^\mu (1-\gamma_5) e$$

* تمرین ۱-۸ جزوه‌ی نوترینوی بس

همان تمرین را برای حالتی که محیط قطبیده است حل کنید.

$\bar{\nu}_L \gamma_0 \nu_L V_c(x)$

$V_c(x) = \sqrt{2}\, G_F N_e(x)$

Dispersion relation   معادله ی پاشندگی

$\mathcal{L} = \bar{\nu}_L i \cancel{\partial} \nu_L - \bar{\nu}_L \gamma_0 \nu_L V_c(x)$

معادله ی اویلر-لاگرانژ ↓

$\cancel{p}\, \nu_L - V_c \gamma_0 \nu_L = 0$

$\left((E - V_c)\gamma^0 - \vec{p}\cdot\vec{\sigma}\right) \nu_L = 0$

$\left((E - V_c)\gamma^0 + \vec{p}\cdot\vec{\sigma}\right) \times$

$\left[(E - V_c)^2 - |\vec{p}|^2\right] \nu_L = 0$

$E = V_c + |\vec{p}|$ ← معادله ی پاشندگی

* نشان دهید که در حضور جمله جرمی (دیراک - مایورانا)

معادله ی پاشندگی به صورت زیر تبدیل می شود .

$E = V_c + \sqrt{\vec{p}^2 + m^2}$

نکته ی لفظی

Majorana   مایورانا

Jarlskog   یارلسکوگ

تقریب فرانسبیتی

$E \simeq V_c + |\vec{p}| + \frac{m^2}{2|\vec{p}|}$.

$$i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \left\{ U \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m^2_{21}}{2E} & 0 \\ 0 & 0 & \frac{\Delta m^2_{31}}{2E} \end{bmatrix} \cdot U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$a = \sqrt{2}\, G_F N_e$

فاز کلی

تقلیل به آنالیز دو نوترینو

$$\frac{\Delta m_{31}^2}{2E} \gg \frac{\Delta m_{21}^2}{2E}$$

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U_{PMNS} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$U_{PMNS} =$$

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$$\times \operatorname{diag}\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1\right)$$

$$\theta_{13} \ll 1 \qquad s_{23} \simeq c_{23} = \frac{\pi}{4}$$

$$\nu_e \overset{?}{\simeq} c_{12}\,\nu_1 + \sin\theta_{12}\,\nu_2$$

$$\nu' = -\sin\theta_{12}\,\nu_1 + \cos\theta_{12}\,\nu_2$$

$$i\frac{d}{dt}\begin{bmatrix} \nu_e \\ \nu' \end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{2}\,G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta_{12} \\ \frac{\Delta m^2}{4E}\sin 2\theta_{12} & \frac{\Delta m^2}{2E}\cos 2\theta_{12} \end{bmatrix}}_{H(t)} \begin{bmatrix} \nu_e \\ \nu' \end{bmatrix}$$

mild time dependence $\longleftrightarrow$ adiabacity

$$U_m(t)^\dagger H(t)\, U_m(t) = \begin{bmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{bmatrix}$$

$$U_m(t) = \begin{bmatrix} \cos\theta_m(t) & \sin\theta_m(t) \\ -\sin\theta_m(t) & \cos\theta_m(t) \end{bmatrix}$$

$$\begin{bmatrix} \nu_e \\ \nu' \end{bmatrix} = U_m(t) \begin{bmatrix} \nu_{m1} \\ \nu_{m2} \end{bmatrix}$$

$$E_{1,2}(t) = \frac{1}{2}\left(\sqrt{2}\,G_F N_e(t) + \frac{\Delta m^2}{2E}\cos 2\theta \pm \sqrt{\left(\sqrt{2}\,G_F N_e(t) - \frac{\Delta m^2}{2E}\cos 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E}\sin 2\theta\right)^2}\right)$$

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}\,G_F N_e(t)}$$

$$\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}\, G_F N_e(t)$$

$$N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E\, G_F \sqrt{2}} \longrightarrow \theta_m = \frac{\pi}{4}$$

$$i\frac{d}{dt}\begin{pmatrix}\nu_{m1}\\ \nu_{m2}\end{pmatrix} = \left\{\begin{pmatrix}E_1(t) & 0\\ 0 & E_2(t)\end{pmatrix} + \begin{pmatrix}0 & i\dot{\theta}_m\\ -i\dot{\theta}_m & 0\end{pmatrix}\right\}\begin{pmatrix}\nu_{m1}\\ \nu_{m2}\end{pmatrix}$$

adiabatic condition $|\dot{\theta}_m| \ll |E_2 - E_1|$

resonance : لحظه خطرناک

منطقهٔ رزنانس

$$\tan 2\theta_m > 1$$

پهنای منطقهٔ رزنانس $= \Delta x$

$$\sqrt{2}\, G_F \frac{dN_x}{dx}\Delta x = \frac{\Delta m^2}{2E}\sin 2\theta$$

$$N\big|_{resonance} = \frac{\Delta m^2 \cos 2\theta}{2E\,\sqrt{2}G_F}$$

$$\Delta x = \frac{\tan 2\theta}{\frac{1}{N}\frac{dN}{dx}} = \frac{\tan 2\theta}{\frac{d\ln N}{dx}}$$

شرط آدیاباتیک $\quad \Delta x\,(E_2 - E_1)\big|_{resonance} \gg 1$

$$\frac{\tan 2\theta}{\frac{d\ln N_e}{dx}\Big|_{res}} \gg \frac{E}{\Delta m^2 \sin 2\theta}$$

بقای انرژی چه؟

resonance = تشدید

$$\nu_{m1}(t) = T\, e^{-i\int_0^t E_1(t')dt'}\,\nu_{m1}(0)$$

$$\nu_{m2}(t) = T\, e^{-i\int_0^t E_2(t')dt'}\,\nu_{m2}(0)$$

$$P(\nu_e \to \nu_e) = \left| \langle \nu_e | T \exp\left(-i \int_0^t H(t')\, dt'\right) | \nu_e \rangle \right|^2 =$$

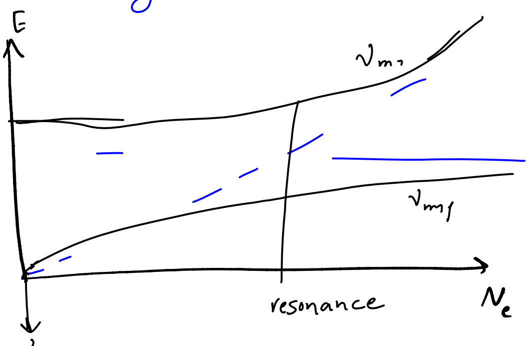
$$\left| \begin{pmatrix} \cos\theta_m(t) & \sin\theta_m(t) \end{pmatrix} \begin{bmatrix} e^{-i\int_0^t E_1(t')dt'} & 0 \\ 0 & e^{-i\int_0^t E_2(t')dt} \end{bmatrix} \begin{bmatrix} \cos\theta_m(0) \\ \sin\theta_m(0) \end{bmatrix} \right|^2 =$$

$$\overline{P}(\nu_e \to \nu_e) = \cos^2\theta_m(t)\, \cos^2\theta_m(0) + \sin^2\theta_m(t)\, \sin^2\theta_m(0)$$

MSW

Wolfenstein, 1978

Mikheyev & Smirnov, 1985



$$\overline{P}(\nu_e \to \nu_e) = (1 - P_{jump}) \left(\cos^2\theta_m(t) \cos^2\theta_m(0) + \sin^2\theta_m(t) \sin^2\theta_m(0) \right) + P_{jump} \left(\sin^2\theta_m(t) \cos^2\theta_m(0) + \cos^2\theta_m(t) \sin^2\theta_m(0) \right)$$

$\theta_m(t) = \theta \leftarrow$ خلاء

$\theta_m(0) = \frac{\pi}{2} \leftarrow$ بسیار غلیظ

$$\overline{P}(\nu_e \to \nu_e) \simeq \sin^2\theta + P_{jump} \cos^2\theta$$

پرش تنها در منطقه تشدید می‌شود:

$$N_e = N_e^{res} + \underbrace{\frac{dN_e}{dx} x}_{\text{خطی}}$$

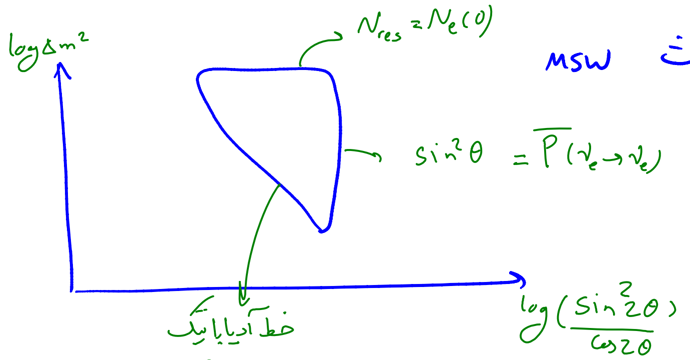
$$P_{j...} = \exp\left(-\pi \;\; \Delta m^2 \sin^2 2\theta \right.$$

$$\underbrace{{}_{jump} - 1 \left(\frac{}{4} \; \frac{}{4E\cos 2\theta} \; \frac{d\ln N_e}{dx}\Big|_{res} \right)}$$

Landau - Zener formula

Landau, 1932

Zener, 1932



$$\frac{\Delta m^2}{4E}\frac{\sin^2 2\theta}{\cos 2\theta} = cte$$

ψ_1 و ψ_2 فرمیون وایل

آنتی دنگ

$$\underbrace{\left(\psi_1^T C \psi_2\right)^T} =$$

$$= + \psi_2^T C \psi_1$$

$$(m_\nu)_{\alpha\beta} = \underbrace{\overbrace{\frac{(m_\nu)_{\alpha\beta} + (m_\nu)_{\beta\alpha}}{2}}^{\text{symmetric}}}_{(M_\nu)_{\alpha\beta}} + \underbrace{\overbrace{\frac{(m_\nu)_{\alpha\beta} - (m_\nu)_{\beta\alpha}}{2}}^{\text{anti-symmetric}}}_{(A_\nu)_{\alpha\beta}}$$

$$(m_\nu)_{\alpha\beta} \; \nu_{L\alpha}^T C \nu_{L\beta} = (M_\nu)_{\alpha\beta} \; \nu_{L\alpha}^T C \nu_{L\beta}$$

ماتریس جرمی مایورانا متقارن است.

$$m_\nu = U_{PMNS} \begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix} U_{PMNS}^T$$

$$U_{PMNS} = V_{23} V_{13} V_{12} \Phi$$

$$V_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \qquad V_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{bmatrix}$$

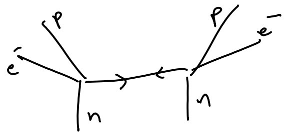
$$V_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ & & \end{bmatrix} \qquad \Phi = \mathrm{diag}(1, e^{i\alpha_2}, e^{i\alpha_3})$$

L 0 0 1 J

۹ پارامتر

Oscillation $\begin{cases} \Delta m_{21}^2 , \Delta m_{31}^2 \\ \theta_{12}, \theta_{13}, \theta_{23} \end{cases}$

δ

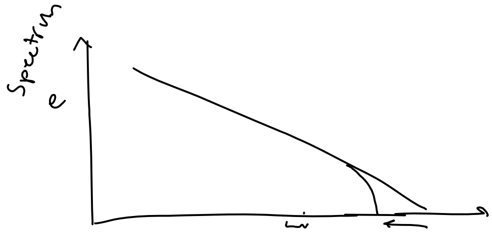


$\mathcal{M}_\alpha$ $(m_\nu)_{ee}$

$= m_1 e^{i\alpha_1} U_{e1}^2 + m_2 e^{i\alpha_2} U_{e2}^2 + m_3 e^{i\alpha_3} U_{e3}^2$

$|\mathcal{M}|^2$

Kurie plot



$m_{eff} = m_1$

Katrin $\longrightarrow m_1 \longrightarrow$ فاز

$\begin{cases} \text{Solar neutrino} \\ \text{KamLAND} \end{cases}$ Δm_{12}^2, θ_{12}

atmospheric Δm_{31}^2, $\theta_{23} \approx \frac{\pi}{4}$

K2K
MINOS

$\begin{cases} \text{CERN - Gransasso} \\ \text{T2K} \end{cases}$ $\nwarrow \nu_\tau$

Nova

CHOOZ $\longrightarrow$ $\begin{cases} \text{Daya Bay} \\ \text{Double-CHOOZ} \\ \text{Reno} \end{cases}$

$\theta_{13} = ?$ $\theta_{13} < 10^\circ$

$\theta_{13} = 8.5^\circ$ $\delta = ??$

8th of March 2012

منابع نوترینو

راکتور $\bar{\nu}_e$ → انرژی ؟

خورشیدی ν_e → " ؟

اتمسفری ν_e , ν_μ " ؟

ابرنواختر

relic neutrino

AGN - GRBs

نوترینوی زمینی

آشکارسازها

Super Kamiokande SK

↑

Kamiokande - IMB

SNO ← D_2O

ICECUBE
BAICAL
ANTARES

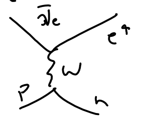
ANITA

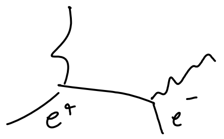
نوترینوهای خورشیدی را چه گونه آشکارسازی کنند ؟

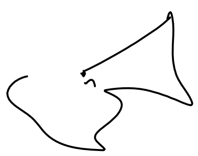
" راکتوری " " " " ؟

KamLAND

$\bar{\nu}_e \longrightarrow e^+$







200 μsec

$n + p \to d + \gamma$

SNO in Canada D_2O

SK مشترک با

ES : $\nu_e + e^- \longrightarrow \nu_e + e^-$

دیاگرام‌ها را بکشید

NC : $\nu + D \to \nu + n + p$

$n + D \leftarrow \gamma + \ldots$

سمی 6 صحبت کنم

$n + H \to D$

خارج

CC: $\nu_e + D \longrightarrow e^- + p + p$

Seesaw

type I ??

type II ?

type III ?

Neutrino Oscillation in Three Generation Scheme

$\Delta m^2_{21}, \quad \Delta m^2_{31}$

$\theta_{12} \quad \theta_{13} \quad \theta_{23}, \delta$

$$\Delta m^2_{31} = \Delta m^2_{atm} \simeq 2 \times 10^{-3}\ eV^2$$

$$\Delta m^2_{21} = \Delta m^2_{solar} \simeq 7 \times 10^{-5}\ eV^2$$

Vacuum neutrino oscillation due to Δm^2_{31}

$$\frac{\Delta m^2_{21}}{E} L = 3.6 \times 10^{-2} \frac{\Delta m^2_{21} / 7 \times 10^{-5}\ eV^2}{E / 1\ GeV} \frac{L}{100\ km} \ll 1$$

$$\frac{\Delta m^2_{31}}{E} L = 1.0 \times \frac{\frac{\Delta m^2_{31}}{2 \times 10^{-3}\ eV^2}}{\frac{E}{1\ GeV}} \frac{L}{100\ km} \gtrsim 1$$

K2K KEK to Kamiokande

$\begin{cases} \text{OPERA} \\ \text{ICARUS} \end{cases}$

MINOS $\longrightarrow$ MINOS + 2013-2016

$$P(\nu_\alpha \to \nu_\beta) = \Big| \underbrace{U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2}}_{\delta_{\alpha\beta} - U^*_{\alpha 3} U_{\beta 3}} + U^*_{\alpha 3} U_{\beta 3} e^{\frac{-i \Delta m^2_{31}}{2E} t} \Big|^2$$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4 (1 - |U_{\mu 3}|^2) |U_{\mu 3}|^2 \sin^2 (\frac{\Delta m^2_{31}}{4E} t)$$

$$= 1 - 4 (1 - \sin^2\theta_{23} \cos^2\theta_{13}) \sin^2\theta_{23} \cos^2\theta_{13} \sin^2 \frac{\Delta m^2_{31} t}{4E}$$

$$\simeq 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31} t}{4E}$$

$$P(\nu_e \to \nu_e) = 1 - 4(1 - |U_{e3}|^2)|U_{e3}|^2 \sin^2(\frac{\Delta m^2_{31}}{4E} t)$$

$$= 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31}}{4E} t \simeq 1$$

CHOOZ $\quad E \sim$ few MeV $\quad P(\nu_e \to \nu_e) \simeq 1$

$L \sim 1\,km$

$$\sin^2 2\theta_{13} \leq 0.2$$

$$P(\nu_\mu \to \nu_\tau) = 4 |U_{\mu 3}|^2 |U_{\tau 3}|^2 \sin^2(\frac{\Delta m^2_{31}}{4E} t)$$

$$= \sin^2 2\theta_{23} \cos^4\theta_{13} \sin^2 \frac{\Delta m^2_{31}}{4E} t$$

$$\simeq \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31}}{4E} t$$

$$P(\nu_\mu \to \nu_e) = 4|U_{e3}|^2 |U_{\mu 3}|^2 \sin^2 \frac{\Delta m^2_{31} t}{4E}$$

$$\simeq \sin^2 2\theta_{13} \sin^2\theta_{23} \sin^2 \frac{\Delta m^2_{31}}{4E} t \simeq 0$$

θ_{13} و Δm^2_{31} نوترینوهای جوی $\quad \left\{ \begin{array}{l} \nu_e \\ \nu_\mu \to \nu_\tau \end{array} \right.$

اثر δ ؟؟

Vacuum oscillation due to Δm^2_{21}

$\frac{\Delta m^2_{31} L}{2E} \gg 1 \quad \leftarrow$ averaging

$$\frac{\Delta m^2_{21}}{E} L = 7.2 \frac{\Delta m^2_{31} / 7\times 10^{-5} eV^2}{\frac{E}{5 MeV}} \left(\frac{L}{100 km}\right) \simeq 1$$

$$\frac{\Delta m^2_{31}}{E} L = 2\times 10^2 \frac{\frac{\Delta m^2_{31}}{2\times 10^{-3} eV^2}}{\frac{E}{5 MeV}} \frac{L}{100 km} \gg 1$$

KamLAND

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\beta 1} U^*_{\alpha 1} + U_{\beta 2} U^*_{\alpha 2} e^{-i\frac{\Delta m^2_{21} t}{2E}} \right|^2$$

$$+ |U_{\beta 3}|^2 |U_{\alpha 3}|^2$$

$$+ |U_{\beta 3}|^2 |U_{\alpha 3}|^2$$

$$P(\nu_\alpha \to \nu_\alpha) = (1 - |U_{\alpha 3}|^2)^2 \, P_{eff}(\nu_\alpha \to \nu_\alpha) + |U_{\alpha 3}|^4$$

$$P_{eff}(\nu_\alpha \to \nu_\alpha) = 1 - 4 \frac{|U_{\alpha 2}|^2 (1 - |U_{\alpha 2}|^2 - |U_{\alpha 3}|^2)}{(1 - |U_{\alpha 3}|^2)^2} \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

$$S = \cos^4\theta_{13} \, S_{eff}(\theta_{12}, \Delta m_{21}^2) + \sin^4\theta_{13}$$

$$\simeq \cos 2\theta_{13} \, S_{eff}(\theta_{12}, \Delta m_{21}^2)$$

$$S_{eff}(\theta_{12}, \Delta m_{21}^2) = 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

برای $\bar{\nu}_e$ هم همینطور

CP violation in neutrino oscillation

$$A_{\alpha\beta}^{CP} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

$$A_{\alpha\beta}^{T} = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$

$$A_{\alpha\beta}^{CP} = A_{\alpha\beta}^{T}$$

CP: $U_{\alpha i} \longrightarrow U_{\alpha i}^{*}$

$$A_{\alpha\beta}^{T} = -A_{\beta\alpha}^{T}$$

$$A_{\alpha\beta}^{CP} = -A_{\beta\alpha}^{CP} \implies A_{\alpha\alpha}^{CP} = 0$$

$$P(\nu_\alpha \to \nu_\alpha) = P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$$

$$\sum_{\beta = e,\mu,\tau} P(\nu_\alpha \to \nu_\beta) = \sum_{\beta = e,\mu,\tau} P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 1$$

$$A_{e\mu}^{CP} = A_{\tau e}^{CP} \qquad A_{\mu\tau}^{CP} = A_{e\mu}^{CP}$$

$$A_{e\mu}^{CP} = A_{\mu\tau}^{CP} = A_{\tau e}^{CP} \equiv A^{CP}$$

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$$A^{CP}_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = -4\sum_{i<j}$$

$$\mathrm{Im}\left[U_{\alpha i} U^*_{\beta i} U_{\beta j} U^*_{\alpha j}\right] \sin\frac{\Delta m^2_{ij} t}{2E}$$

$$J_{\alpha\beta,ij} = \mathrm{Im}\left[U_{\alpha i} U^*_{\beta i} U_{\beta j} U^*_{\alpha j}\right]$$

$$J_{\alpha\beta,ij} = -J_{\beta\alpha,ij} \qquad J_{\alpha\beta,ji} = -J_{\alpha\beta,ij}$$

$$J_{\alpha\beta,12} + J_{\alpha\beta,22} + J_{\alpha\beta,32} = J_{\alpha\beta,12} + J_{\alpha\beta,32} = 0$$

$$J_{\alpha\beta,13} = J_{\alpha\beta,32}$$

همین طور

همین طور

$$J_{e\mu,ij} = J_{\mu\tau,ij} = J_{\tau e,ij}$$

Jarlskog پارامتر

$$J \equiv J_{e\mu,12}$$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta$$

$$A^{CP} = -4J\left\{\sin\frac{\Delta m^2_{12} t}{2E} + \sin\frac{\Delta m^2_{23} t}{2E} + \sin\frac{\Delta m^2_{31} t}{2E}\right\}$$

Re-phasing invariant

تغییرناپذیر

$$\nu_\alpha \to U_{\alpha\beta}\,\nu_\beta \qquad \to \bar{l}_\alpha \gamma^\mu \frac{1-\gamma_5}{2} \nu_\alpha W_\mu$$

$$l_{L\alpha} \to U_{\alpha\beta}\, l_{L\beta}$$

$$m_\nu^\dagger m_\nu \longrightarrow U^\dagger m_\nu^\dagger m_\nu U$$

$$m_l^\dagger m_l \longrightarrow U^\dagger m_l^\dagger m_l U$$

$$\bar{l}_{R\beta}\, m_l\, l_{L\beta}$$

$$[m_\nu^\dagger m_\nu, m_l^\dagger m_l] \longrightarrow U^\dagger [m_\nu^\dagger m_\nu, m_l^\dagger m_l] U$$

$\mathrm{Det}\left[[m_\nu^\dagger m_\nu , m_l^\dagger m_l] \right]$ ← ناوردا

ناوردا تحت چیه؟

* تمرین: به کار گرفتن پارامتریسیون استاندارد

$$\mathrm{Det}\left[[m_\nu^\dagger m_\nu , m_l^\dagger m_l] \right]$$

را محاسبه کنید.

Observing CP-violation

۱) $\Delta m^2_{21} \neq 0$ یا $\Delta m^2_{31} \neq 0$

۲) $\theta_{ij} \neq 0$ or $\sin\delta \neq 0$

۳) no time averaging

Resonant Matter Oscillation

اثر ماده + سه نسل → حلّ تحلیلی ندارد

$$S = P(\nu_e \to \nu_e) = \cos^4\theta_{13}\, S_{eff}(\theta_{12}, \Delta m^2_{21}; a_{eff}) + \sin^4\theta_{13}$$

$$a_{eff} \equiv \cos^2\theta_{13}\, a(x) = \cos^2\theta_{13} \sqrt{2}\, G_F N_e(x)$$

Lim, 1987; Smirnov, 1992; Shi and Schramm 1992

$$\frac{\Delta m^2_{21}}{2E} , \sqrt{2}\, G_F N_e \ll \frac{\Delta m^2_{31}}{2E}$$

$$H \simeq \mathrm{diag}\left(0, 0, \frac{\Delta m^2_{31}}{2E}\right)$$

$$\tilde{\nu}_e \equiv \cos\theta_{12}\, \nu_1 + \sin\theta_{12}\, \nu_2$$

$$\tilde{\nu}_\mu = -\sin\theta_{12}\, \nu_1 + \cos\theta_{12}\, \nu_2$$

$$\xi \equiv \begin{pmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \\ \nu_3 \end{pmatrix} = \begin{pmatrix} \cos\theta_{12}\nu_1 + \sin\theta_{12}\nu_2 \\ -\sin\theta_{12}\nu_1 + \cos\theta_{12}\nu_2 \\ \nu_3 \end{pmatrix}$$

$$\xi = \begin{pmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \\ \tilde{\nu}_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta_{13}\,\nu_e - \sin\theta_{13}\, e^{-i\delta}\, \tilde{\nu}_\tau \\ \tilde{\nu}_\mu \\ \sin\theta_{13}\, e^{i\delta}\, \nu_e + \cos\theta_{13}\, \tilde{\nu}_\tau \end{pmatrix}$$

$$\bar{\nu}_\tau \qquad \sin\theta_{13} e^{i\delta}\nu_e + \cos\theta_{13}\,\tilde{\nu}_\tau$$

$$\tilde{\nu}_\mu \equiv \cos\theta_{23}\,\nu_\mu - \sin\theta_{23}\,\nu_\tau \simeq \frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)$$

$$\tilde{\nu}_\tau \equiv \sin\theta_{23}\,\nu_\mu + \cos\theta_{23}\,\nu_\tau \simeq \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau)$$

$$\theta_{23} \simeq \frac{\pi}{4}$$

فقط $\bar{\nu}_e$ اثر ماده را حس می‌کند.

$$i\frac{d\xi}{dt} = \left\{ V_{12}\begin{pmatrix} 0 & & \\ & \Delta_{12} & \\ & & \Delta_{13}\end{pmatrix}V_{12}^\dagger + V_{13}^\dagger \begin{pmatrix} a(t) & & \\ & 0 & \\ & & 0\end{pmatrix} V_{13} \right\}\xi$$

$$= \left\{ \begin{pmatrix} \Delta_{12}S_{12}^2 + a\,C_{13}^2 & \Delta_{12}S_{12}C_{12} & 0 \\ \Delta_{12}S_{12}C_{12} & \Delta_{12}C_{12}^2 & 0 \\ 0 & 0 & \Delta_{13} + a\,S_{13}^2 \end{pmatrix} + \begin{bmatrix} 0 & 0 & a\,S_{13}C_{13} \\ 0 & 0 & 0 \\ a\,S_{13}C_{13} & 0 & 0 \end{bmatrix} \right\}\xi$$

$$\Delta_{12} \equiv \frac{\Delta m_{21}^2}{2E} \qquad \Delta_{13} \equiv \frac{\Delta m_{31}^2}{2E}$$

$$C_{ij} = \cos\theta_{ij} \qquad S_{ij} = \sin\theta_{ij} \qquad \frac{\Delta m_{21}^2}{2E},\ \sqrt{2}\,G_F N_e \ll \frac{\Delta m_{31}^2}{2E}$$

$$i\frac{d\xi}{dt} \simeq \begin{pmatrix} \Delta_{12}S_{12}^2 + a\,C_{13}^2 & \Delta_{12}S_{12}C_{12} & 0 \\ \Delta_{12}S_{12}C_{12} & \Delta_{12}C_{12}^2 & 0 \\ 0 & 0 & \Delta_{13} + a\,S_{13}^2 \end{pmatrix}\xi$$

$$a_{eff} = \cos^2\theta_{13}\,a(t)$$

$$S = \left| C_{13}^2 A_{eff}(\bar{\nu}_e \to \bar{\nu}_e) + S_{13}^2 e^{-i(\Delta_{13} + a S_{13}^2)t} \right|^2$$

$$\simeq \cos 2\theta_{13}\, S_{eff}(\theta_{12}, \Delta m_{21}^2; a_{eff})$$

$$e^{-i\Delta_{13}t} \longrightarrow 0$$

$$\frac{a\sin\theta_{13}}{\frac{\Delta m_{31}^2}{2E}} \lesssim 10^{-2} \qquad \text{خطای ترتیب}$$

$$\pi^+ \to \mu^+ + \nu_\mu$$

$\pi^+ \to \mu^+ + \nu_\mu$

$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$

$$R = \frac{(\nu_\mu + \bar{\nu}_\mu)_{obs} / (\nu_e + \bar{\nu}_e)_{obs}}{(\nu_\mu + \bar{\nu}_\mu)_{pred} / (\nu_e + \bar{\nu}_e)_{pred}} \sim 0.6$$

ν_e خیر
با انتظار سازگار است.

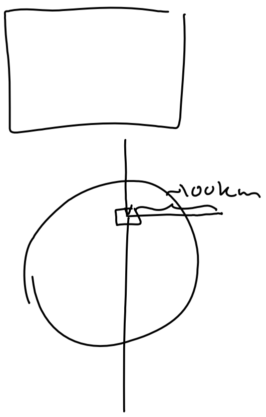
ν_μ کمتر
از آنچه باید مشاهده

$$|\Delta m^2_{31}| = (1.3 - 3.0) \times 10^{-3}\ eV^2$$

$$\sin^2 2\theta_{23} > 0.9$$

Sub-GeV

Multi-GeV



Solar neutrino oscillation

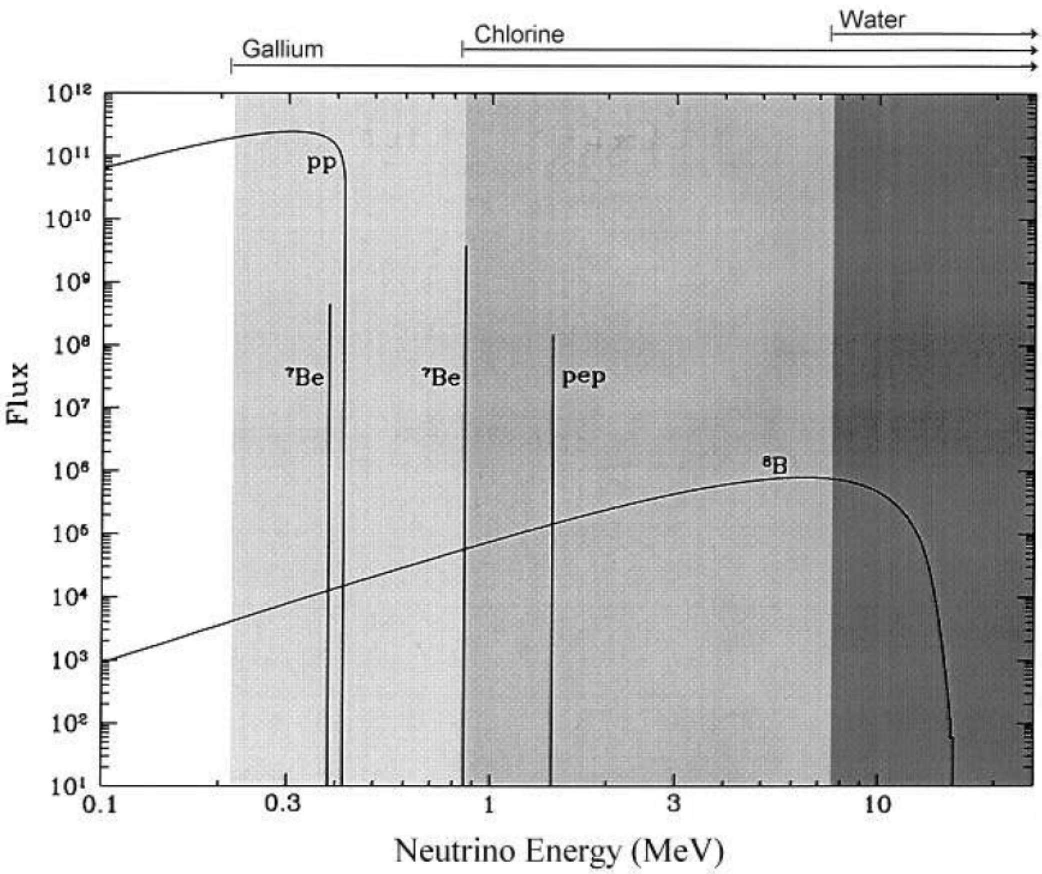
$p + p \to D + e^+ + \nu_e$ $\qquad \langle E_\nu \rangle = 0.26\ MeV$

⋮

$e^- + {}^7Be \to {}^7Li + \nu_e$ $\qquad E_\nu = 0.86\ MeV$

⋮

${}^8B \to {}^8Be + e^+ + \nu_e$ $\qquad \langle E_\nu \rangle = 7.2\ MeV$



$4e^- + 4p \rightarrow {}^4He + 2\nu_e + \gamma$ (26.73 MeV)

R. Davis   Cl experiment

C_2Cl_4 Homestake mine

(

Cl experiment

$\nu_e + {}^{37}Cl \longrightarrow {}^{37}Ar + e^-$

$E_{th} = 0.81$ MeV

B , Be

SK   (۲

$\nu + e^- \rightarrow \nu + e^-$ $E_{th} = 5$ MeV

Ga experiment   SAGE , GNO   (۳

$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$

pp   Be   B

SNO

$\nu_e + D \to e^- + p\,p$

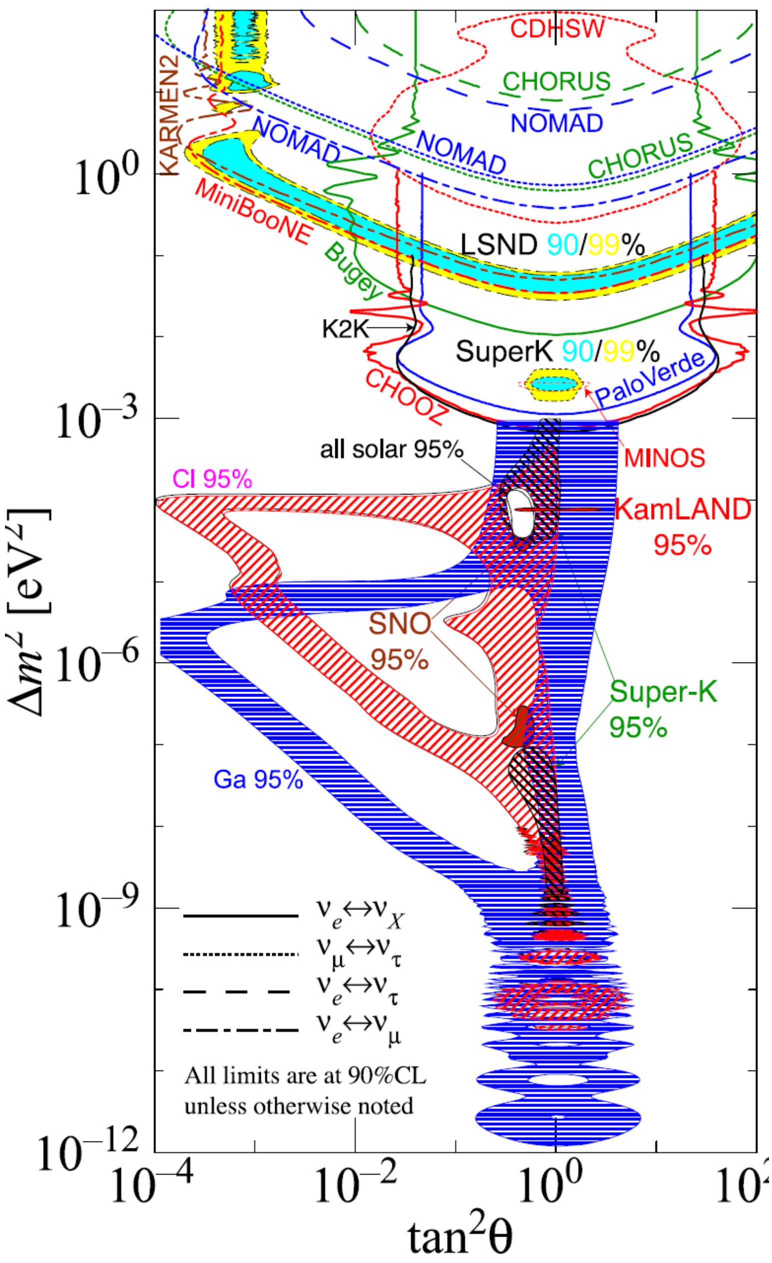
$\nu + D \to \nu + n + p$

↑ B neutrinos

radio-chemical   Cl & Ga experiments

observed / predicted < 1

NC   SNO معجز



$$\varphi_{SK} = \varphi_{SSM}\{S + (1-S)\underbrace{R_\sigma}\}$$

$R_\sigma \sim \frac{1}{6}$

$$\varphi_{SNO}(CC) = \varphi_{SSM} \times S$$

$\varphi_{SNO}(CC) = \varphi_{SSM} \times S$

هم
ν_μ, ν_e

ES ؟

$\varphi_{SK} \simeq 2.4 \times 10^{6}\ cm^{-2}\, sec^{-1}$

$\varphi_{SNO}(CC) \simeq 1.8 \times 10^{6}\ cm^{-2}\, sec^{-1}$

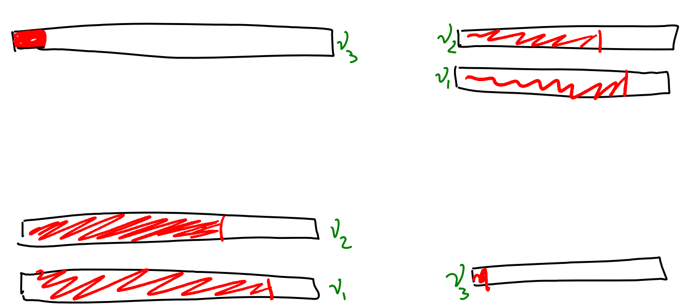
$\varphi_{SSM} = 5.4 \times 10^{6}\ cm^{-2}\, sec^{-1}$

$\cancel{\nu_e \to \nu_s}$

$\Delta m^2_{21}\ ,\ \Delta m^2_{31}$ علامت

normal

inverted



neutrino mass scheme

normal / inverted

hierarchical / quasi-degenerate

بنابه تعریف $m_2 > m_1$

normal $m_3 > m_2, m_1$

inverted $m_1, m_2 > m_3$

$$m_2 - m_1 = \frac{m_2^2 - m_1^2}{m_2 + m_1}$$

$m_1 > 0.1\,eV \to \Delta m = m_2 - m_1 < 10^{-3}\,eV$

$m_1 = 0 \quad m_2 - m_1 = \sqrt{m_2^2 - m_1^2}\ ,\ m_3 - m_1 = \sqrt{m_3^2 - m_1^2}$

$m_1 = 0 \qquad \dfrac{m_2 - m_1}{m_3 - m_1} = \dfrac{\sqrt{\Delta m^2_{sol}}}{\sqrt{\Delta m^2_{atm}}} \ll 1$

$m_1 > 0.1\,eV \qquad \dfrac{m_3 - m_1}{m_3 + m_1}\ ,\ \dfrac{m_2 - m_1}{m_1 + m_2} \ll 1$

نمی‌دانیم که کدام‌یک از این scheme ها درست است.

در واقع کیهان‌شناسی quasi-degenerate را زیر سوال برده.

نوسان در خلاء

$$P(\nu_\alpha \longrightarrow \nu_\beta) = \left| U_{\alpha 1} U_{\beta 1}^{*} + U_{\alpha 2} U_{\beta 2}^{*} e^{i\Delta_{12}} + U_{\alpha 3} U_{\beta 3}^{*} e^{i\Delta_{13}} \right|^2$$

یکی از $\theta_{ij} = 0$ یا $\delta = 0$ یا π

تحت $\Delta_{ij} \to -\Delta_{ij}$ $\Longrightarrow$ $P(\nu_\alpha \to \nu_\beta)$ ناوردا

کلی

$$\left\{ \begin{array}{l} \Delta_{ij} \to -\Delta_{ij} \\ \delta \to -\delta \end{array} \right. \qquad P(\nu_\alpha \to \nu_\beta) =$$

در خلاء — ناوردا

با این همه معنی دارد که بپرسیم $m_3 < m_2$ یا $m_3 > m_2$

اثر ماده در خورشید:

Δ_{12} علامت

برای آن که علامت Δ_{13} را یابیم باید به اثرات subdominant ماده، در آزمایش‌های long baseline و مشاهدات نوترینوهای جوی هم حساس شویم

INO ?

Indian Neutrino Observatory