۲۰۱۰/۱۰/۳۰
۰۱:۲۷ ب.ظ

$$J_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu}$$

$$\vec{K}^{i} = J^{0i} = L^{0i} + S^{0i}$$

$$\vec{J}^{i} = \varepsilon^{ijk} J_{jk}$$

$$[S_{\mu\nu}, P_\alpha] = 0$$

$$[S_{\mu\nu}, L_{\alpha\beta}] = 0$$

$$[S_{\mu\nu}, S_{\alpha\beta}] \neq 0$$

$$\left.\begin{array}{l} [S^{0i}, \vec{S}^{k}] \neq 0 \\ [k^{i}, \vec{S}^{k}] \neq 0 \end{array}\right\} \text{در حالت کلی}$$

اما اگر جت خود الحاقی باشد J تغییر می‌کند.

$$[Q, \varphi] = i\delta\varphi \qquad \left([Q, \varphi] = i\delta\varphi\right)^{\dagger}$$

$$Q\varphi - \varphi Q = i\delta\varphi \xrightarrow{\text{Conjugate}} \varphi^{\dagger}Q^{\dagger} - Q^{\dagger}\varphi^{\dagger} = -i\delta\varphi^{\dagger}$$

$$[Q^{\dagger}, \varphi^{\dagger}] = +i\delta\varphi^{\dagger}$$

در مورد بارهای معمولی که سروکار داریم. اما باز هم این $\leftarrow Q = Q^{\dagger}$

ابهام در ترتیب را حل نمی‌کند.

$$\underset{\overline{AB}}{\text{کلاسیک}} \xrightarrow{\text{کوانتمی}} \begin{array}{c} \frac{AB + B^{\dagger}A^{\dagger}}{2} \\ \frac{BA + A^{\dagger}B^{\dagger}}{2} \end{array}$$

$$\cos\alpha\, AB + \sin\alpha\, BA + H.C.$$

مدل استاندارد

$$SU(3) \times SU(2)_L \times U(1)_Y$$

Glashow - Weinberg - Salam

(GWS)

Glashow, 1961; Weinberg, 1967; Salam 1968

در برهمکنش های ضعیف

$$\left\{\begin{array}{l} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu \\ \pi^- \to \mu^- \bar{\nu}_\mu \\ n \to p + e^- + \bar{\nu}_e \end{array}\right.$$

left-handed leptons

right-handed anti leptons

$$J_\mu(x) \equiv J_\mu^{+}(x) = \bar{\nu}_{eL}(x)\gamma_\mu e_L(x) = \bar{\nu}_e(x)\gamma_\mu \frac{1-\gamma_5}{2} e(x)$$

$$J_\mu^{\dagger}(x) \equiv J_\mu^{-}(x) = \bar{e}_L \gamma_\mu \nu_{eL} = \frac{1}{2}\bar{e}\gamma_\mu(1-\gamma_5)\nu_e$$

$$L = \frac{1-\gamma_5}{2}\begin{pmatrix}\nu_e \\ e^-\end{pmatrix} = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L$$

$$\tau^{+} = \frac{\tau^1 + i\tau^2}{2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\tau^{-} = \frac{\tau^1 - i\tau^2}{2} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$J_\mu^{+} = \bar{L}\gamma_\mu \tau^{+} L$$

$$J_\mu^{-} = \bar{L}\gamma_\mu \tau^{-} L$$

مشابه موارد بالا و با به کارگیری ماتریس τ^3 می‌توان نوشت

$$J_\mu^{3} = \bar{L}\gamma_\mu \frac{\tau^3}{2} L = \frac{1}{2}\bar{\nu}_e\gamma_\mu\nu_e - \frac{1}{2}\bar{e}_L\gamma_\mu e_L$$

$W_\mu^{\pm} \qquad W_\mu^3$

$$J_\mu^i(x) = \bar{L} \gamma_\mu \frac{\tau^i}{2} L$$

$$T^i = \int J_0^i(x)\, d^3x$$

$$[T^i, T^j] = i\, \varepsilon^{ijk}\, T^k$$

Levi - Civita تانسور

$$\varepsilon^{123} = 1$$

$$R = \frac{1+\gamma_5}{2} e = e_R$$

$$J_\mu^3 \neq J_\mu^{em}$$ وسوسه

$$J_\mu^{em} = -\bar{e}\gamma^\mu e = -\bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R$$

$$J_\mu^{em} = q \bar{\psi} \gamma_\mu \psi$$

$$Q = \int J_0^{em} d^3x = -\int \bar{e} \gamma_0 e\, d^3x =$$

$$-\int (e_L^\dagger e_L + e_R^\dagger e_R)\, d^3x$$

$$[Q, T^3] = ? \qquad [Q, T^1] = ? \qquad [Q, T^2] = ?$$

$SU(2)_L$ و $U(1)_{em}$ همزمان نمی‌توانند تقارن های مدل باشند.

$SU(2)_L : \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ دوتایی $\qquad\qquad U(1)_{em} : \begin{pmatrix} \nu_e \\ e \end{pmatrix}$ دوتایی نیست

$U(1)_Y$

$$\frac{Y}{2} = Q - T^3 = \int d^3x \left(-\frac{1}{2} \nu_{eL}^\dagger \nu_{eL} - \frac{1}{2} e_L^\dagger e_L - e_R^\dagger e_R\right)$$

$$[Q - T^3, T^i] = 0$$

$U(1)_Y$ و $SU(2)$ تقارن های همزمان مدل می‌توانند باشند.

$$Q = T^3 + \frac{Y}{2}$$

ذرات	Q	(T, T^3)	Y
ν_e, ν_μ, ν_τ	0	$(\frac{1}{2}, +\frac{1}{2})$	-1
e_L, μ_L, τ_L	-1	$(\frac{1}{2}, -\frac{1}{2})$	-1
e_R, μ_R, τ_R	-1	0	-2

نکته ی تاریخی (دقت زیاد تا اشتباه نشود) !!!

Nakano - Nishijima - Gell-Mann رابطه ی

{ Nakano & Nishijima, 1953

(Gell-Mann 1953

$Y = B + S$

p دیوورد $= ?$

K^+ دیوورد $= ?$

$K^+ = |u\bar{s}\rangle$

$SU(2)$ ایزواسپین با $SU(2)$ پیمانه‌ای فرق دارد.

به دیدگاه مدرن بازگردیم.

$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad R = e_R$

$SU(2)_L: \quad L \to L' = e^{-i\frac{\alpha^i \tau^i}{2}} L \,, \quad R \to R' = R$

$U(1)_Y: \quad L \to L' = e^{i\beta/2} L \,, \quad R \to R' = e^{i\beta} R$

$\alpha^i = \alpha^i(x) \qquad \beta = \beta(x)$

$SU(2)_L \times U(1)_Y$ لاگرانژی ناوردا است

$$\mathcal{L}_F = \bar{L}\, i\gamma^\mu \left(\partial_\mu - ig \frac{\vec{\tau}}{2}\cdot\vec{A}_\mu + \frac{i}{2} g' B_\mu\right) L$$
$$+ \bar{R}\, i\gamma^\mu \left(\partial_\mu + ig' B_\mu\right) R$$

$$D_\mu = \partial_\mu - ig \frac{\vec{\tau}}{2}\cdot\vec{A}_\mu - ig' \frac{Y}{2} B_\mu$$

$$\cancel{m\bar{e}e = m\bar{e}_R e_L + m\bar{e}_L e_R}$$

matter field L, R
gauge field $\vec{A}_\mu, B_\mu$

$$\mathcal{L}_G = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\, \varepsilon_{ijk} A^j_\mu A^k_\nu$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

No mass term for B_μ or $\vec{A}_\mu$

در دنیای واقعی این میدان‌ها جرم دارند.

$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$

$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \quad \leftarrow Q = T_3 + \frac{Y}{2} \to Y_\varphi = ?$

$$\mathcal{L}_S = (D_\mu\phi)^\dagger D^\mu\phi - V(\phi^\dagger\phi)$$

کردآن

$$D_\mu\phi = \left(\partial_\mu - ig \frac{\vec{\tau}\cdot\vec{A}_\mu}{2} - ig' B_\mu\right)\phi$$

$$V(\phi^\dagger\phi) = m^2 \phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$\lambda > 0$

کتاب یک اشتباه تایپی داره :

$$\mathcal{L}_Y = -G_e\left(\bar{L}\Phi R + \bar{R}\Phi^{\dagger}L\right) + h.c.$$

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_Y$$

$SU(2)_L \times U(1)_Y$ شکست خودبخود تقارن

$$m^2 = -\mu^2 \qquad \mu^2 > 0$$

$$\varphi_0 = \langle 0|\varphi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\frac{\tau^3}{2}\varphi_0 = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = -\frac{\varphi_0}{2}$$

$$Y\varphi_0 = \varphi_0$$

بنابراین

$$e^{-i\alpha^3\frac{\tau^3}{2}}\varphi_0 \neq \varphi_0 \qquad e^{-i\beta\frac{Y}{2}}\varphi_0 \neq \varphi_0$$

اما

$$Q = T^3 + \frac{Y}{2}$$

$$Q\varphi_0 = \left(T^3 + \frac{Y}{2}\right)\varphi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix} = 0$$

$$e^{-i\varepsilon Q}\varphi_0 = \varphi_0$$

نتیجه تقارن متناظر با T^3 و Y شکسته اما تقارن متناظر با Q حفظ.

به زبان دیگر

$$\varphi \longrightarrow \varphi' = e^{iT^3\alpha}\varphi\, e^{-iT^3\alpha}$$

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \rightarrow \begin{bmatrix} e^{i\alpha}\varphi_1 \\ e^{-i\alpha}\varphi_2 \end{bmatrix}$$

$$\langle 0|\varphi|0\rangle \neq \langle 0|\varphi'|0\rangle$$

$$T^3|0\rangle \neq 0$$

اما

$$Q|0\rangle = 0$$

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = e^{i\frac{\vec{\tau}\cdot\vec{\xi}}{2v}}\begin{bmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{bmatrix}$$

$$U(\xi) = e^{-i\frac{\vec{\tau}\cdot\vec{\xi}}{2v}}$$

پیمانه ی یکانی

$$\varphi' = U(\xi)\varphi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

$$L' = U(\xi)L$$

$$\vec{A}_\mu' = U(\xi)\vec{A}_\mu U(\xi)^{-1} - \frac{i}{g}\left(\partial_\mu U(\xi)\right)U^{\dagger}(\xi)$$

$$R' = R$$

$$B_\mu' = B_\mu$$

$$\mathcal{L}_F = \bar{L}'i\gamma^\mu\left(\partial_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{A}_\mu' + \frac{i}{2}g'B_\mu'\right)L'$$

$$\mathcal{L}_F = \bar{L}' i\gamma^\mu (\partial_\mu - ig \frac{\vec{\tau}}{2}\cdot \vec{A}'_\mu + \frac{i}{2} g' B'_\mu) L'$$

$$+ \cdots$$

$$\mathcal{L}_s = (D_\mu \varphi)' (D^\mu \varphi)'$$

$$(D_\mu \varphi)' = (\partial_\mu - ig \frac{\vec{\tau}}{2}\cdot \vec{A}'_\mu - \frac{ig' B'_\mu}{2}) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

جملات درجه‌ی ۲

$$\mathcal{L}_{mass} = \frac{v^2}{2} (0 \quad 1)\left(g \frac{\vec{\tau}}{2}\cdot \vec{A}'_\mu + \frac{g'}{2} B'_\mu\right)\left(g \frac{\vec{\tau}}{2}\cdot \vec{A}'_\mu + \frac{g'}{2} B'_\mu\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$\frac{v^2}{8} \left(g^2 A'^1_\mu A'^{1\mu} + g^2 A'^2_\mu A'^{2\mu} + (g A'^3_\mu - g' B'_\mu)^2 \right)$$

$$W_\mu^\pm = \frac{A^1_\mu \mp i A^2_\mu}{\sqrt{2}}$$

$$\frac{v^2}{8} g^2 (A'^1_\mu A'^{1\mu} + A'^2_\mu A'^{2\mu}) = \frac{m_W^2}{2} W^+_\mu W^{-\mu}$$

$$m_W = \frac{gv}{2} \qquad (W^+_\mu)^\dagger = W^-_\mu$$

$$\frac{v^2}{8} (A'^3_\mu \quad B'_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} A'^{3\mu} \\ B'^\mu \end{pmatrix}$$

$$\downarrow \text{ قطری کردن}$$

$$\frac{v^2}{8} (Z_\mu \quad A_\mu) \begin{bmatrix} g^2 + g'^2 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$= \frac{v^2}{8} (g^2 + g'^2) \; Z_\mu Z^\mu + 0 \, A_\mu A^\mu$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A'^3_\mu \\ B'_\mu \end{pmatrix}$$

$\theta_W =$ زاویه‌ی واینبرگ یا زاویه‌ی اختلاط ضعیف

$$\tan\theta_W = \frac{g'}{g}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \qquad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$m_Z = \frac{v}{2}\sqrt{g^2 + g'^2}$$

$$m_Z = \frac{m_W}{\cos\theta_W} \quad \longleftarrow$$ پیش‌بینی !

در پیمانه یکانی

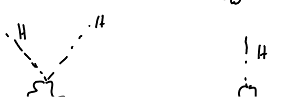
$$V(\varphi^\dagger \varphi) = -\frac{\mu^2 v^2}{4} + \frac{1}{2}(2\mu^2) H^2 + \lambda v H^3$$

$$+ \frac{\lambda}{4} H^4 \qquad m_H = \sqrt{2\mu^2}$$

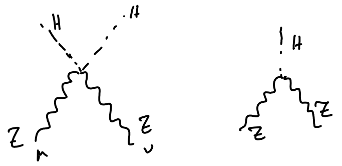
$$\mathcal{L}_s = (D_\mu \phi)(D^\mu \phi) - V(\varphi^\dagger \varphi) =$$

$$\frac{1}{2}\partial_\mu H \partial^\mu H - \frac{M_H^2}{2} H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$

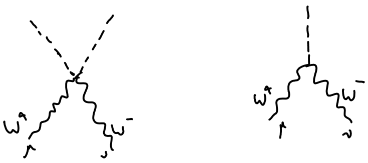
$$+ \frac{g^2}{8}(H^2 + 2Hv)\left[\frac{Z_\mu Z^\mu}{\cos^2\theta_W} + 2 W^+_\mu W^{\mu -}\right]$$



$+\frac{g^2}{8}(H^2+2Hv)\left[\frac{Z_\mu Z^\mu}{\cos^2\theta_W}+2W_\mu^+ W^{-\mu}\right]$



$i\frac{g^2}{2}\frac{\eta_{\mu\nu}}{\cos^2\theta_W}$ $\qquad$ $\frac{ig^2\eta_{\mu\nu}v}{2\cos^2\theta_W}$

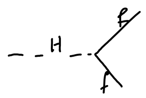


$\frac{ig^2}{2}\eta_{\mu\nu}$ $\qquad$ $\frac{ig^2 v}{2}$

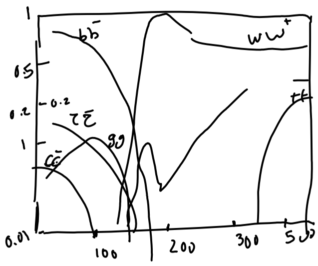
$$\mathcal{L}_y = -G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L)$$

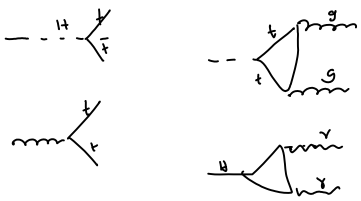
$$= -\frac{G_e v}{\sqrt{2}}\bar{e}e - \frac{G_e}{\sqrt{2}}H\bar{e}e$$

$$m_e = \frac{G_e v}{\sqrt{2}}$$



$-i\frac{m_f}{v}$





نکته اول شکست خود به خود تقارن

تقدم و تأخر زمانی

در انرژی پایین $\qquad U(1)_{em}$ و β-decay
weak interactions

$E \ll m_{EW} \simeq m_W$

$E > m_{EW}$ $\qquad W^+$, W^-, $U(1)_{em}$

در کیهان‌شناسی اول و آخر فرق دارد.

$\mu^2(T)$

همه ی بحث‌های ما در دمای صفر است .

دقت کنید که دمای صفر و انرژی صفر یکی نیستند .

در LHC دما ؟

انرژی ؟

نکته ی دوم

$SU(2) \times U(1)$ یا $U(2)$

$e^{i\beta} \quad e^{i\frac{\tau^i}{2}\alpha^i} \begin{bmatrix} - \\ - \end{bmatrix}$

دو جور ضریب جفت شدگی داریم

g و g'

! نه چهار جور !! و نه یک جور !!

$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \longrightarrow$ یک Hypercharge دارند

اما e_R و e_L هایپرچارج متفاوت دارند .

نکته ی سوم

بالای ص ۶۱

Ghost در پیمانه ی یکانی لزوماً ضروری نیست !

CC and NC

CC ↓ مخفف Charged current

NC ↓ مخفف Neutral current

kinetic

$$\mathcal{L} = \bar{L}\, i\gamma_\mu \partial^\mu L + \bar{R}\, i\gamma_\mu \partial^\mu R =$$

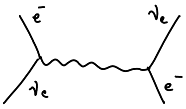
$$\bar{e}\, i\gamma^\mu \partial_\mu e + \bar{\nu}_{eL}\, i\gamma^\mu \partial_\mu \nu_{eL}$$

$$\mathcal{L}_{CC} = g\left(J_\mu^{1} A^{1\mu} + J_\mu^{2} A^{2\mu} \right) =$$

$$\frac{g}{\sqrt{2}}\left(J_\mu^{+} W_\mu^{-} + J_\mu^{+} W^{+\mu} \right)$$

$$J_\mu^{\pm} = \bar{L}\, \gamma_\mu \tau^{\pm} L \qquad \tau^{\pm} = \frac{\tau^1 \pm i\tau^2}{2}$$

$$J_\mu^{+} = \frac{1}{2}\, \bar{\nu}_e \gamma_\mu (1-\gamma_5)\, e$$



$$M = (-)\frac{g^2}{2} J^{+\mu} \frac{i\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2 + i\varepsilon} J^{-\nu}$$

از کجا آمد؟

$q^2 \ll m_W^2$

$M = -i\,\underline{g^2} \quad J^{+\mu} J^{-}$

$2 m_W^2$

$$\mathcal{L}_{cc}^{eff} = -\frac{G_F}{\sqrt{2}}\, 4 J^{+\mu} J^-_\mu = -\frac{G_F}{\sqrt{2}} (\bar{\nu}_e \gamma^\mu (1-\gamma_5) e)$$

$$(\bar{e} \gamma_\mu \underline{(1-\gamma_5)} \nu_e)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2} \qquad G_F = 1.16639 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$$m_W = \frac{g v}{2} \qquad v = \frac{1}{(\sqrt{2} G_F)^{1/2}} = 246\ \mathrm{GeV}$$

$$\frac{v}{\sqrt{2}} \simeq 170\ \mathrm{GeV} \simeq m_t$$

$$\mathcal{L}^{eff}_{cc} = -\frac{G_F}{\sqrt{2}} (\bar{\nu}_e \gamma^\mu (1-\gamma_5) e)(\bar{\mu} \gamma_\mu (1-\gamma_5) \nu_\mu)\ [illegible] ,$$

$$\mathcal{L}_{NC} = g J^3_\mu A^{3\mu} + \frac{g'}{2} J^Y_\mu B^\mu =$$

$$(g \sin\theta_W J^3_\mu + g' \cos\theta_W \frac{J^Y_\mu}{2}) A^\mu$$

$$+ \underbrace{(g \cos\theta_W J^3_\mu - g' \sin\theta_W \frac{J^Y_\mu}{2})}_{\frac{g}{\cos\theta_W} J^Z_\mu} Z^\mu$$

$$J^{em}_\mu = J^3_\mu + \frac{J^Y_\mu}{2}$$

$$g \sin\theta_W J^3_\mu + g' \cos\theta_W \frac{J^Y_\mu}{2} = g' \cos\theta_W J^{em}_\mu$$
$$+ (g \sin\theta_W - g' \cos\theta_W) J^3_\mu$$

$$e = g' \cos\theta_W = g \sin\theta_W$$

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \qquad e = \frac{g g'}{\sqrt{g^2 + g'^2}}$$

$$\frac{g}{\cos\theta_W} J^Z_\mu = g \cos\theta_W J^3_\mu - g' \sin\theta_W \frac{J^Y_\mu}{2} = \frac{g}{\cos\theta_W} (J^3_\mu - \sin^2\theta_W J^{em}_\mu)$$

$$L = \begin{pmatrix} f \\ f' \end{pmatrix}_L \qquad R^f = f_R \qquad R^{f'} = f'_R$$

$$J^Z_\mu = J^3_\mu - \sin^2\theta_W\, J^{em}_\mu =$$
$$\bar{L} \gamma_\mu \frac{\tau^3}{2} L - \sin^2\theta_W (Q_f \bar{f} \gamma_\mu f + Q_{f'} \bar{f}' \gamma_\mu f')$$
$$= a^f_L \bar{f}_L \gamma_\mu f_L + a^f_R \bar{f}_R \gamma_\mu f_R + a^{f'}_L \bar{f}'_L \gamma_\mu f'_L + a^{f'}_R \bar{f}'_R \gamma_\mu f'_R$$

$$a^f_L = \frac{1}{2} - q_f \sin^2\theta_W \qquad a^{f'}_L = -\frac{1}{2} - q_{f'} \sin^2\theta_W$$

$$a^f_R = -q_f \sin^2\theta_W \qquad a^{f'}_R = -q_{f'} \sin^2\theta_W$$

$$J^Z_\mu = \bar{f} \gamma_\mu (C^f_V - C^f_A \gamma_5) f + \bar{f}' \gamma_\mu (C^{f'}_V - C^{f'}_A \gamma_5) f'$$

$$C^f_V = \frac{1}{2}(a^f_L + a^f_R) = \frac{1}{4} - Q_f \sin^2\theta_W$$
$$C^f_A = \frac{1}{2}(a^f_L - a^f_R) = \frac{1}{4}$$
$$C^{f'}_V = \frac{1}{2}(a^{f'}_L + a^{f'}_R) = -\frac{1}{4} - Q_{f'} \sin^2\theta_W$$
$$C^{f'}_A = \frac{1}{2}(a^{f'}_L - a^{f'}_R) = -\frac{1}{4}$$

واپاشی بوزون Z°

$$\mathcal{L}^{Z}_{NC} = \frac{g}{\cos\theta_W} \; J^{Z}_{\mu} Z^{\mu}$$

$$\frac{ig}{\cos\theta_W} \; \gamma_\mu (C_V^f - C_A^f \gamma_5)$$

دستگاه مرکز جرم $\quad P_Z = (m_Z , 0, 0, 0)$

$$P_f = \left(\frac{m_Z}{2} , \vec{k} \right)$$

$$P_{\bar f} = \left(\frac{m_Z}{2} , -\vec{k} \right) \qquad m_f \ll m_Z$$

در روش قطبش خاص برای Z درنظر بگیریم و حل کنیم

$$\varepsilon^\mu(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \varepsilon^\mu(0,1) = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix} \quad \varepsilon^\mu(0,-1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}$$

$$\Pi^{\mu\nu}(p) = \sum_{\sigma} e^{\mu}(p,\sigma) \, e^{\nu *}(p,\sigma)$$

روی همه اسپین‌ها جمع ببندیم و تقسیم بر ۳ کنیم.

$$\Pi^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underbrace{\eta^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}}_{\text{تانسور وایبرگ}}$$

$$\frac{p^\mu p^\nu}{m} - \eta^{\mu\nu}$$ مال ما باید این گونه باشد

تانسور وایبرگ $\quad \eta^{\mu\nu} = \mathrm{diag}(1, 1, 1, -1)$

مال ما $\quad \mathrm{diag}(1, -1, -1, -1)$

(t, x, y, z) تانسور ما

(x, y, z, t) ~ وایبرگ

$$\mathcal{M} = \frac{ig}{\cos\theta_W} \; \bar f' \gamma_\mu (C_V^f - C_A^f \gamma_5) f \; Z^\mu$$

$$|\mathcal{M}|^2 = \left| \frac{g}{\cos\theta_W} \right|^2 Z^\mu Z^\nu \; \bar f' \gamma_\mu (C_V^f - C_A^f \gamma_5) f f^\dagger$$

$$(C_V^{f*} - C_A^f \gamma_5^*) \; \gamma_\nu^\dagger \gamma^{\circ\dagger} f'$$

$$\gamma_\nu^\dagger = \begin{cases} \gamma_\nu & \nu = 0 \\ -\gamma_\nu & \nu = 1,2,3 \end{cases} \Rightarrow \gamma_\nu^\dagger \gamma^{\circ\dagger} = \gamma^\circ \gamma_\nu$$

$$|\mathcal{M}|^2 = \left| \frac{g}{\cos\theta_W} \right|^2 Z^\mu Z^\nu \; \bar f' \gamma_\mu (C_V^f - C_A^f \gamma_5) f \bar f (C_V^{f*} + C_A^{f*} \gamma_5)$$

$$\gamma_\nu f' = \left| \frac{g}{\cos\theta_W} \right|^2 Z^\mu Z^\nu$$

$$\mathrm{Tr}\Big\{ \underbrace{f' \bar f'}_{\cancel{P}_{f'}} \gamma_\mu (C_V^f - C_A^f \gamma_5) \underbrace{f \bar f}_{\cancel{P}_f} (C_V^{f*} + C_A^{f*} \gamma_5) \gamma_\nu \Big\}$$

$$= \left| \frac{g}{\cos\theta_W} \right|^2 \left(\frac{P_Z^\mu P_Z^\nu}{m_Z^2} - \eta^{\mu\nu} \right) P_{f'}^\alpha P_f^\beta$$

$$\Big\{ \left(|C_V^f|^2 + |C_A^f|^2 \right) \Big(\mathrm{Tr}\{ \gamma_\alpha \gamma_\mu \gamma_\beta \gamma_\nu \} + 2\mathrm{Re}[C_V^f C_A^{f*}]$$

$$\mathrm{Tr}\{ \gamma_\nu \gamma_\alpha \gamma_\mu \gamma_\beta \gamma_5 \} \Big)$$

$$P_Z = P_f + P_{\bar f}, \qquad X_\mu \gamma^\mu X_\nu \gamma^\nu = X_\mu X_\nu \left[\frac{\{\gamma^\mu,\gamma^\nu\}}{2} + \frac{[\gamma^\mu,\gamma^\nu]}{2}\right] = X\cdot X$$

$$\Downarrow$$

$$\cancel{P}_{\bar f}\ \cancel{P}_Z\ P_f = 0 + O(m_f^2)$$

$$\underbrace{\mathrm{Tr}\{\gamma_\nu\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_5\}}_{\eta^{\mu\nu}\,\times} = 4i\,\varepsilon_{\nu\alpha\mu\beta} \qquad = 0$$

$$\mathrm{Tr}\{\gamma_\alpha\gamma_\beta\} = 4\eta_{\alpha\beta}$$

$$\mathrm{Tr}\{\underbrace{\gamma_\mu\gamma_\nu}_{2\eta_{\mu\nu}-\gamma_\nu\gamma_\mu}\gamma_\rho\gamma_\sigma\} = 8\eta_{\mu\nu}\eta_{\rho\sigma} - \mathrm{Tr}\{\gamma_\nu\underbrace{\gamma_\mu\gamma_\rho}_{2\eta_{\mu\rho}-\gamma_\rho\gamma_\mu}\gamma_\sigma\} =$$

$$= 8\eta_{\mu\nu}\eta_{\rho\sigma} - 8\eta_{\mu\rho}\eta_{\nu\sigma} + \mathrm{Tr}\{\gamma_\nu\gamma_\rho\underbrace{\gamma_\mu\gamma_\sigma}_{2\eta_{\mu\sigma}-\gamma_\sigma\gamma_\mu}\}$$

$$= 8\eta_{\mu\nu}\eta_{\rho\sigma} - 8\eta_{\mu\rho}\eta_{\nu\sigma} + 8\eta_{\mu\sigma}\eta_{\nu\rho} - \mathrm{Tr}\{\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\mu\}$$

$$\mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\} = 4(\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})$$

$$\overline{|M|^2} = -\frac{1}{3}\left|\frac{g}{\cos\theta_W}\right|^2 \eta^{\mu\nu}\,\mathrm{Tr}\{\cancel{P}_{\bar f}\gamma_\mu \cancel{P}_f \gamma_\nu\}$$

$$(|C_V^f|^2 + |C_A^f|^2) = -\left|\frac{g}{\cos\theta_W}\right|^2 \frac{4}{3}\times\Big(2P_{\bar f}\cdot P_f$$

$$-2P_f\cdot P_{\bar f}\ \underbrace{\eta^{\mu\nu}\eta_{\mu\nu}}_{4}\Big)(\ ) = \frac{8}{3}\left|\frac{g}{\cos\theta_W}\right|^2 \underbrace{P_f\cdot P_{\bar f}}_{\frac{m_Z^2}{2}}(\ )$$

$$= \frac{4}{3}\left|\frac{g}{\cos\theta_W}\right|^2 m_Z^2\,(|C_V^f|^2 + |C_A^f|^2)$$

$$d\Gamma = \frac{1}{2m_Z}\int \frac{d^3P_f}{(2\pi)^3}\,\frac{d^3P_{\bar f}}{(2\pi)^3}\,\frac{1}{2E_f}\,\frac{1}{2E_{\bar f}}$$

$$\overline{|M|^2}\,(2\pi)^4\,\delta(m_Z - |P_f| - |P_{\bar f}|)\,\delta^3(\vec P_f + \vec P_{\bar f})$$

$$= \frac{1}{2m_Z}\,\frac{1}{(2\pi)^2}\int \frac{d\Omega\,|P_f|^2\,dP_f}{4|P_f|^2}\,\delta(m_Z - 2|P_f|)\,\overline{|M|^2}$$

$$= \frac{\overline{|M|^2}}{m_Z\ 16\pi} = \frac{2}{3}\frac{G_F}{\sqrt2}\,m_Z^3\,(|C_V^f|^2 + |C_A^f|^2)$$

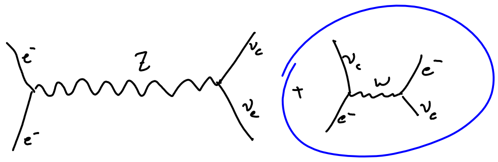
$$\frac{g^2}{8m_Z^2\cos\theta_W} = \frac{G_F}{\sqrt2}$$

$$e^{i(A+B)} = 1 + i(A+B) - \frac{(A+B)^2}{2!} + \dots$$

$$= 1 + i(A+B) - \frac{A^2 + B^2 + 2AB}{2!}$$

$$\int \mathcal{L}\,d^4x = A + B$$

$$A = \int d^4x\,\frac{g}{\cos\theta_W}\,J_\mu^Z Z^\mu$$



$$M = -\frac{i}{2!}\,\frac{g^2}{M_Z^2\cos^2\theta_W}\,J_\mu^Z J^{Z\mu}$$

$$J_\mu^Z = a_L^e \bar{e}_L \gamma_\mu e_L + a_R^e \bar{e}_R \gamma_\mu e_R + a_L^\nu \bar{\nu}_L \gamma_\mu \nu_L$$

$$a^f = T^3 - q_f \sin^2\theta_W$$

$$\downarrow$$

$$a_L^e = -\frac{1}{2} + \sin^2\theta_W \simeq -0.27$$

$$a_R^e = -\sin^2\theta_W = -0.23$$

$$a_L^\nu = \frac{1}{2} = 0.5$$

$$\mathcal{L}_{eff} = \frac{G_N}{\sqrt{2}}\, 4\, J_\mu^Z J^{Z\mu}$$

$$\frac{G_N}{\sqrt{2}} = \frac{g^2}{8 m_Z^2 \cos^2\theta_W}$$

$$\mathcal{L}^{eff} = -\frac{G_F}{\sqrt{2}}\, 4 \left(J_\mu^+ J^{-\mu} + \rho\, J_\mu^Z J^{Z\mu}\right)$$

$$\rho = \frac{G_N}{G_F} = \frac{m_W^2}{M_Z^2\cos^2\theta_W} = 1$$

مشاهده $\rho = 1 \implies$ Higgs doublet

$\varphi \to$ n-plet $\quad \langle \varphi \rangle = v_h \chi_h \quad$ (در مورد مدل استاندارد $\chi = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$)

$$(D_\mu\varphi)^\dagger D_\mu \varphi = \dots + $$

$$\frac{v_h^2}{2}\chi_h^\dagger \left(g \underbrace{\vec{T}_h}\cdot \vec{A}_\mu + g' \frac{Y_h}{2} B_\mu\right)\left(g \vec{T}_h \cdot \vec{A}_\mu + g' \frac{Y_h}{2} B_\mu\right)\chi_h$$

ماتریس‌های جبر در نمایشی که φ عضو آن است

$$Q_h = T_{h3} + \frac{Y_h}{2} \quad \leftarrow$$ این رابطه بازهم برقرار است.

$$Q_h = 0 \to T_{h3} = -\frac{Y_h}{2}$$

$$\mathcal{L}_{mass} = \frac{v_h^2}{2} g^2 \chi_h^\dagger \left(A_\mu^1 A^{1\mu} T_{h1}^2 + A_\mu^2 A^{2\mu} T_{h2}^2\right)\chi_h$$

$$+ \frac{g^2 v_h^2}{2} \underbrace{\left(A_\mu^3 A^{3\mu} + \frac{g'^2}{g^2} B_\mu B^\mu - 2\frac{g'}{g} A_{3\mu} B^\mu\right)}_{\left(A_\mu^3 + \frac{g'}{g} B_\mu\right)^2 = \frac{Z_\mu^2}{\cos^2\theta_W}} \chi_h^\dagger T_{h3}^2 \chi_h$$

نکته مهم

$$\chi_h^\dagger T_{h1}^2 \chi_h = \chi_h^\dagger T_{h2}^2 \chi_h$$

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} = \frac{\sum_h v_h^2 T_{h\perp}^2}{\sum_h v_h^2 T_{h3}^2}$$

بیش از یک هیگز

$$T_{h\perp}^2 = \frac{1}{2}\left(T_{h1}^2 + T_{h2}^2\right) = \frac{1}{2}\left(\vec{T}_h^2 - T_{h3}^2\right) = \frac{1}{2}\left[I_h(I_h+1) - T_{h3}^2\right]$$

اگر همه هیگزها در یک نمایش باشند

$$\rho = \frac{1}{2}\,\frac{I_h(I_h+1) - T_{h3}^2}{T_{h3}^2}$$

$I_h = \frac{1}{2}\,,\; T_{h3} = \pm\frac{1}{2} \longrightarrow \rho = 1$

$I_h = 1\,,\; \begin{Bmatrix} -1 \\ +1 \\ 0 \end{Bmatrix} \longrightarrow \frac{1}{2},\; \frac{1}{2},\; \infty \quad (m_Z = 0)$

و تصحیح کوانتمی می‌گیرد.

سه نسل لپتونی

$L_\alpha \qquad R_\alpha$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \; e_R \;\Big|\; \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \; \mu_R \;\Big|\; \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \; \tau_R$$

$$\mathcal{L}_K = \sum_\alpha \bar{L}'_\alpha\, i\gamma^\mu \partial_\mu L'_\alpha + \sum_\alpha \bar{R}'_\alpha\, i\gamma^\mu \partial_\mu R'_\alpha$$

$$\mathcal{L}^{CC}_{gauge} = \sum_\alpha \frac{g}{\sqrt{2}} \left(\bar{L}'_\alpha \gamma_\mu \tau^- L'_\alpha W^{-\mu} + \bar{L}'_\alpha \gamma_\mu \tau^+ L'_\alpha W^{+\mu} \right)$$

$$\mathcal{L}^{NC+em}_{gauge} = \left(g\sin\theta_W \sum_\alpha \bar{L}'_\alpha \gamma_\mu \frac{\tau^3}{2} L'_\alpha + g'\cos\theta_W \frac{Y_L}{2} \sum_\alpha \bar{L}'_\alpha \gamma_\mu L'_\alpha \right) A^\mu$$

$$+ \left(g\cos\theta_W \sum_\alpha \bar{L}'_\alpha \gamma_\mu \frac{\tau^3}{2} L'_\alpha - g'\sin\theta_W \frac{Y_L}{2} \sum_\alpha \bar{L}'_\alpha \gamma_\mu L'_\alpha - g'\sin\theta_W \frac{Y_R}{2} \sum_\alpha \bar{R}'_\alpha \gamma_\mu R'_\alpha \right) Z^\mu$$

$$- \sum_\alpha e \bar{R}'_\alpha \gamma^\mu R'_\alpha A_\mu$$

$$\mathcal{L}_Y = - \sum_{\alpha\beta} G_{\alpha\beta}\, \bar{L}'_\alpha \phi R'_\beta + \text{H.c.}$$

$$G = U^\dagger G^{diag} V \qquad G^{diag}_{\alpha\beta} = G_\alpha \delta_{\alpha\beta}$$

$$R = V R' \qquad L = U L'$$

$$\mathcal{L}_Y = -\sum_\alpha G_\alpha \bar{L}_\alpha \phi R_\alpha + \text{H.c.}$$

$$\sum_\alpha \bar{R}_\alpha \gamma^\alpha R_\alpha = \sum_\alpha \bar{R}'_\alpha \gamma^\alpha R'_\alpha$$

No lepton flavor violation
in the SM

$$\sum_{\alpha\beta} F_{\alpha\beta}\, \bar{L}_\alpha i\sigma_2 \varphi^* \nu_{R\beta} + \text{H.c.} \quad \longleftarrow \text{flavor violation}$$

$$\sum_\alpha Z'_\mu f_\alpha \bar{L}_\alpha \gamma^\mu \frac{\tau^3}{2} L_\alpha \qquad f_\alpha \neq f_\beta \leftarrow \alpha \neq \beta$$

تعمیم به کوارک ها

quark family	Q	(T, T^3)	Y
u_L, c_L, t_L	$+\frac{2}{3}$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$
d_L, s_L, b_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$
u_R, c_R, t_R	$+\frac{2}{3}$	0	$\frac{4}{3}$
d_R, s_R, b_R	$-\frac{1}{3}$	0	$-\frac{2}{3}$

quark family	Q	(T, T^3)	Y
u_L, c_L, t_L	$+\frac{2}{3}$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$
d_L, s_L, b_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$
u_R, c_R, t_R	$+\frac{2}{3}$	0	$\frac{4}{3}$
d_R, s_R, b_R	$-\frac{1}{3}$	0	$-\frac{2}{3}$

$$SU(3)_c \times \underbrace{SU(2) \times U(1)}_{\text{Color-blind}}$$

کوارک‌ها رنگ دارد

$$q_i \underset{(r\ b\ g)}{} \Longrightarrow q$$

$$Q_{Li} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \qquad U_{Ri} \qquad D_{Ri} \qquad (i = 1, 2, 3)$$

$$\mathcal{L}_F = \sum_{i=1}^{3} \bar{Q}_{Li}\, i\gamma^\mu \left(\partial_\mu - ig\frac{\vec{\tau}\cdot\vec{A}}{2} - \frac{i}{6} g' B_\mu\right) Q_{Li}$$
$$+ \sum_{i=1}^{3} \bar{U}_{Ri}\, i\gamma^\mu \left(\partial_\mu - i\frac{2}{3} g' B_\mu\right) U_{Ri}$$
$$+ \sum_{i=1}^{3} \bar{D}_{Ri}\, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{3} B_\mu\right) D_{Ri}$$

$$\mathcal{L}_Y = -\sum_{i,j} \left(\Gamma_{ij}^{(D)} \bar{Q}_{Li} \phi D_{Rj} + \Gamma_{ij}^{(U)} \bar{Q}_{Li} \tilde{\Phi} U_{Rj} + \text{H.c.} \right)$$

$$\tilde{\phi} = i\tau_2 \varphi^* = \begin{bmatrix} \varphi^{0*} \\ -\varphi^- \end{bmatrix} \qquad Y_\varphi = 1 \qquad Y_{\tilde{\varphi}} = -1$$

$$\begin{cases} \varphi \xrightarrow{SU(2)} U\varphi \\ \varphi^* \longrightarrow U^*\varphi^* \end{cases}$$
$$\tilde{\varphi} \longrightarrow U\tilde{\varphi}$$

$$\tau_1 = \tau_1^* \quad -\tau_2 = \tau_2^* \quad \tau_3 = \tau_3^*$$
$$\tau_{1,3} \cdot \tau_2 = -\tau_2 \tau_{1,3}$$
$$\tau_2 \tau_2 = \tau_2 \tau_2$$

$SU(N)$

$$\tilde{q} = \varepsilon^{i_1 \cdots i_n} q^{i_1 *}_{j_1} \cdots q^{i_n *}_{j_n} \qquad q_i = \begin{bmatrix} - \\ - \\ - \\ - \\ - \end{bmatrix}$$

$$q_i \xrightarrow{SU(N)} U q_i$$

$$\tilde{q} \xrightarrow{SU(N)} U\tilde{q}$$

$$q_i^j \longrightarrow U_{jk}\, q_i^k \qquad \mathrm{Det}[U] = \mathrm{Det}[U^*] = 1$$

$$SU(2) \qquad \varepsilon^{ij} = i\sigma^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\varphi \to \frac{v+H}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \bar{Q}_{Li} \phi D_{Rj} = \frac{v}{\sqrt{2}} \bar{D}_{Li} D_{Rj} + \frac{H}{\sqrt{2}} \bar{D}_{Li} D_{Rj}$$

$$\tilde{\varphi} \to \frac{v+H}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \bar{Q}_{Li} \tilde{\phi} U_{Rj} = \frac{v}{\sqrt{2}} \bar{U}_{Li} U_{Rj} + \frac{H}{\sqrt{2}} \bar{U}_{Li} U_{Rj}$$

ماتریس جرمی کوارک‌ها

$$\mathcal{L}^{(m_q)} \ldots \sum \bar{D}\, M_{ij}^D\, D \ldots - \sum \bar{U}\, M_{ij}^U\, U \ldots + \text{H.c}$$

$$M^{(D)}_{ij} = \Gamma^{D}_{ij}\frac{v}{\sqrt{2}} \qquad M^{(U)}_{ij} = \Gamma^{U}_{ij}\frac{v}{\sqrt{2}}$$

$$\left.\begin{aligned} \mathcal{U}_D^{\dagger} M^{D} \mathcal{V}_D &= M^{D} \\ \mathcal{U}_U^{\dagger} M^{U} \mathcal{V}_U &= M^{(U)} \end{aligned}\right\} \text{قطری}$$

$$\begin{cases} D'_L = \mathcal{U}_D^{\dagger} D_L \\ D'_R = \mathcal{V}_D^{\dagger} D_R \end{cases} \qquad \begin{cases} U'_L = \mathcal{U}_U^{\dagger} U_L \\ U'_R = \mathcal{V}_U^{\dagger} U_R \end{cases}$$

$$\mathcal{L}^{(m_q)} = -\bar{D}'_L M^{(D)} D'_R - \bar{D}'_R M^{(D)\dagger} D'_L - \bar{U}'_L M^{(U)} U'_R - \bar{U}'_R M^{(U)} U'_L$$

$$M^{(D)} = \begin{bmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{bmatrix} \qquad M^{(U)} = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{bmatrix}$$

$$D' = \begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} \qquad U' = \begin{bmatrix} u' \\ c' \\ t' \end{bmatrix}$$

$$Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$$

$$Q \longrightarrow ?$$

$$\sum_{\alpha} \bar{Q}_{\alpha} \gamma_{\mu} \frac{\tau^{3}}{2} Q_{\alpha} = ?$$

$$\bar{U}_L \gamma^{\mu} U_L \qquad \bar{U}_R \gamma^{\mu} U_R$$

$$\bar{D}_L \gamma^{\mu} D_L \qquad \bar{D}_R \gamma^{\mu} D_R$$

$$\bar{Q} \gamma_{\mu} \frac{\tau^{\pm}}{2} Q \longrightarrow ?$$

$$\mathcal{L}^{(q)}_{cc} = \frac{g}{\sqrt{2}} \sum_i \bar{U}_{L_i} \gamma^{\mu} D_{L_i} W^{+}_{\mu} + \frac{g}{\sqrt{2}} \sum_i \bar{D}_{L_i} \gamma^{\mu} U_{L_i} W^{-}_{\mu}$$

$$\Downarrow$$

$$\mathcal{L}^{q}_{cc} = \frac{g}{\sqrt{2}} \bar{U}'_L \mathcal{U}_U^{\dagger} \gamma^{\mu} \mathcal{U}_D D'_L W^{+}_{\mu} + \frac{g}{\sqrt{2}} \bar{D}'_L \mathcal{U}_D^{\dagger} \gamma^{\mu} \mathcal{U}_U U'_L W^{-}_{\mu}$$

$\cancel{\mathcal{V}^{\dagger}_{D}}$ کتاب اشتباه املایی دارد

$$V_{CKM} = \mathcal{U}_U^{\dagger} \mathcal{U}_D$$

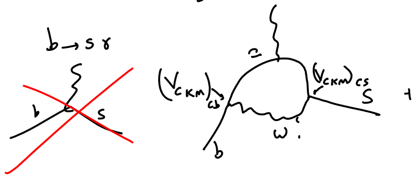
نتیجه برهمکنش های Z_{μ} و فوتون F را حفظ می‌کنند

اما CC می‌کشند

در حد درخت $F.C\,NC$ نداریم

ولی $FCCC$ داریم.

اما در حد حلقه می‌توانیم داشته باشیم



J_μ^Z

$$f = \begin{cases} u \\ c \\ t \end{cases} \qquad f' = \begin{cases} d \\ s \\ b \end{cases} \qquad Z_\mu \text{ برهمکنش}$$

$$a^f = T^3 - q \sin^2\theta_W$$

$$C_V^u = \frac{1}{2}(a_L^u + a_R^u) = \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W - \frac{2}{3}\sin^2\theta_W\right) = \frac{1}{4} - \frac{2}{3}\sin^2\theta_W$$

$$= C_V^c = C_V^t$$

$$C_A^u = \frac{1}{2}(a_L^u - a_R^u) = \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W + \frac{2}{3}\sin^2\theta_W\right) = \frac{1}{4}$$

$$= C_A^t = C_A^c$$

$$C_V^d = \frac{1}{2}(a_L^d + a_R^d) = \frac{1}{2}\left(\frac{-1}{2} + \frac{1}{3}\sin^2\theta_W + \frac{1}{3}\sin^2\theta_W\right) = \frac{-1}{4} + \frac{\sin^2\theta}{3}$$

$$= C_V^b = C_V^s$$

$$C_A^d = \frac{1}{2}(a_L^d - a_L^d) = -\frac{1}{4} = C_A^b = C_A^s$$

$Z \to e\bar{e}$? $\quad Z \to \mu\bar{\mu}$? $\quad Z \to \tau\bar{\tau}$?

$Z \to c\bar{c}$? $\quad Z \to u\bar{u}$? $\quad Z \to d\bar{d}$?

$Z \to t\bar{t}$? $\quad Z \to b\bar{b}$?

$Z \to$ invisible

CC & FC

$$\mathcal{L}_{CC}^{(a)} = \frac{g}{2\sqrt{2}} \bar{U}' \gamma^\mu (1-\gamma_5) V_{CKM} D' W_\mu^+ + h.c.$$

$n = 3$ ~ نسل

$\frac{n(n-1)}{2}$ ← زاویه

فاز غیرفیزیکی

$\frac{n(n+1)}{2}$ ← فاز $- (2n-1) = \frac{(n-1)(n-2)}{2}$

↓

آیا می‌توان میدان‌ها را rephase کرد؟

$n=2 \qquad n=3$

تنها دو نسل $n=2$

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} (\bar{u} \;\; \bar{c}) \gamma^\mu \frac{1-\gamma_5}{2} V_{CKM} \begin{pmatrix} d \\ s \end{pmatrix} W_\mu^+ + H.c.$$

$$V_{CKM} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

واپاشی K^+ را چه گونه حساب کنیم؟

$K^+ \to \mu^+ \nu_\mu$

مکانیزم GIM

1960

$Q_L^{(1)} = \begin{pmatrix} u \\ d_c \end{pmatrix}_L \qquad Q_L^{(2)} = \begin{pmatrix} c \\ s_c \end{pmatrix}_L$

$(d_c) \quad (\cos\theta_c \quad \sin\theta_c) (d)$

$$Q_L^{(1)} = \begin{pmatrix} u \\ d_c \end{pmatrix}_L \qquad Q_L^{(2)} = \begin{pmatrix} c \\ s_c \end{pmatrix}_L$$

$$\begin{pmatrix} d_c \\ s_c \end{pmatrix}_L = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_L$$

$$\bar{Q}_L^{(1)} \gamma_\mu \frac{\tau^3}{2} Q_L^{(1)} = \frac{1}{2} \bar{u}_L \gamma_\mu u_L - \cos^2\theta_c \, \bar{d}_L \gamma_\mu d_L$$

$$- \sin^2\theta_c \, \bar{s}_L \gamma_\mu s_L - \sin\theta_c \cos\theta_c \, (\bar{d}_L \gamma_\mu s_L + \bar{s}_L \gamma_\mu d_L)$$

$$\mathcal{L}^{em} = -\frac{1}{3} e \, (\bar{d} \gamma_\mu d + \bar{s} \gamma_\mu s) A^\mu$$

$$\left.\begin{array}{l} d \to d\cos\theta_c + s\sin\theta_c \\ s \to -d\sin\theta_c + d\cos\theta_c \end{array}\right\}$$ $\mathcal{L}_{em}$ فرق نمی‌کند.

اما $\quad Z^\mu \quad \bar{d}_L \gamma_\mu s_L$

اگر $Q_L^{(2)}$ را کنار بگذاریم

سهم Q^2 این را حذف می‌کند.

$$K_L^0 \to \mu^+\mu^- \qquad K^+ \to \pi^+ \nu \bar{\nu}$$

اما:

$$\begin{cases} Br(K_L^0 \to \mu^+\mu^-) = (7.25 \pm 0.16) \times 10^{-9} \\ Br(K^+ \to \pi^+ \bar{\nu}\nu) = (1.6^{+1.8}_{-0.8}) \times 10^{-10} \end{cases}$$

Glashow, Iliopoulos & Maiani (GIM) 1970

$$Q^{(2)} = \begin{pmatrix} c \\ s_c \end{pmatrix}_L = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}_L$$

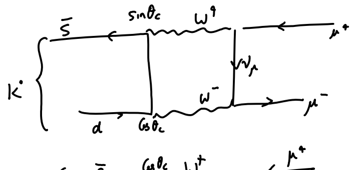
$$Q_L^{(2)} \gamma_\mu \frac{\tau^3}{2} Q_L^{(2)} = \frac{1}{2} \big(\bar{c}_L \gamma_\mu c_L - \cos^2\theta_c \, \bar{s}_L \gamma_\mu s_L$$

$$+ \sin^2\theta_c \, \bar{d}_L \gamma_\mu d_L + \sin\theta_c \cos\theta_c (\bar{d}_L \gamma_\mu s_L + \bar{s}_L \gamma_\mu d_L) \big)$$

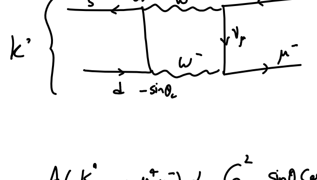
$$\Downarrow$$

$$\bar{Q}_L^{(1)} \gamma_\mu \frac{\tau^3}{2} Q_L^{(1)} + \bar{Q}_L^{(2)} \gamma_\mu \frac{\tau^3}{2} Q_L^{(2)} =$$

$$\frac{1}{2} (\bar{u}_L \gamma_\mu u_L + \bar{c}_L \gamma_\mu c_L - \bar{d}_L \gamma_\mu d_L - \bar{s}_L \gamma_\mu s_L)$$



$$G_F \propto \sin\theta_c \cos\theta_c \left(1 + O\left(\frac{m_u^2}{m_W^2}\right)\right)$$



$$-G_F \propto \sin\theta_c \cos\theta_c \left(1 + O\left(\frac{m_c^2}{m_W^2}\right)\right)$$

$$A(K_L^0 \to \mu^+\mu^-) \propto G_F^2 \sin\theta_c \cos\theta_c \, (m_c^2 - m_u^2)$$

Gaillard & Lee, 1974

$Br(K_L^0 \to \mu\mu) \longrightarrow m_c \simeq 1.5$ GeV

1974 $\longleftarrow J/\psi$

PDG پارامتریزاسیون استاندارد

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & \cdots & s_{23}c_{13} \\ \cdots & \cdots & c_{23}s_{13} \end{pmatrix}$$

دوران حول محور 3

$$V_{CKM} = R_1(\theta_2)\, R_3(\theta_1)\, C(0,0,\delta)\, R_1(\theta_3)$$

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \qquad C(0,0,\delta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{bmatrix}$$

$\vdots$

Wolfenstein پارامتریزاسیون

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$A, \rho, \eta \sim 1 \qquad \lambda = \sin\theta_c \simeq 0.22$

تجربی

$Br(b \to s\gamma) \gg Br(b \to d\gamma)$

شاهد پذیرهای فیزیکی نباید به پارامتریزاسیون ربط داشته باشند.

پتانسیل هیگز

$$\bar{Q}_{Li}\, \phi\, D_{Rj} = \frac{(v+H)}{\sqrt{2}} \bar{D}_{Li}\, D_{Rj}$$

$$\bar{Q}_{Li}\, \tilde{\phi}\, U_{Rj} = \frac{(v+H)}{\sqrt{2}}\, U_{Li} U_{Rj}$$

$$\mathcal{L}_{Hqq} = -\frac{H}{v}\left(m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b + m_c \bar{c}c + m_t \bar{t}t\right)$$

BSM

$\phi_1 \quad \phi_2 \qquad \langle \phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

$v^2 = v_1^2 + v_2^2$

$G^{(1)}_{ij} \, \bar{Q}_{Li} \phi_1 D_{Rj} + G^{(2)}_{ij} \, \bar{Q}_{Li} \phi_2 D_{Rj}$

قطری ← $v_1 G^{(1)}_{ij} + v_2 G^{(2)}_{ij}$ پایه جرم

در این پایه لزوماً $G^{(1)}_{ij}$ و $G^{(2)}_{ij}$ قطری نیستند

$\downarrow$

FC

Weyl فرمیون وایل

$\psi = \underbrace{\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}}_{\text{اسپینور دیراک}} \qquad \underbrace{\psi_L \qquad \psi_R}_{\text{اسپینور وایل}}$

$J_{\mu\nu} \propto [\gamma_\mu, \gamma_\nu]$ تحت تبدیل لورنتس

$J_{0i} \propto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix} - \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = 2 \begin{bmatrix} -\sigma^i & 0 \\ 0 & \sigma^i \end{bmatrix}$

$J_{ij} \propto \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{bmatrix} - \begin{bmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix} \propto \varepsilon^{ijk} \begin{bmatrix} \sigma^k & \\ & \sigma^k \end{bmatrix}$

تحت تبدیل لورنتس $\psi_L \to \psi'_L$ و $\psi_R \to \psi'_R$ اما یکسان تبدیل نمی‌شوند.

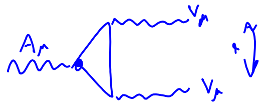
$\psi_L^\dagger \psi_L \quad \to$ ناوردای لورنتس نیست

$\psi_R^\dagger \psi_R \quad \to$ 〃 〃 〃

$\psi_R^\dagger \psi_L \quad$ 〃 〃 هست.

بی هنجاری

منبع بعش ۱۹۰۲ و ۲۰۰۲ پکین



$\bar{\psi} \gamma^\mu \gamma^5 \psi$

$\partial_\mu \overbrace{j^{\mu 5}} = \frac{-e^2}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$

triangle anomaly = γ_5 anomaly = axial anomaly

= Adler - Bell - Jackiw (ABJ) anomaly

Adler 1969 Bell - Jackiw 1969

Bardeen 1969

a b c

$A_\mu^a \quad A_\nu^b \quad A_\lambda^c$

$$\mathrm{tr}\left[\gamma^5 t^a \{t^b, t^c\}\right]$$

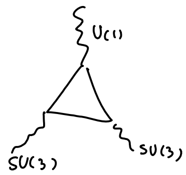
over all Fermions

QED QCD $\rightarrow$ no anomaly

$\mathrm{tr}[\tau^a] = \mathrm{tr}[\lambda] = 0$ SU(2) ها SU(3) ها

Chiral theory $\rightarrow$ dangerous

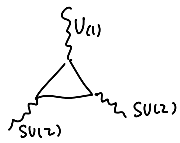
One generation
One color
$\times 3 \times 3$



$$\mathrm{tr}\left\{\gamma^5 t^a \{t^b, t^c\}\right\} = $$

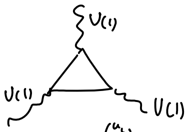
$$= \sum_q Y_q = \underset{u_R}{\frac{2}{3}} - \underset{d_R}{\frac{1}{3}} - 2 \times \frac{1}{6} = 0$$

d_L, u_L left



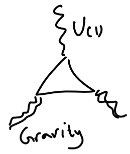
$$\mathrm{tr}\left\{\gamma^5 Y_{fL} \underbrace{\{t^a, t^b\}}_{\delta_{ab}}\right\}$$

$$= \left(-\left(-\tfrac{1}{2}\right) - \underset{\text{رنگ}}{3}\left(\tfrac{1}{6}\right)\right) \times 2 = 0$$



$$\mathrm{tr}(Y^3) = -2\underset{\binom{\nu_L}{e_L}}{\left(-\tfrac{1}{2}\right)^3} + \underset{e_R}{(-1)^3}$$

$$-3\Big[\underset{\binom{u_L}{d_L}}{2\left(\tfrac{1}{6}\right)^3} - \underset{u_R}{\left(\tfrac{2}{3}\right)^3} - \underset{d_R}{\left(-\tfrac{1}{3}\right)^3}\Big] = 0$$



$$\mathrm{tr}[Y] = -2\left(-\tfrac{1}{2}\right) + (-1) - 3\left[2 \times \tfrac{1}{6} - \left(\tfrac{2}{3}\right) - \left(-\tfrac{1}{3}\right)\right] = 0$$

ور برید. آیا شرط کنسل شدن همه‌ی ابرچاج‌ها را به ما می‌دهد؟

به خاطر داشته باشیم که:

(۱) بی‌هنجاری مربوط به تئوری های کایرال است.
اگر اسکالر اضافه کنیم مشکلی نیست.
اگر زوج فرمیون کایرال (فرمیون وایل) با ابرچاج مخالف اضافه کنیم باز هم جای نگرانی نیست.

(۲) عدم وابستگی به جرم

(۳) One-loop exact!

* جریان مربوط به عدد باریونی و L و B-L را بنویسید و بی‌هنجاری های این جریان‌ها را در چارچوب مدل استاندارد به دست آورید

عدد باریونی کوارک؟ L ذرات مخلوق