String Theory Final Exam

Starting date: 16 Bahman 1396, Due date: 21 Bahman

U-duality group of M-theory on T^3 is known to be $SL(3, Z) \times SL(2, Z)$. Verify and examine this.

1) Start from 11d SUGRA and reduce it on T^3. In the 8d, we then have a gravity theory with a three-form, some two-forms, some vector gauge fields and some scalars. Write out the spectrum of this 8d theory explicitly and show how they are arranged in the representations of the U-duality group.

2) Next, consider M2,M5-branes of the M-theory. When compactified on T^3 we get some different branes in the 8d theory. How does these branes fall into representations of the U-duality group?

3) What is the moduli space of M-theory/T^3?

4) We know that M-theory in the Discrete Light-Cone Quantization, in the sector with N units of the light-cone momentum, is described by the BFSS matrix model, which is a $0 + 1$ dimensional $U(N)$ super-Yang-Mills (SYM) theory. In this context verify that the matrix theory exhibits U-duality. To this end, show

   4-1) Matrix theory on T^3 is described by the $3 + 1$ dimensional SYM on the dual T^3.

   4-2) How is the $SL(3, Z) \times SL(2, Z)$ U-duality group realized in $3 + 1$ SYM/T^3?

Bosonic Closed Strings on T^2 ***and on*** T^3***.*** We know that closed string theory compactified on S^1 at the "selfdual radius" $R^2 = \alpha'$ acquires extra massless states, which have non-zero winding modes. Among these massless states there are vector modes which then yield non-Abelian gauge theory in the low-energy effective theory; for the theory on S^1 at selfdual radius this is a $SU(2) \times SU(2)$ gauge theory.

5-1) Explore the same phenomenon for the "selfdual" points of the bosonic theory on T^2 and T^3. What is the gauge group we get here? What is the gauge theory of 24 or 23 dimensional bosonic theory (obtained from 26 dim. theory on T^2 or T^3).

5-2) What are the other massless states of this 24 or 23 dimensional theory (than the vector states)? List all the other massless scalar, tensor and form fields.

5-3) We know that the 26 dim. theory can have D_p-branes. Analyze D-branes of the theory compactified on T^3. What happens to the branes at the selfdual point? Is there anything special about them?

D0-brane interactions. Work out the interaction amplitude of two D0-branes moving at relative velocity v. Analyze the $v \to 0$ and $v \to 1$ limit of the interaction.

D_p-branes and decoupling. Consider N parallel D_p-branes (*Hint: see hep-th/9802042*).

7-1) Write out the corresponding supergravity solution.

7-2) Take the Near Horizon (NH) limit of this solution for generic p.

7-3) Explore the behavior of the dilaton field for various p's; where does the strong coupling regime happen in these solutions? Find an appropriate theory or duality frame where the theory is trustable in those strongly coupled regions and write the solution in that duality frame. Do this for each and every p.

7-4) Analyzing a massless scalar field on the NH geometry, discuss when the NH limit is a decoupling limit.

All the best,
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